



# **2C09**

## **Design for seismic and climate changes**

### **Lecture 18: Seismic analysis of inelastic MDOF systems II**

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European Erasmus Mundus Master Course  
**Sustainable Constructions**

**under Natural Hazards and Catastrophic Events**

520121-1-2011-1-CZ-ERA MUNDUS-EMMC

# Lecture outline

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**18.1 Modelling of material nonlinearity for structural analysis under seismic action**

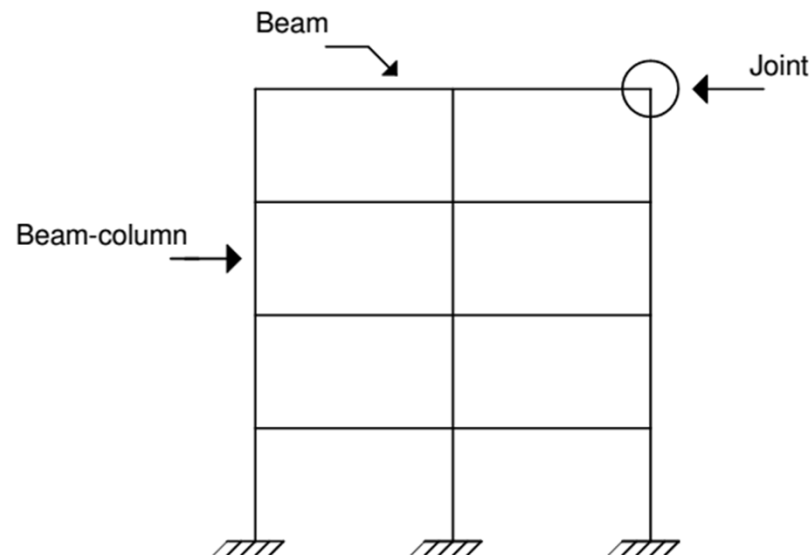
**18.2 Concentrated plasticity**

**18.3 Distributed plasticity**

# Structural model

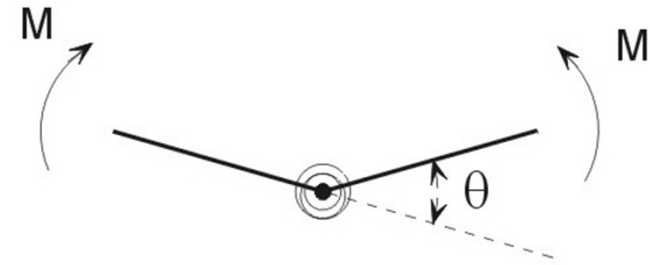
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- Many structures can be idealised through line members (beams, columns, braces) connected in nodes
- Modelling of inelastic behaviour of structural components is necessary in order to perform a inelastic structural analysis
- Though principles of modelling of nonlinear behaviour are general, there may differences in the way they are implemented in different structural analysis programs
- Software:
  - SAP 2000,
  - Opensees,
  - Seismostruct,
  - Drain,
  - Ruaumoko,
  - Idarc, etc.

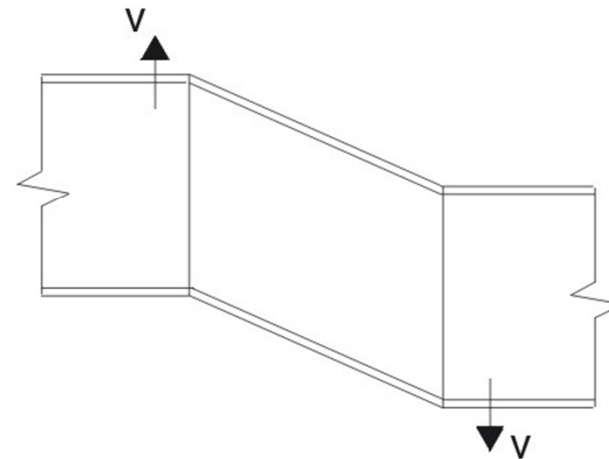


# Dissipative zones

- Different plastic mechanisms are possible, depending on the type of structural action developed
- Typical examples:
  - Bending in beams and beam-column joints in moment-resisting frames
  - Shear in links
  - Axial deformations in braces
  - Bending and axial forces in columns



Plastic hinge in bending ( $M-\theta$ )



Shear mechanism of a panel

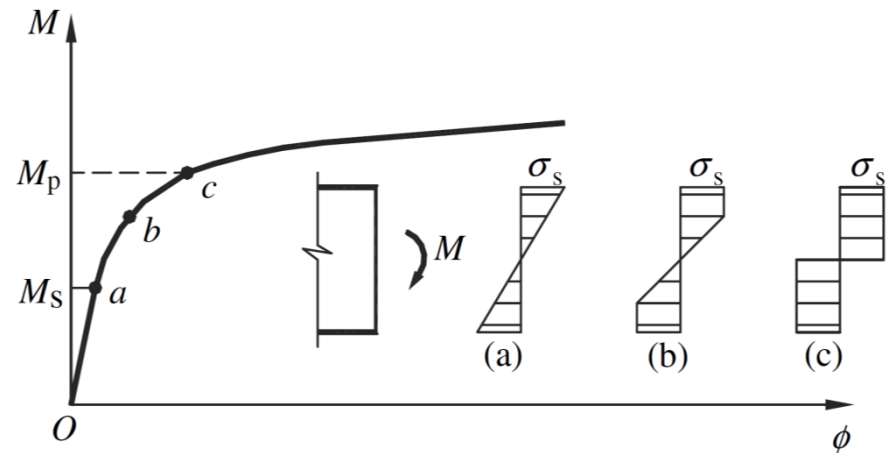
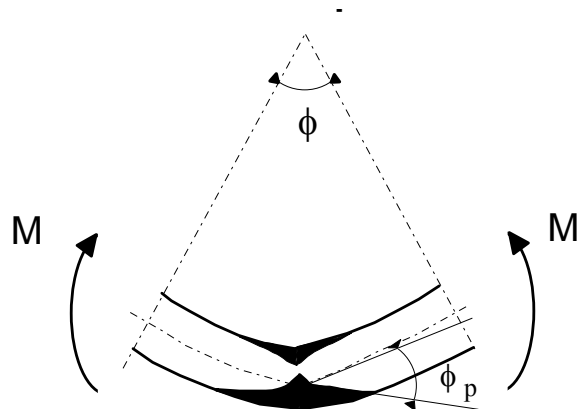


Plastic tension of a bar



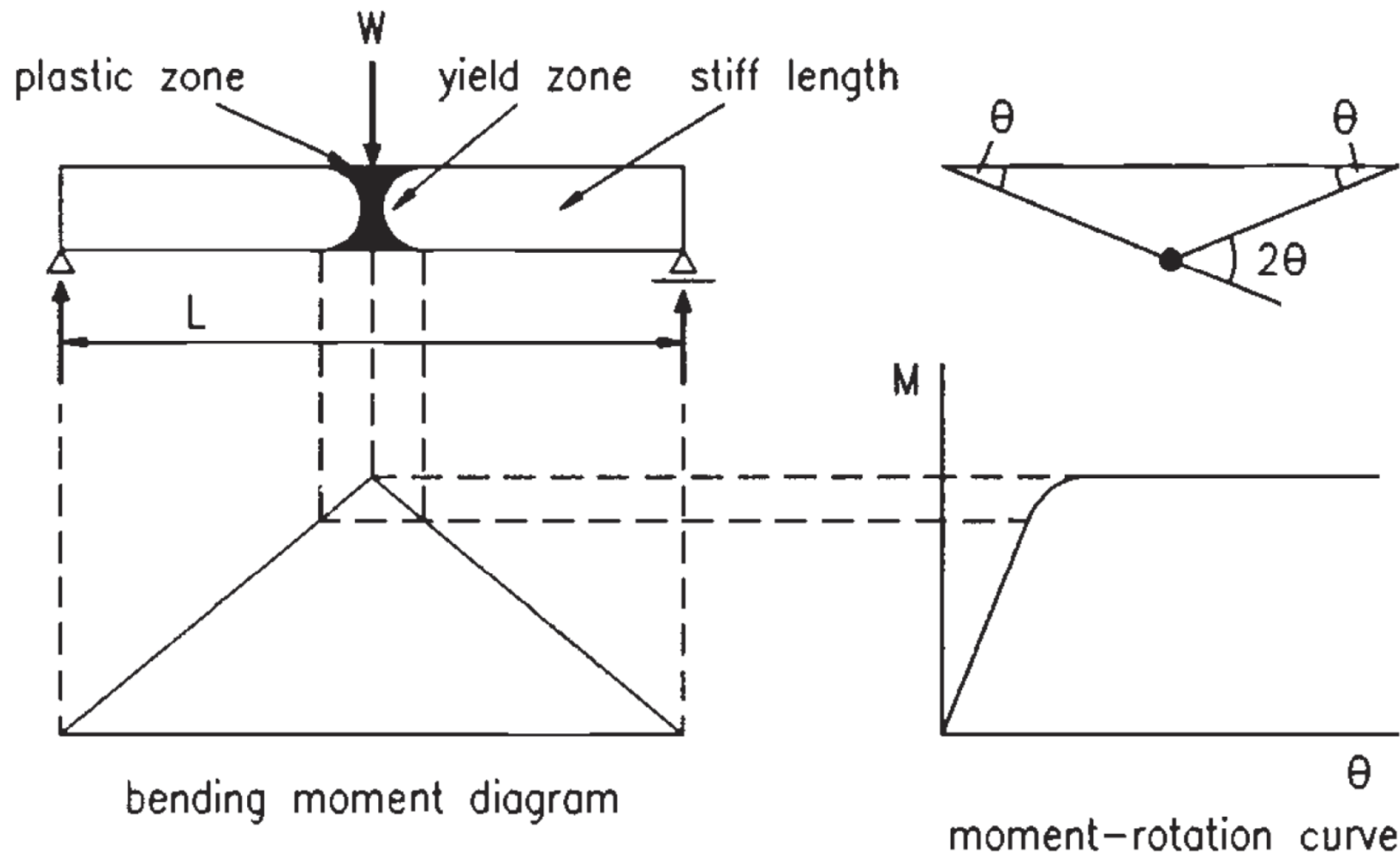
# Plastic deformations in bending

- Maximum stresses in a cross-section subjected to bending occur at extreme fibers of the cross-section
- When the yield stress is reached at the extreme fiber of the cross-section, the elastic moment resistance is reached
- If the moment keeps increasing, plastic deformations advance toward the neutral axis
- When all of the cross-section yields, the plastic moment of the cross-section is attained



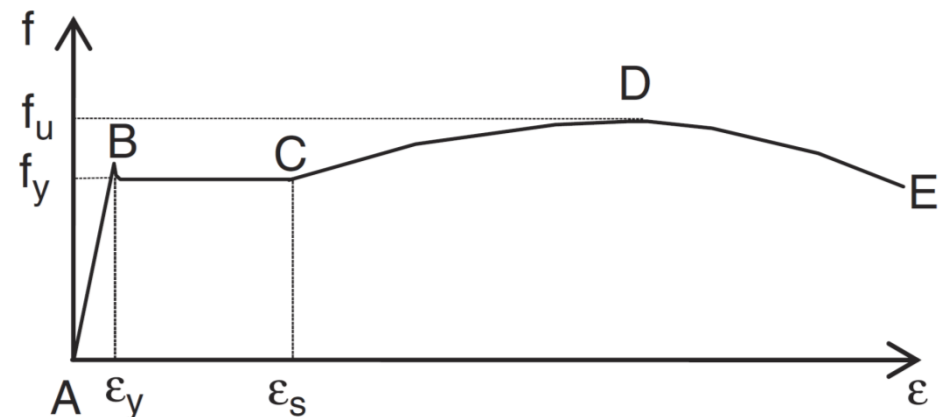
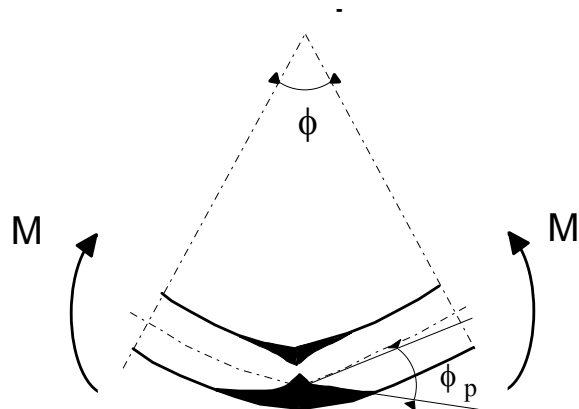
# Plastic deformations in bending

- Plastic deformations are spread over a finite length of the member



# Plastic deformations in bending

- Due to strain hardening:
  - Moment in the plastic section will increase beyond the plastic capacity of the cross-section
  - Plastic deformations will occur in sections adjacent to the one where maximum moment occurs
- Plastic deformations in the cross-section occur progressively from the extreme fiber toward the neutral axis
- Member yielding occurs progressively from the maximum moment cross-section toward adjacent ones (due to strain hardening and moment gradient)

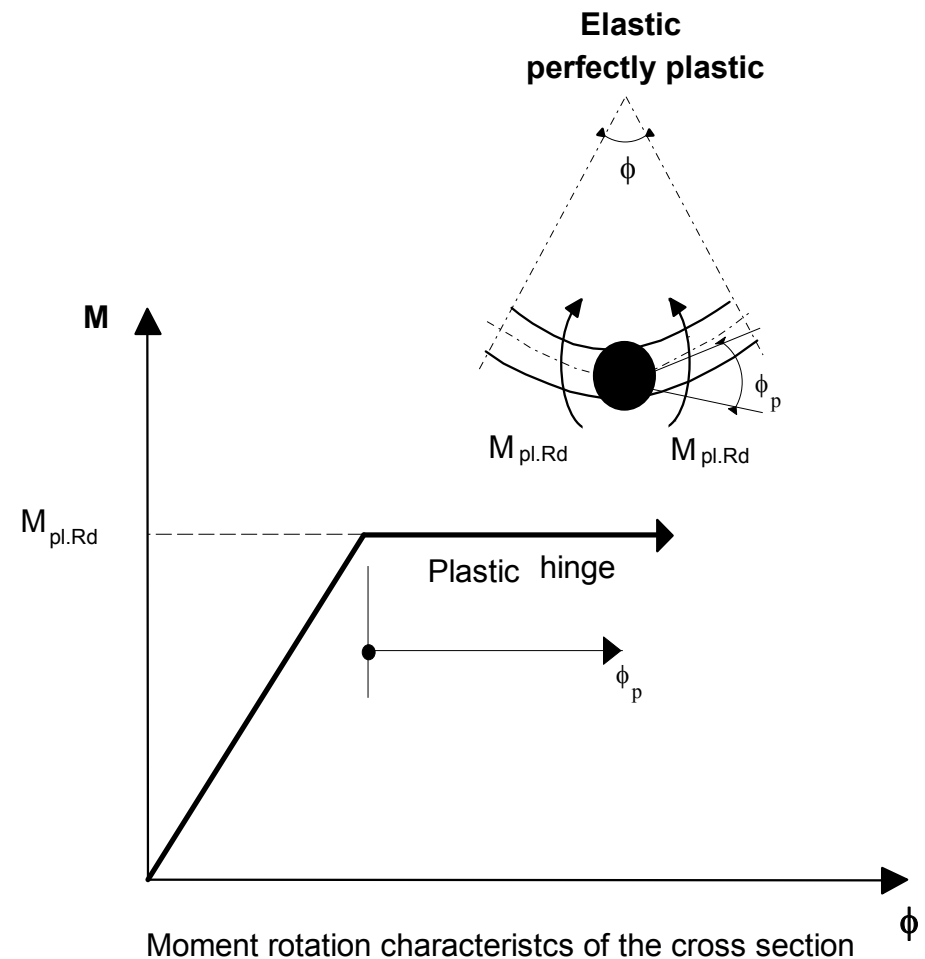


# Plastic deformations in bending

- As a simplification of plastic deformations in bending, the plastic hinge concept is often used:
  - Elastic deformations before the attainment of plastic moment
  - Plastic deformations after the attainment of plastic moment
  - Plastic deformations concentrated at a single cross-section

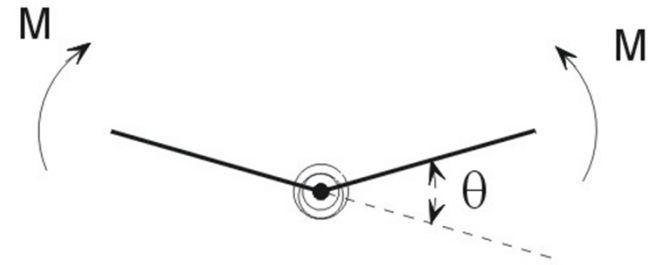


**Concentrated plasticity**

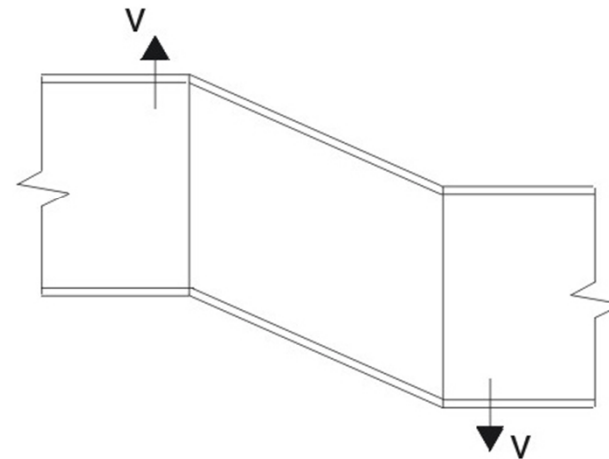


# Concentrated plasticity

- Concentrated plasticity model can be generalised for all types of action (bending, shear, axial)
- The term "plastic hinge" is often used to denote plastic deformations in tension or shear, though the term "hinge" is not correct in this case



Plastic hinge in bending ( $M-\theta$ )



Shear mechanism of a panel

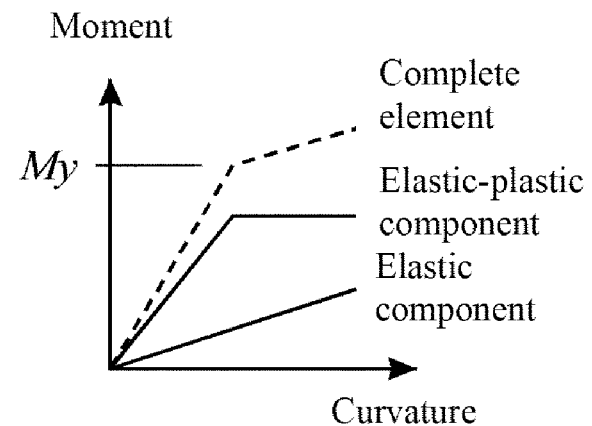
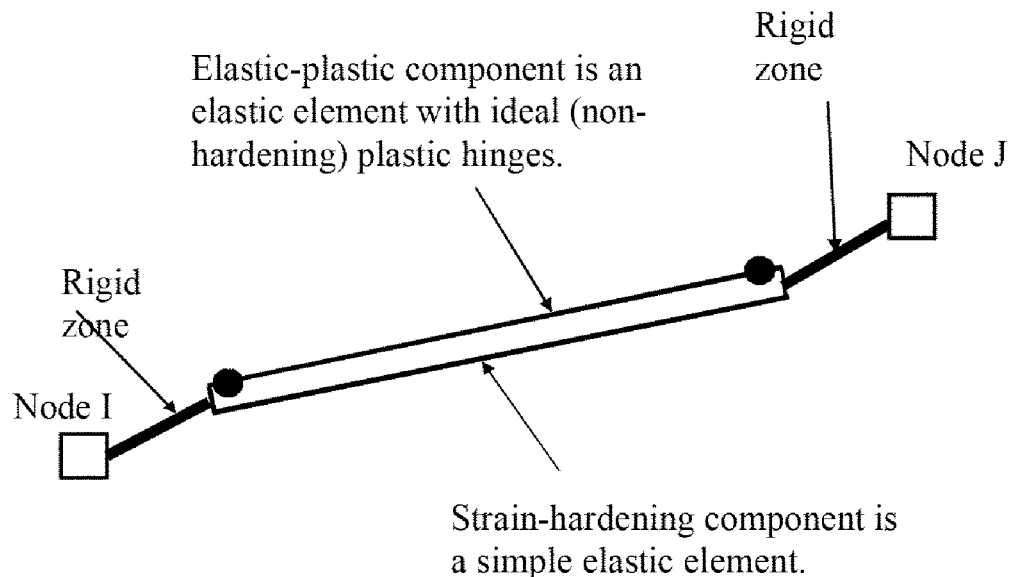
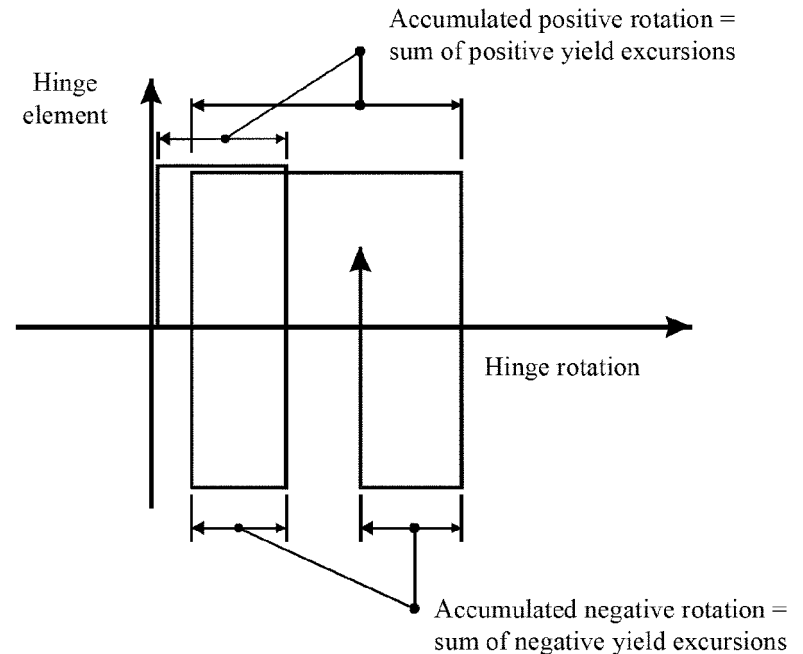


Plastic tension of a bar

# Beam with plastic hinges

## ■ Element modelling:

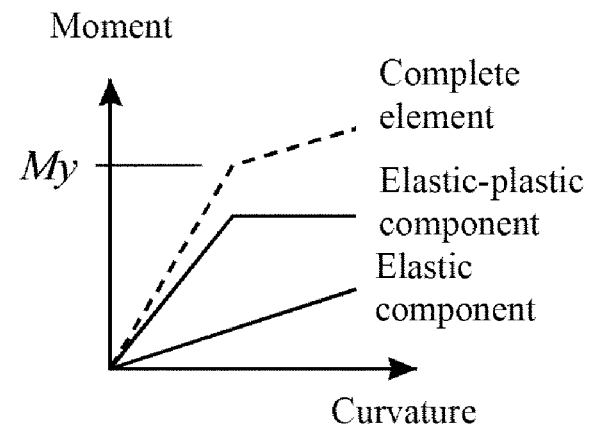
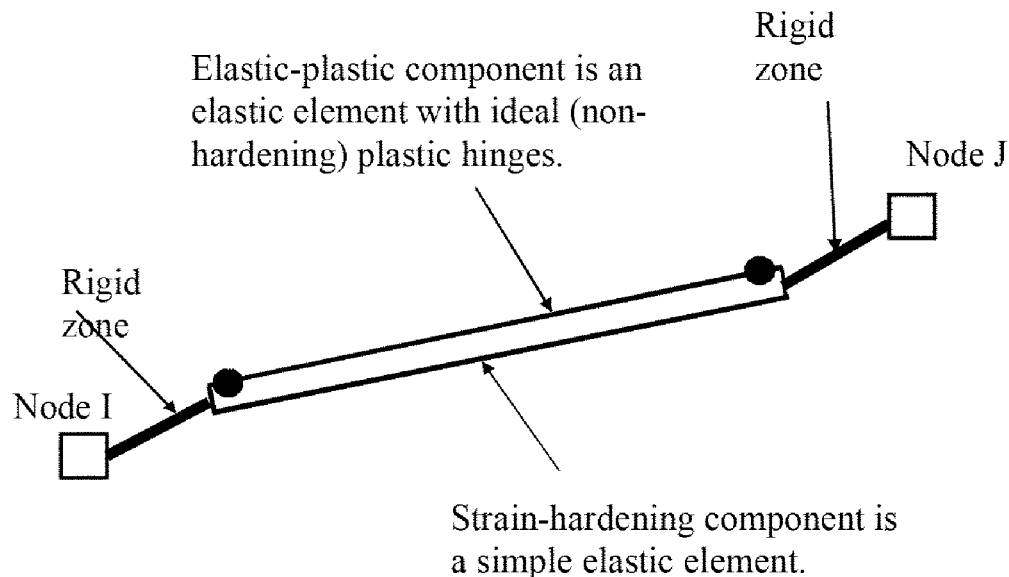
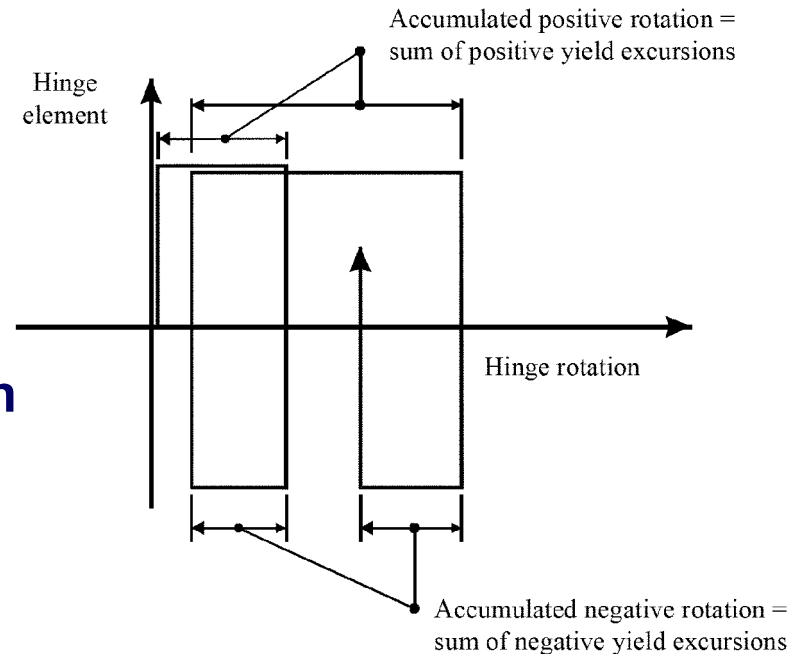
- Elastic beam
- Rigid-plastic hinges at the member ends: plastic deformations are concentrated in zero-length plastic hinges and occur only due to bending



# Beam with plastic hinges

## ■ Element modelling:

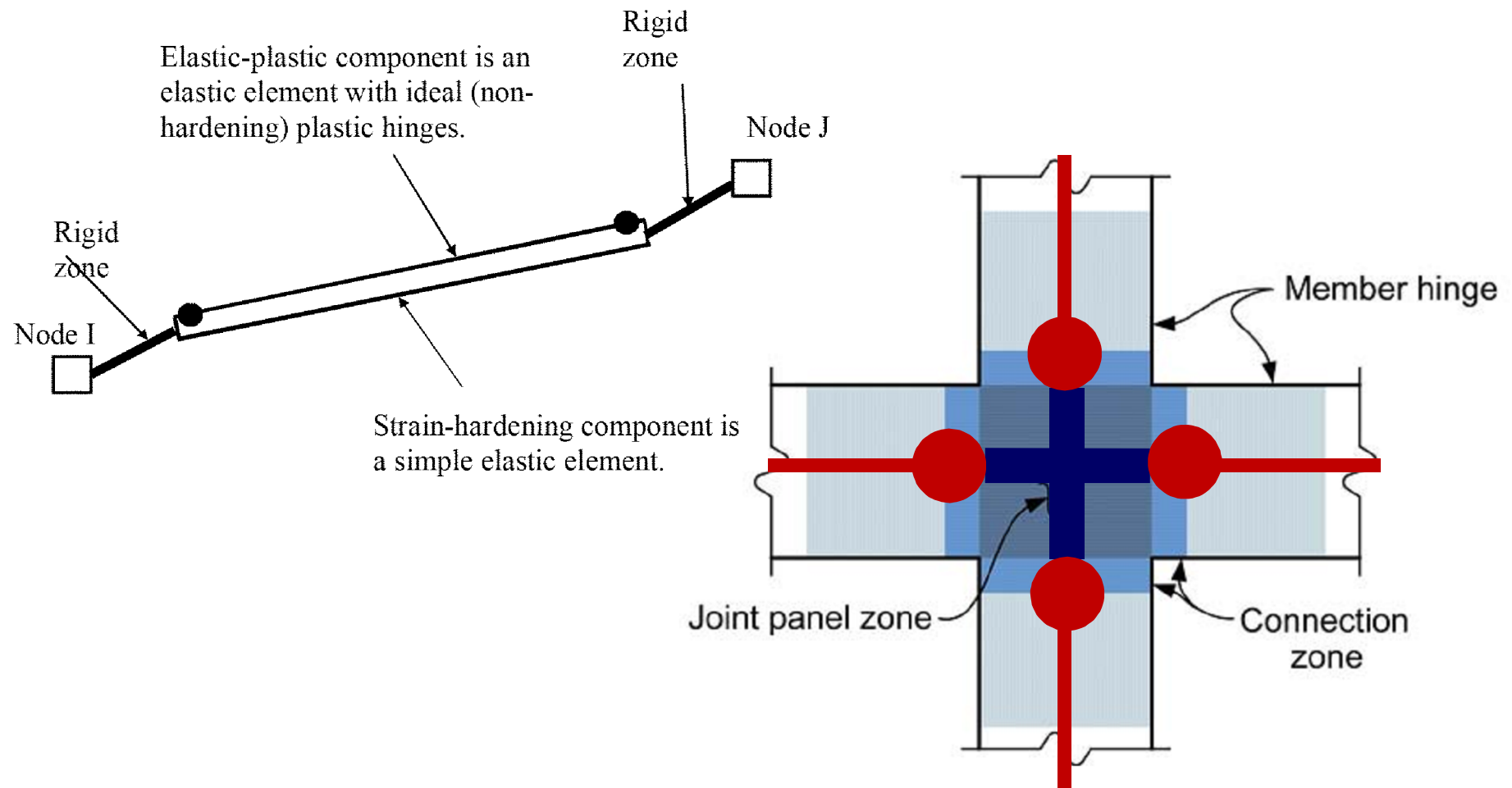
- Optional rigid zones at the end of the member allows modelling of finite dimensions of the beam-column joint
- Strain hardening modelled using an elastic element in parallel with the elasto-plastic one



# Beam with plastic hinges

- **Element modelling:**

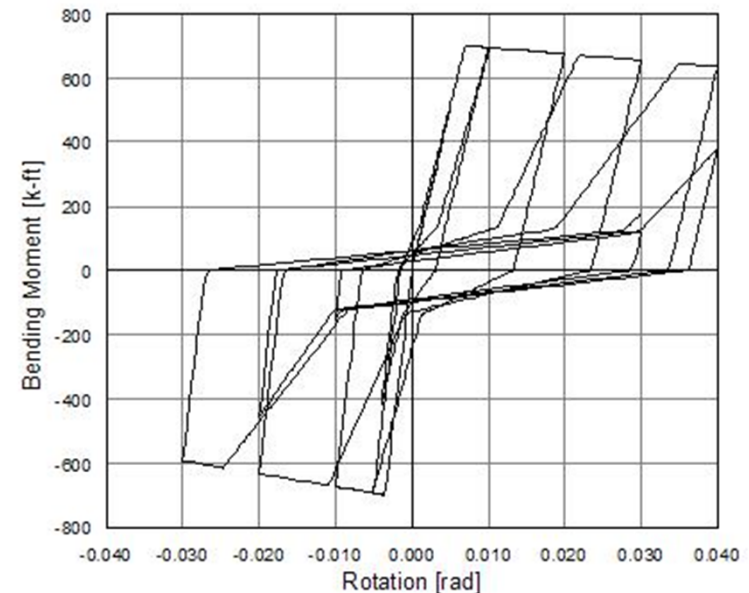
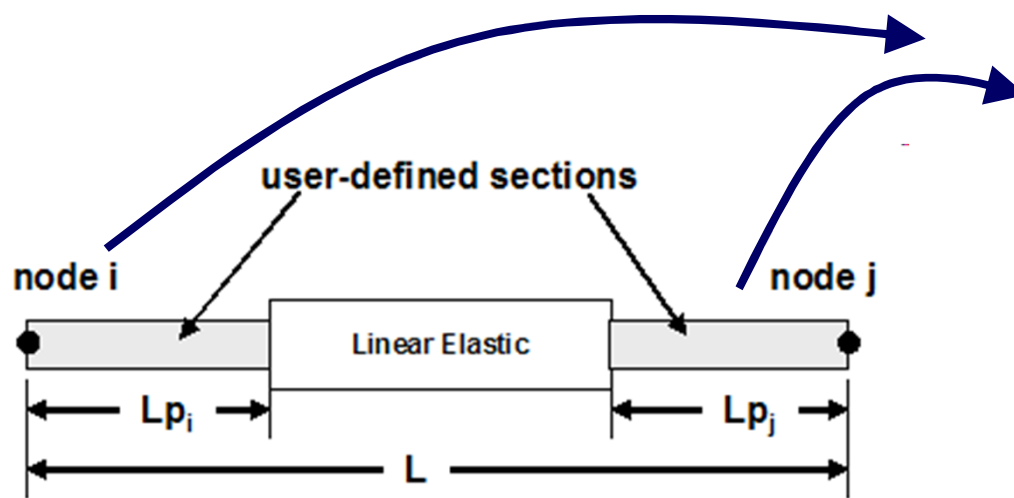
- Optional rigid zones at the end of the member allows modelling of finite dimensions of the beam-column joint





# Beam with plastic hinges

- **Element modelling (alternative):**
  - Elastic beam element
  - Plastic hinges of finite length at the beam ends
- **Possibility to model different hysteretic characteristics of the cyclic response**
  - Different hysteresis models exists
  - Modelling of strength and stiffness degradation possible

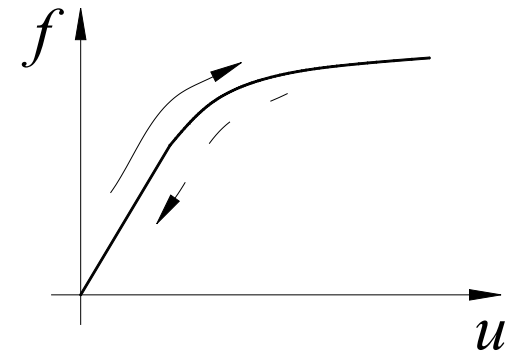


# Beam with plastic hinges

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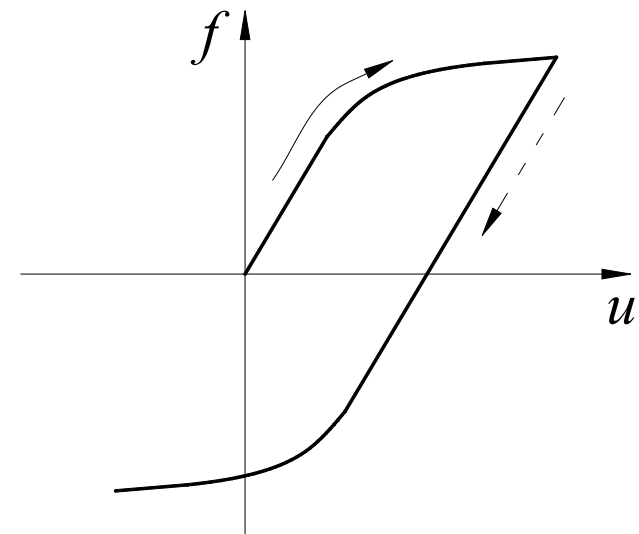
- **Path-independent models:**

- The current value of the force depends only on the current value of deformation
- The material follows the same path during loading and unloading
- Are generally appropriate for nonlinear static analysis



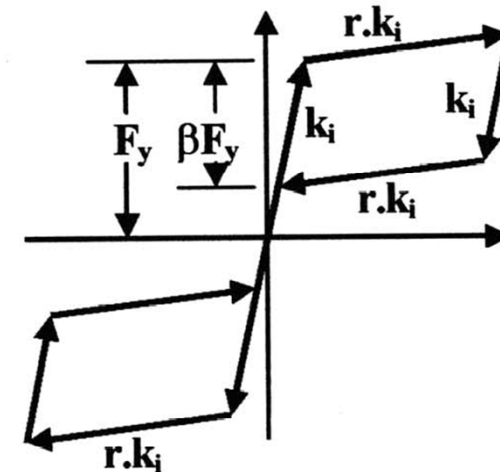
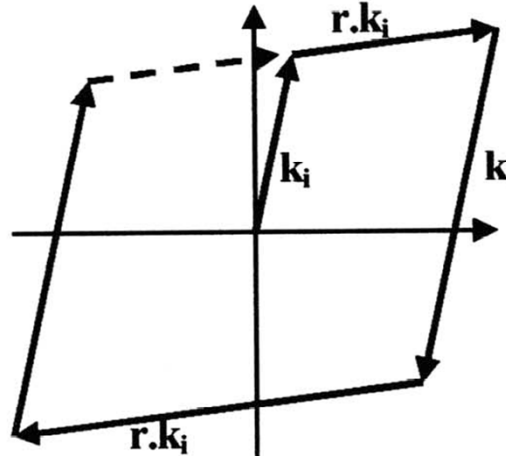
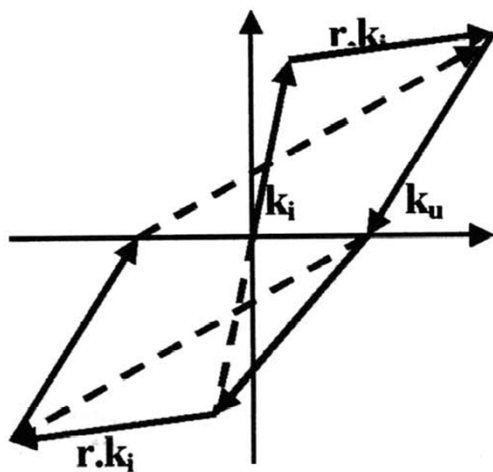
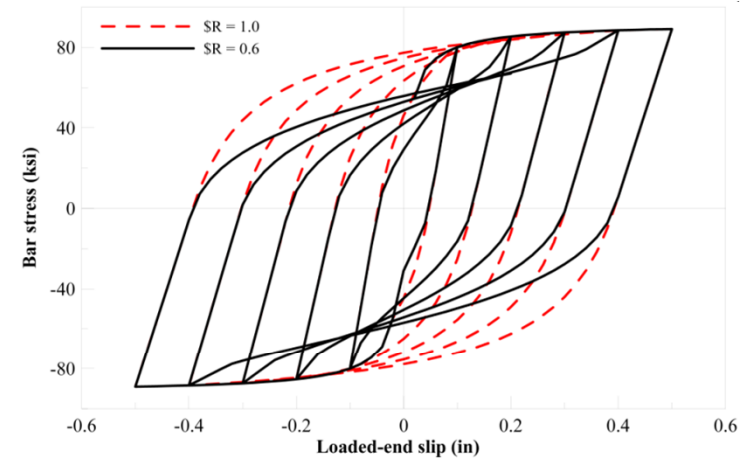
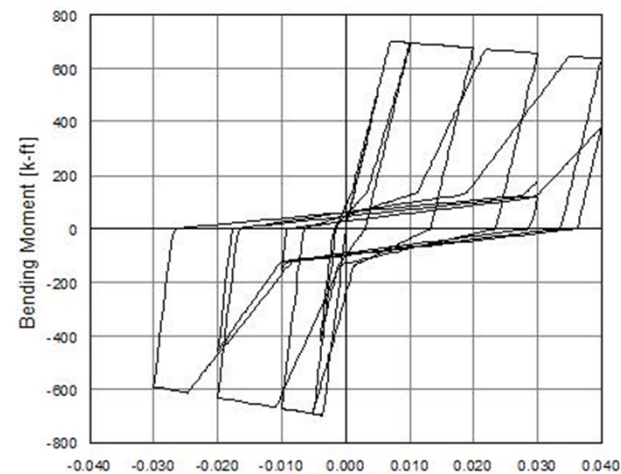
- **Path-dependent models:**

- The current value of the force depends on the current value of deformation, as well as on the loading history
- Necessary in dynamic time-history analysis



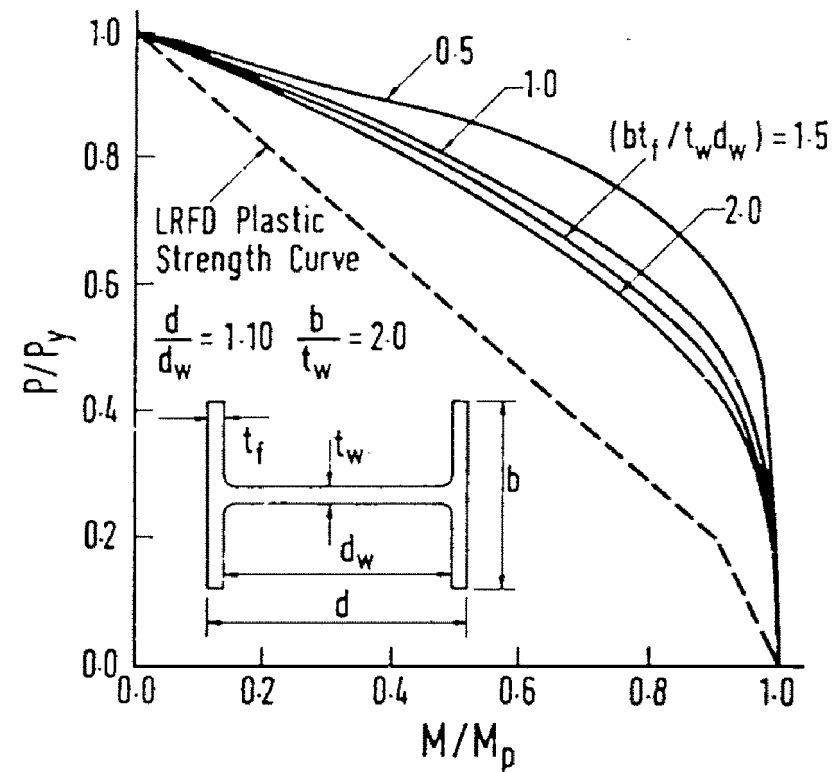
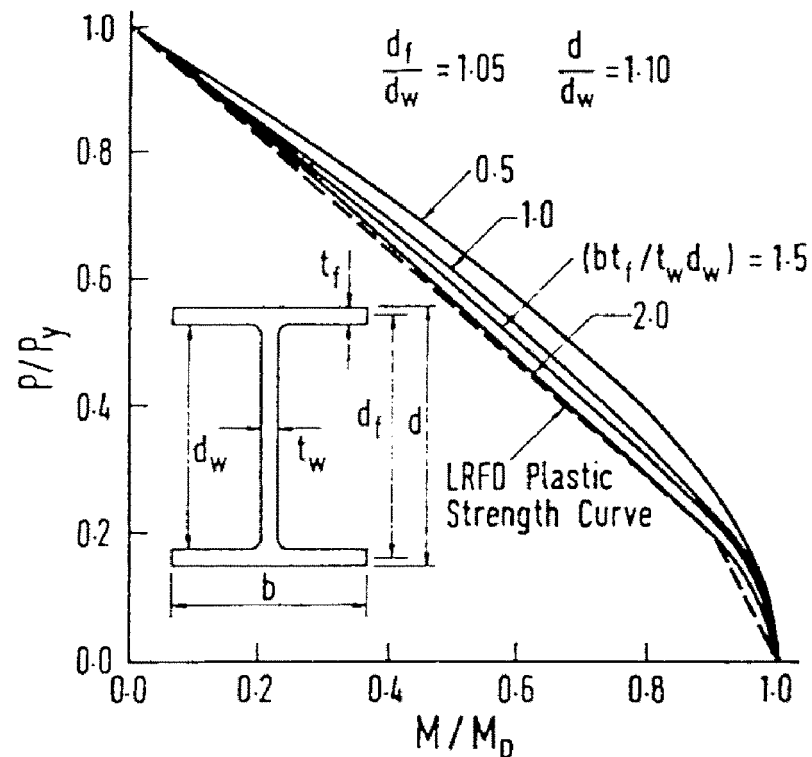
# Beam with plastic hinges

- Examples of hysteretic models for plastic hinges, adapted to different types of materials, joints or members



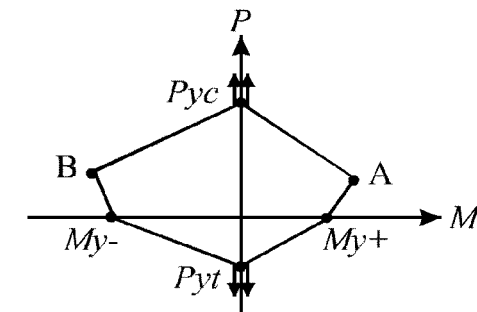
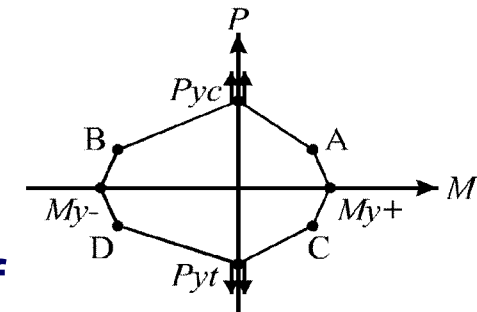
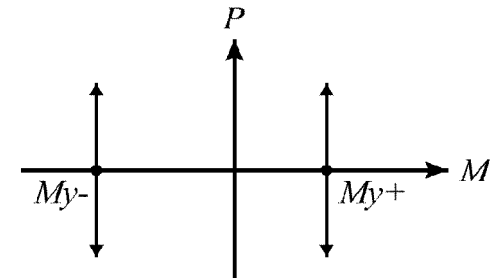
# Beam with plastic hinges

- Axial force affects the moment capacity of the cross-section  $\Rightarrow$  it is necessary to account for the axial force – bending moment interaction for members subject to bending moments and axial forces



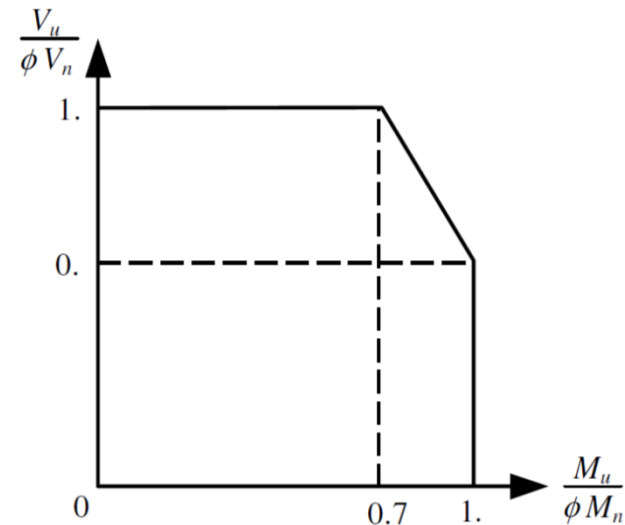
# Beam with plastic hinges

- Bending moment – axial force interaction in plastic hinges (concentrated plasticity): bending moment capacity affected by the axial force, but only plastic rotations are assumed to occur
  - Interaction neglected: elements with small axial force
  - Interaction curve (surface): steel members subjected to bending moment and axial force ( $A=0.1P_y$  for bending about the strong axis of double T cross-sections)
  - Interaction curve (surface): reinforced concrete members subjected to bending moment and axial force

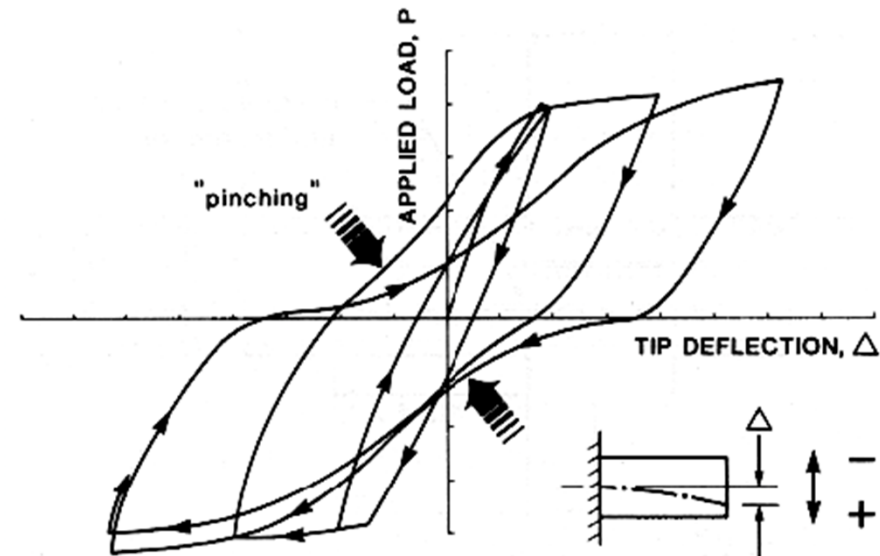


# Beam with plastic hinges

- Interaction between bending moment and shear force can generally be neglected in steel members (condition in EN1993-1-1:  $V_{Ed}/V_{Rd} \leq 0,5$ )

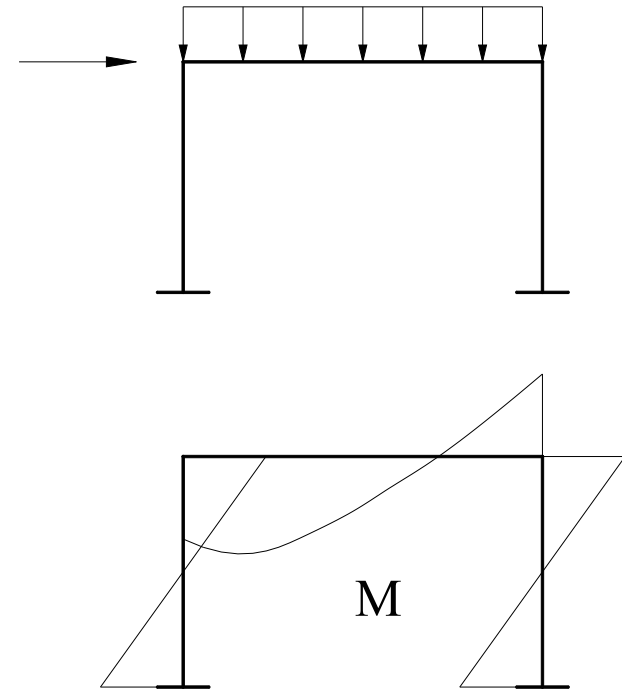


- R.C. members:
  - Shear force leads to brittle failure
  - Large shear force reduces ductility in bending and generates pinching of the hysteretic curves



# Beam with plastic hinges

- **Position of plastic hinges are pre-defined:**
  - Usually at the element ends
  - Some programs allow defining plastic hinges at arbitrary positions along the member
- **Plastic hinges should be modelled at cross-sections of maximum bending moment**
- **Plastic hinges at element ends:**
  - Adequate in case of columns
  - May be incorrect in case of beams with large gravitational loading and/or large spans
- **Solution: finer meshing of the element**



# Beam with plastic hinges

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- **Advantages:**

- Numeric efficiency (relatively low computational resources necessary)
- Different hysteretic models can be implemented
- "Complete" output is available (forces and deformations), which are easy to post-process (rotations in plastic hinges can be compared directly to rotation capacities in codes)

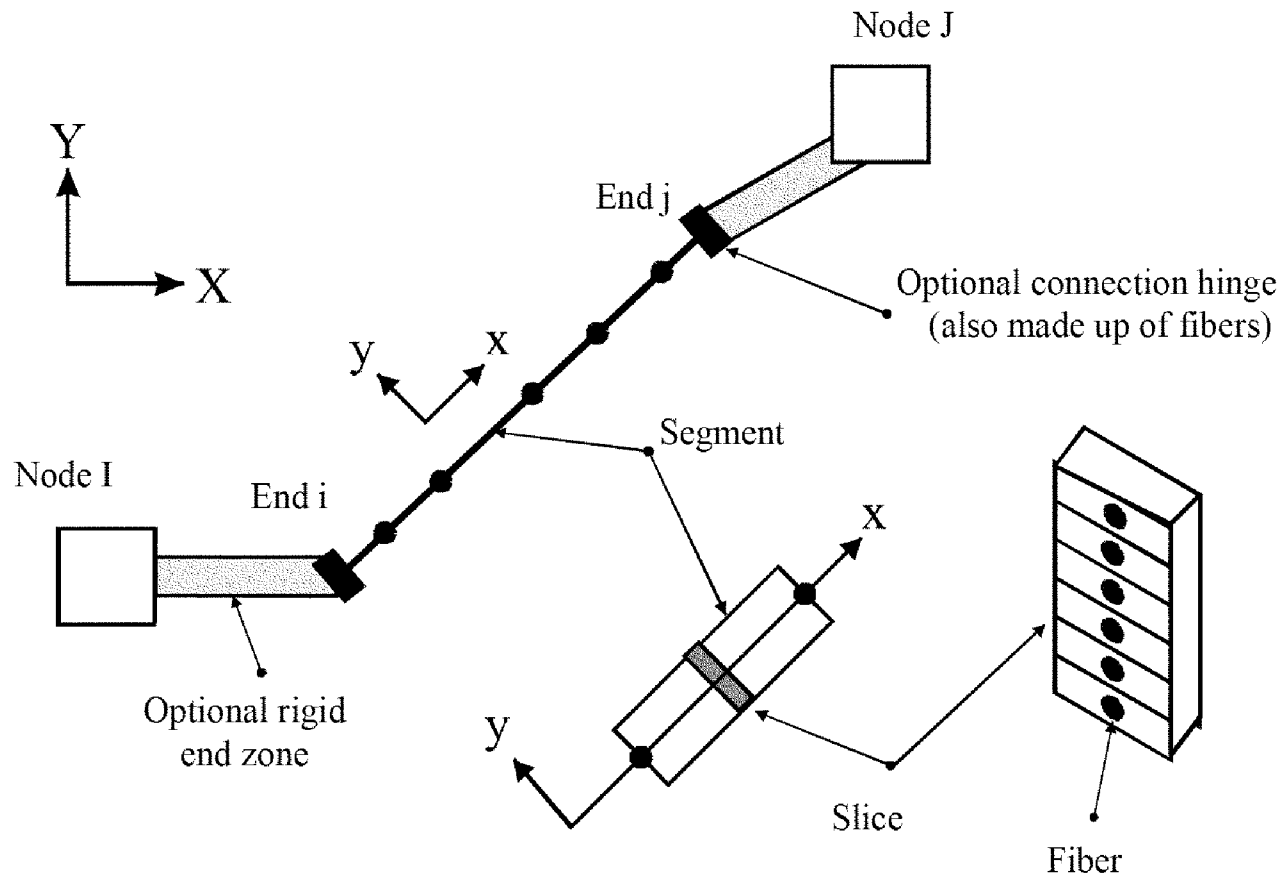
- **Disadvantages:**

- Pre-defined location of plastic hinges
- Neglecting of plastic axial deformations
- Neglecting in some cases of bending moment – axial force interaction or bending moment – shear force interaction



# Beam with distributed plasticity

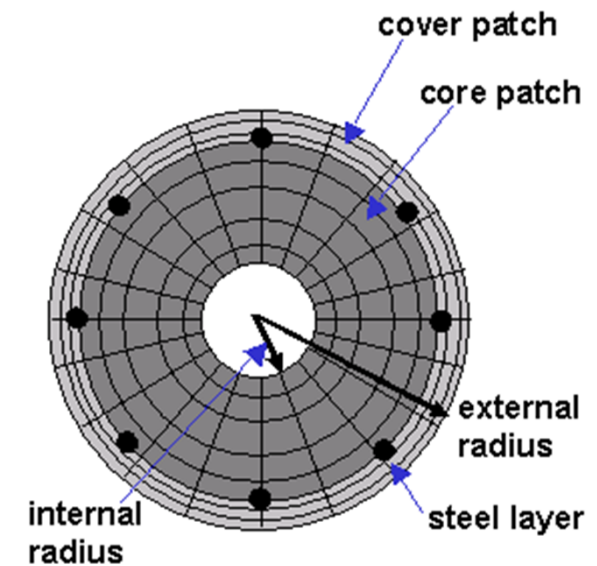
- As an alternative to concentrated plasticity models, distributed plasticity can be used:
  - accounts for progressive yielding in the cross-section
  - accounts for progressive yielding along the element



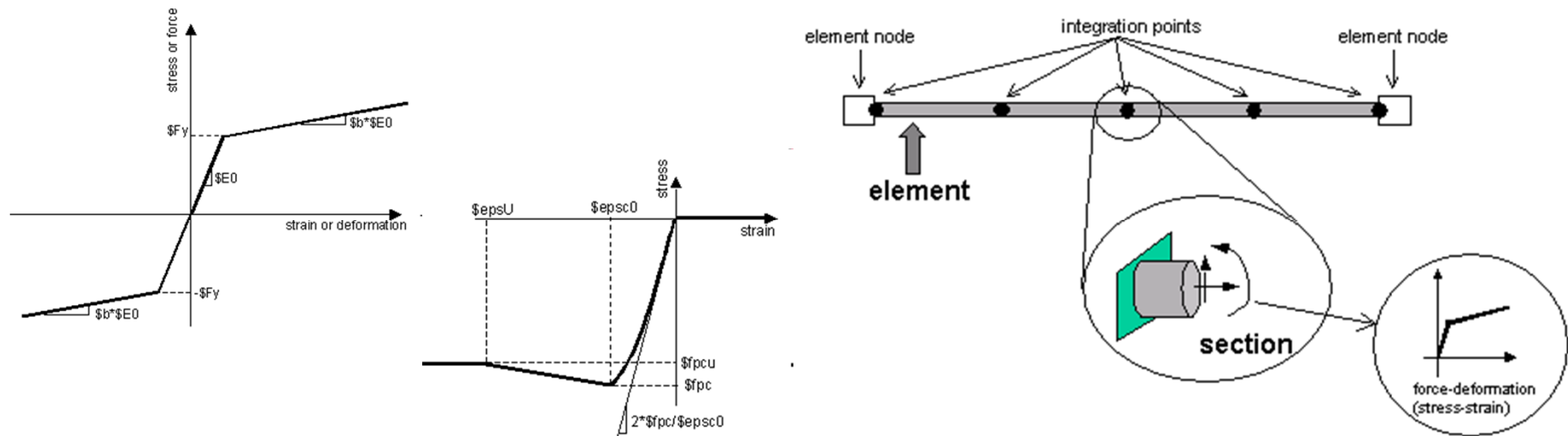
# Beam with distributed plasticity

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- **Element modelling:**
  - Element is subdivided into a number of segments
  - Response is monitored in the center of each segment (properties and response quantities are constant on each segment)
  - Cross-section of each segment is discretized into a number of fibers (defined by location and characteristics: stress-strain relationship)

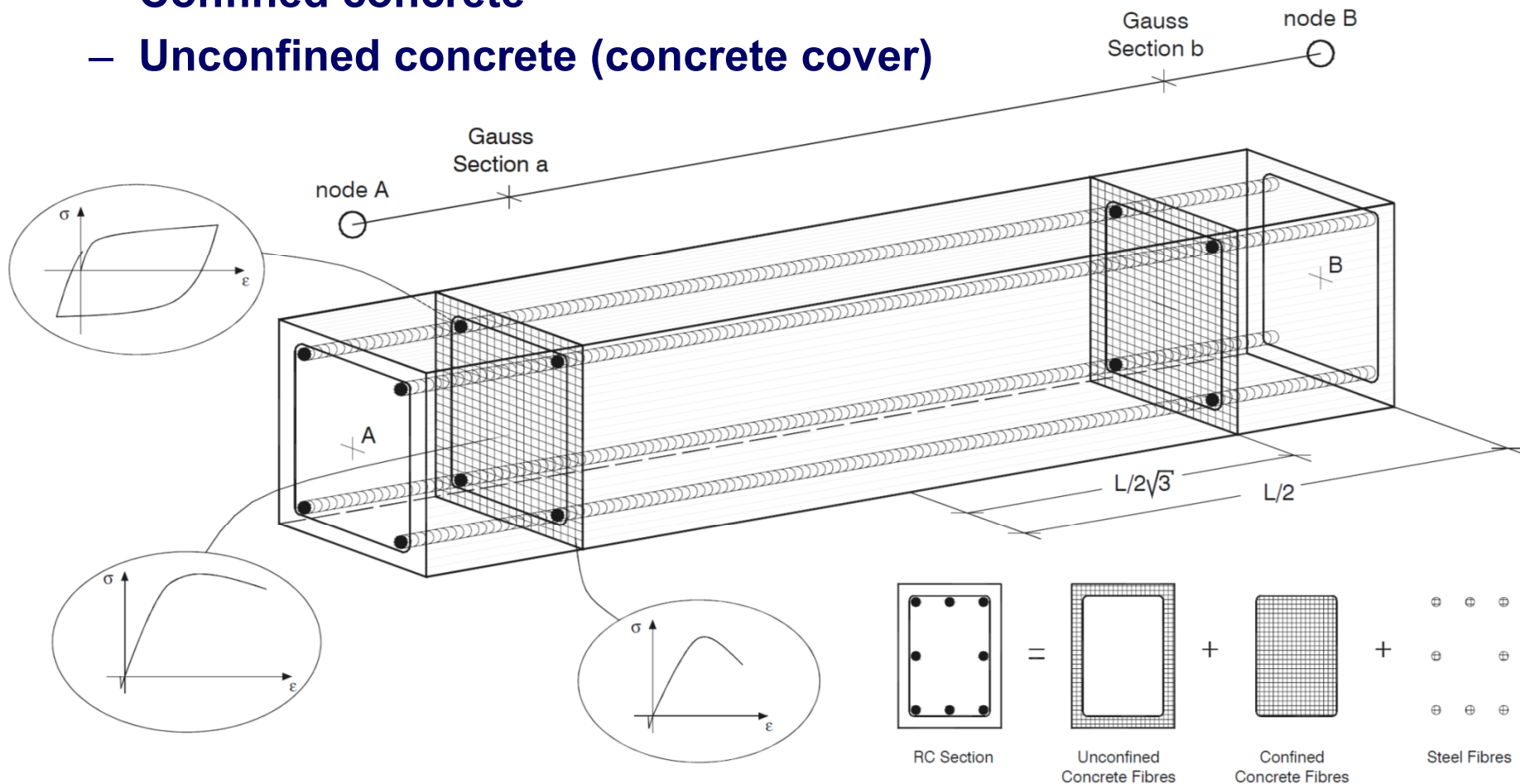


- **Element modelling (alternative):**
  - Element is subdivided into a number of segments
  - Response is monitored at integration points (properties and response quantities have a linear or parabolic variation between integration points)
  - Cross-section of each segment is discretized into a number of fibers (defined by location and characteristics: stress-strain relationship), or through a force-deformation relationship (in this case the plasticity is distributed along the element only)



# Beam with distributed plasticity

- Example of modelling a r.c. beam – cross-section discretized in fibers representing:
  - Longitudinal reinforcement bars
  - Confined concrete
  - Unconfined concrete (concrete cover)



# Beam with distributed plasticity

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- **Advantages:**

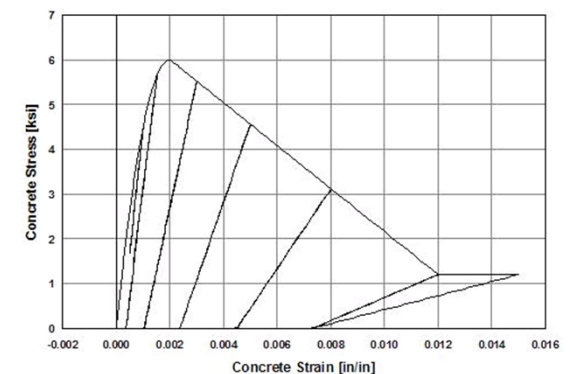
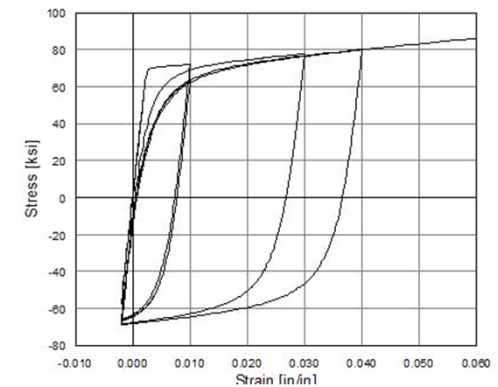
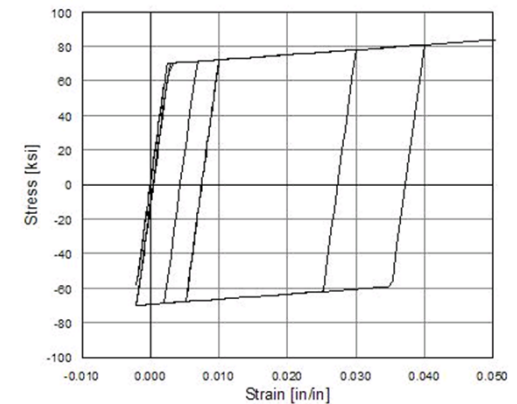
- Allows a more realistic modelling of the element: progressive yielding in the cross-section and along the element
- Allows for a "natural" modelling of interaction between the bending moment and axial force

- **Disadvantages :**

- Relatively high computational resources necessary
- Response may be sensible to the number of fibers and segments used: calibration needed
- Neglecting in some cases of bending moment – shear force interaction
- Obtained results may be difficult to post-process
  - When plastic deformations are provided in terms of strain and not rotations
  - Large amount of data
- Empirical modelling of cyclic response difficult

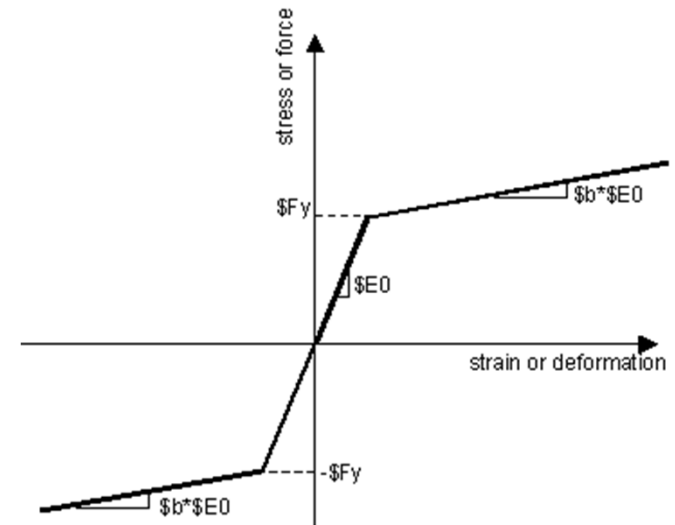
# Material models

- When distributed plasticity is employed, materials models are necessary
- A large number of material models are available
- Examples (models implemented in OPENSEES):
  - Elasto-plastic model for steel
  - Nonlinear model for steel
  - Kent-Scott-Park concrete model

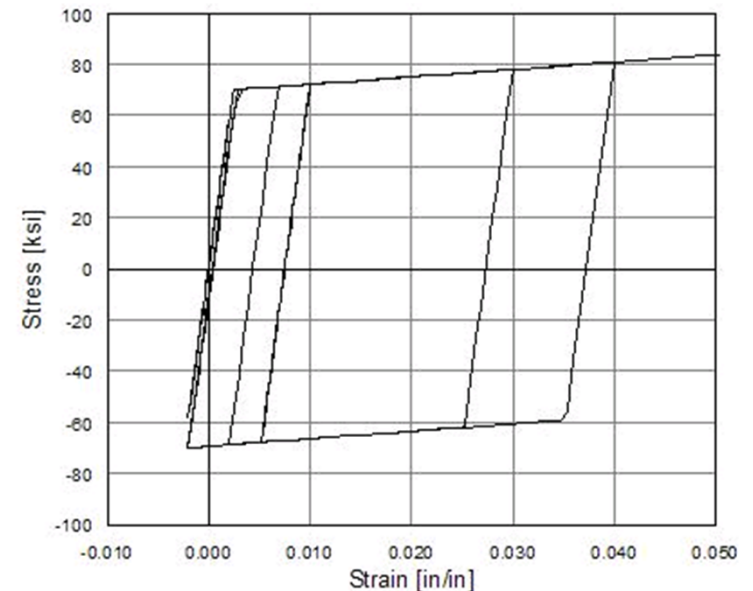


# Material models: steel

- **Elasto-plastic model with kinematic and isotropic hardening for steel**

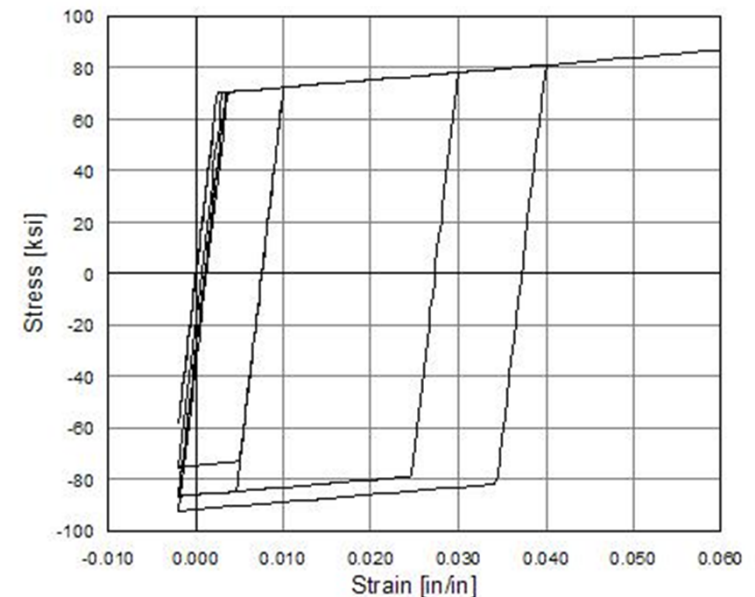
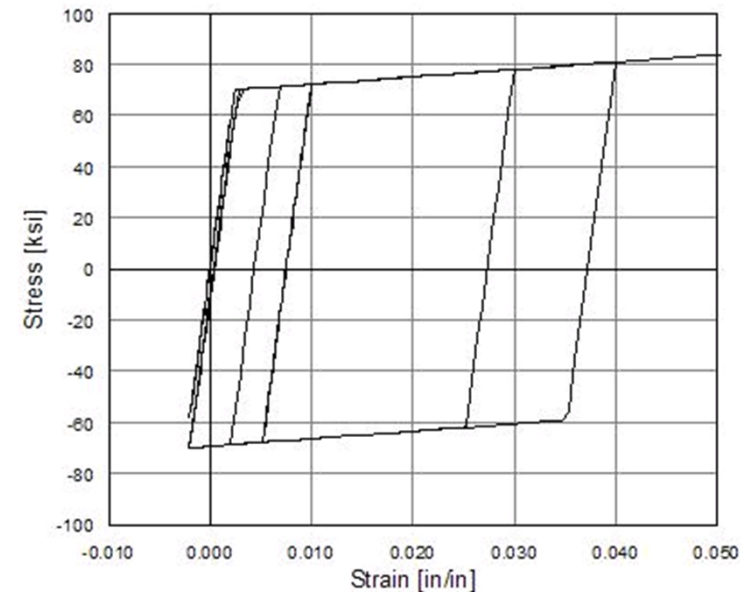


- **Bilinear model:**
  - Elastic response (linear stress-strain relationship) before attaining the yield stress
  - Plastic deformations with strain hardening (linear stress-strain relationship) after the yield stress was reached



# Material models: steel

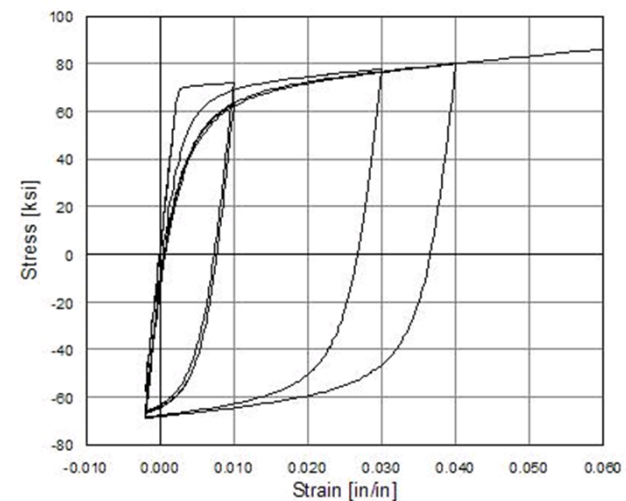
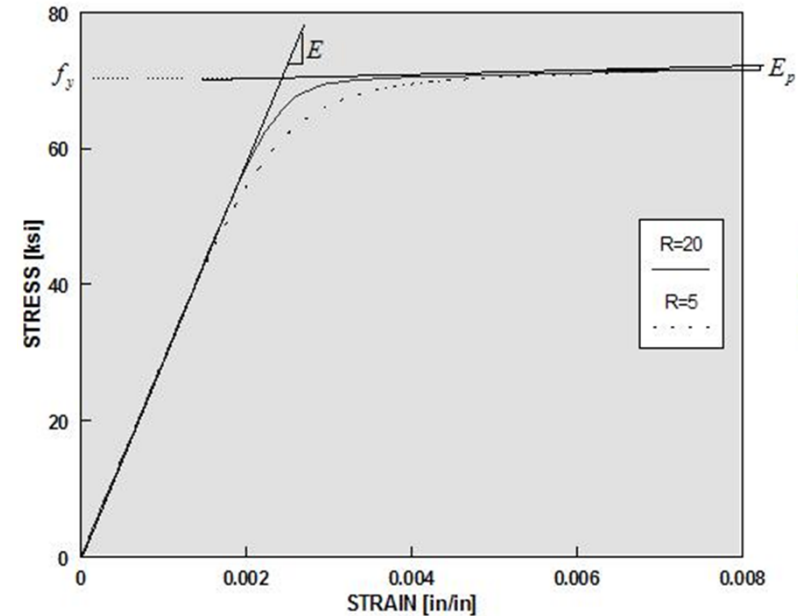
- **Kinematic hardening:**
  - Represents the slope of the stress-strain diagram after the yield stress was reached
  - Yield strength decreases when the direction of strain is changed (Bauschinger effect). The total elastic deformation at reloading is twice the one under direct loading.
- **Isotropic hardening: increase of compression yield envelope as proportion of yield strength after a plastic strain in tension (and vice-versa)**





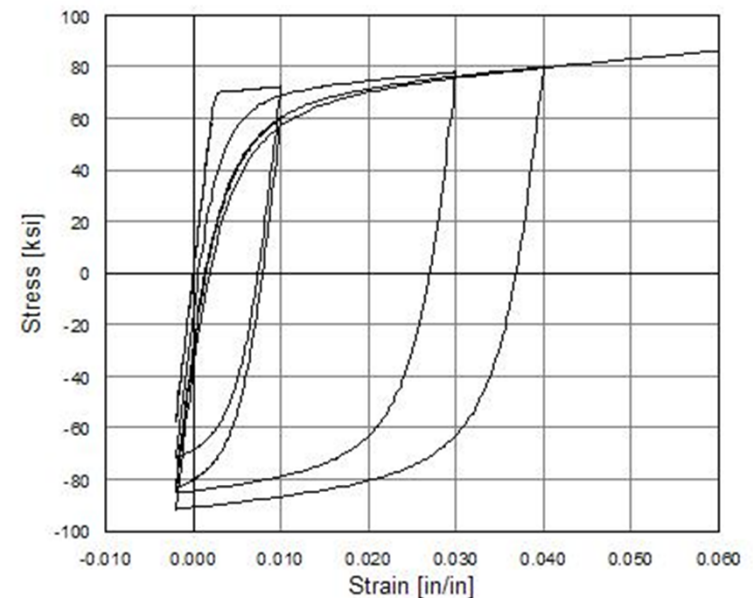
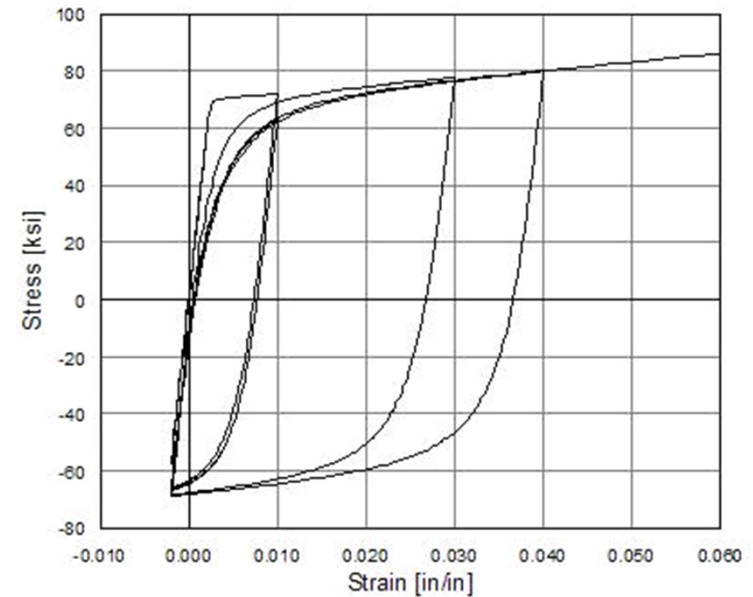
# Material models: steel

- **Nonlinear Giuffre-Menegotto-Pinto model for steel with kinematic and isotropic strain hardening**
- **Nonlinear model:**
  - Initial elastic response
  - Strain hardening after reaching the yield stress
  - Smooth transition from elastic to plastic branch



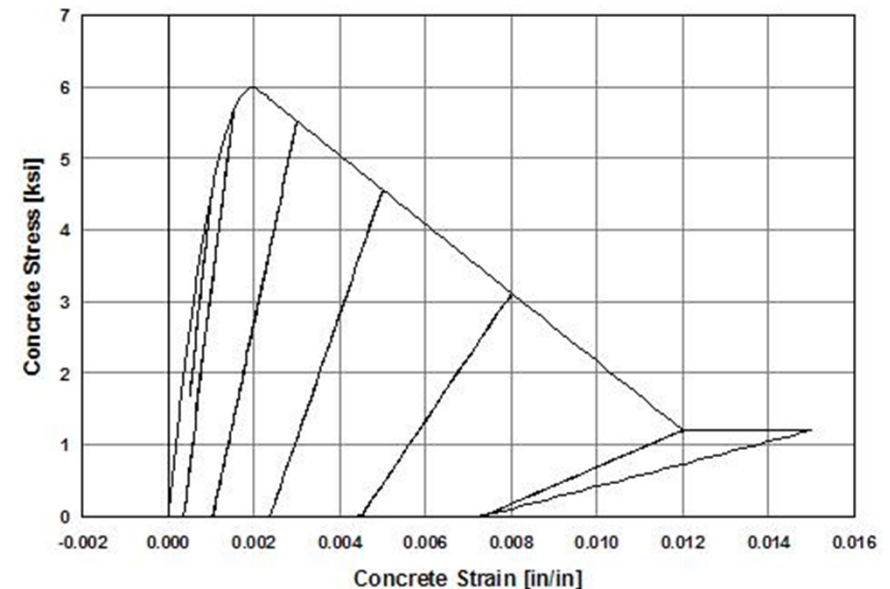
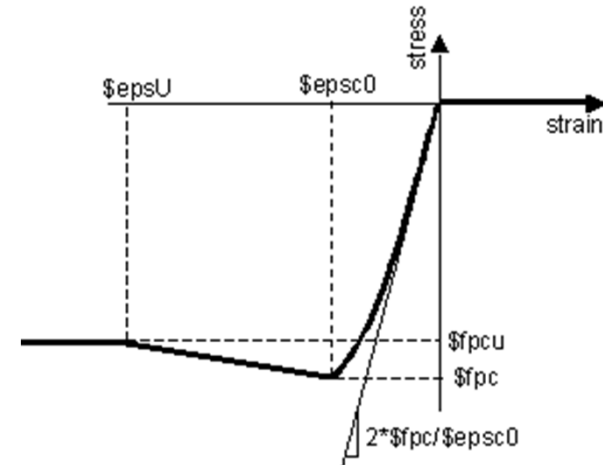
# Material models: steel

- Nonlinear Giuffre-Menegotto-Pinto model with kinematic strain hardening
- Nonlinear Giuffre-Menegotto-Pinto model with kinematic and isotropic strain hardening



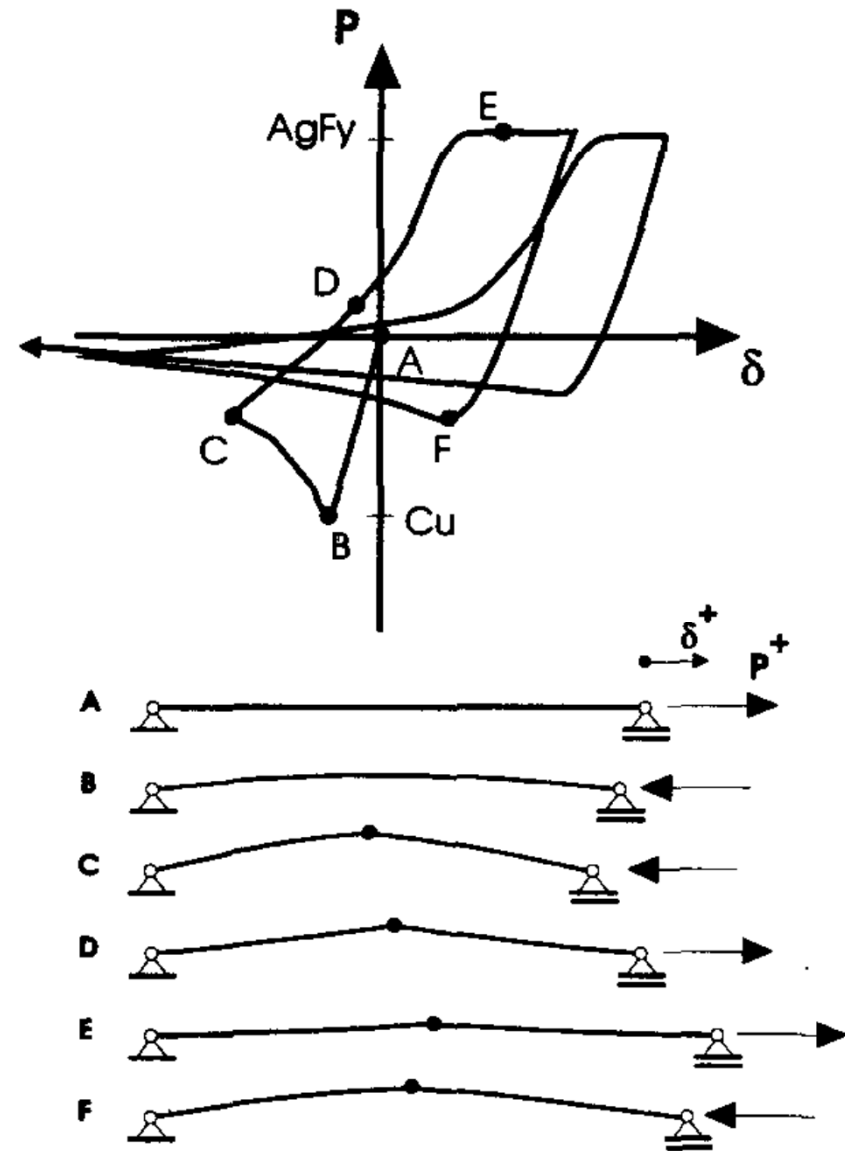
# Material models: concrete

- Kent-Scott-Park model for concrete
- Nonlinear model:
  - Parabolic stress-strain deformation in compression up to the compression strength is reached
  - Negative slope afterwards up to a residual strength of 10-20% of the compressive one
  - Tensile strength neglected
  - Degraded linear unloading/reloading stiffness



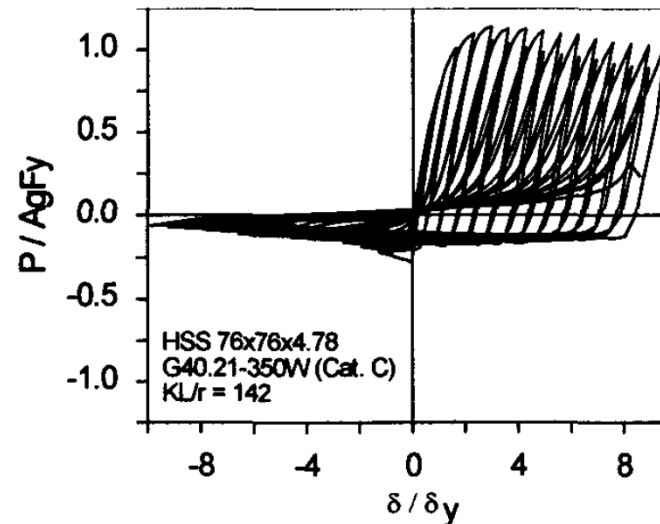
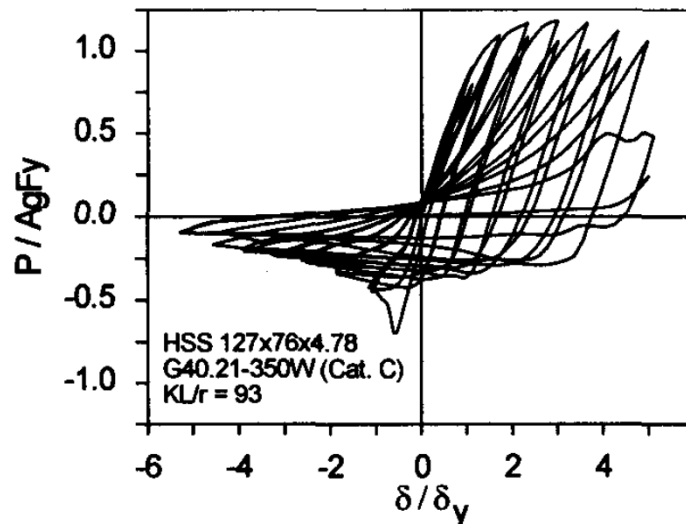
# Brace modelling

- Behaviour of braces in the inelastic range characterised by
  - Yielding of the brace in tension
  - Buckling in compression



# Brace modelling

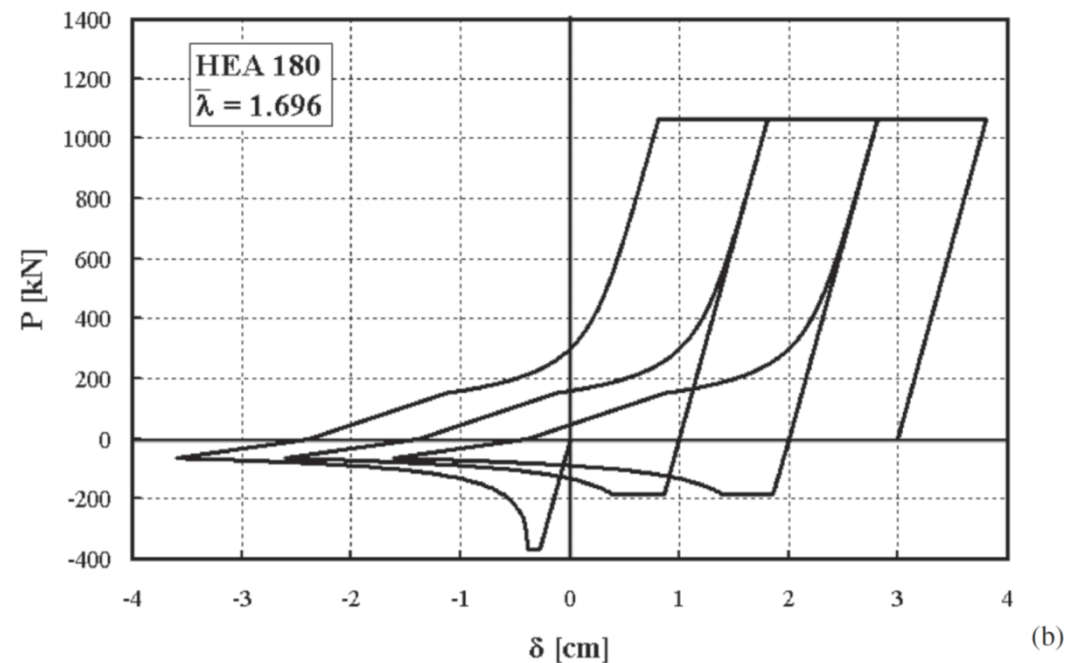
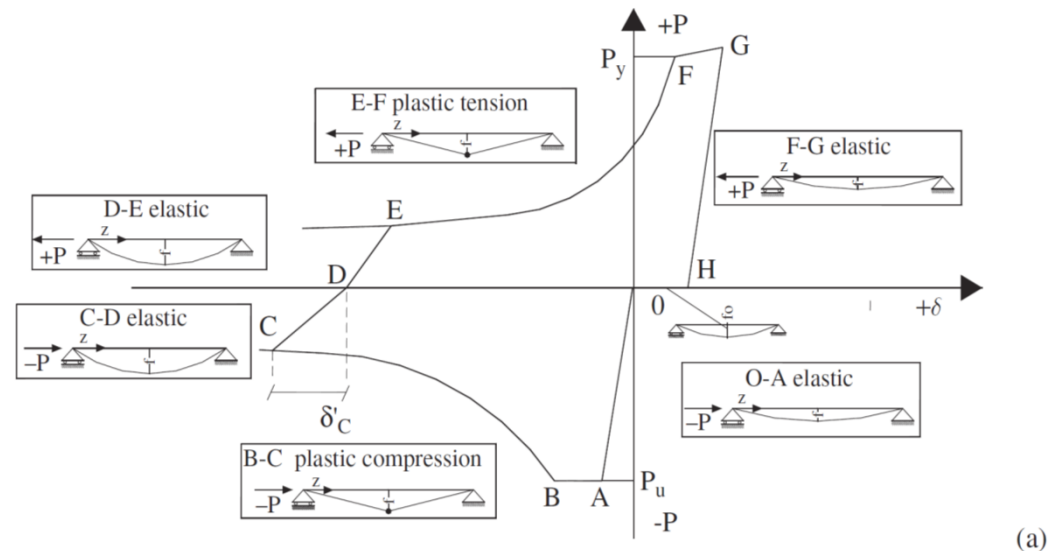
- Force deformation relationship is affected on the slenderness of the member



- Cyclic response of the member can be modelled using:
  - Phenomenological models (truss elements with concentrated plasticity)
  - Physical theory models (beam-column elements with distributed plasticity)

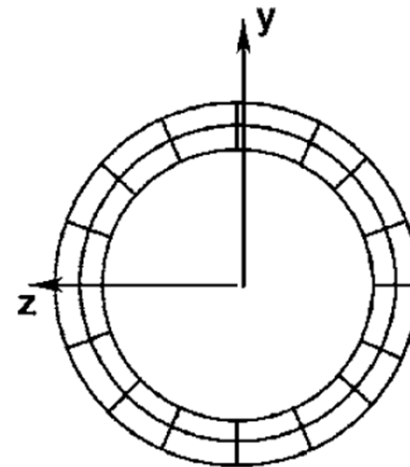
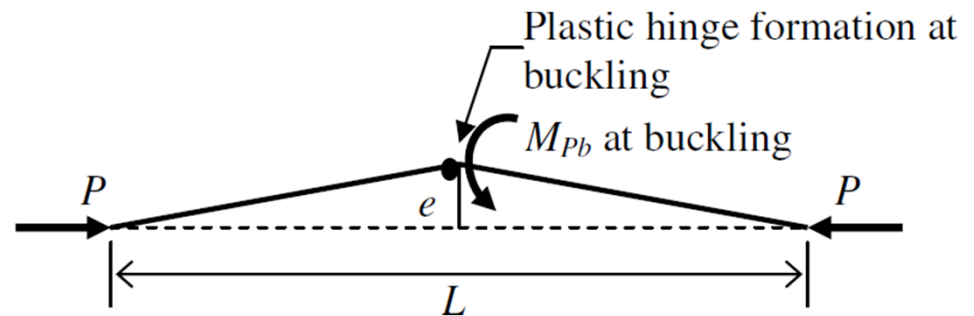
# Brace modelling

- Phenomenological models (truss elements with concentrated plasticity)
- A single plastic "hinge" at the middle of the element, describing the relationship between the axial force and axial deformation



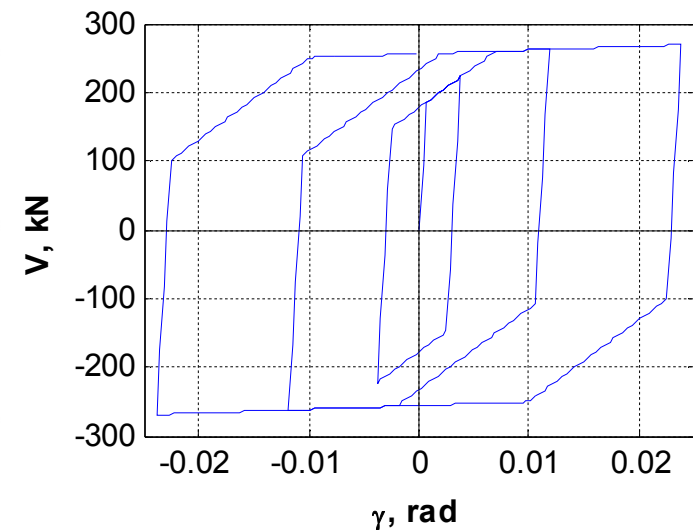
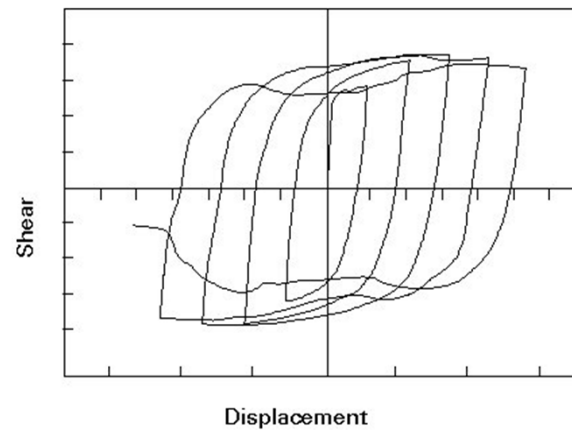
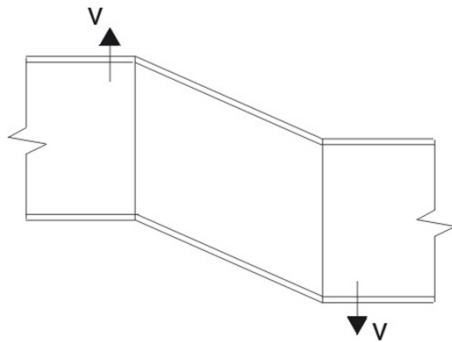
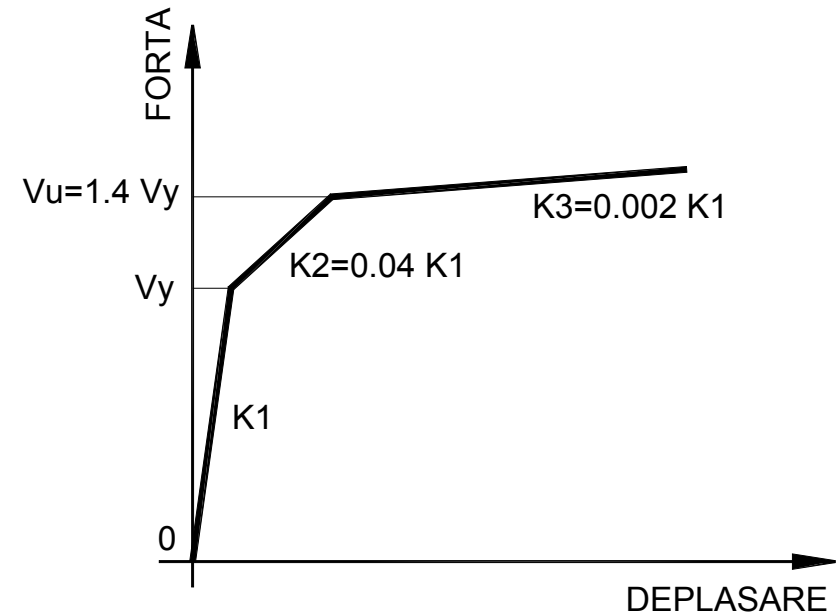
# Brace modelling

- Physical theory models
- Element with distributed plasticity able to reproduce
  - Bending deformations
  - Axial deformations
  - Bending moment – axial force interaction
- Modelling initial member imperfections and adopting a geometrically nonlinear analysis



# Modelling of links

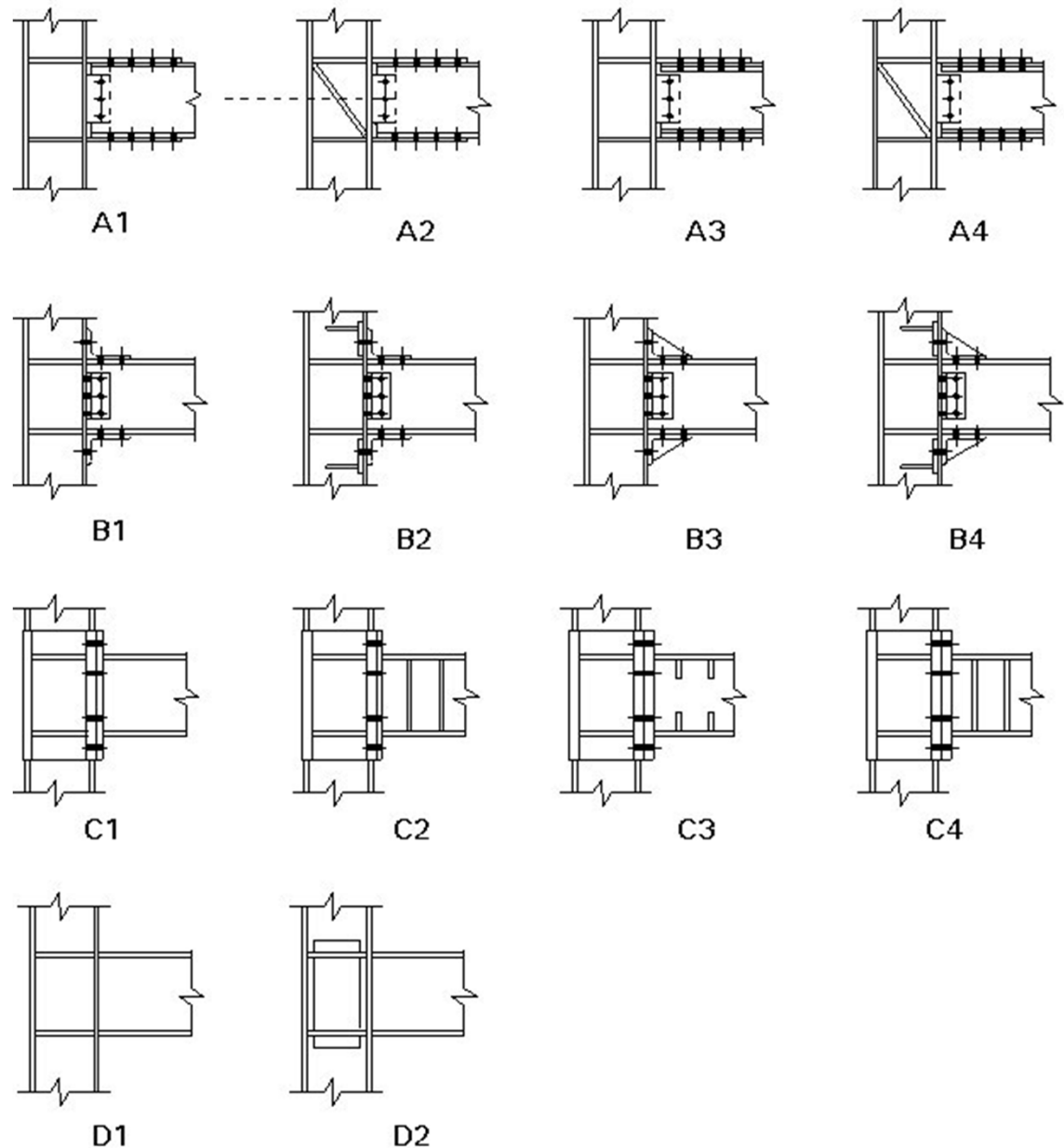
- **Short links:**
  - Subjected predominantly to shear (V)
  - A single plastic "hinge" at the middle of the member
- **Long links: bending (M)**
- **Intermediate links: M-V interaction**





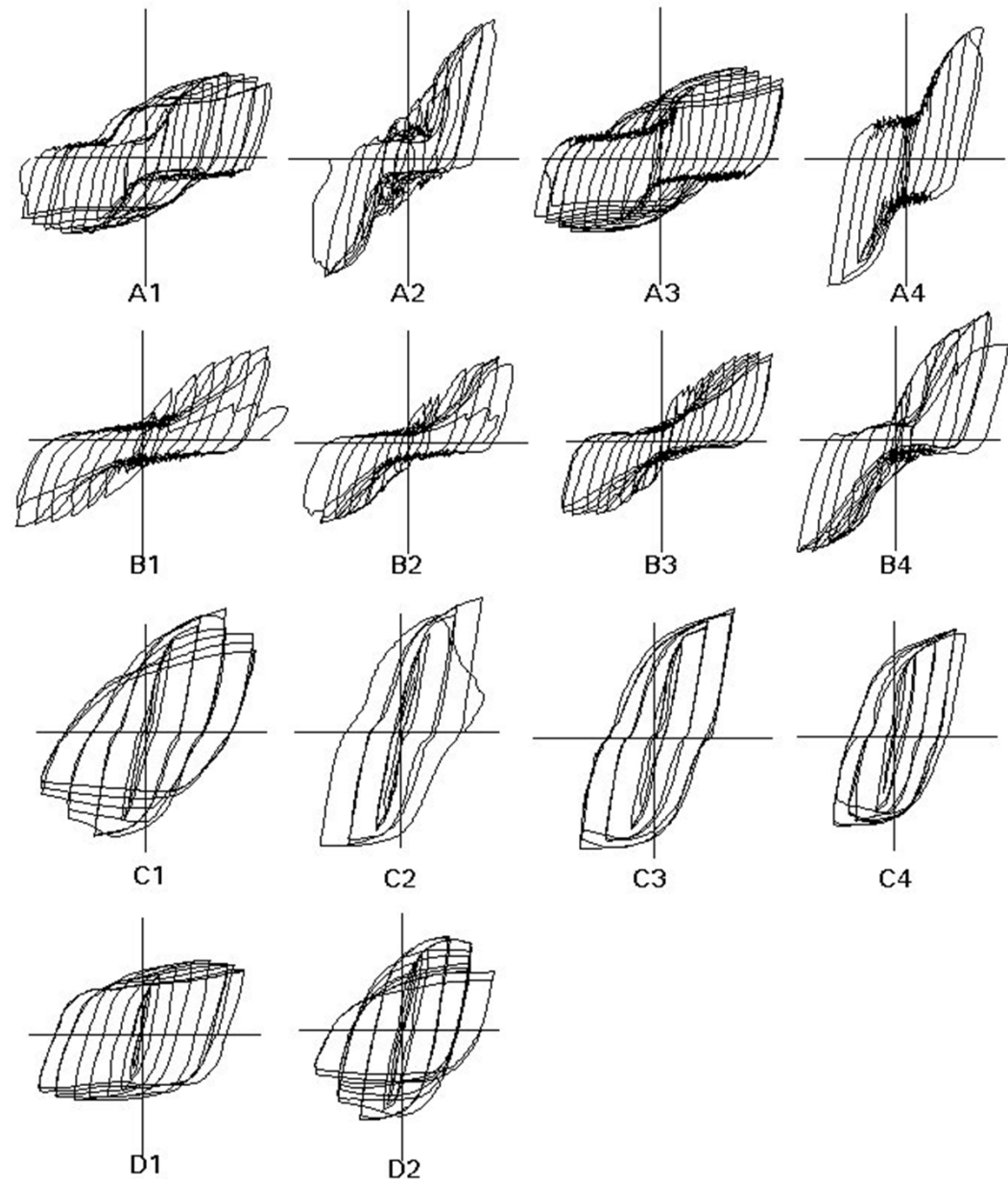
# Modelling of beam-column joints

- Large number of joint types and the complex state of stress and strain in connections lead to very different characteristics of the moment-rotation curves



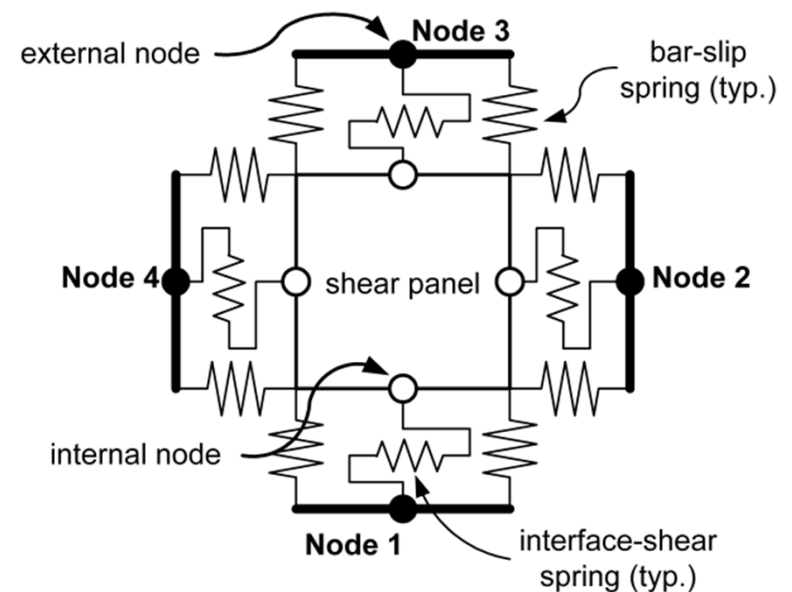
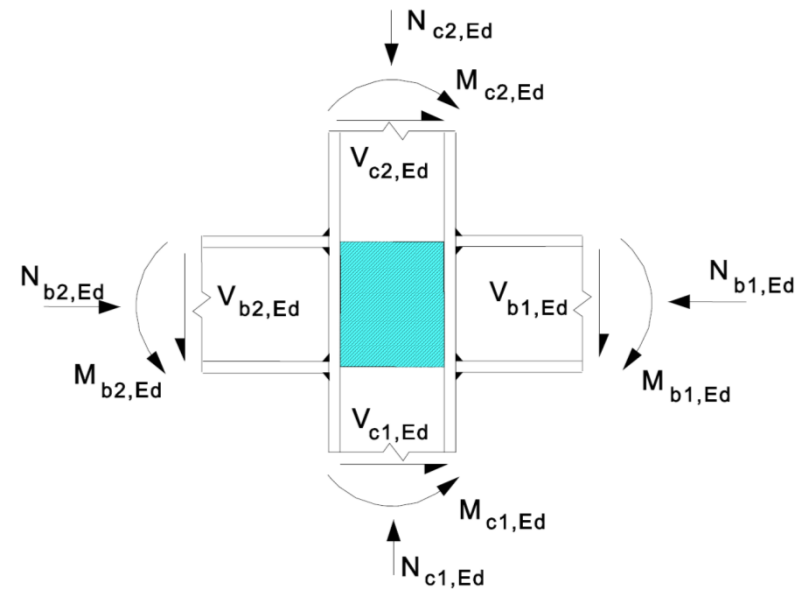
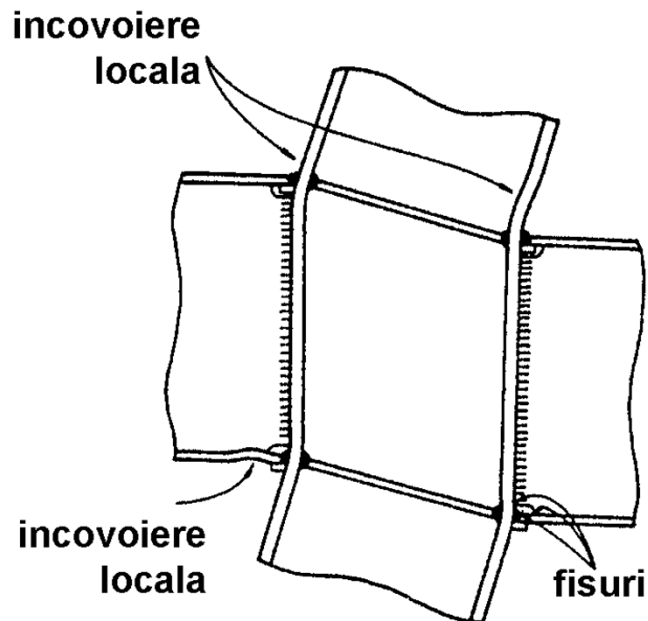
# Modelling of beam-column joints

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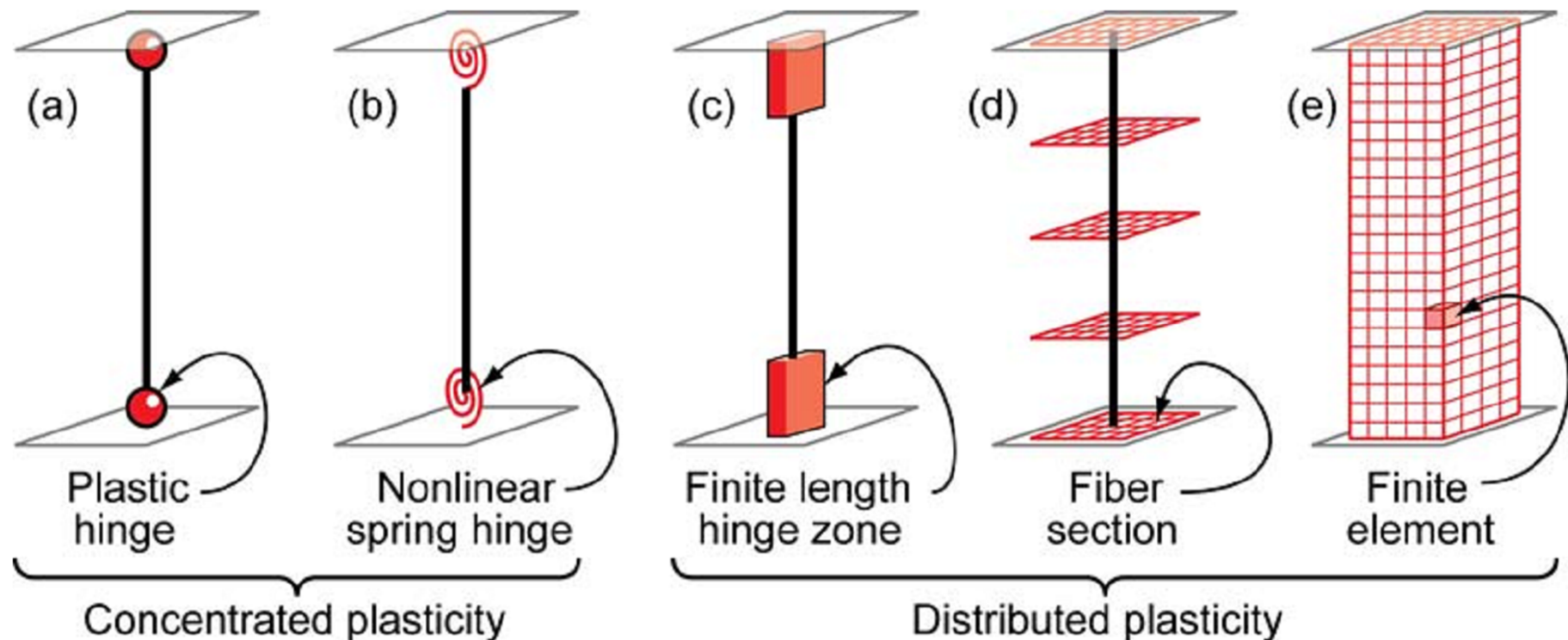
# Modelling of beam-column joints

- Column web panel subjected to shear
- Beam – column connection subjected to bending
- Detailed modelling of the joint region might be necessary



# Modelling of plasticity: overview

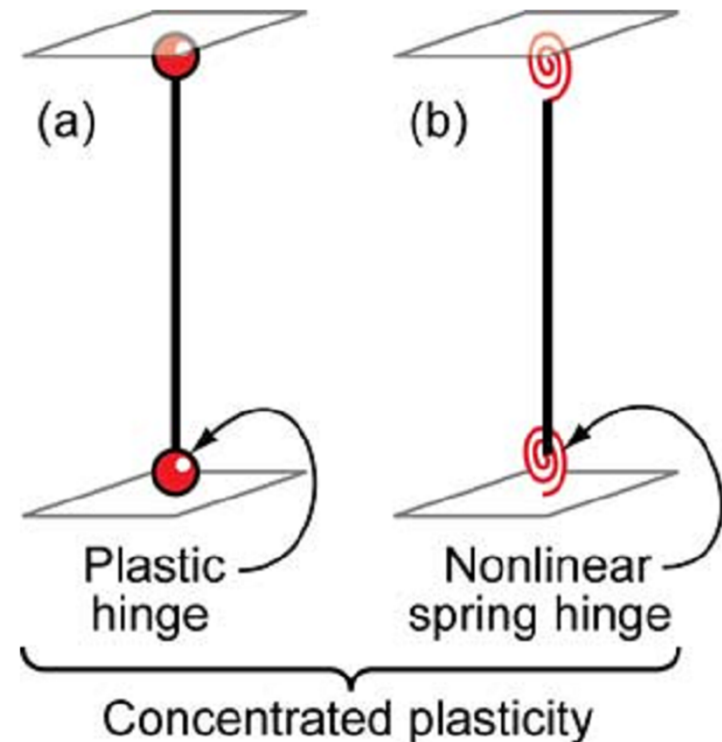
- Inelastic structural component models can be differentiated by the way that plasticity is distributed through the member cross sections and along its length.



# Modelling of plasticity: overview

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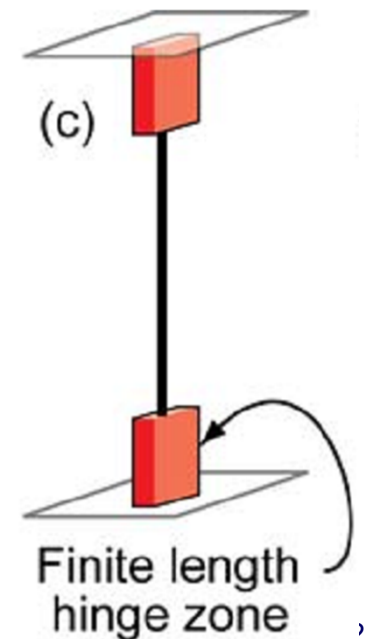
- The simplest models concentrate the inelastic deformations at the end of the element, such as through a rigid-plastic hinge (a) or an inelastic spring with hysteretic properties (b). By concentrating the plasticity in zero-length hinges with moment-rotation model parameters, these elements have relatively condensed numerically efficient formulations.



# Modelling of plasticity: overview

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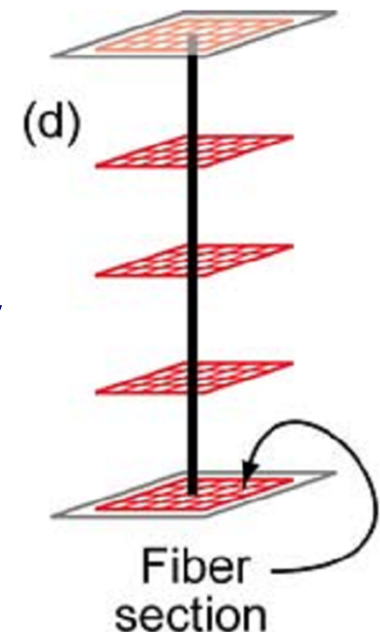
- The finite length hinge model is an efficient distributed plasticity formulation with designated hinge zones at the member ends. Cross sections in the inelastic hinge zones are characterized through either nonlinear moment-curvature relationships or explicit fiber-section integrations that enforce the assumption that plane sections remain plane. The inelastic hinge length may be fixed or variable, as determined from the moment curvature characteristics of the section together with the concurrent moment gradient and axial force. Integration of deformations along the hinge length captures the spread of yielding more realistically than the concentrated hinges, while the finite hinge length facilitates calculation of hinge rotations.



## Modelling of plasticity: overview

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- The fiber formulation models distribute plasticity by numerical integrations through the member cross sections and along the member length. Uniaxial material models are defined to capture the nonlinear hysteretic axial stress-strain characteristics in the cross sections. The plane-sections-remain-plane assumption is enforced, where uniaxial material “fibers” are numerically integrated over the cross section to obtain stress resultants (axial force and moments) and incremental moment curvature and axial force-strain relations. The cross section parameters are then integrated numerically at discrete sections along the member length, using displacement or force interpolation functions.

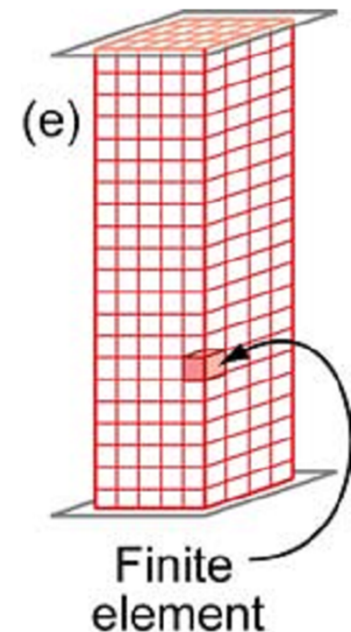




# Modelling of plasticity: overview

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- The most complex models discretize the continuum along the member length and through the cross sections into small (micro) finite elements with nonlinear hysteretic constitutive properties that have numerous input parameters. This fundamental level of modeling offers the most versatility, but it also presents the most challenge in terms of model parameter calibration and computational resources. As with the fiber formulation, the strains calculated from the finite elements can be difficult to interpret relative to acceptance criteria that are typically reported in terms of hinge rotations and deformations.





## References / additional reading

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- **EN 1998-1:2004. "Eurocode 8: Design of structures for earthquake resistance - Part 1: General rules, seismic actions and rules for buildings".**
- **Gregory G. Deierlein, Andrei M. Reinhorn, Michael R. Willford (2010). "Nonlinear Structural Analysis for Seismic Design. A Guide for Practicing Engineers". EHRP Seismic Design Technical Brief No. 4. NIST GCR 10-917-5.**



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