2C09
Design for seismic and climate changes

Lecture 08: Seismic response of SDOF systems

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13/03/2014

European Erasmus Mundus Master Course
Sustainable Constructions
under Natural Hazards and Catastrophic Events
520121-1-2011-1-CZ-ERA MUNDUS-EMMC

Universitatea Politehnica Timişoara

Lecture outline

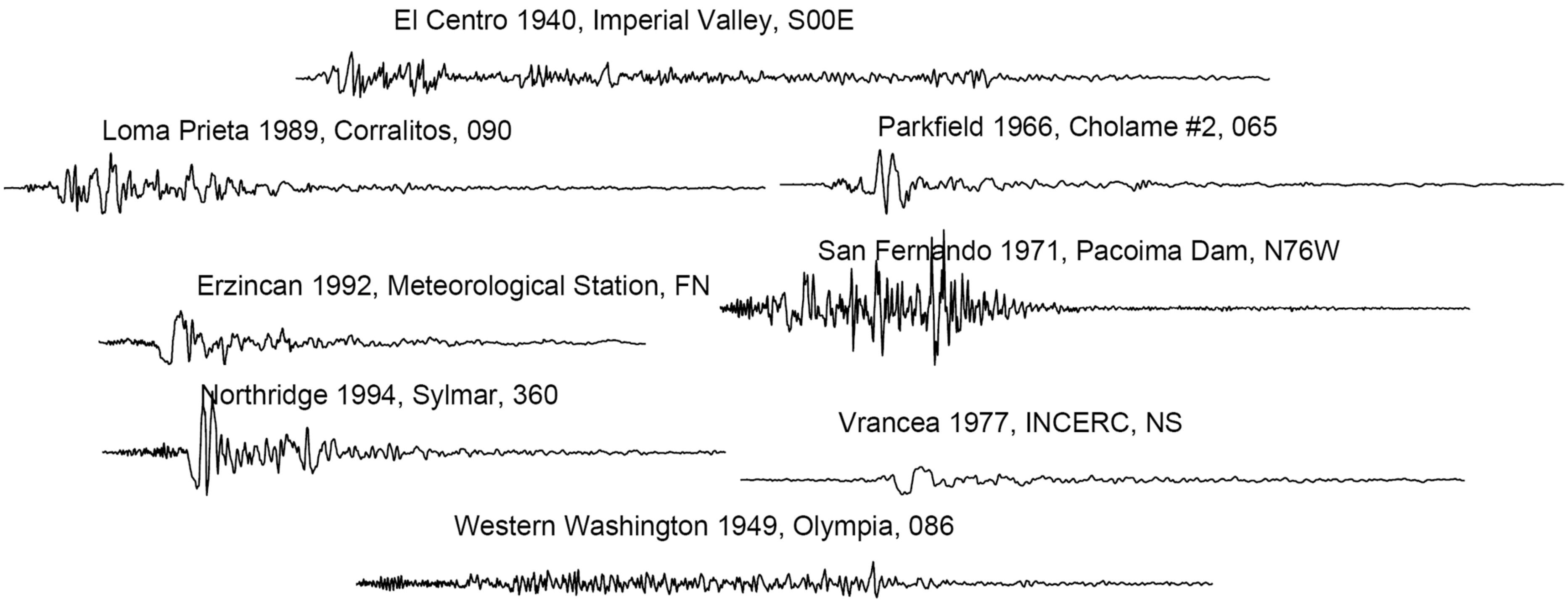
8.1 Time-history response of linear SDOF systems.

8.2 Elastic response spectra.

8.3 Time-history response of inelastic SDOF systems.

Seismic action

- **Ground acceleration: accelerogram**



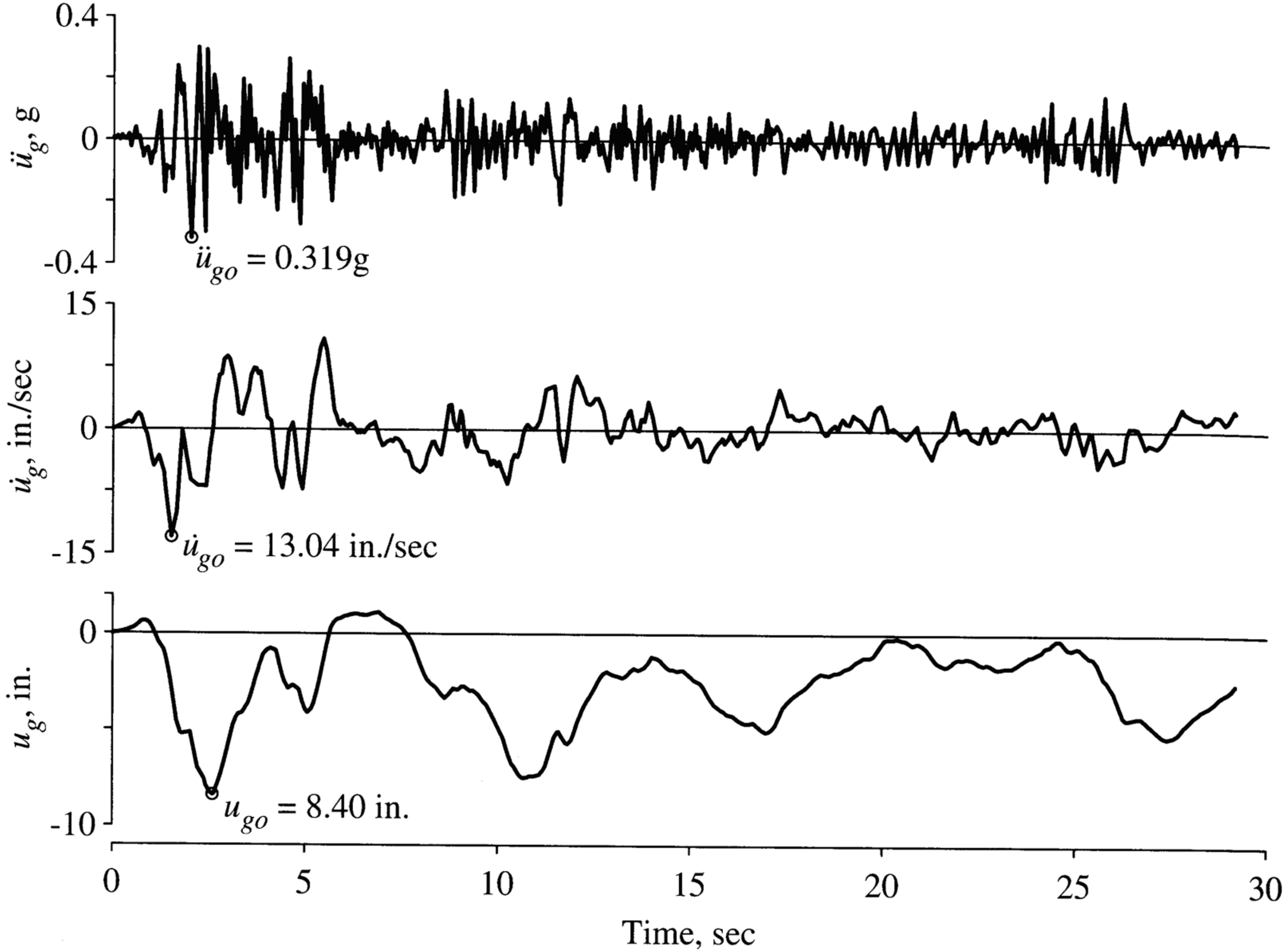
- **Properties of a SDOF system (m, c, k) +** $\ddot{u}_g(t) \quad \Rightarrow$

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g$$

- **Relative displacement, velocity and acceleration of a SDOF system**

Seismic action

- **North-south component of the El Centro, California record during Imperial Valley earthquake from 18.05.1940**



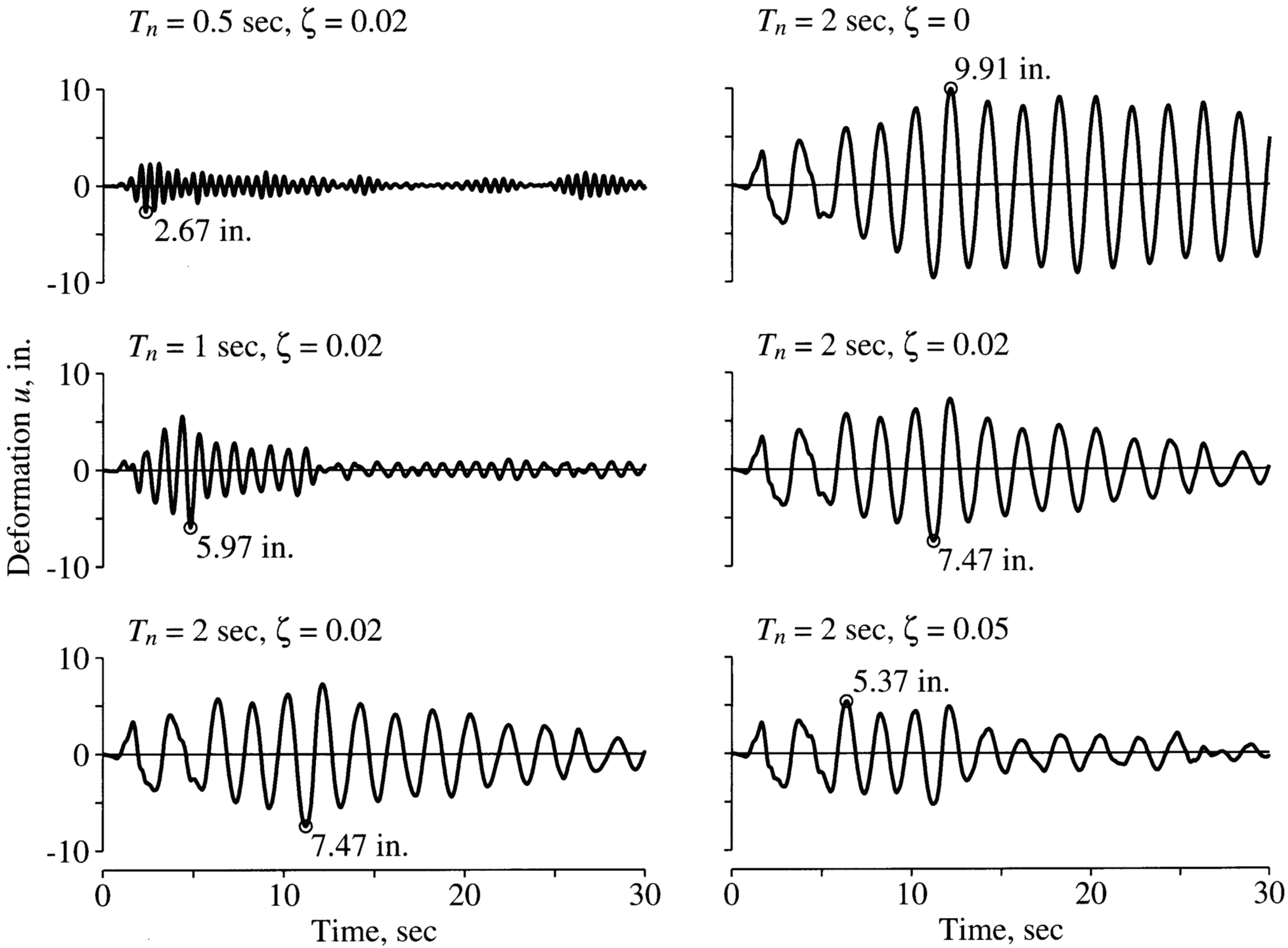
Determination of seismic response

- **Equation of motion:** $m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g$
- **/m:** $\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g$
- **Numerical methods**
  - **central difference method**
  - **Newmark method**
  - **...**

$$\left.\begin{array}{l}\text{central difference method}\\ \text{Newmark method}\\ \ldots\end{array}\right\} \Rightarrow \quad u \equiv u\left(t, T_n, \xi\right)$$

- **Response depends on:**
  - **natural circular frequency ω_n (or natural period T_n)**
  - **critical damping ratio ξ**
  - **ground motion $\ddot{u}_g$**

Seismic response



Elastic response spectra

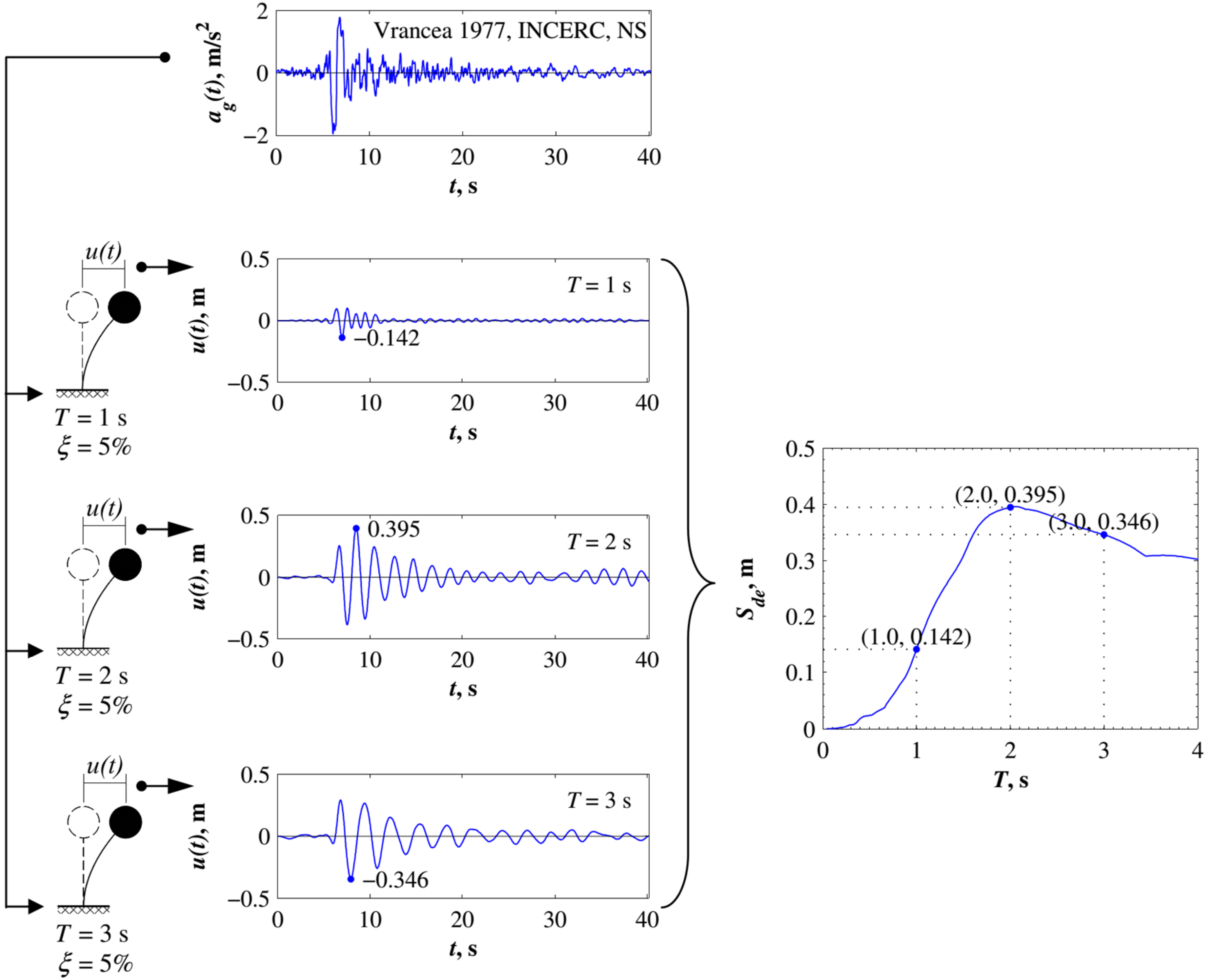
- **Response spectrum: representation of peak values of seismic response (displacement, velocity, acceleration) of a SDOF system versus natural period of vibration, for a given critical damping ratio**

$$u_0\left(T_n,\xi\right)=\max_t\left|u\left(t,T_n,\xi\right)\right|$$

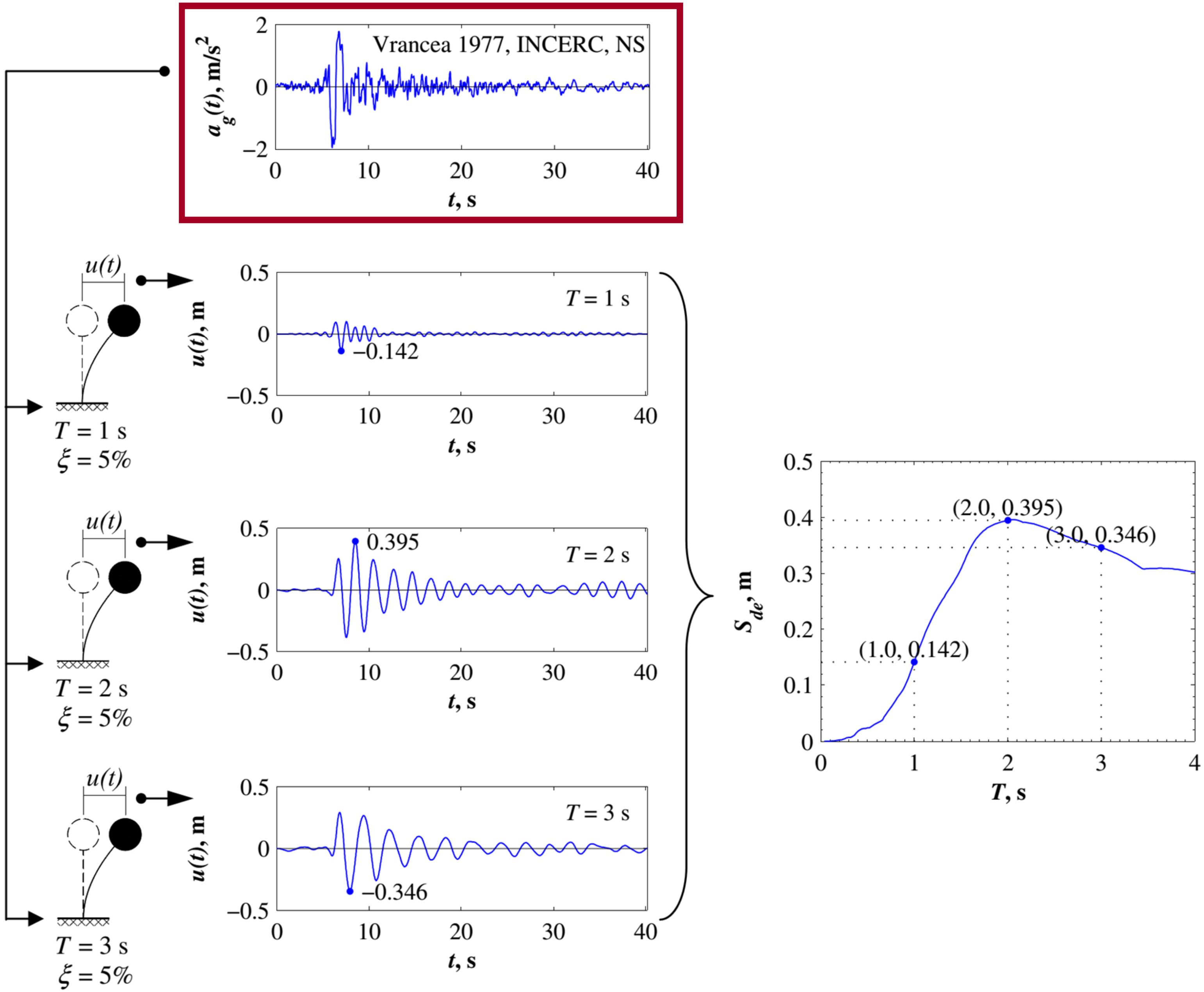
$$\dot{u}_0\left(T_n,\xi\right)=\max_t\left|\dot{u}\left(t,T_n,\xi\right)\right|$$

$$\ddot{u}_0^t\left(T_n,\xi\right)=\max_t\left|\ddot{u}^t\left(t,T_n,\xi\right)\right|$$

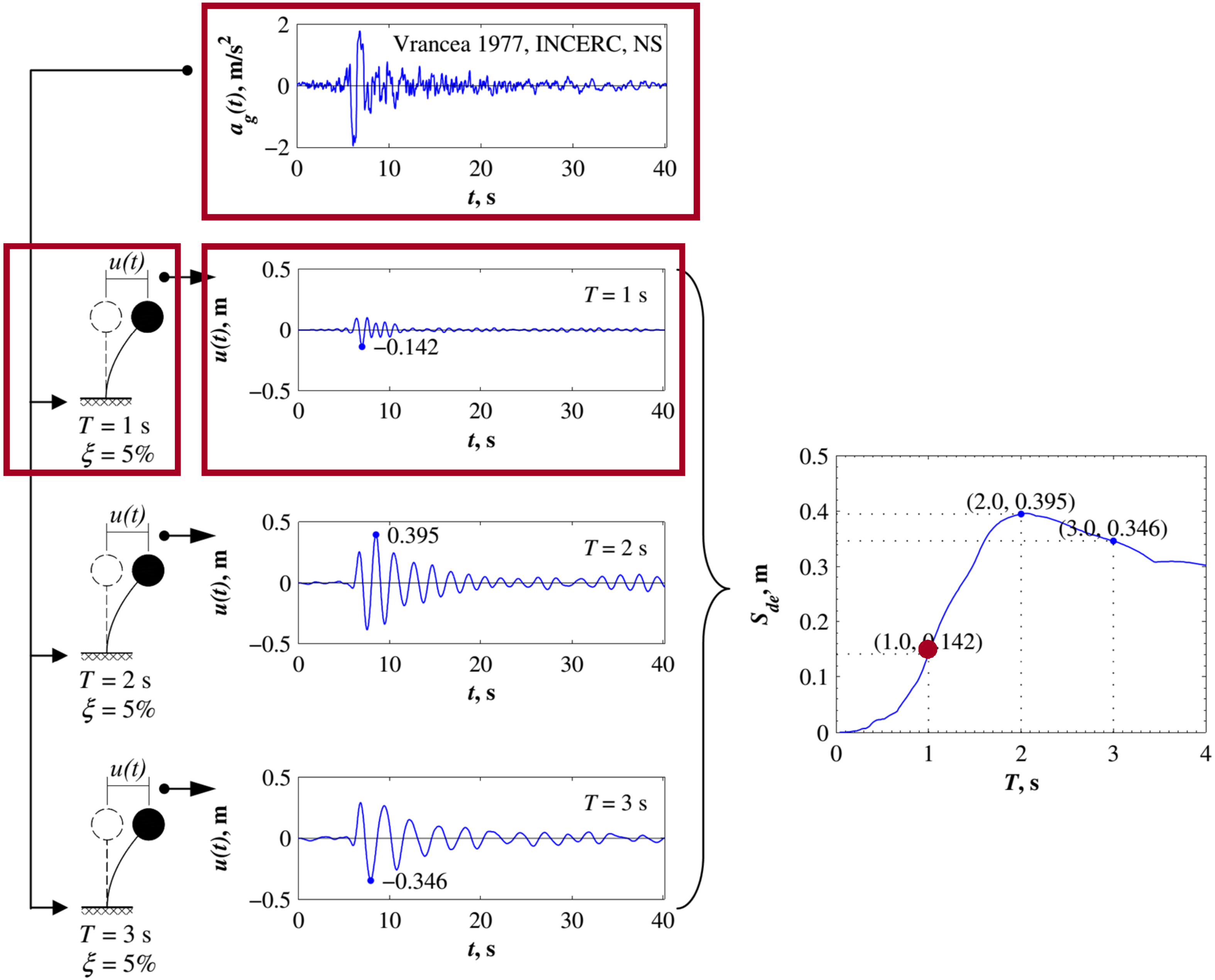
Elastic displacement response spectrum: $D \equiv u_0$



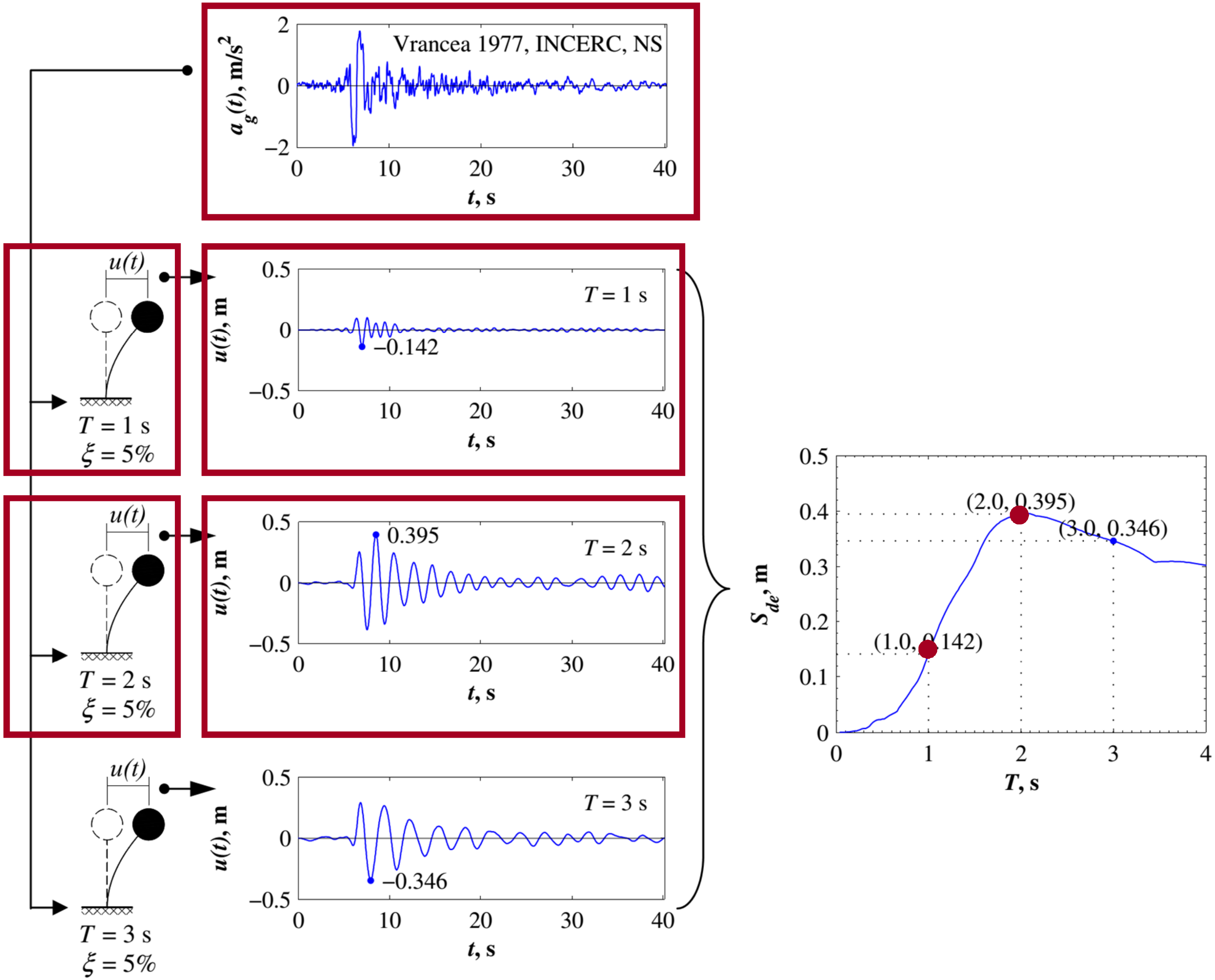
Elastic displacement response spectrum: $D \equiv u_0$



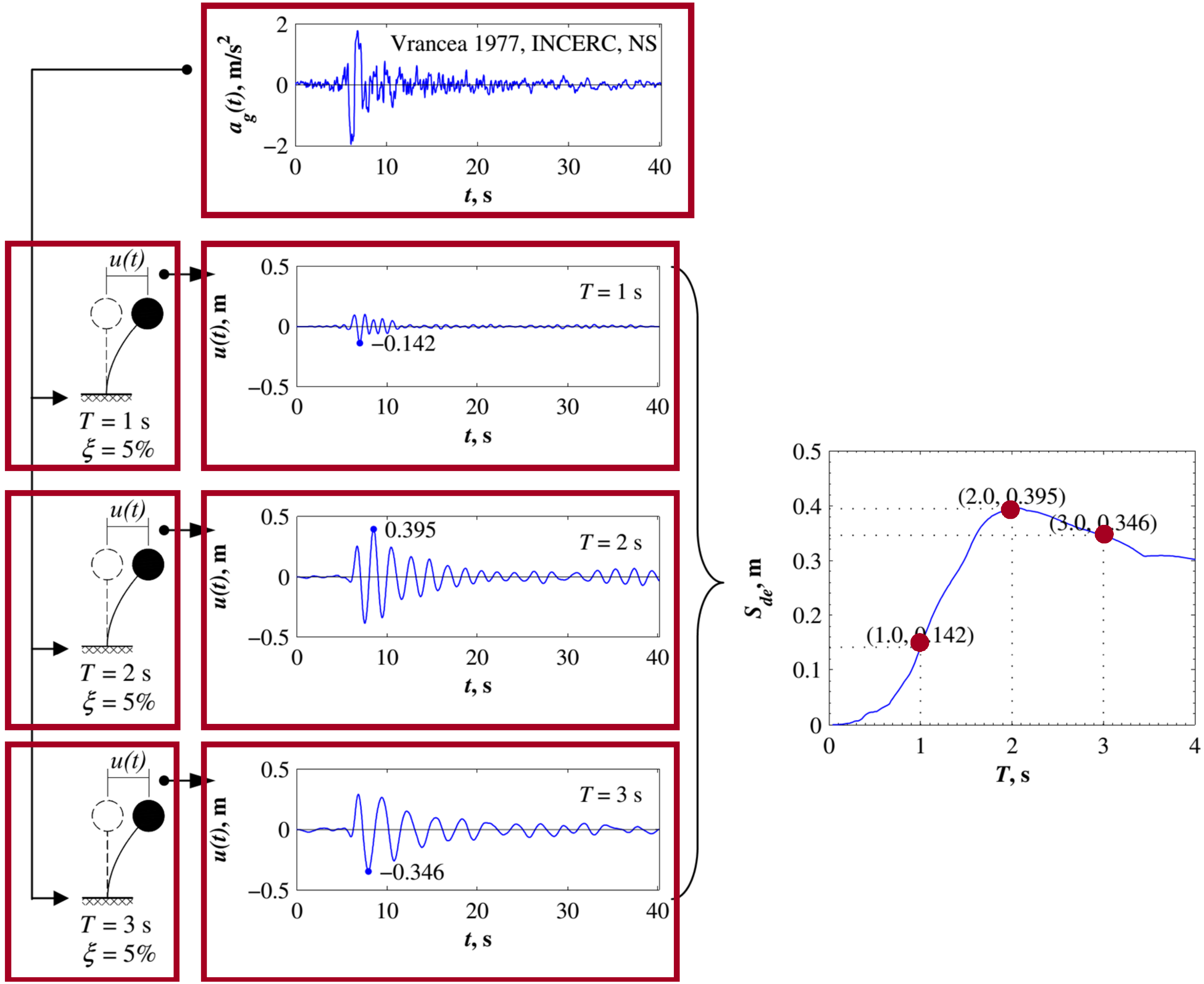
Elastic displacement response spectrum: $D \equiv u_0$



Elastic displacement response spectrum: $D \equiv u_0$



Elastic displacement response spectrum: $D \equiv u_0$



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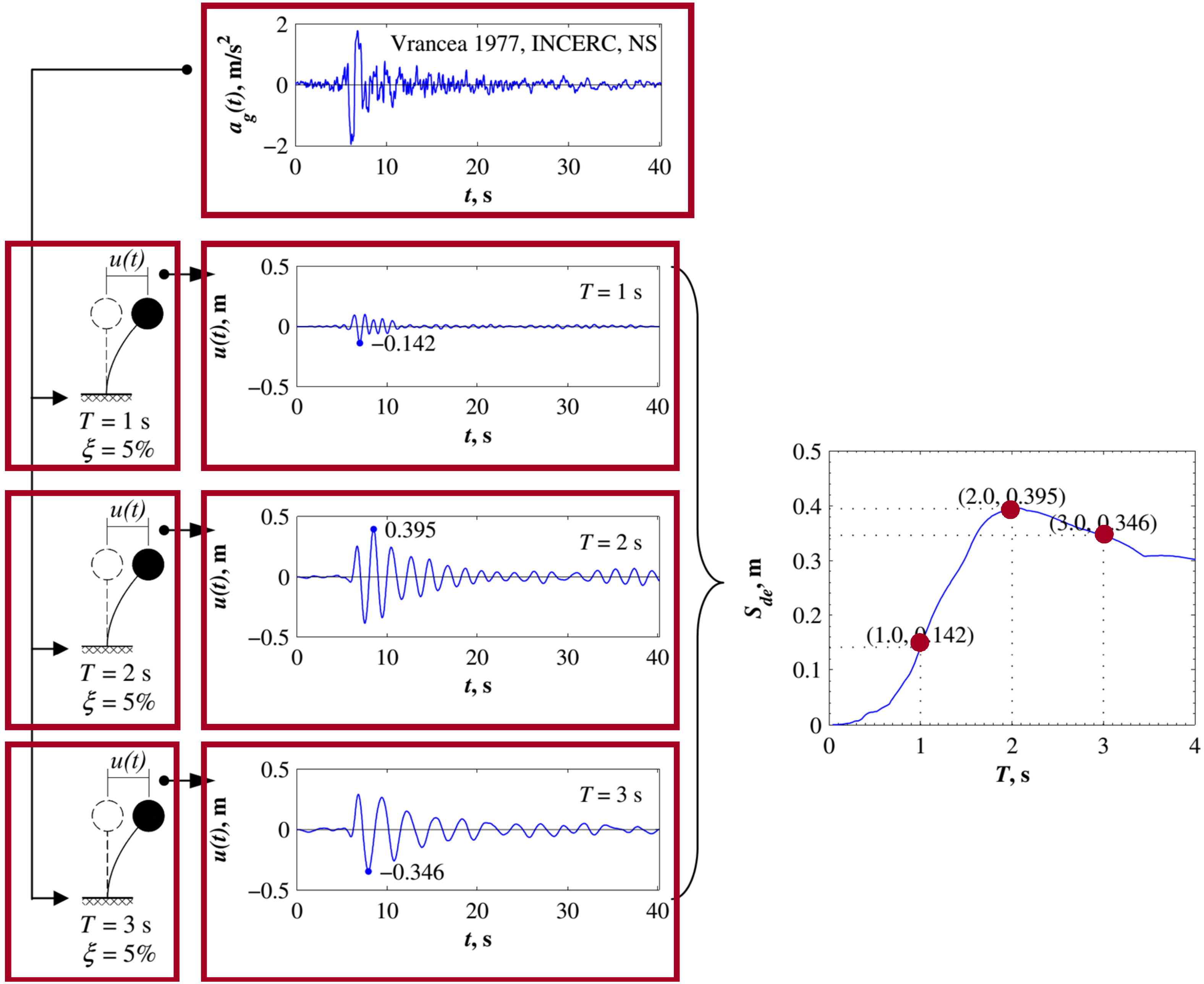


Pseudo-velocity and pseudo-acceleration

- **Spectral pseudo-velocity:**
  - **units of velocity**
  - **different from peak velocity**

$$V = \omega_n D = \frac{2\pi}{T_n} D$$

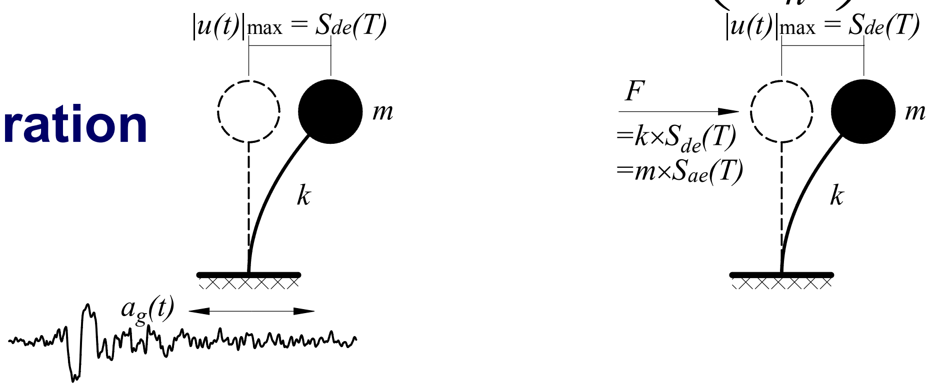
- **Strain energy**

$$E_{S0} = \frac{k u_0^2}{2} = \frac{kD^2}{2} = \frac{k\left(V/\omega_n\right)^2}{2} = \frac{mV^2}{2}$$

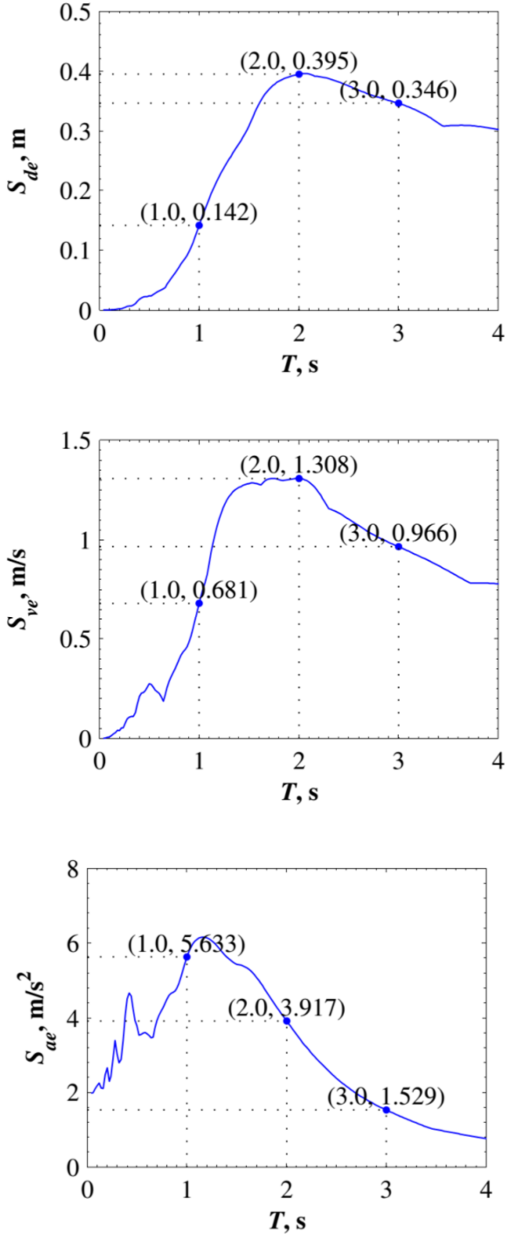
- **Spectral pseudo-acceleration:**

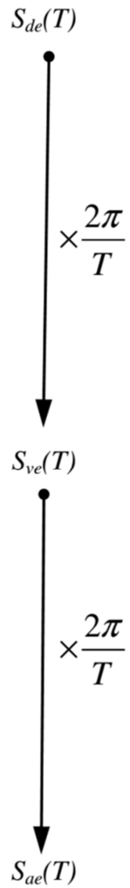
$$f_{S0} = k u_0 = m\omega_n^2 u_0 = mA \;\Rightarrow\; A = \omega_n^2 u_0 = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

  - **units of acceleration**
  - **different from peak acceleration**



Pseudo-velocity and pseudo-acceleration





$$D$$

$$V = \omega_n D = \frac{2\pi}{T_n} D$$

$$A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

Combined D-V-A spectrum

- **Displacement, pseudo-velocity and pseudo-acceleration spectra:**
  - **same information**
  - **different physical meaning**

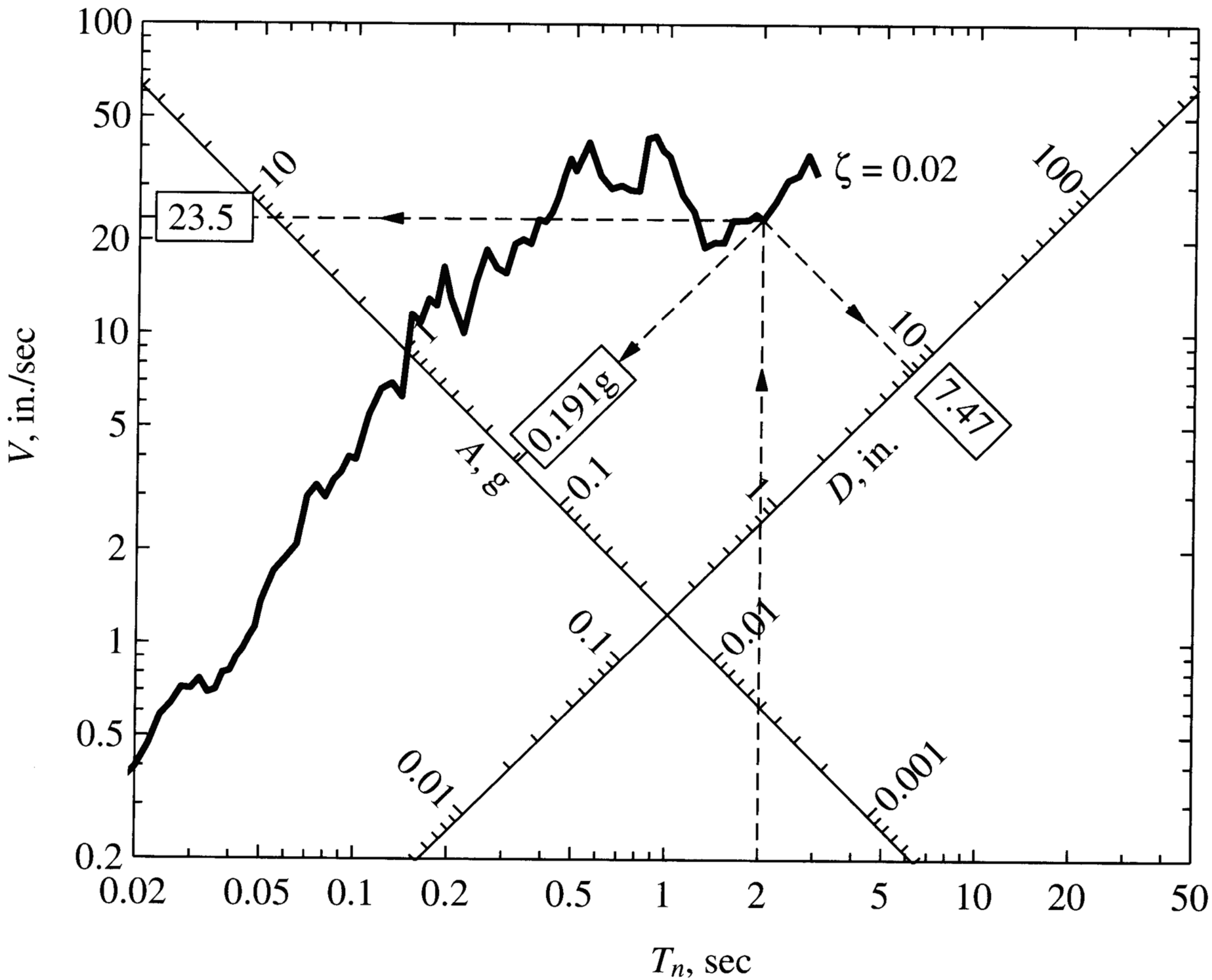
$$\frac{A}{\omega_n} = V = \omega_n D \quad or \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D$$

$$\left(T_n / 2\pi\right) A = V \Longrightarrow \lg T_n + \lg A - \lg 2\pi = \lg V$$

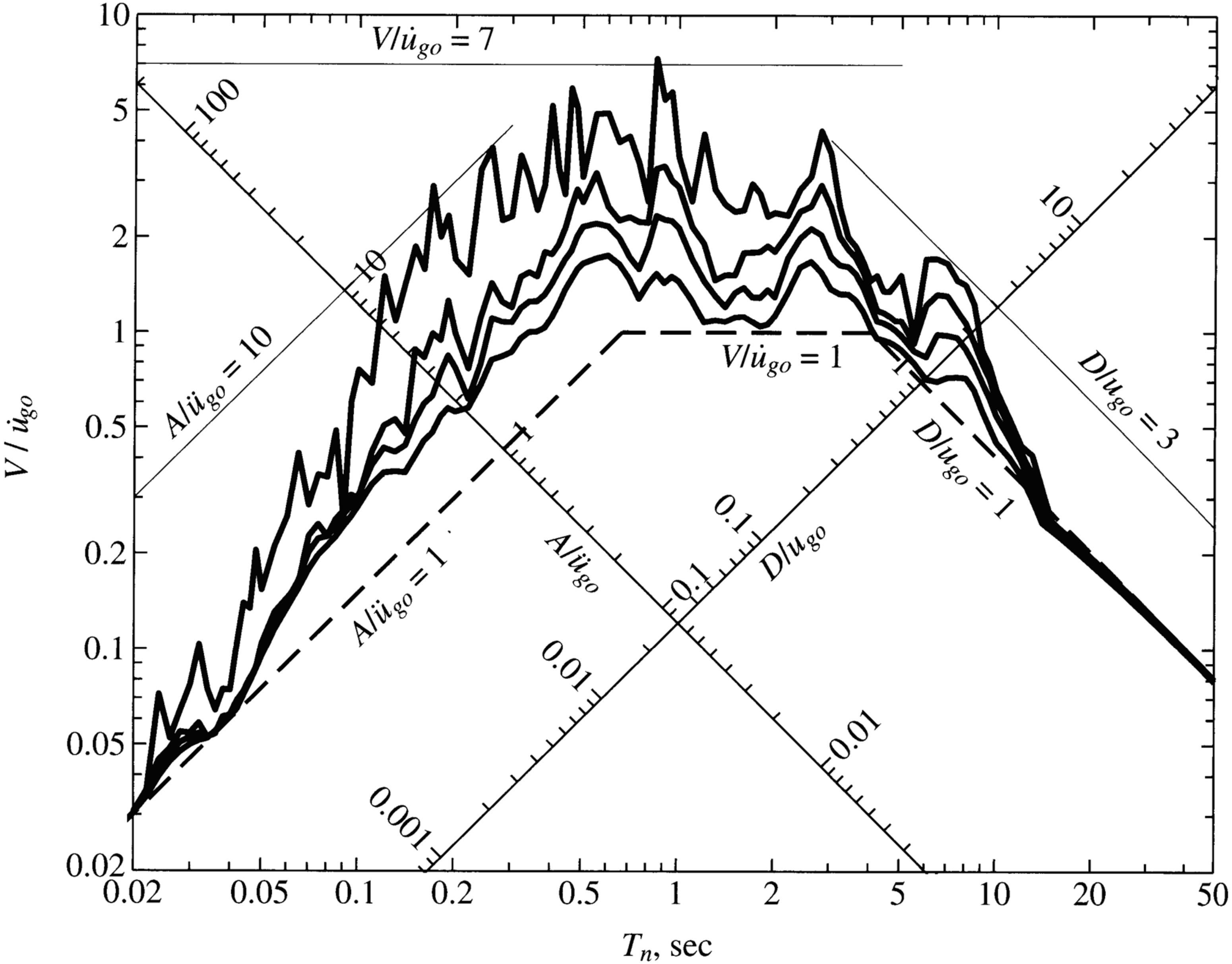
- **A line inclined at +45º for lg*A* - lg2π = const. $\Rightarrow$ spectral pseudo-acceleration: an axis inclined to -45º**

- **Similarly, spectral displacement: an axis inclined to +45º**

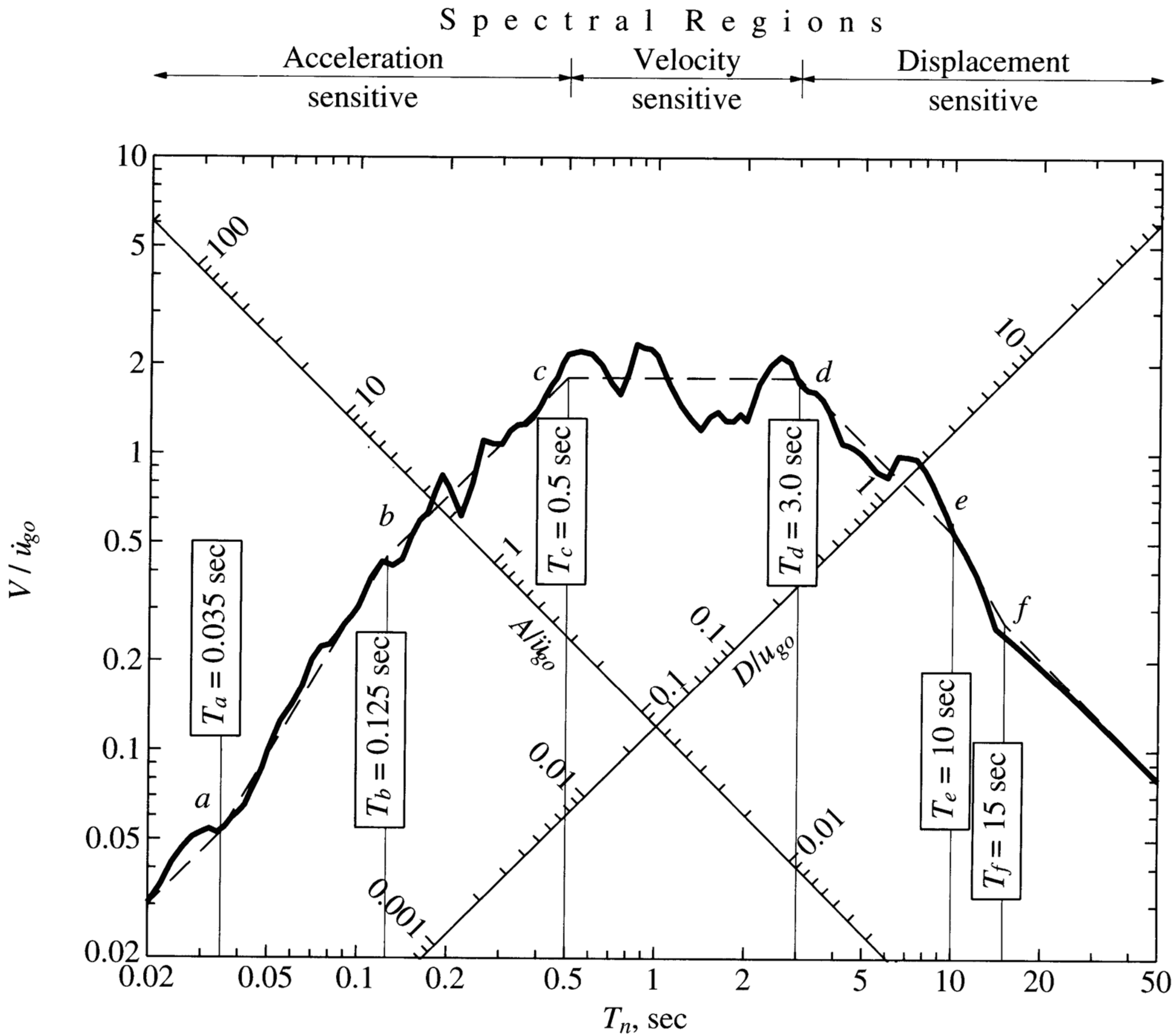
Combined D-V-A spectrum



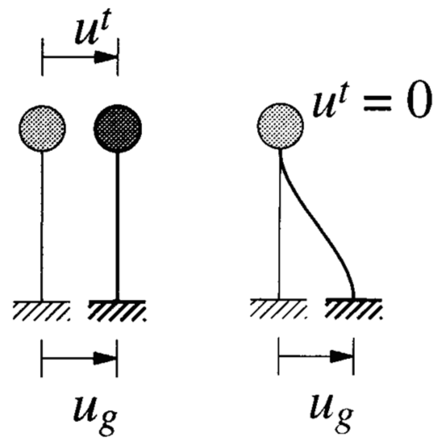
Characteristics of elastic response spectra



Characteristics of elastic response spectra



Characteristics of elastic response spectra



- For $T_n < T_a$
  - pseudo-acceleration A is close to $\ddot{u}_{g0}$
  - spectral displacement D is small
- For $T_n > T_f$
  - spectral displacement D is close to u_{g0}
  - spectral pseudo-acceleration A is small
- Between T_a and $T_c \Rightarrow A > \ddot{u}_{g0}$
- Between T_b and $T_c \Rightarrow A$ can be considered constant

- Between T_d and $T_f \Rightarrow D > u_{g0}$
- Between T_d and $T_e \Rightarrow D$ can be considered constant

- Between T_c and $T_d \Rightarrow V > \dot{u}_{g0}$
- Between T_c and $T_d \Rightarrow V$ can be considered constant

Characteristics of elastic response spectra

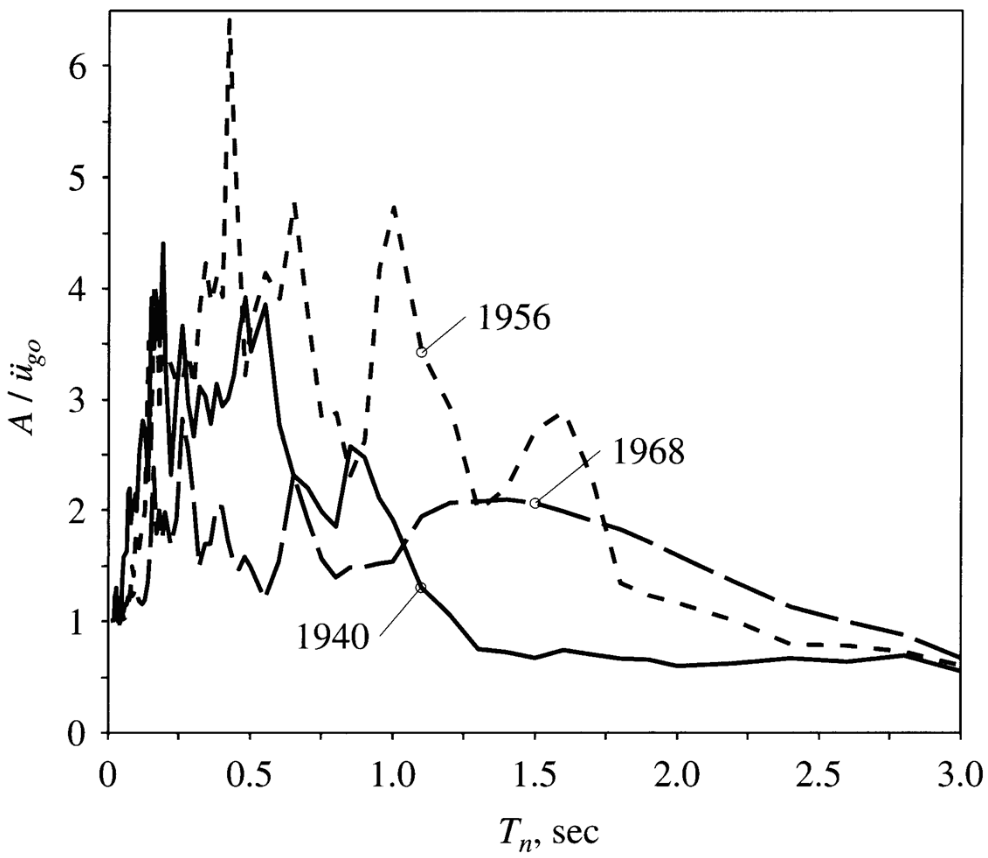
- $T_n > T_d \Rightarrow$ **response region sensible to displacements**
- $T_n < T_c \Rightarrow$ **response region sensible to accelerations**
- $T_c < T_n < T_d \Rightarrow$ **response region sensible to velocity**

- **Larger damping:**
  - smaller values of displacements, pseudo-velocity and pseudo-acceleration
  - more "smooth" spectra

- **Effect of damping:**
  - insignificant for $T_n \to 0$ and $T_n \to \infty$,
  - important for $T_b < T_n < T_d$

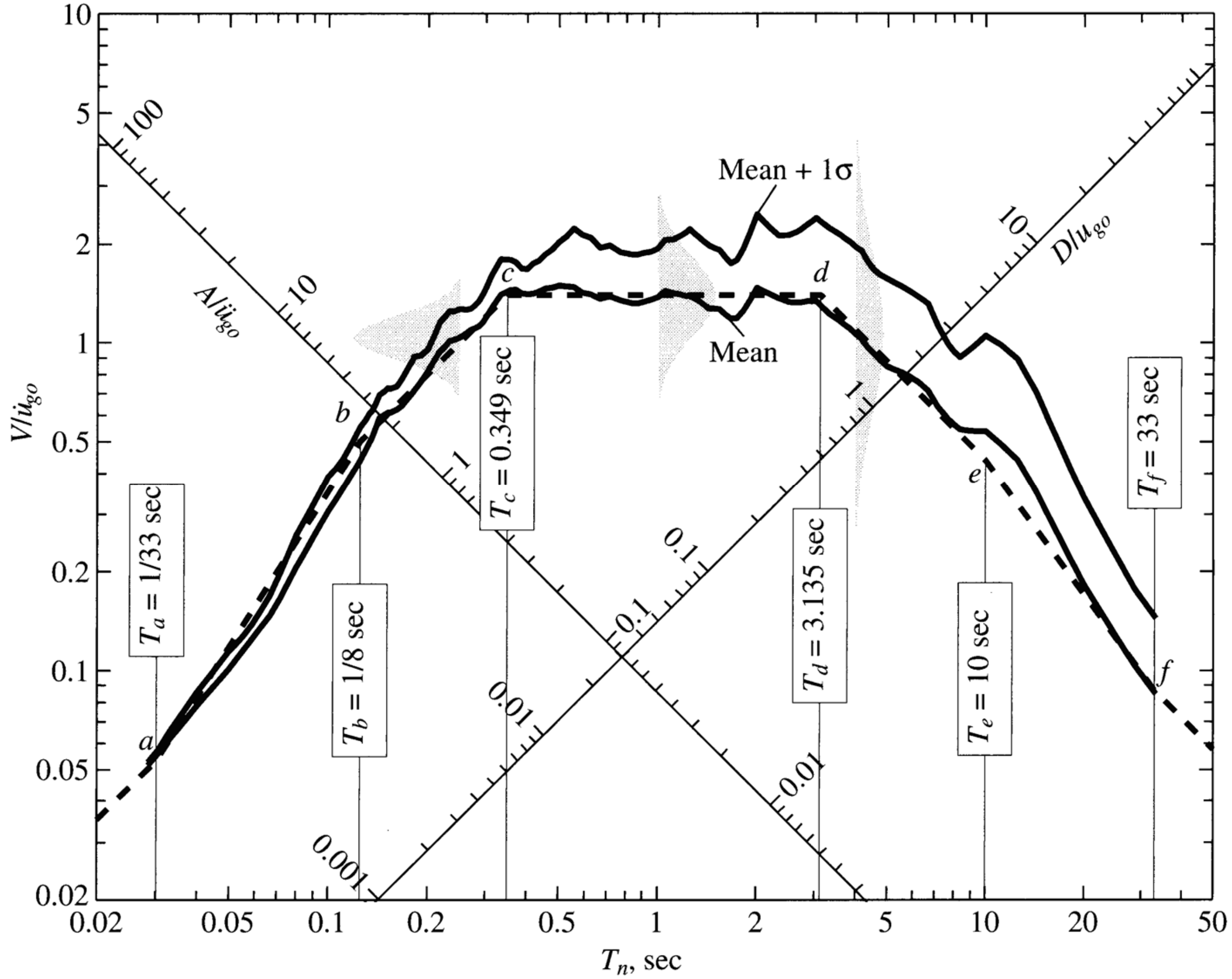
Elastic design spectra

- **Spectra of past ground motions:**
  - jagged shape
  - variation of response for different earthquakes
  - areas where previous data is not available

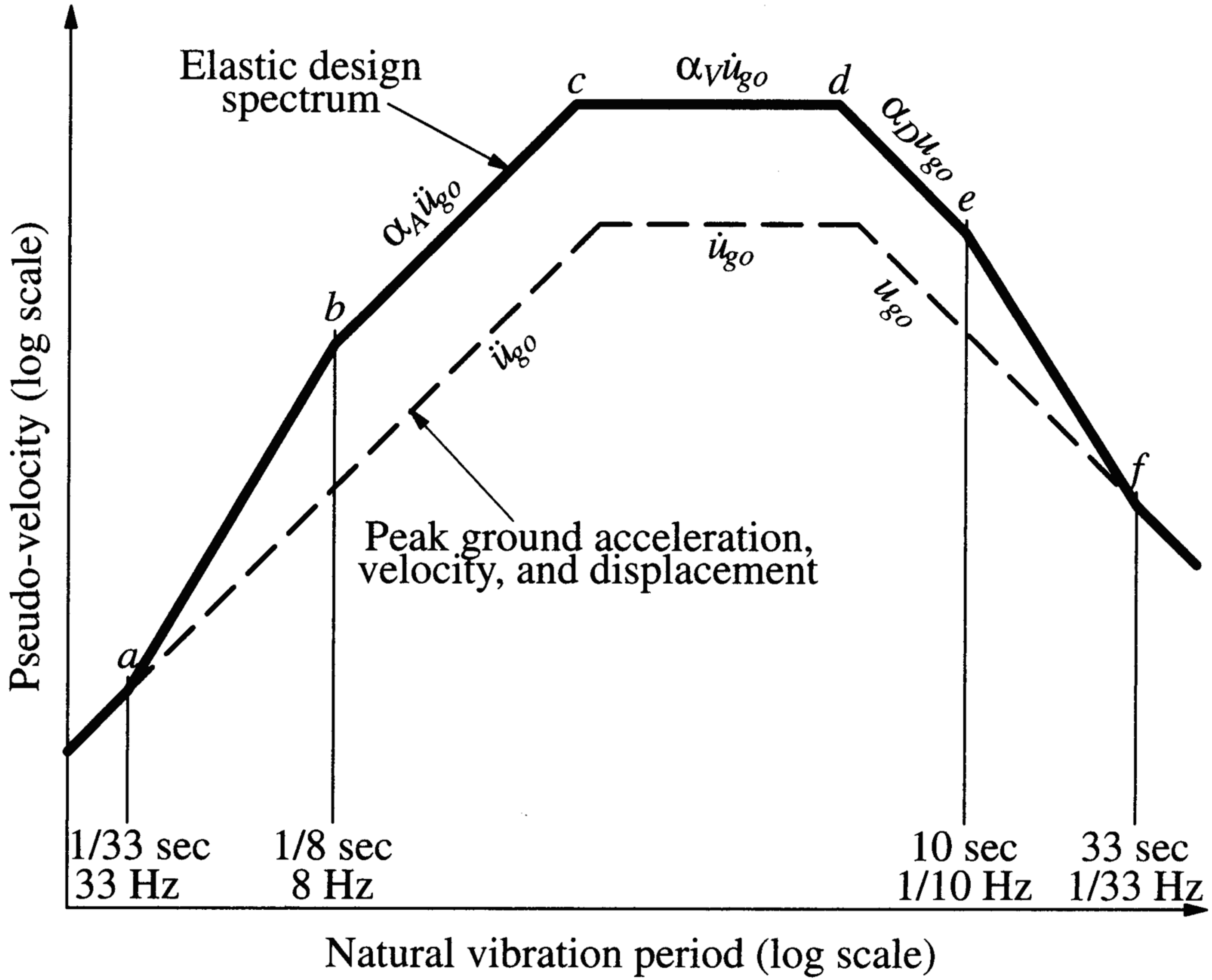


Elastic design spectra

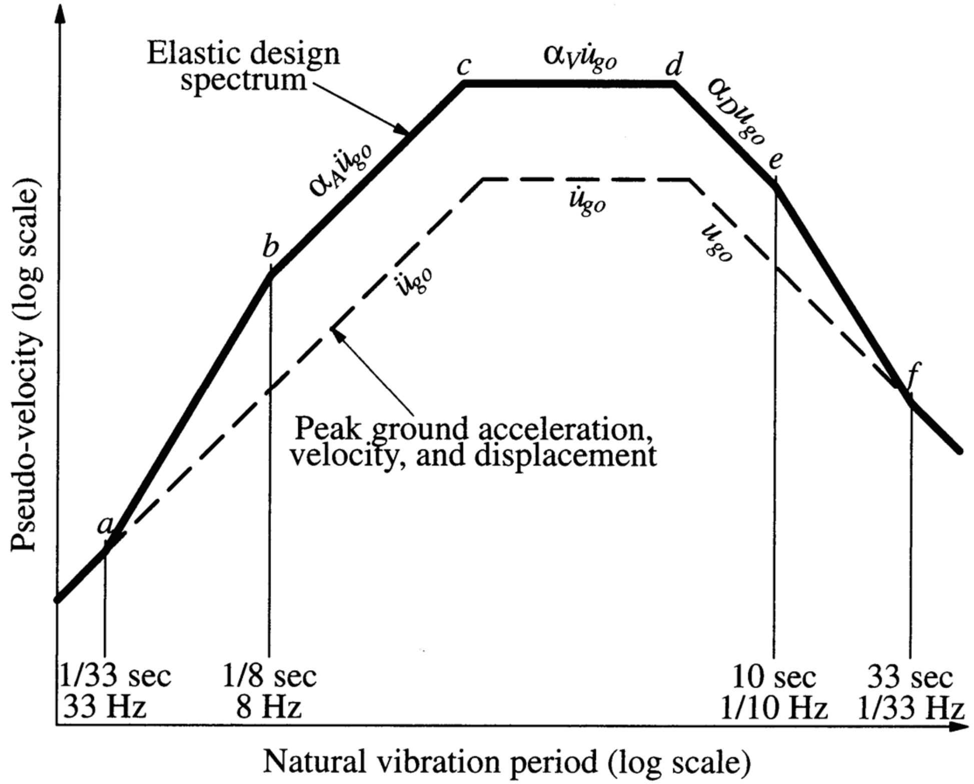
- idealized "smooth" spectra
- based on statistical interpretation (median; median plus standard deviation) of several records characteristic for a given site



Elastic design spectra



Elastic design spectra



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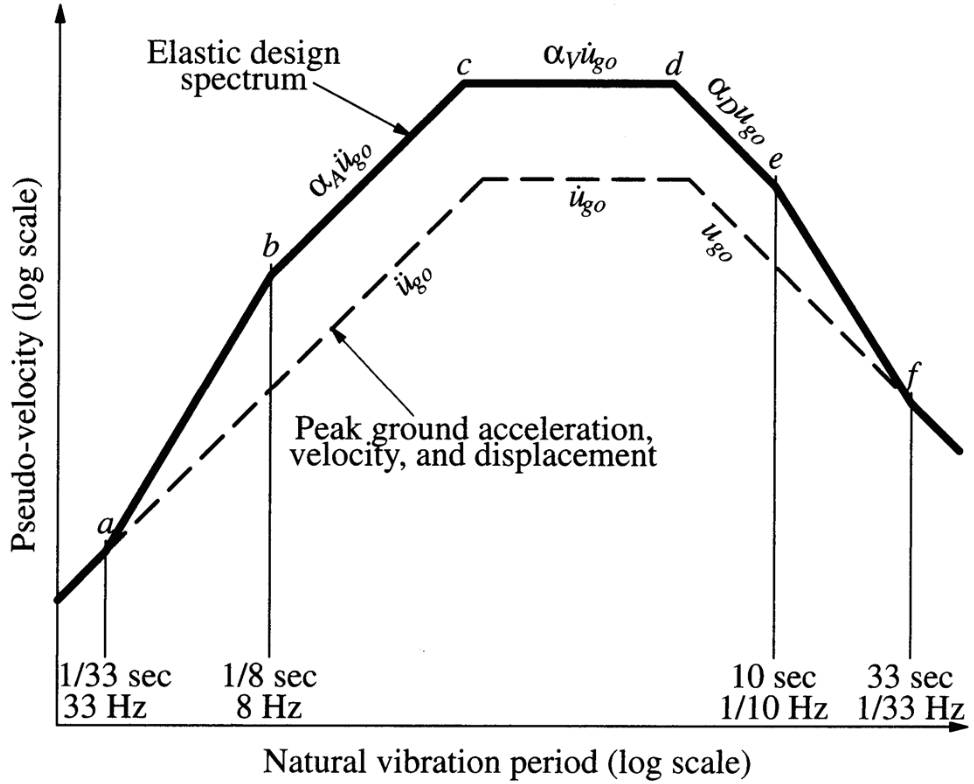

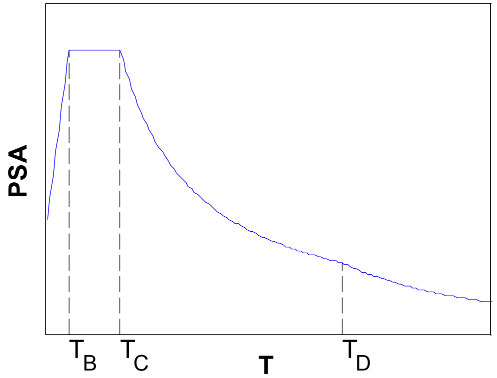

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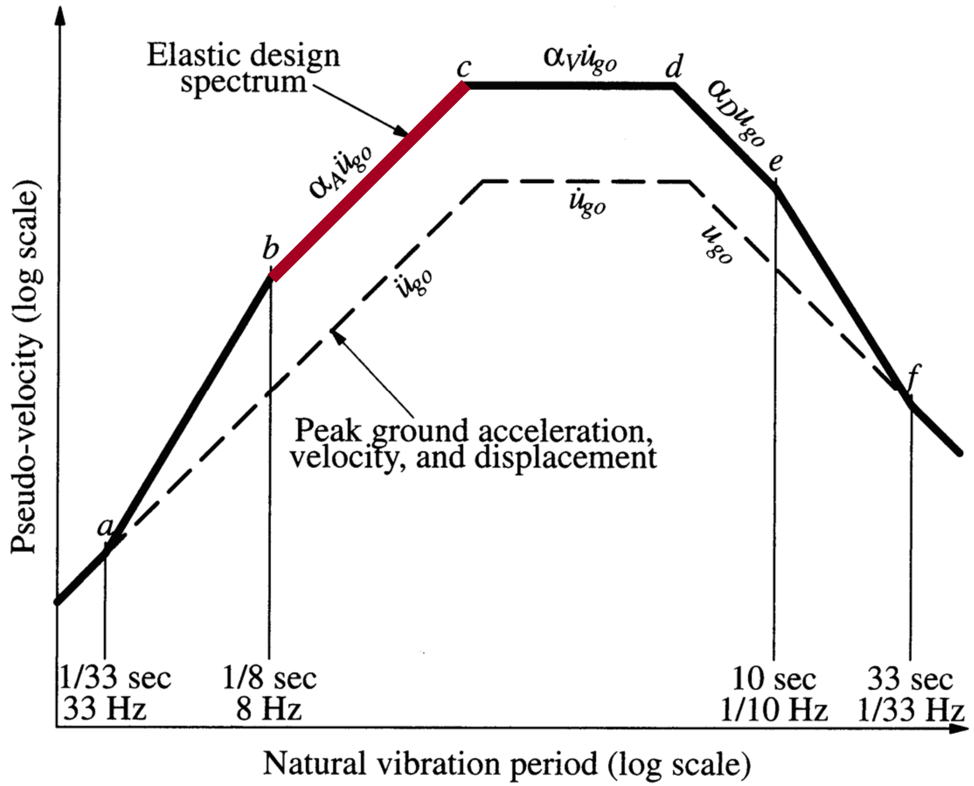

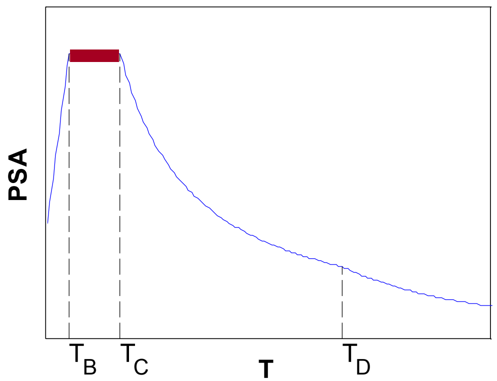

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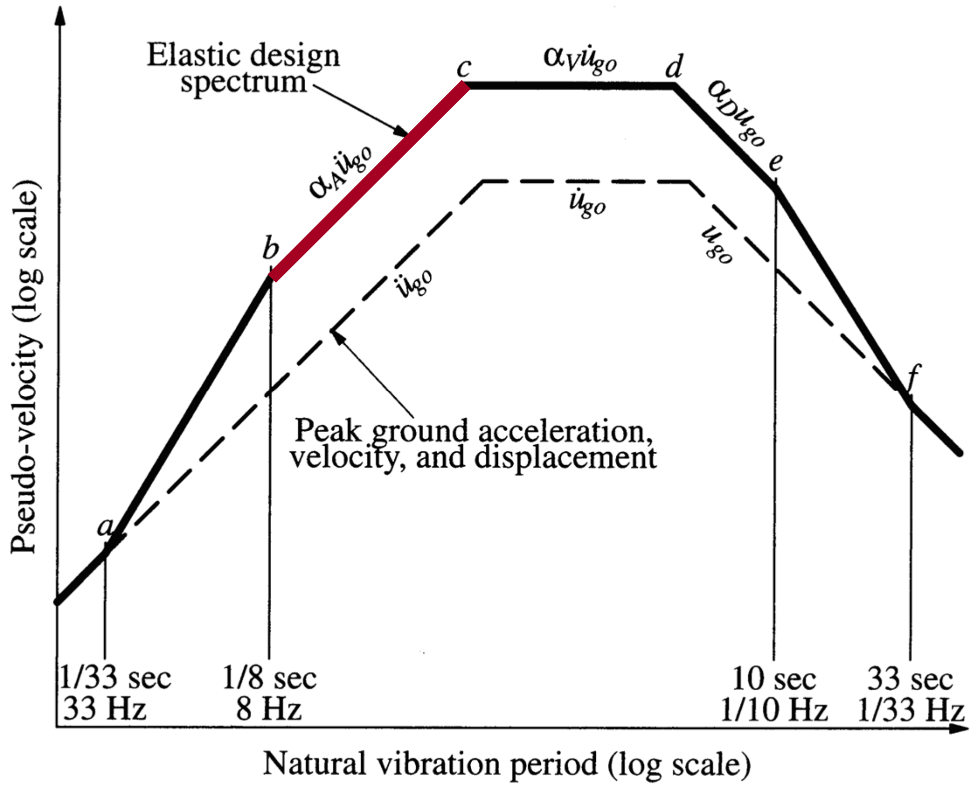

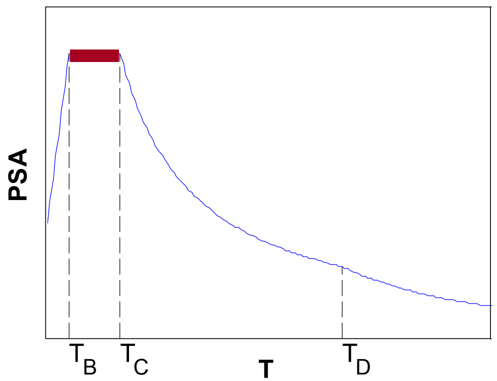

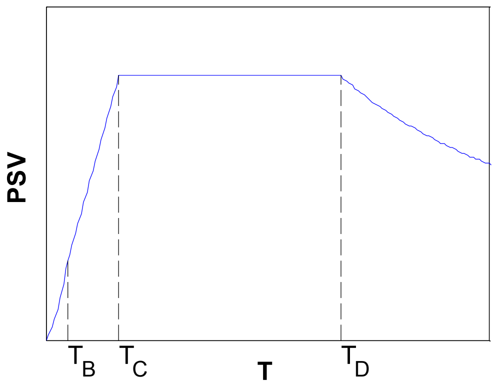

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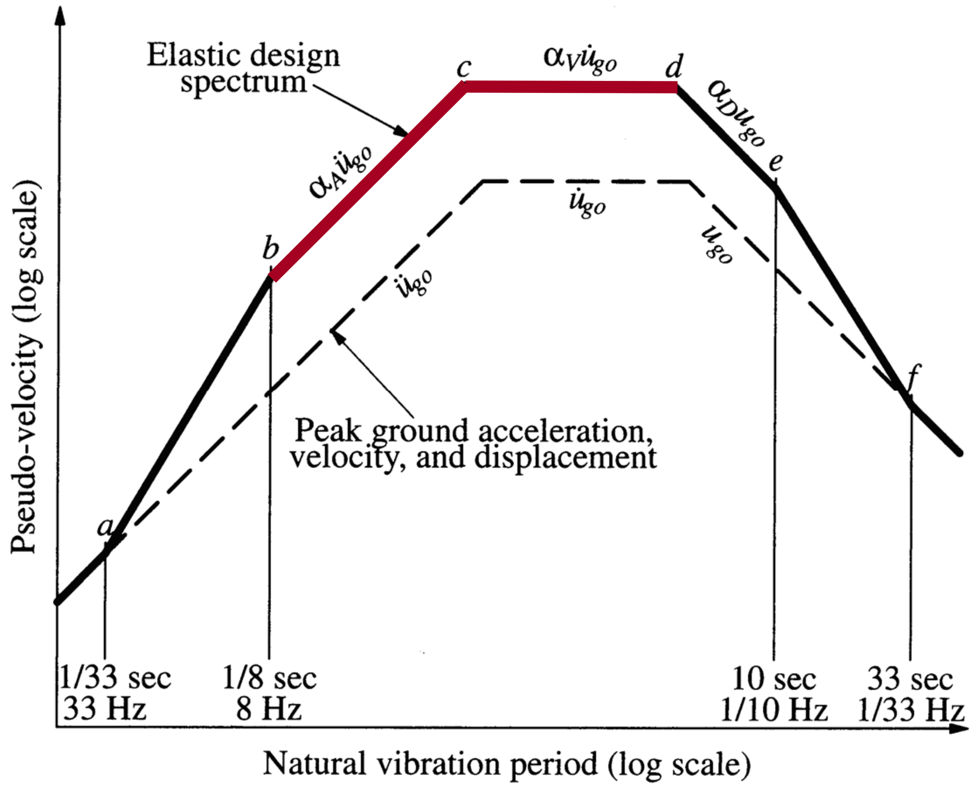

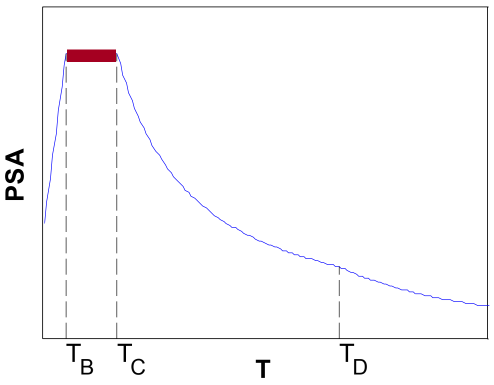

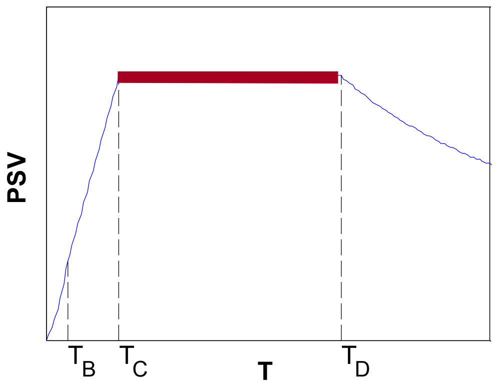

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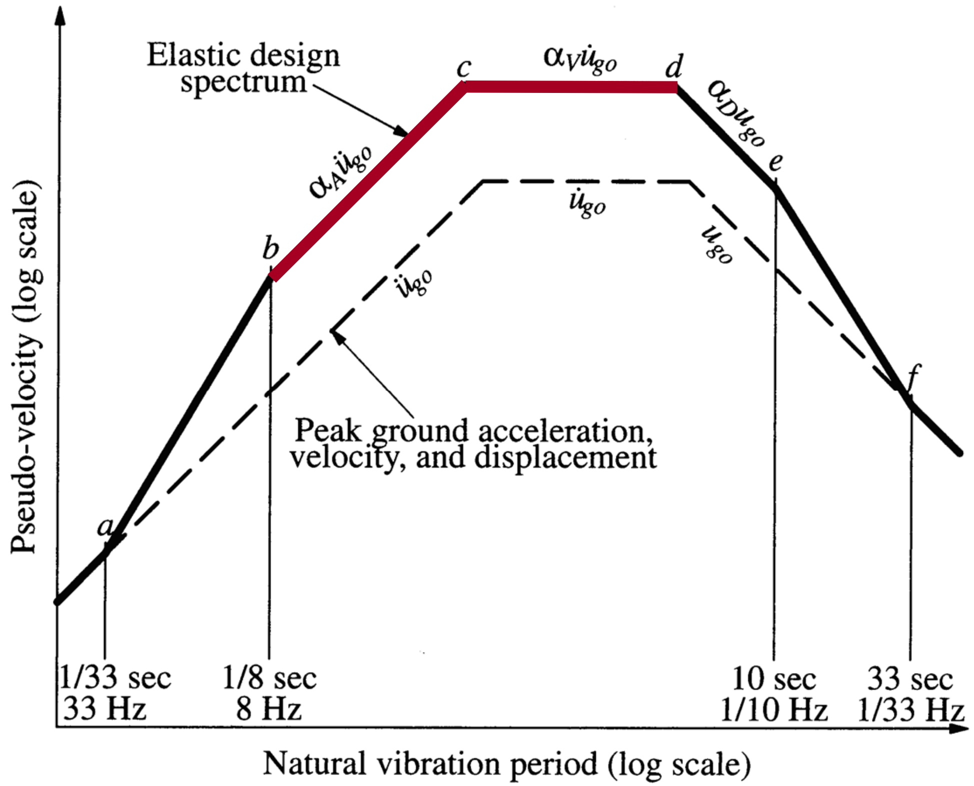
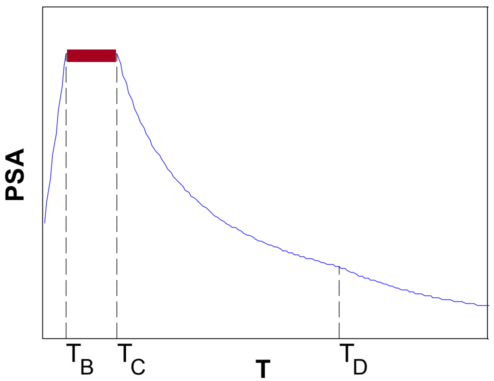
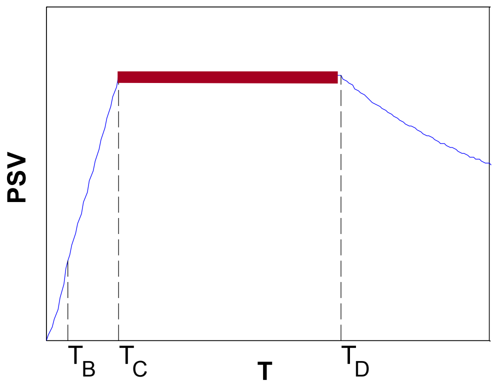
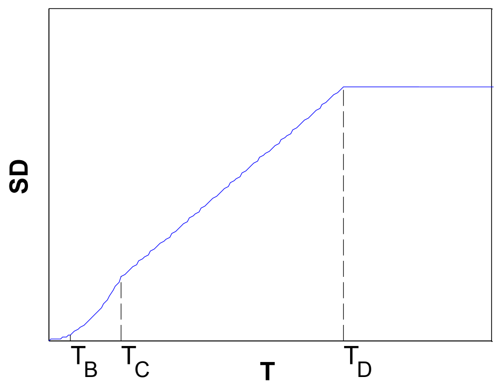
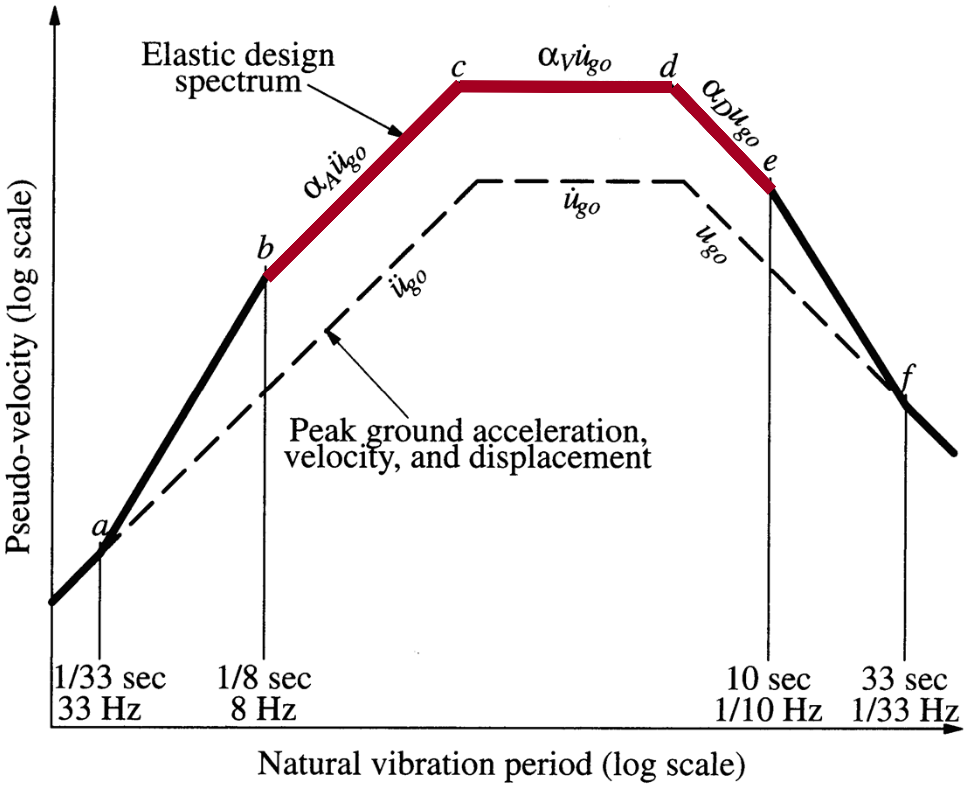


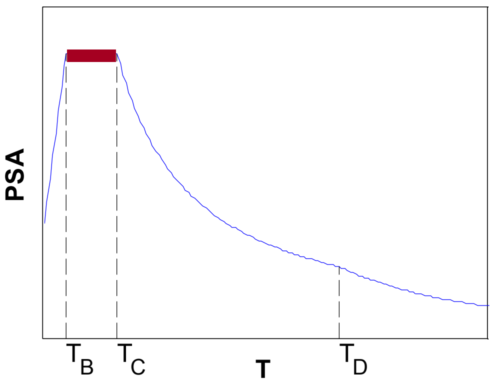


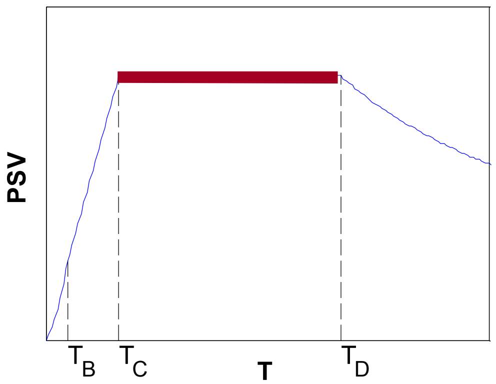


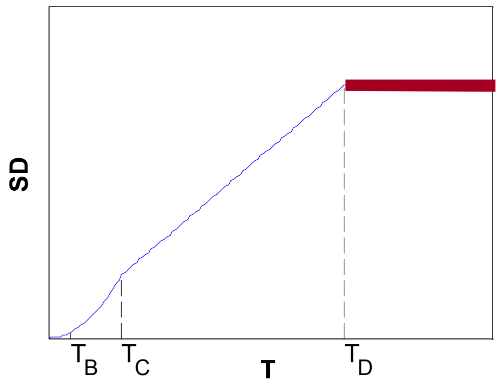


Elastic design spectra



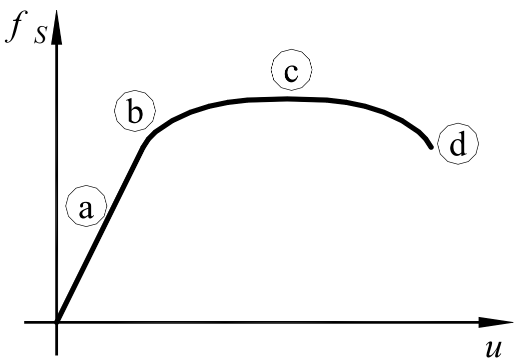


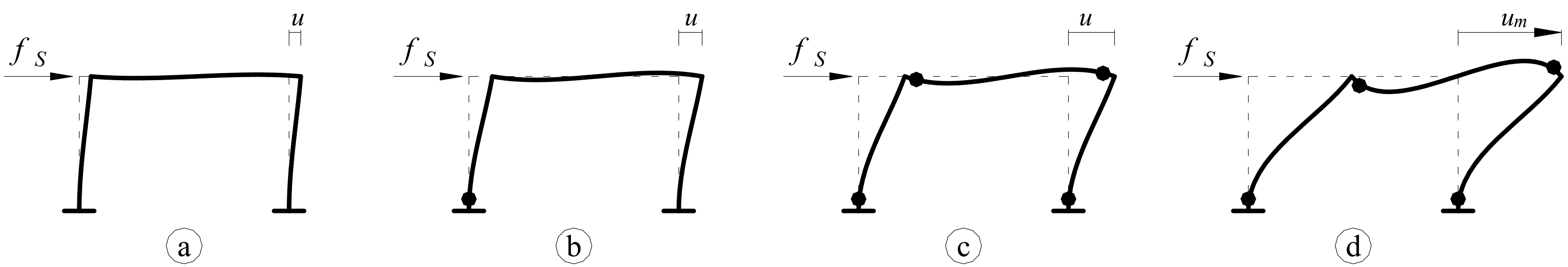




Inelastic response of SDOF systems

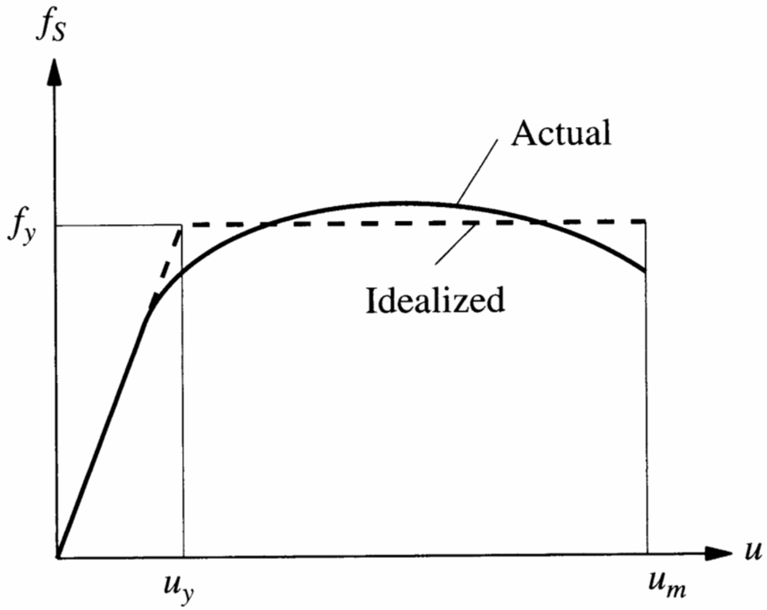
- **Most structures designed for seismic forces lower than the ones assuring an elastic response during the design earthquake**
  - design of structures in the elastic range for rare seismic events considered uneconomical
  - in the past, structures designed for a fraction of the forces necessary for an elastic response, survived major earthquakes



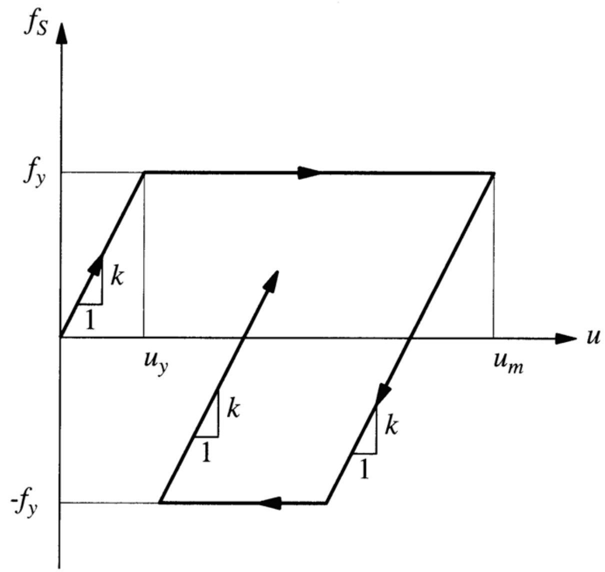


Inelastic response of SDOF systems

- **Elasto-plastic system:**
  - **stiffness *k***
  - **yield force f_y**
  - **yield displacement u_y**
- **Elasto-plastic idealization: equal area under the actual and idealised curves up to the maximum displacement u_m**



- **Cyclic response of the elasto-pla**



Corresponding elastic system

- **Corresponding elastic system:**
  - **same stiffness**
  - **same mass**
  - **same damping**

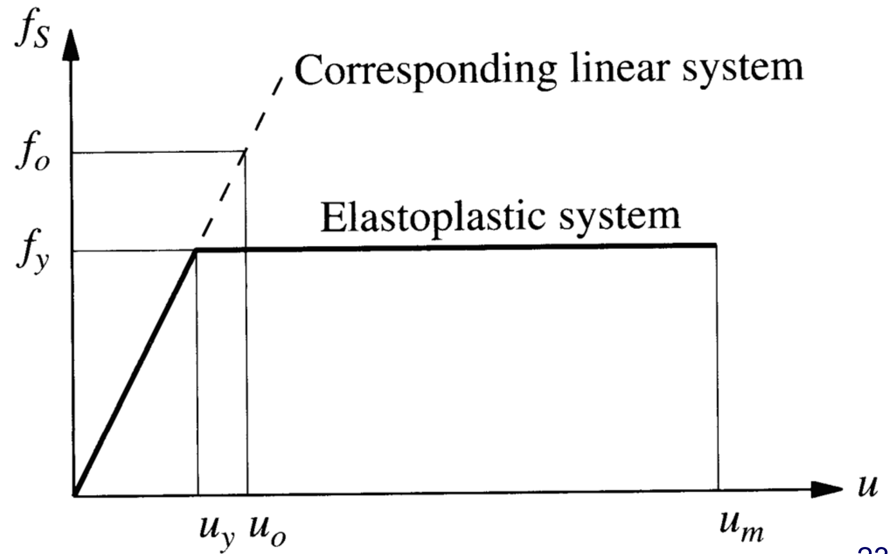
  } **the same period of vibration (at small def.)**

- **Inelastic response:**
  - **yield force reduction factor R_y**

$$R_y = \frac{f_0}{f_y} = \frac{u_0}{u_y}$$

  - **ductility factor**

$$\mu = \frac{u_m}{u_y}$$



Equation of motion

- **Equation of motion:** $m\ddot{u} + c\dot{u} + f_S\left(u,\dot{u}\right) = -m\ddot{u}_g$

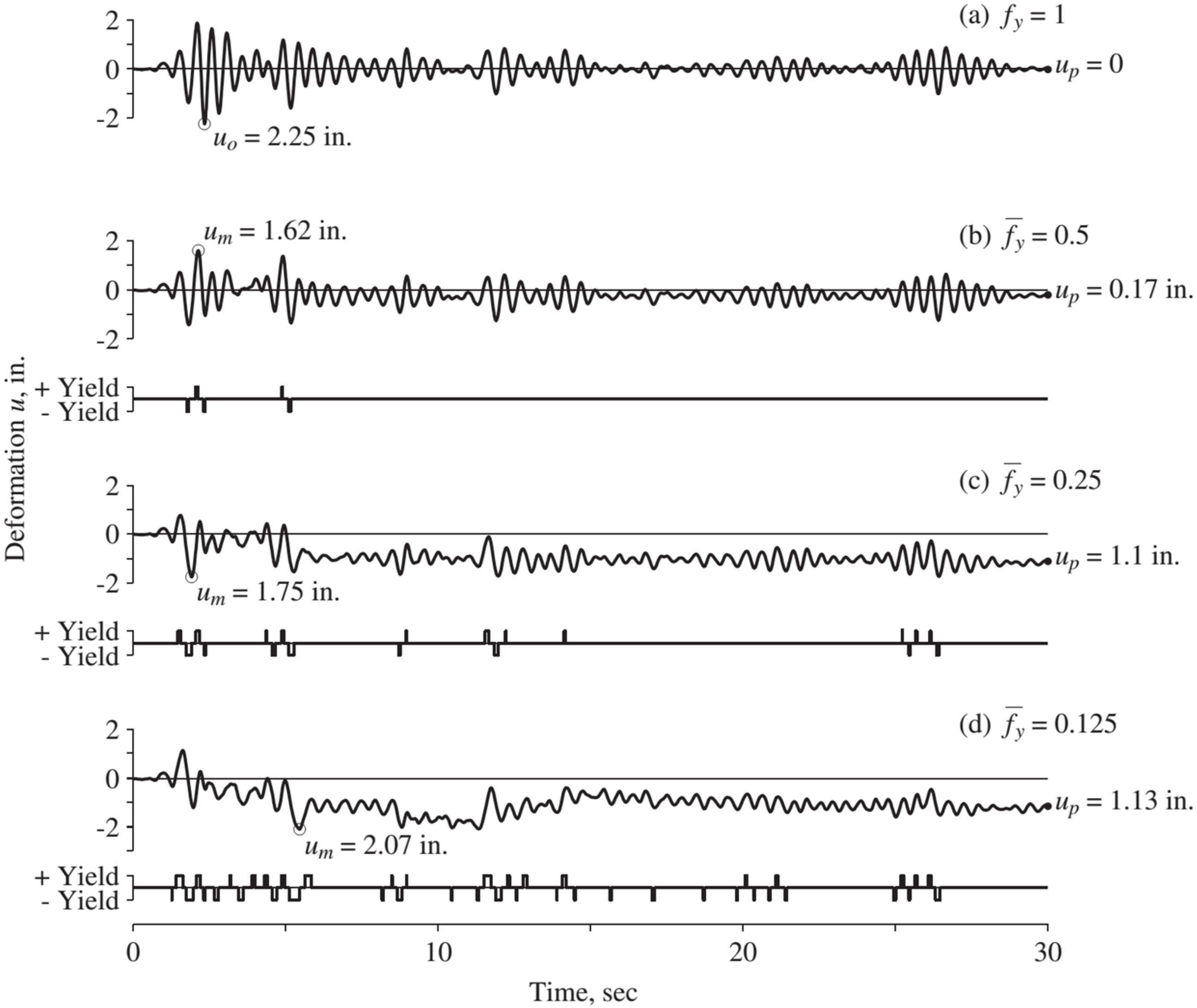
- **/m** $\Rightarrow$ $\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u_y \tilde{f}_S\left(u,\dot{u}\right) = -\ddot{u}_g$

  $\tilde{f}_S\left(u,\dot{u}\right) = f_S\left(u,\dot{u}\right)/f_y$

- **Seismic response of an inelastic SDOF system depends on:**
  - **natural circular frequency of vibration** ω_n
  - **critical damping ratio** ξ
  - **yield displacement** u_y
  - **force-displacement shape** $\tilde{f}_S\left(u,\dot{u}\right)$

Effects of inelastic force-displacement relationship

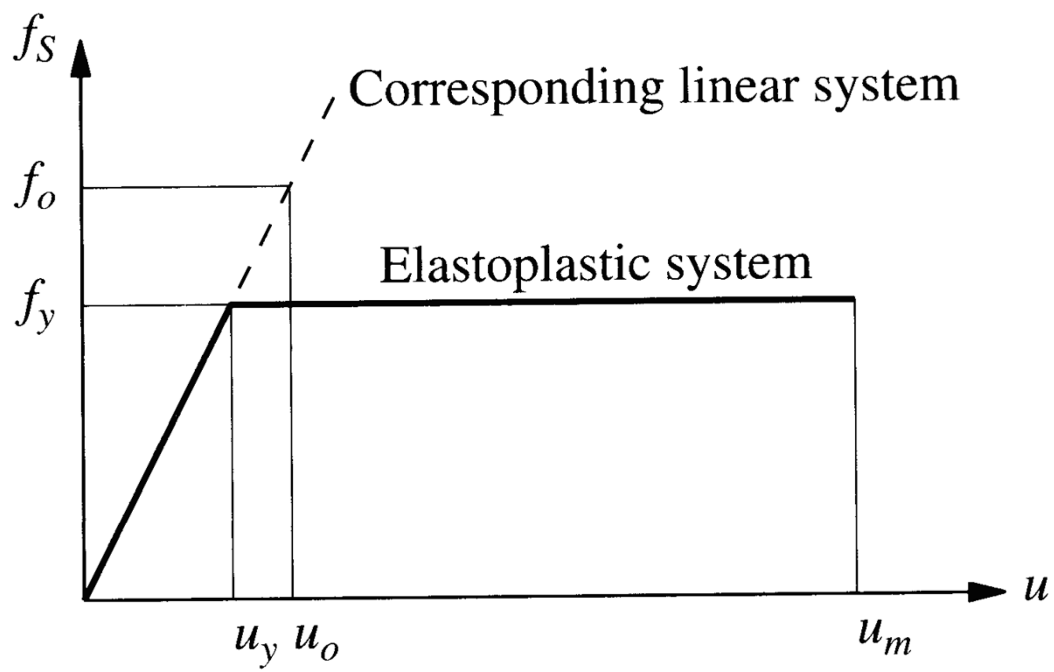
- **4 SDOF systems (El Centro):**
  - $T_n = 0.5$ sec
  - $\xi = 5\%$
  - $R_y = 1, 2, 4, 8$
- **Elastic system:**
  - vibr. about the initial position of equilibrium
  - $u_p=0$
- **Inelastic syst.:**
  - vibr. about a new position of equilibrium
  - $u_p \neq 0$



Elastic ⇔ inelastic

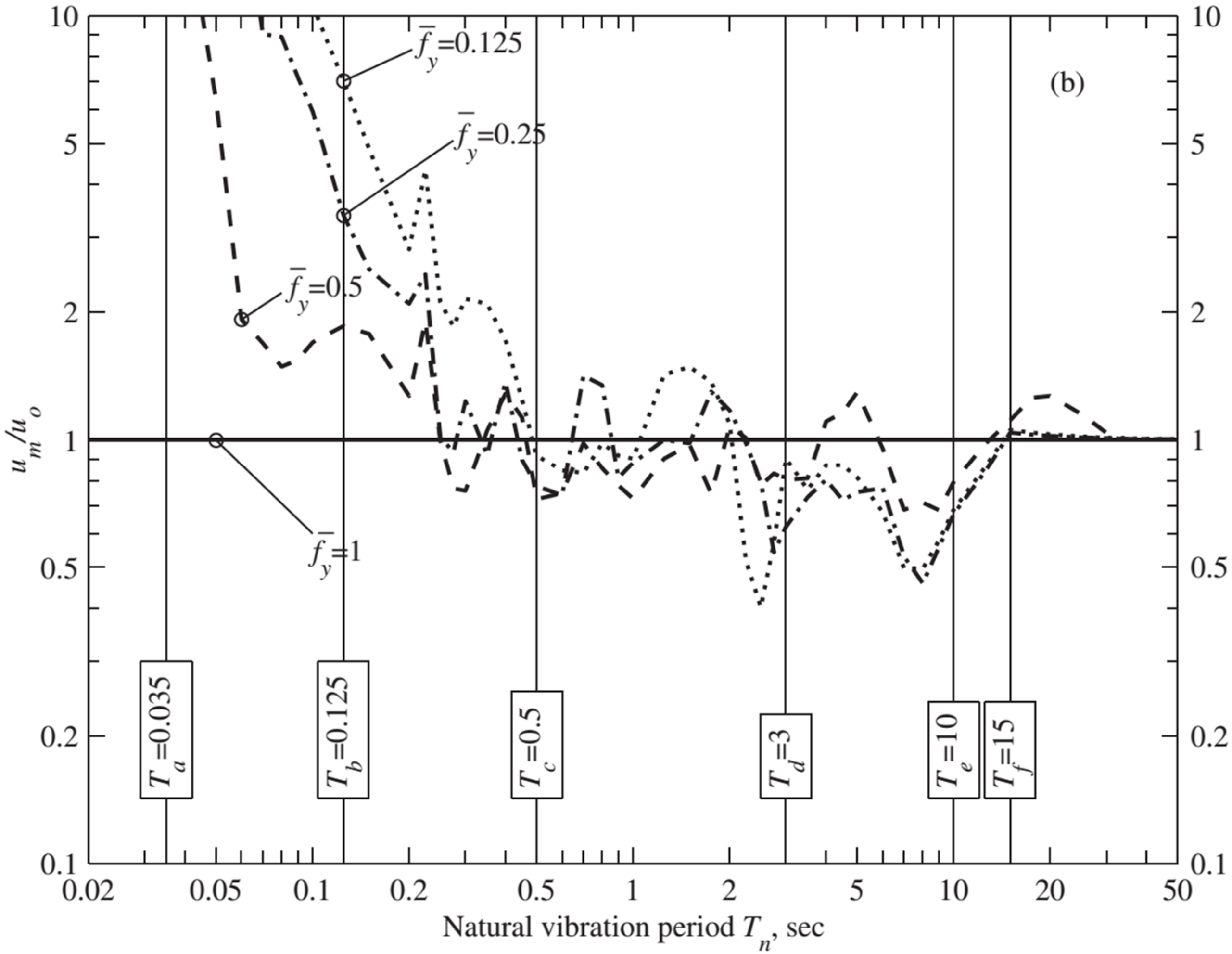
- **Design of a structure responding in the elastic range:** $f_0 \leq f_{Rd}$
- **Design of a structure responding in the inelastic range:** $u_m \leq u_{Rd}$ $\mu \leq \mu_{Rd}$

ductility demand   **ductility capacity**



u_m/u_0 ratio

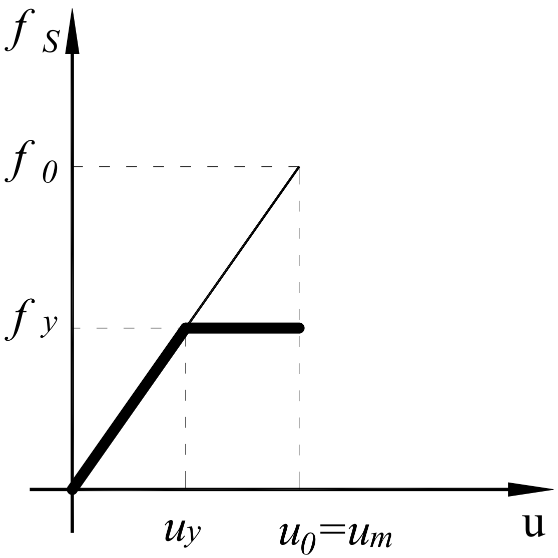
- **El Centro ground motion**
  - $\xi = 5\%$
  - $R_y = 1, 2, 4, 8$
- $T_n > T_f$
  - u_m independent of R_y
  - $u_m \cong u_0$
- $T_n > T_c$
  - u_m depends on R_y
  - $u_m \cong u_0$
- $T_n < T_c$
  - u_m depends on R_y
  - $u_m > u_0$

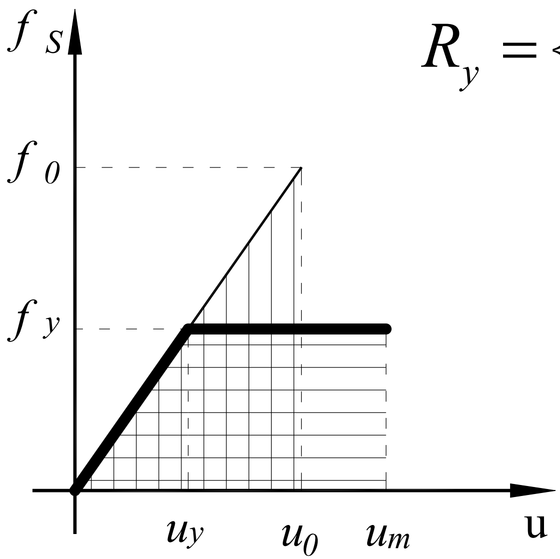


R_y - μ relationship: idealisation

- **T_n in the displacement- and velocity-sensitive region:**
  - "equal displacement" rule $u_m/u_0=1 \Rightarrow R_y=\mu$
- **T_n in the acceleration-sensitive region:**
  - "equal energy" rule $u_m/u_0>1 \Rightarrow$ $R_y = \sqrt{2\mu - 1}$
- **$T_n<T_a$:**
  - small deformations, elastic response $\Rightarrow R_y=1$

$$R_y = \begin{cases} 1 & T_n < T_a \\ \sqrt{2\mu - 1} & T_b < T_n < T_{c'} \\ \mu & T_n > T_c \end{cases}$$





R_y - μ relationship: idealisation

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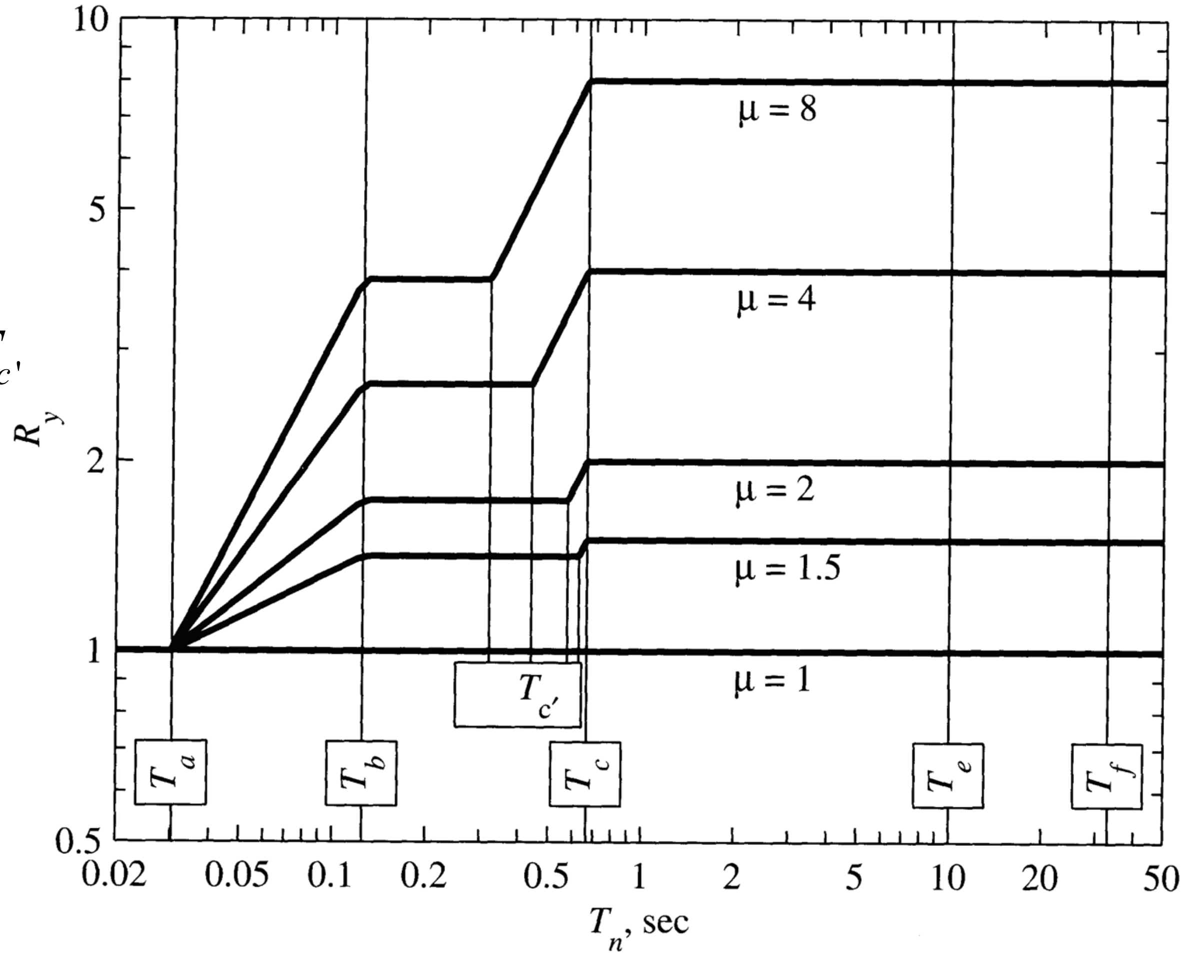

References / additional reading

- Anil Chopra, "Dynamics of Structures: Theory and Applications to Earthquake Engineering", Prentice-Hall, Upper Saddle River, New Jersey, 2001.
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