2C09
Design for seismic and climate changes

Lecture 08: Seismic response of SDOF systems

Aurel Stratan, Politehnica University of Timisoara
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Lecture outline

8.1 Time-history response of linear SDOF systems.

8.2 Elastic response spectra.

8.3 Time-history response of inelastic SDOF systems.
### Seismic action

#### Ground acceleration: accelerogram

El Centro 1940, Imperial Valley, S00E

Loma Prieta 1989, Corralitos, 090

Parkfield 1966, Cholame #2, 065

Erzincan 1992, Meteorological Station, FN

San Fernando 1971, Pacoima Dam, N76W

Northridge 1994, Sylmar, 360

Vrancea 1977, INCERC, NS

Western Washington 1949, Olympia, 086

#### Properties of a SDOF system $(m, c, k) + \ddot{u}_g(t) \Rightarrow$

$$m\dddot{u} + c\ddot{u} + ku = -m\dddot{u}_g$$

#### Relative displacement, velocity and acceleration of a SDOF system
Seismic action

- North-south component of the El Centro, California record during Imperial Valley earthquake from 18.05.1940
Determination of seismic response

- **Equation of motion:**
  \[ m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g \]

- /m:
  \[ \ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g \]

- **Numerical methods**
  - central difference method
  - Newmark method
  - ...

- **Response depends on:**
  - natural circular frequency \( \omega_n \) (or natural period \( T_n \))
  - critical damping ratio \( \xi \)
  - ground motion \( \ddot{u}_g \)
Seismic response

$T_n = 0.5 \text{ sec, } \zeta = 0.02$

$T_n = 1 \text{ sec, } \zeta = 0.02$

$T_n = 2 \text{ sec, } \zeta = 0$

$T_n = 2 \text{ sec, } \zeta = 0.02$

$T_n = 2 \text{ sec, } \zeta = 0.05$
Elastic response spectra

- Response spectrum: representation of peak values of seismic response (displacement, velocity, acceleration) of a SDOF system versus natural period of vibration, for a given critical damping ratio

\[
\begin{align*}
    u_0(T_n, \xi) &= \max_t |u(t, T_n, \xi)| \\
    \dot{u}_0(T_n, \xi) &= \max_t |\dot{u}(t, T_n, \xi)| \\
    \ddot{u}_0^t(T_n, \xi) &= \max_t |\ddot{u}^t(t, T_n, \xi)|
\end{align*}
\]
Elastic displacement response spectrum: $D=\mathbf{u}_0$
Elastic displacement response spectrum: $D = u_0$
Elastic displacement response spectrum: $D \equiv u_0$
Elastic displacement response spectrum: $D = u_0$
Elastic displacement response spectrum: $D = u_0$
Elastic displacement response spectrum: $D = u_0$
Pseudo-velocity and pseudo-acceleration

- **Spectral pseudo-velocity:**
  - units of velocity
  - different from peak velocity

\[ V = \omega_n D = \frac{2\pi D}{T_n} \]

- **Strain energy**

\[ E_{S0} = \frac{k u_0^2}{2} = \frac{k D^2}{2} = \frac{k (V/\omega_n)^2}{2} = \frac{m V^2}{2} \]

- **Spectral pseudo-acceleration:**
  - units of acceleration
  - different from peak acceleration

\[ f_{S0} = k u_0 = m \omega_n^2 u_0 = mA \quad \Rightarrow \quad A = \omega_n^2 u_0 = \omega_n^2 D = \left( \frac{2\pi}{T_n} \right)^2 D \]
Pseudo-velocity and pseudo-acceleration

\[ V = \omega_n D = \left( \frac{2\pi}{T_n} \right)^2 D \]

\[ A = \omega_n^2 D = \left( \frac{2\pi}{T_n} \right)^2 D \]
Combined D-V-A spectrum

- Displacement, pseudo-velocity and pseudo-acceleration spectra:
  - same information
  - different physical meaning

\[
\frac{A}{\omega_n} = V = \omega_n D \quad \text{or} \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D
\]

\[
\left(\frac{T_n}{2\pi}\right) A = V \quad \Rightarrow \quad \lg T_n + \lg A - \lg 2\pi = \lg V
\]

- A line inclined at +45° for \( \lg A - \lg 2\pi = \text{const.} \Rightarrow \) spectral pseudo-acceleration: an axis inclined to -45°

- Similarly, spectral displacement: an axis inclined to +45°
Combined D-V-A spectrum
Characteristics of elastic response spectra
Characteristics of elastic response spectra

Spectral Regions

Acceleration sensitive

Velocity sensitive

Displacement sensitive

\[ V / u_{g0} \]

\[ T_n, \text{ sec} \]

- \( T_a = 0.035 \text{ sec} \)
- \( T_b = 0.125 \text{ sec} \)
- \( T_c = 0.5 \text{ sec} \)
- \( T_d = 3.0 \text{ sec} \)
- \( T_e = 10 \text{ sec} \)
- \( T_f = 15 \text{ sec} \)
Characteristics of elastic response spectra

- For $T_n < T_a$
  - pseudo-acceleration $A$ is close to $\ddot{u}_g^0$
  - spectral displacement $D$ is small

- For $T_n > T_f$
  - spectral displacement $D$ is close to $u_g^0$
  - spectral pseudo-acceleration $A$ is small

- Between $T_a$ and $T_c \Rightarrow A > \ddot{u}_g^0$

- Between $T_b$ and $T_c \Rightarrow A$ can be considered constant

- Between $T_d$ and $T_f \Rightarrow D > u_g^0$

- Between $T_d$ and $T_e \Rightarrow D$ can be considered constant

- Between $T_c$ and $T_d \Rightarrow V > \dot{u}_g^0$

- Between $T_c$ and $T_d \Rightarrow V$ can be considered constant
Characteristics of elastic response spectra

- $T_n > T_d \Rightarrow$ response region sensible to displacements
- $T_n < T_c \Rightarrow$ response region sensible to accelerations
- $T_c < T_n < T_d \Rightarrow$ response region sensible to velocity

- Larger damping:
  - smaller values of displacements, pseudo-velocity and pseudo-acceleration
  - more "smooth" spectra

- Effect of damping:
  - insignificant for $T_n \to 0$ and $T_n \to \infty$,
  - important for $T_b < T_n < T_d$
Elastic design spectra

- Spectra of past ground motions:
  - jagged shape
  - variation of response for different earthquakes
  - areas where previous data is not available
Elastic design spectra

- idealized "smooth" spectra
- based on statistical interpretation (median; median plus standard deviation) of several records characteristic for a given site
Elastic design spectra
Elastic design spectra

Elastic design spectrum

Peak ground acceleration, velocity, and displacement

Pseudo-velocity (log scale)

Natural vibration period (log scale)

1/33 sec 1/8 sec 10 sec 33 sec
33 Hz 8 Hz 1/10 Hz 1/33 Hz
Elastic design spectra

Elastic design spectrum

Peak ground acceleration, velocity, and displacement

Pseudo-velocity (log scale)

Natural vibration period (log scale)

PSA

B C T D
Elastic design spectra

Elastic design spectrum

Peak ground acceleration, velocity, and displacement

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PSA
Elastic design spectra

Elastic spectrum

Peak ground acceleration, velocity, and displacement

Natural vibration period (log scale)

Pseudo-velocity (log scale)

1/33 sec 1/8 sec 10 sec 33 sec
33 Hz 8 Hz 1/10 Hz 1/33 Hz

PSA

PSV
Elastic design spectra

Peak ground acceleration, velocity, and displacement

Natural vibration period (log scale)

Pseudo-velocity (log scale)
Elastic design spectra

Elastic design spectrum

Peak ground acceleration, velocity, and displacement

Pseudo-velocity (log scale)

Natural vibration period (log scale)

PSA

PSV

SD

20
Elastic design spectra

Elastic design spectrum

Peak ground acceleration, velocity, and displacement

Natural vibration period (log scale)

Pseudo-velocity (log scale)

PSA

PSV

SD
Inelastic response of SDOF systems

- Most structures designed for seismic forces lower than the ones assuring an elastic response during the design earthquake
  - design of structures in the elastic range for rare seismic events considered uneconomical
  - in the past, structures designed for a fraction of the forces necessary for an elastic response, survived major earthquakes
Inelastic response of SDOF systems

- **Elasto-plastic system:**
  - stiffness $k$
  - yield force $f_y$
  - yield displacement $u_y$

- **Elasto-plastic idealization:**
  equal area under the actual and idealised curves up to the maximum displacement $u_m$

- **Cyclic response of the elasto-plastic system:**

![Diagram of cyclic response](image)
Corresponding elastic system

- Corresponding elastic system:
  - same stiffness
  - same mass
  - same damping

- Inelastic response:
  - yield force reduction factor $R_y$
    \[ R_y = \frac{f_0}{f_y} = \frac{u_0}{u_y} \]
  - ductility factor
    \[ \mu = \frac{u_m}{u_y} \]
Equation of motion

- **Equation of motion:** \[ m\ddot{u} + c\dot{u} + f_s(u,\ddot{u}) = -m\ddot{u}_g \]

- \( /m \Rightarrow \ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u_y\tilde{f}_s(u,\ddot{u}) = -\ddot{u}_g \)
  \[ \tilde{f}_s(u,\ddot{u}) = f_s(u,\ddot{u})/f_y \]

- **Seismic response of an inelastic SDOF system depends on:**
  - natural circular frequency of vibration \( \omega_n \)
  - critical damping ratio \( \xi \)
  - yield displacement \( u_y \)
  - force-displacement shape \( \tilde{f}_s(u,\ddot{u}) \)
Effects of inelastic force-displacement relationship

- **4 SDOF systems (El Centro):**
  - $T_n = 0.5$ sec
  - $\xi = 5\%$
  - $R_y = 1, 2, 4, 8$

- **Elastic system:**
  - vibr. about the initial position of equilibrium
  - $u_p = 0$

- **Inelastic syst.:**
  - vibr. about a new position of equilibrium
  - $u_p \neq 0$
Elastic $\Leftrightarrow$ inelastic

- Design of a structure responding in the elastic range:
  \[ f_0 \leq f_{Rd} \]

- Design of a structure responding in the inelastic range:
  \[ u_m \leq u_{Rd} \quad \mu \leq \mu_{Rd} \]

ductility demand  ductility capacity

![Diagram showing corresponding linear system and elastoplastic system]
El Centro ground motion
- $\xi = 5\%$
- $R_y = 1, 2, 4, 8$

- $T_n > T_f$
  - $u_m$ independent of $R_y$
  - $u_m \approx u_0$

- $T_n > T_c$
  - $u_m$ depends on $R_y$
  - $u_m \approx u_0$

- $T_n < T_c$
  - $u_m$ depends on $R_y$
  - $u_m > u_0$
$R_y - \mu$ relationship: idealisation

- $T_n$ in the displacement- and velocity-sensitive region:
  - "equal displacement" rule $u_m/u_0=1 \Rightarrow R_y=\mu$
- $T_n$ in the acceleration-sensitive region:
  - "equal energy" rule $u_m/u_0>1 \Rightarrow R_y = \sqrt{2\mu - 1}$
- $T_n<T_a$:
  - small deformations, elastic response $\Rightarrow R_y=1$

\[
R_y = \begin{cases} 
1 & \text{if } T_n < T_a \\
\sqrt{2\mu - 1} & \text{if } T_b < T_n < T_c \\
\mu & \text{if } T_n > T_c
\end{cases}
\]
$R_y - \mu$ relationship: idealisation

$$R_y = \begin{cases} 
1 & T_n < T_a \\
\sqrt{2\mu - 1} & T_b < T_n < T_c' \\
\mu & T_n > T_c 
\end{cases}$$
References / additional reading
