ADVANCED DESIGN OF STEEL AND COMPOSITE STRUCTURES

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Sustainable Constructions
under Natural Hazards and Catastrophic Events
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Module B – Design of industrial buildings using non-uniform members

B.3 – Design of non-uniform members

1 – Introduction
2 – Non-uniform members – approaches and problems
4 – Design resistance of non-uniform members (clause 6.3.4)
5 – Example
Introduction
Tapered steel members are used in steel structures

- Structural efficiency → optimization of cross section capacity → saving of material
- Aesthetical appearance

Multi-sport complex – Coimbra, Portugal

Construction site in front of the Central Station, Europaplatz, Graz, Austria
Tapered members are commonly used in steel frames:

- industrial halls, warehouses, exhibition centers, etc.

- Adequate verification procedures are then required for these types of structures!
Introduction

However, there are several difficulties in performing the stability verification of structures composed of non-uniform members;

- Guidelines are inexistent or not clear for the designer.

- Due to this reason, simplifications that are not mechanically consistent are adopted. These may be either too conservative or even Unconservative!
Non-uniform members

Approaches and Problems
Non-uniform members – approaches and problems

- **Prismatic members** – Clauses 6.3.1 to 6.3.3
  - Developed for prismatic members
  - Sinusoidal imperfections

\[
\delta_0(x) = e_0 \sin\left(\frac{\pi x}{L}\right)
\]

\[
M''(x) = EI\delta'' \propto \sin\left(\frac{\pi x}{L}\right)
\]

- Ayrton-Perry type equation:
  Is maximum at mid span:

\[
\varepsilon(x) = \frac{N}{N_{Rk}} + \frac{M''(x)}{M_{y,Rk}}
\]

OK!
Non-uniform members – approaches and problems

- Non-prismatic members

- Analytical expressions for the elastic critical load are not readily available;

- The choice of the critical section for the application of the buckling resistance formulae is not straightforward.
Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply

- Cross section utilization due to applied (first order) forces is not constant anymore

\[
\frac{N}{N_{Rk}} \quad \text{Not Constant}
\]
Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply ???

\[ \delta_0(x) = \epsilon_0 \sin \left( \frac{\pi x}{L} \right) \]

\[ M''(x) = EI\delta'' \propto \sin \left( \frac{\pi x}{L} \right) \]

- Ayrton-Perry type equation:
  Is it maximum at mid span ???

First yield criteria:
\[ \varepsilon(x) = \frac{N}{N_{Rk}} + \frac{M''(x)}{M_{y,Rk}} \]

KO!
Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply

- Position of the critical cross-section – not at mid span
  - Account for 2nd order effects; iterative procedure, not practical;

- 1st order critical cross section is considered!
Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply

- Variation of cross section class

- Definition of an equivalent class for the member
Non-uniform members – approaches and problems

- Non-uniform members – 2\textsuperscript{nd} order analysis with imperfections

- Definition of local imperfections:
  - Same problem: $e_0/L$ calibrated for prismatic members with sinusoidal imperfections

<table>
<thead>
<tr>
<th>Buckling curve acc. to EC3-1-1, Table 6.1</th>
<th>Elastic analysis</th>
<th>Plastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_0/L$</td>
<td>$e_0/L$</td>
</tr>
<tr>
<td>$a_0$</td>
<td>1/350</td>
<td>1/300</td>
</tr>
<tr>
<td>$a$</td>
<td>1/300</td>
<td>1/250</td>
</tr>
<tr>
<td>$b$</td>
<td>1/250</td>
<td>1/200</td>
</tr>
<tr>
<td>$c$</td>
<td>1/200</td>
<td>1/150</td>
</tr>
<tr>
<td>$d$</td>
<td>1/150</td>
<td>1/100</td>
</tr>
</tbody>
</table>
Non-uniform members – approaches and problems

- Non-uniform members – 2nd order analysis with imperfections
- Definition of local imperfections?

Auvent de la Gare Routière – Ermont

Barajas Airport, Madrid, Spain

Italy pavilion, World Expo 2010 – Shanghai
Non-uniform members – approaches and problems

General method allows the verification of the resistance to lateral and lateral torsional buckling for structural components such as:

– single members, built-up or not, uniform or not, with complex support conditions or not, or

– plane frames or subframes composed of such members,
Non-uniform members – approaches and problems

Non-uniform members – GENERAL METHOD (clause 6.3.4)

1. In-plane resistance
   - In-Plane GMNIA calculations
   - $\alpha_{ult,k}$

2. Out-of-plane elastic critical load
   - LEA calculations
   - $\alpha_{cr,op}$

3. Buckling curve
   - $\bar{\lambda}_{op} = \sqrt{\alpha_{ult,k} / \alpha_{cr,op}}$
   - $X_{LT}$

   $X_{op} = \min(X, X_{LT})$
   $X_{op} = \text{Interpolated}(X, X_{LT})$

   $\chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1$
Non-uniform members – approaches and problems

Non-uniform members – GENERAL METHOD (clause 6.3.4)

1. In-plane resistance
   - In-Plane GMNIA calculations
   - $\alpha_{\text{ult},k}$

2. Out-of-plane elastic critical load
   - $\alpha_{\text{cr},op}$
   - Buckling curve
   - $\lambda_{op} = \sqrt{\alpha_{\text{ult},k} / \alpha_{\text{cr},op}}$

3. $x_{op}$
   - Minimum $(x, x_{LT})$
   - Interpolated $(x, x_{LT})$
   - $x_{op} \geq 1$

Minimum load amplified of the design loads to reach the characteristic resistance of the most critical cross-section of the structural component, without lateral-torsional buckling, but accounting for all effects of the in-plane geometrical deformations and imperfection (global and local): $N_{Rk} / N_{Ed}$
Non-uniform members – approaches and problems

Non-uniform members – GENERAL METHOD (clause 6.3.4)

Minimum amplified for the in-plane the design loads to reach the elastic critical resistance of the structural component, with respect to lateral and lateral-torsional buckling, without accounting for in-plane flexural buckling: \( N_{cr} / N_{Ed} \).

FE can be used to determine \( \alpha_{ult,k} \) and \( \alpha_{cr,op} \).

1. In-plane resistance
   - In-Plane GMNIA calculations
   - \( \alpha_{ult,k} \)

2. Out-of-plane elastic critical load
   - LEA calculations
   - \( \alpha_{cr,op} \)
   - Buckling curve
   - \( \lambda_{op} = \sqrt{\alpha_{ult,k} / \alpha_{cr,op}} \)

3. \( X_{op} = \begin{cases} \text{Minimum} & (X, X_{LT}) \\ \text{Interpolated} & (X, X_{LT}) \end{cases} \)
   - \( \chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1 \)
Non-uniform members – approaches and problems

Non-uniform members – GENERAL METHOD (clause 6.3.4)

1. In-plane resistance
   In-Plane GMNIA calculations
   \[ \alpha_{\text{ult,k}} \]

2. Out-of-plane elastic critical load
   LEA calculations
   \[ \alpha_{\text{cr,op}} \]

\[ \bar{\lambda}_{\text{op}} = \sqrt{\alpha_{\text{ult,k}} / \alpha_{\text{cr,op}}} \]

3. Buckling curve
   \[ X \quad X_{LT} \]
   \[ X_{op} = \text{Minimum} (X, X_{LT}) \]
   \[ X_{op} = \text{Interpolated} (X, X_{LT}) \]

\[ X_{op} \alpha_{\text{ult,k}} / \gamma_{M1} \geq 1 \]

Global non-dimensional slenderness
Non-uniform members – approaches and problems

- Flexural buckling
- Lateral torsional buckling

Each calculated for the global non-dimensional slenderness $\lambda_{op}$.

Reduction factor for:

1. In-plane resistance
   - In-Plane GMNIA calculations
   - $\alpha_{ult,k}$
2. Out-of-plane elastic critical load
   - LEA calculations
   - $\alpha_{cr,op}$
3. Buckling curve
   - $\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}}$
   - $X_{op} = \min (X, X_{LT})$
   - $X_{op} = \text{Interpolated } (X, X_{LT})$

$\chi_{op} \frac{\alpha_{ult,k}}{\gamma_{M1}} \geq 1$
Non-uniform members – approaches and problems

Non-uniform members – GENERAL METHOD (clause 6.3.4)

1. In-plane resistance
   - In-Plane GMNIA calculations
     - $\alpha_{\text{ult},k}$

2. Out-of-plane elastic critical load
   - LEA calculations
     - $\alpha_{\text{cr,op}}$
   - $\bar{\lambda}_{\text{op}} = \sqrt{\alpha_{\text{ult},k} / \alpha_{\text{cr,op}}}$

3. Buckling curve
   - $\chi_{\text{op}} = \min (X, X_{LT})$
   - $\chi_{\text{op}} = \text{Interpolated} (X, X_{LT})$

$\chi_{\text{op}}$ is the reduction factor for the non-dimensional slenderness $\lambda_{\text{op}}$, to take account of lateral and lateral torsional buckling.
Non-uniform members – approaches and problems

Non-uniform members – GENERAL METHOD (clause 6.3.4)

1. In-plane resistance
   - In-Plane GMNIA calculations
   - \( \alpha_{ult,k} \)

2. Out-of-plane elastic critical load
   - LEA calculations
   - \( \alpha_{cr,op} \)
   - \( \lambda_{op} = \sqrt{\alpha_{ult,k} / \alpha_{cr,op}} \)

3. Buckling curve
   - Interpolated \( (x, x_{LT}) \)
   - Minimum \( (x, x_{LT}) \)

   \( x_{op} \geq 1 \)
Non-uniform members – example

\[ N = 80 \text{ kN} \]

\[ p = 12 \text{ kN/m} \]

\[ IPE 360 \text{ mod.} \]

\( (h = 200 \text{ mm}) \)

\[ M = 73.5 \text{ kNm} \]

\[ N = \text{cte} = 80.0 \text{ kN} \]
Non-uniform members – example
Non-uniform members – example

Verification of the cross-section

- The cross-section resistance is checked using clauses 6.2.8 (bending and shear) and 6.2.9 (bending and axial force) for each class of cross-section.

- The utilization ratio $\alpha$ of the cross-section is given by the ratio between the norm of the applied internal forces and the norm of the bending and axial resistance along the same load value:

$$\alpha = \frac{\sqrt{N_{Ed}^2 + M_{y,Ed}^2}}{\sqrt{N_{max}^2 + M_{y,max}^2}} \leq 1$$

Class 1 or 2: being $N_{max}$ and $M_{y,max}$ the values obtained in the interaction curve.

Class 3: $\alpha = \frac{N_{Ed} + M_{y,Ed}}{Af_y + Wely f_y} \leq 1$
Non-uniform members – example

Verification of the cross-section

Critical cross-section at $x = 4.14$ m (Class 1)
Non-uniform members – example

General method (cl. 6.3.4 using the properties of the critical cross-section)

In-plane buckling resistance

\[ \alpha_{ult,k} = \left( \frac{N_{Ed}}{\chi_y N_{Rk}/\gamma M_1} + k_{yy} \frac{M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma M_1} \right)^{-1} = \frac{1}{0.55} = 1.819 \]

(being \( \chi_{Lt} = 1 \))

Out-of-plane buckling resistance

Equations to calculate \( \alpha_{cr,op} \) are difficult to apply!

Numerically, \( \alpha_{cr,op} = 1.482 \)

Buckling resistance

\[ \overline{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 1.108 \]
Non-uniform members – example

General method (cl. 6.3.4 using the properties of the critical cross-section)

For \( x = 4.14 \) m, buckling curves for out-of-plane buckling and lateral-torsional buckling are curve \( c \).

\[
\overline{\lambda_{op}} = \begin{cases} 
\text{curve } c & \Rightarrow \chi_z = 0.48 \\
\text{curve } c & \Rightarrow \chi_{LT} = 0.48
\end{cases}
\]

\[
\chi_{op} = \min(\chi_z; \chi_{LT}) = \text{interp}(\chi_z; \chi_{LT}) = 0.48
\]

So, \( \chi_{op} \frac{\alpha_{ult,k}}{\gamma_{M1}} = \frac{0.48 \times 1.819}{1} = 0.873 < 1 \), therefore buckling resistance is not verified!

The ratio of utilization is \( 1/0.873 = 1.15 \) (15% higher than permitted)!
Non-uniform members – example

General method (cl. 6.3.4 using the properties of the critical cross-section)

<table>
<thead>
<tr>
<th>Method</th>
<th>Util. ratio</th>
<th>Diff (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMNIA</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>Clause 6.3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = x_{cr} + \text{eq. properties for critical loads}$</td>
<td>1.22</td>
<td>39</td>
</tr>
<tr>
<td>$x = 0$</td>
<td>1.16</td>
<td>32.8</td>
</tr>
<tr>
<td>$X = x_{cr}$</td>
<td>1.23</td>
<td>40.8</td>
</tr>
<tr>
<td>$X = L$</td>
<td>1.51</td>
<td>72.7</td>
</tr>
<tr>
<td>General method (theoretical) - $x_{cr}$</td>
<td>1.15</td>
<td>30.8</td>
</tr>
</tbody>
</table>
REFERENCES


REFERENCES


SOFTWARE


**SemiComp Member Design** – Design resistance of prismatic beam-columns”, Greiner et al, RFCS, 2011.
http://www.steelconstruct.com


www.cmm.pt  www.steelconstruct.com
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