2C09
Design for seismic and climate changes

Lecture 17: Seismic analysis of inelastic MDOF systems I

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European Erasmus Mundus Master Course

Sustainable Constructions
under Natural Hazards and Catastrophic Events

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Lecture outline

17.1 Methods of structural analysis

17.2 Linear dynamic analysis

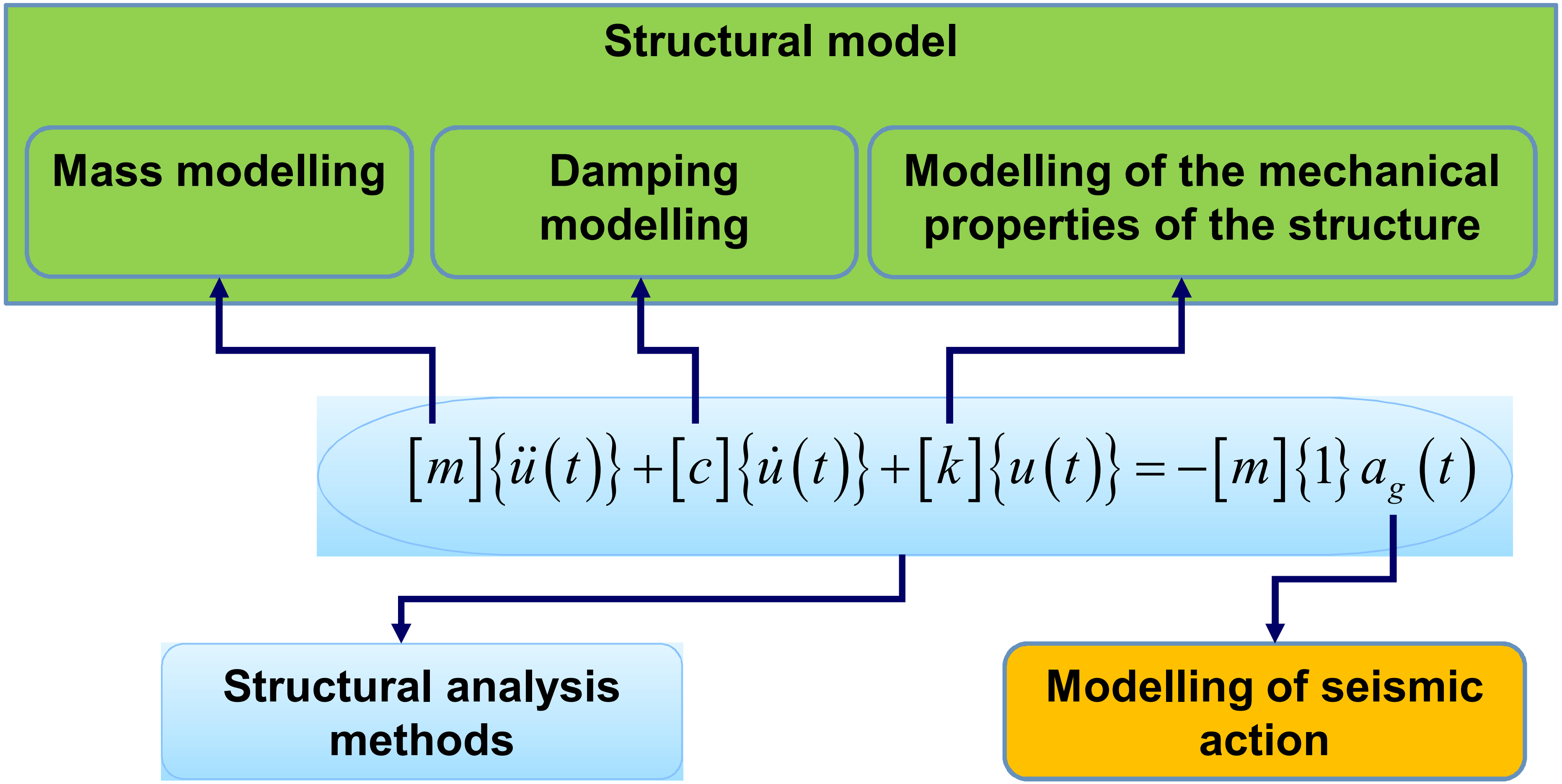
17.3 Nonlinear static analysis.

17.4 Target displacement for nonlinear static analysis.

17.5 Nonlinear dynamic analysis.

17.6 Incremental dynamic analysis.

Structural analysis under seismic action



Structural analysis methods under seismic action

- Elastic analysis
  - Lateral force method (LFM)
  - Modal response spectrum analysis (MRS)
  - Linear dynamic analysis
    - Modal analysis
    - Direct integration of the equation of motion
- Inelastic analysis
  - Nonlinear static analysis (pushover)
  - Nonlinear dynamic analysis

Conventional design: Lateral force method (LFM); Modal response spectrum analysis (MRS)

Advanced design: Linear dynamic analysis; Nonlinear static analysis (pushover); Nonlinear dynamic analysis

- Generally, the structural model should be representative for distribution within the structure of:
  - stiffness and strength,
  - mass and
  - damping

Analysis methods and modelling requirements

Analysis methods		
Material model	Modelling of dynamic effects	
	Static	Dynamic
Linear	Lateral force method	Linear dynamic analysis
Non-linear	Pushover analysis	Nonlinear dynamic analysis

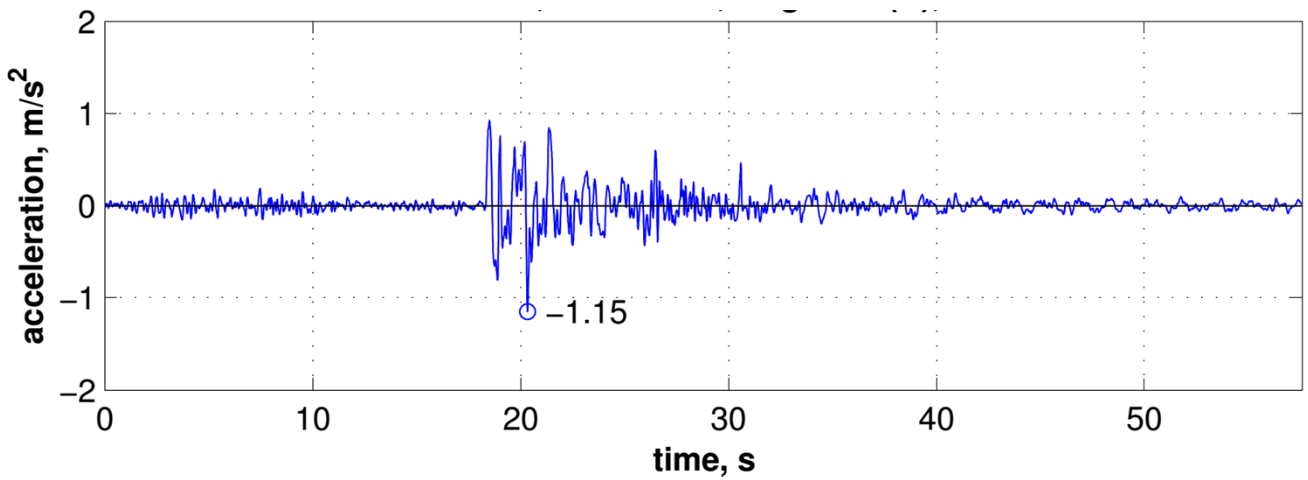
Modelling requirements		
Material model	Modelling of dynamic effects	
	Static	Dynamic
Linear	stiffness (and mass*)	stiffness, mass and damping
Non-linear	stiffness+strength (and mass**)	stiffness+strength, mass and damping

* Only if natural period of vibration is determined using methods in structural dynamics

** Only if target displacement is determined

Linear dynamic analysis

- **Linear elastic response at the material, cross-section and member levels**
- **Infinitely elastic response of the structure**
- **Equilibrium is formulated on the undeformed structure**
- **Modelling of seismic action: accelerograms digitized at time steps of 0.005 – 0.02 sec**

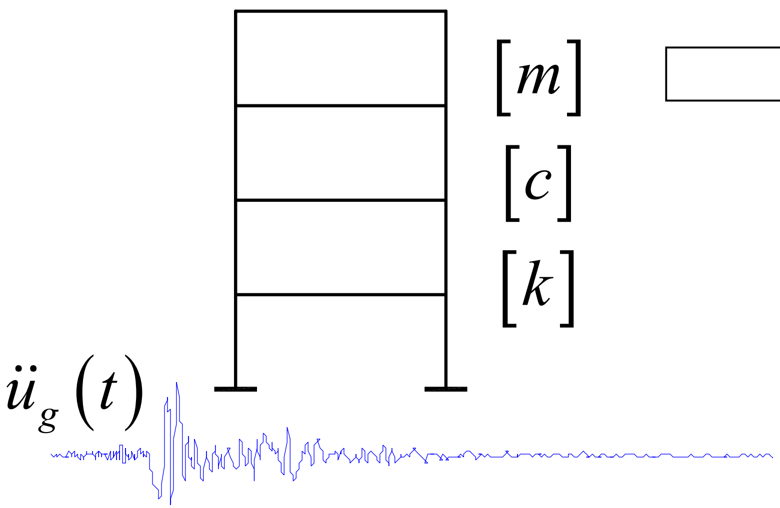


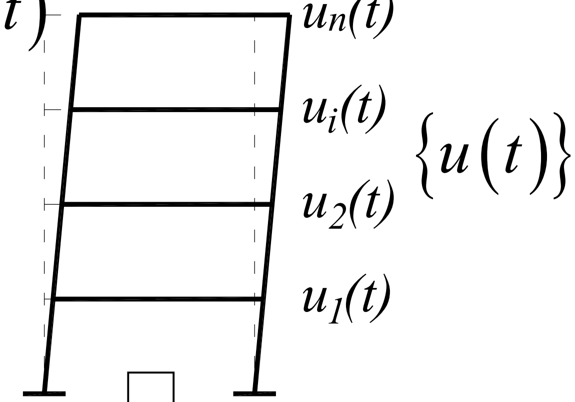
- **Purpose: to determine design forces for structural components and to compute displacements and story drifts**

Linear dynamic analysis: direct analysis

- Time history response is obtained through direct integration (numerical methods) of the equations of motion
- For a system with *N* degrees of freedom, there are *N* coupled differential equations to be solved numerically

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+[k]\{u\}=-[m]\{1\}\ddot{u}_g(t)$$

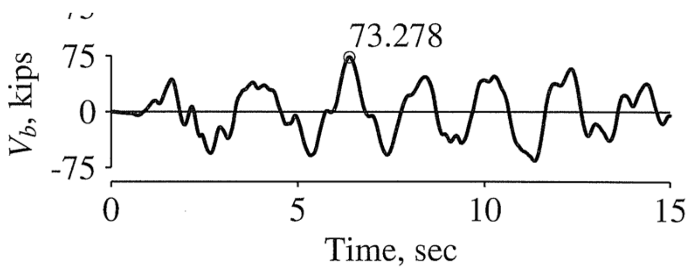




$\{M(t)\}$

$\{N(t)\}$

$\{V(t)\}$



Linear dynamic analysis

- **Advantages of linear dynamic analysis over lateral force method and modal response spectrum analysis:**
  - it is more accurate mathematically,
  - signs of response quantities (such as tension or compression in a brace) are not lost as a result of the combination of modal responses, and
  - story drifts are computed more accurately.

- **The main disadvantages of linear dynamic analysis are:**
  - the need to select and scale an appropriate suite of ground motions, and
  - analysis is resource-intensive
  - large amount of results $\Rightarrow$ a cumbersome and time-consuming post-processing of results.

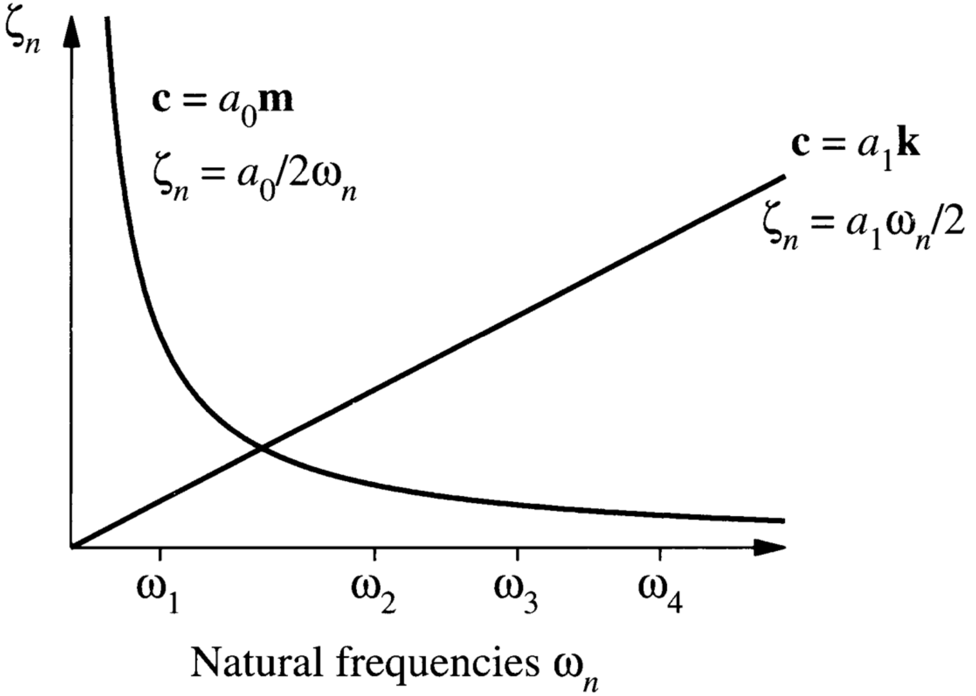
Linear dynamic analysis: modal analysis

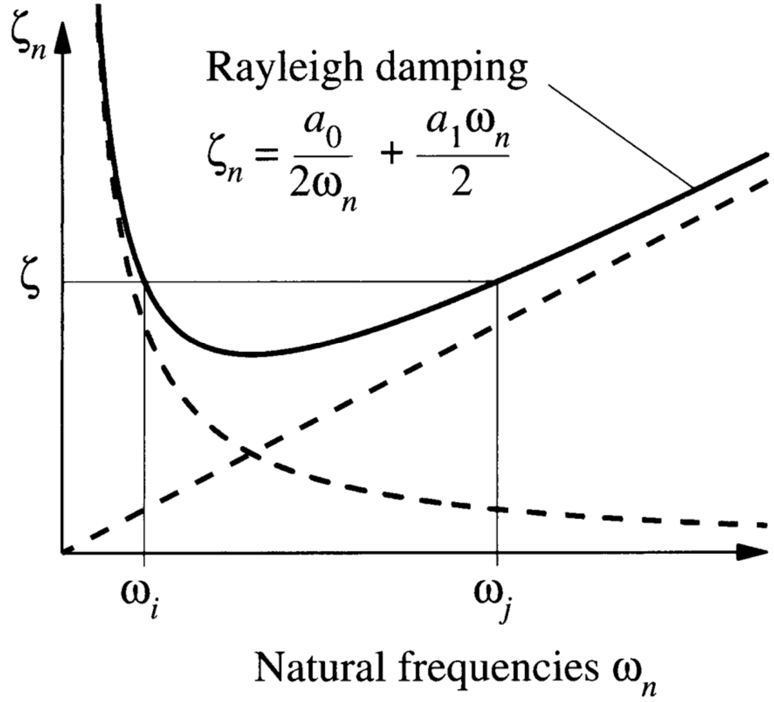
- **The equilibrium equations are transformed, by change of coordinates, into a number of SDOF systems. The maximum number of SDOF systems that can be formed is equal to the number of degrees of freedom in the structure.**
- **The SDOF equations are solved individually, and then the computed displacement histories are transformed back to the original coordinates and superimposed to obtain the system response history.**
- **A limited number of modes may be used to produce reasonably accurate results. While some accuracy is sacrificed where fewer modes are used, the computer resources required to perform the analysis are significantly less than those required for direct analysis.** **The number of modes required for a "reasonably" accurate analysis can be determined using the 90% effective modal mass rule.**

Linear dynamic analysis: modelling of damping

- **Direct analysis: Rayleigh damping** $[c] = a_0[m] + a_1[k]$
  - **mass proportional** $[c] = a_0[m]$
  - **stiffness proportional** $[c] = a_1[k]$
  - **usually ξ = 5%**

$$a_0 = \xi \frac{2\omega_i \omega_j}{\omega_i + \omega_j} \qquad a_1 = \xi \frac{2}{\omega_i + \omega_j}$$

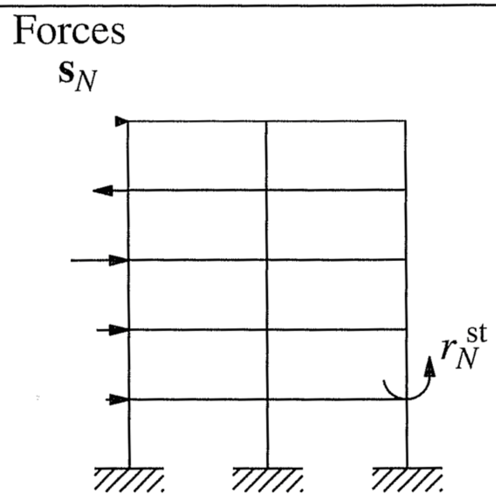
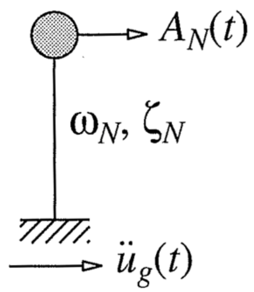




Linear dynamic analysis: modelling of damping

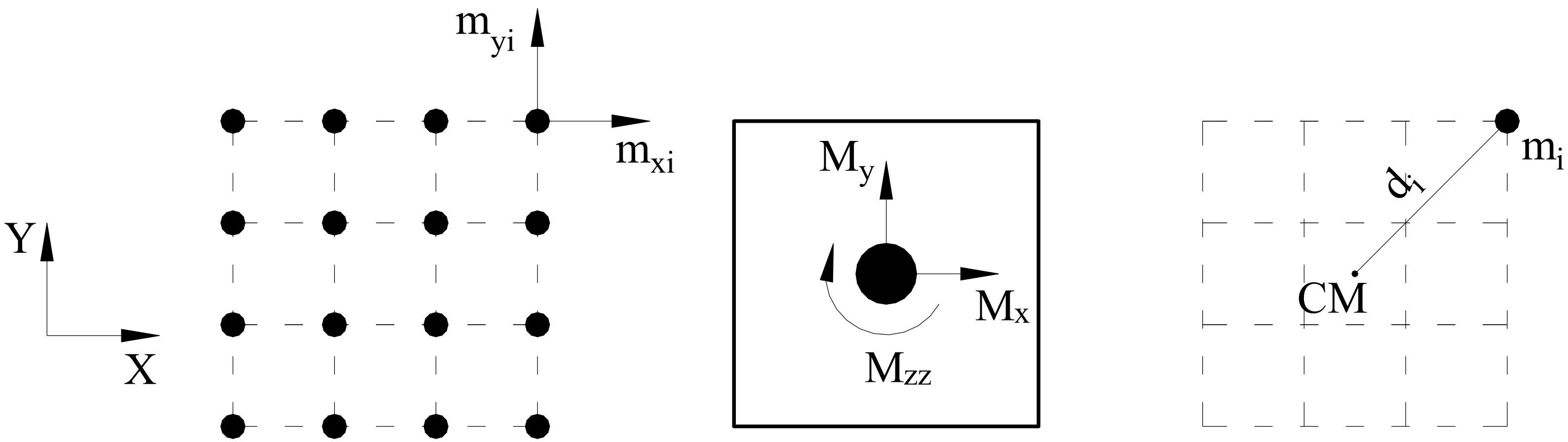
- **Modal analysis: damping assigned directly to each mode that is included in the response.**

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n \ddot{u}_g\left(t\right)$$

Mode	Static Analysis of Structure	Dynamic Analysis of SDF System	Modal Contribution to Dynamic Response
N			$r_N(t) = r_N^{st} A_N(t)$
Total response			$r(t) = \sum_{n=1}^{N} r_n(t)$

Modelling of mass distribution

- Mass is usually lumped in order to reduce the number of dynamic degrees of freedom
- For structures with flexible diaphragms, lumped masses in nodes should approximate the real distribution of mass
- For structures with rigid diaphragms, the mass can be lumped in the centre of mass of the storey
  - Two translational components $M_x = M_y = \sum m_i$
  - Mass moment of inertia (rotational component) $M_{zz} = \sum m_i d_i^2$



Accelerograms: selection

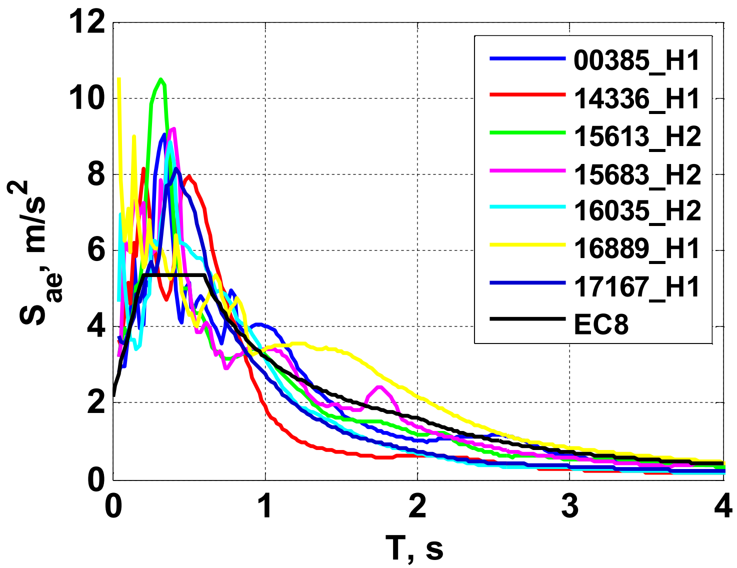
- **Artificial accelerograms**, matching the code elastic response spectra. The duration of accelerograms should be consistent with the magnitude and other relevant features of the seismic event.

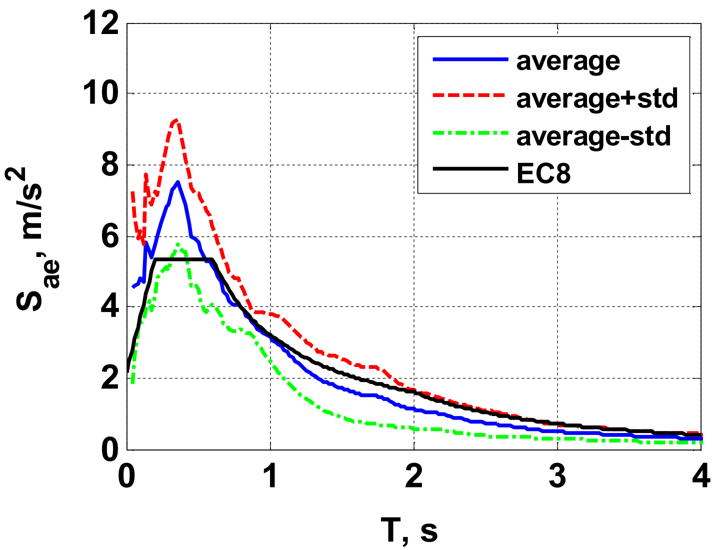
- **Recorded accelerograms**, provided the samples are qualified to the seismogenetic features of the source and to the soil conditions at the site.

- **Simulated accelerograms**, generated through a physical simulation of source and travel path mechanisms, complying with the requirements for recorded accelerograms.

Accelerograms: scaling

- **Eurocode 8 – for any selection procedure, the following should be observed:**
  - PGA of individual time-histories should not be smaller than the codified PGA atop of soil layers ($a_g \cdot S$)
  - In the range of periods $0.2T_1$-$2T_1$ no value of the mean spectrum, calculated from all time histories, should be less than 90% of the corresponding value of the code elastic response spectrum (lower limit ($0.2T_1$) accounts for higher modes of vibration, while upper limit (1.5-$2.0T_1$) accounts for "softening" of the structure due to inelastic response)





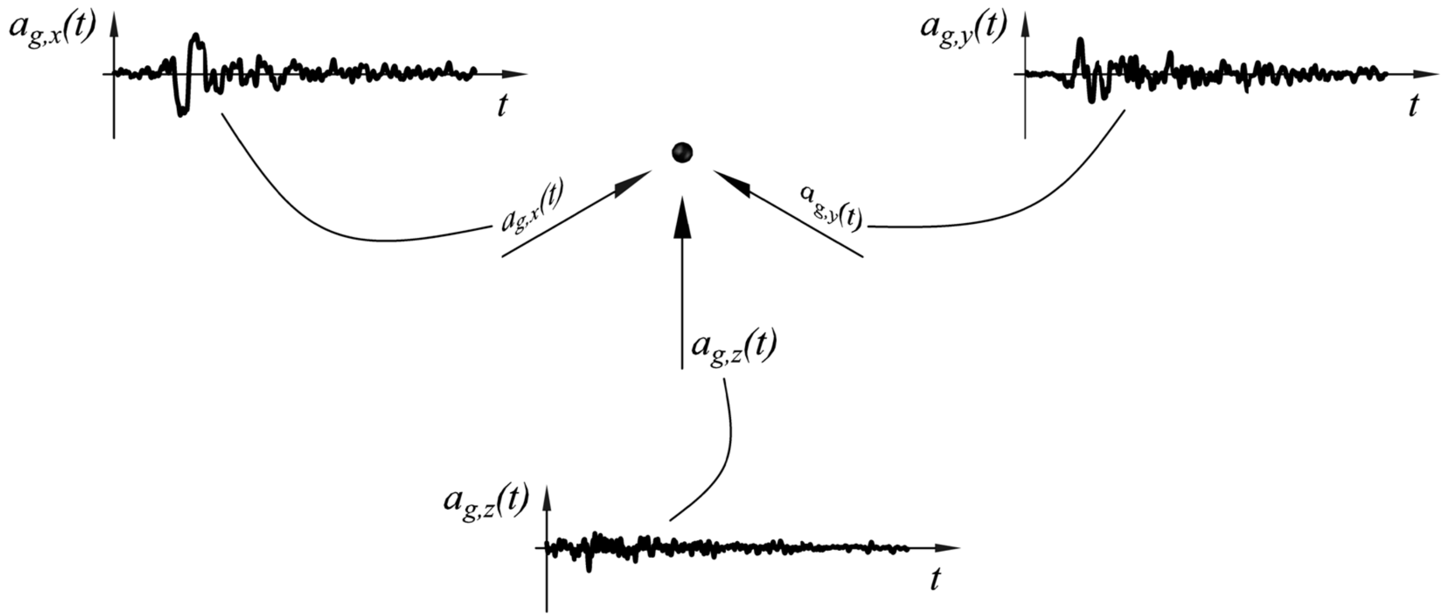
Accelerograms: number

- **Due to uncertainties related to characterisation of seismic motion, a large enough number of accelerograms should be used in a dynamic analysis**
- **At least three accelerograms $\Rightarrow$ seismic evaluation based on peak values of response**
- **At least seven accelerograms $\Rightarrow$ seismic evaluation based on mean values of response**

Linear dynamic analysis: components of seismic action

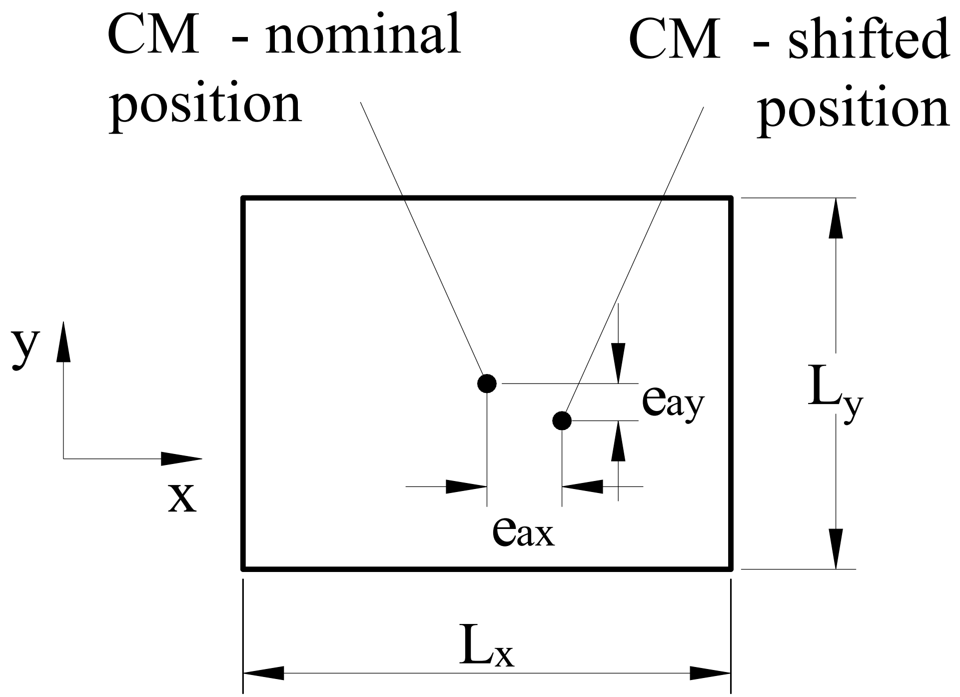
3D model of the structure
+
Linear dynamic analysis
+
Simultaneous application of non-identical accelerograms along the main directions of the structure

Spatial character of the seismic action accounted for directly



Linear dynamic analysis: accidental eccentricity

- **For spatial models (3D): the accidental torsional effects accounted for by shifting the centre of mass from its nominal location with the value of the eccentricity in each of the two horizontal directions**
- **Accidental eccentricity $e_{ai} = \pm 0.05\, L_i$ (EN 1998-1)**



Linear dynamic analysis: accidental eccentricity

- **For planar models (2D): the accidental torsional effects may be accounted for by multiplying the action effects in the individual load resisting elements resulting from analysis by a factor δ**

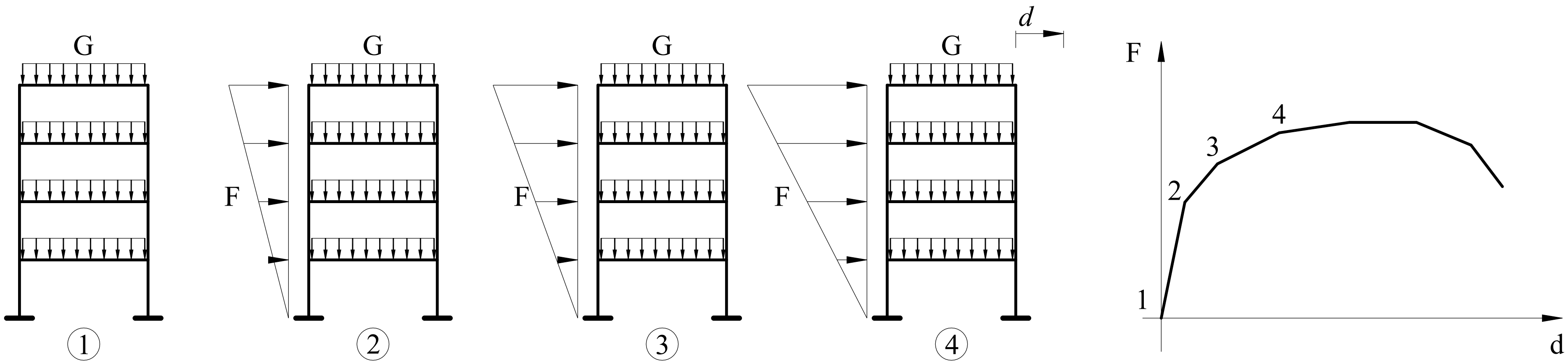
$$\delta = 1 + 1.2\frac{x}{L_e}$$

- x **is the distance of the element under consideration from the centre of mass of the building in plan, measured perpendicularly to the direction of the seismic action considered;**
- L_e **is the distance between the two outermost lateral load resisting elements, measured perpendicularly to the direction of the seismic action considered.**

Nonlinear static analysis (pushover)

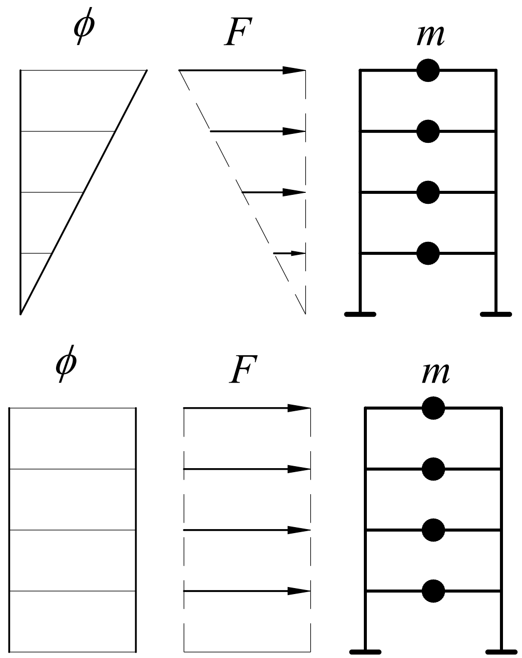
- **Nonlinear static analysis under constant gravity loading and monotonically increasing lateral forces (whose distribution represents the inertia forces expected during ground shaking)**
- **Control elements:**
  - **base shear force**
  - **Control displacement (top displacement)**

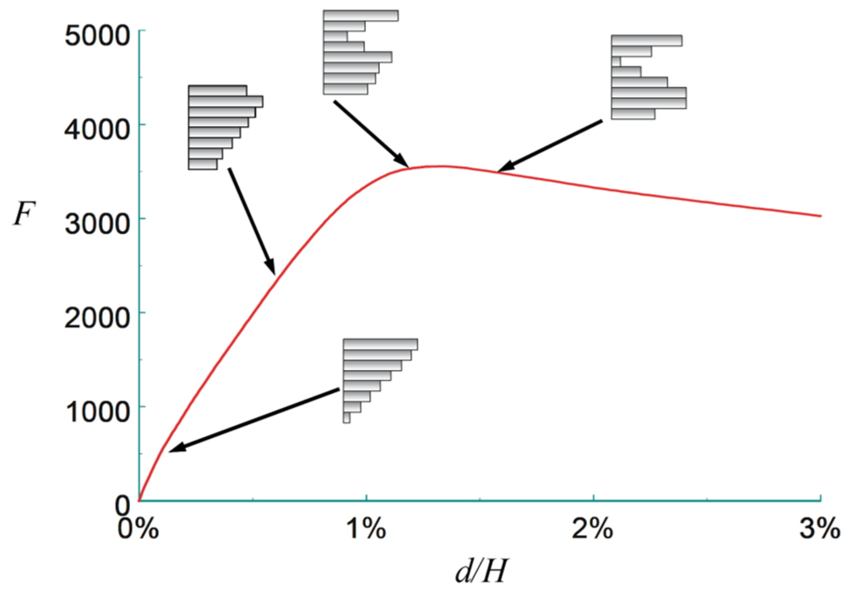
  } **pushover (capacity) curve**
- **Provides the capacity of the structure, and does not give directly the demands associated with a particular level of seismic action**



Nonlinear static analysis (pushover)

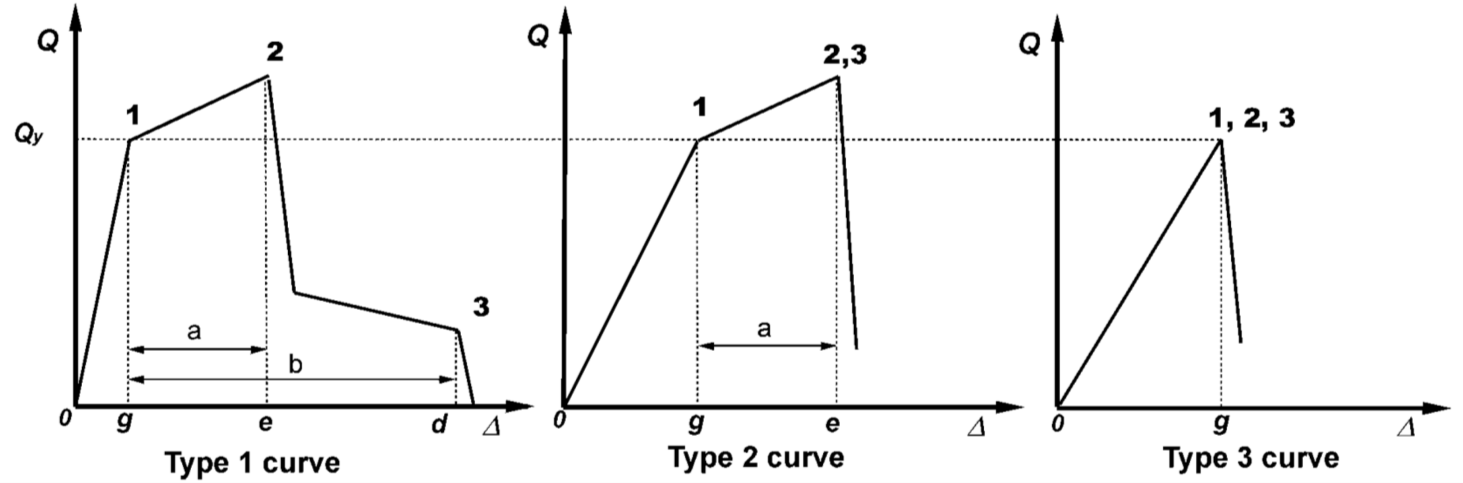
- **Assumes that response is governed by a single mode of vibration, and that it is constant during the analysis**
- **Distribution of lateral forces (applied at storey masses):**
  - **modal (usually first mode – inverted triangle)**
  - **uniform: lateral forces proportional to storey masses**
  - **"adaptive" distributions possible, but less common, requiring specialised software**



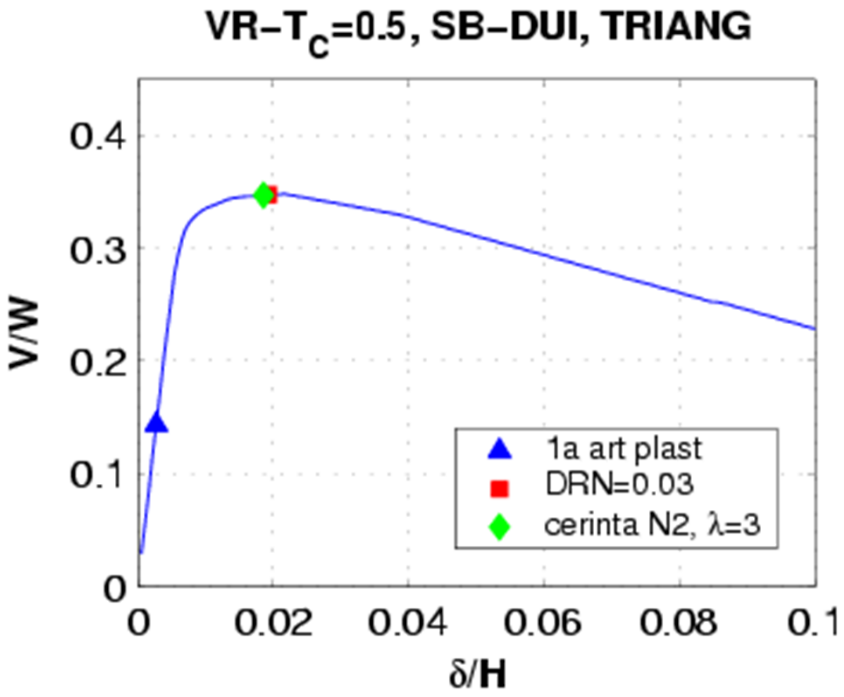


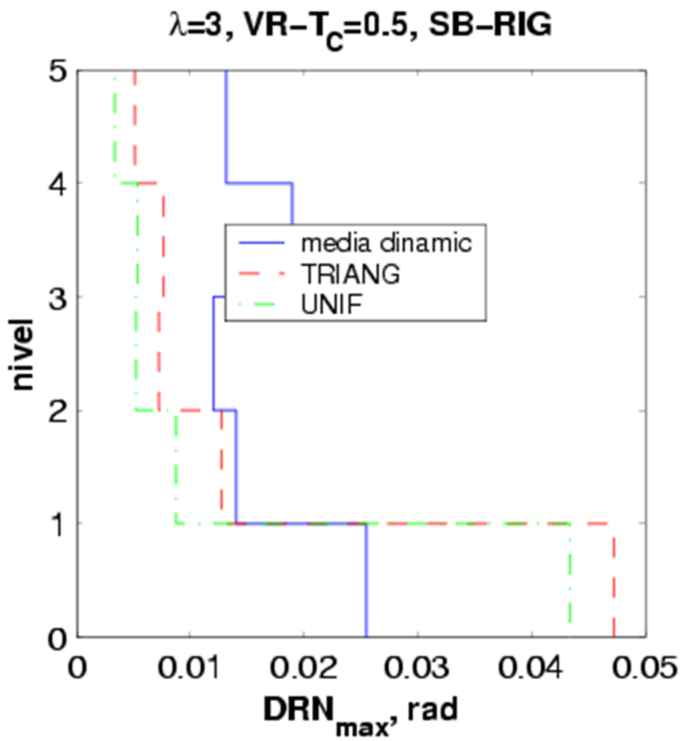
Nonlinear static analysis (pushover)

- Applicable to low-rise regular buildings, where the response is dominated by the fundamental mode of vibration.
- Application of loading:
  - Gravity loading: force control
  - Lateral forces: displacement control
- Modelling of structural components: inelastic monotonic force-deformation obtained from envelopes of cyclic response



Nonlinear static analysis (pushover)



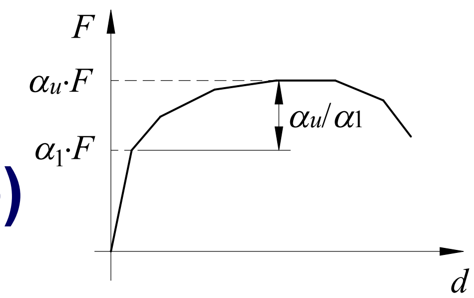


Pushover curve

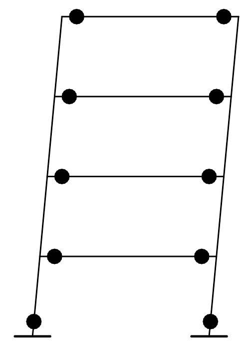
Distribution of interstorey drifts

Nonlinear static analysis (pushover)

- Represents a direct evaluation of overall structural response, not only on an element by element basis
- Allows evaluation of inelastic deformations – the most relevant response quantity in the case of inelastic response
- Allows evaluation of the plastic mechanism and redundancy of the structure (α_u/α_1 ratio)



- "Local" checks:
  - Interstorey drifts
  - Strength demands in non-dissipative components
  - Ductility of dissipative components



- "Global" checks – failure at the structure level
  - Failure to resist further gravity loading
  - Failure of the first vertical element essential for stability of the structure

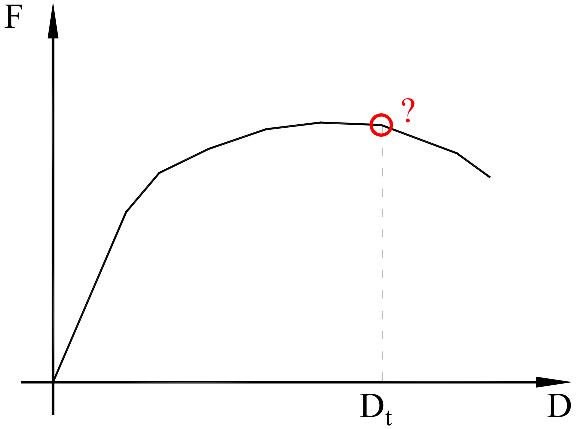
Target displacement: the N2 method

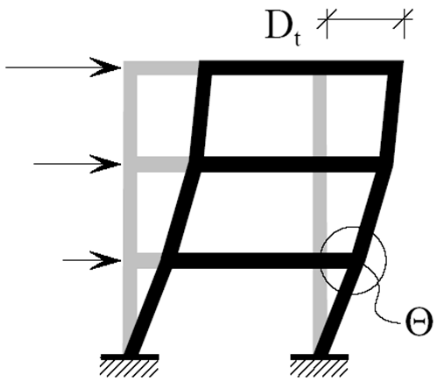
- Nonlinear static analysis ⇒ "capacity curve"
- Evaluation of seismic performance:
  - Plastic deformation demands
  - Stresses

  for a given level of seismic hazard

  ⬇

  corresponding displacement demand?



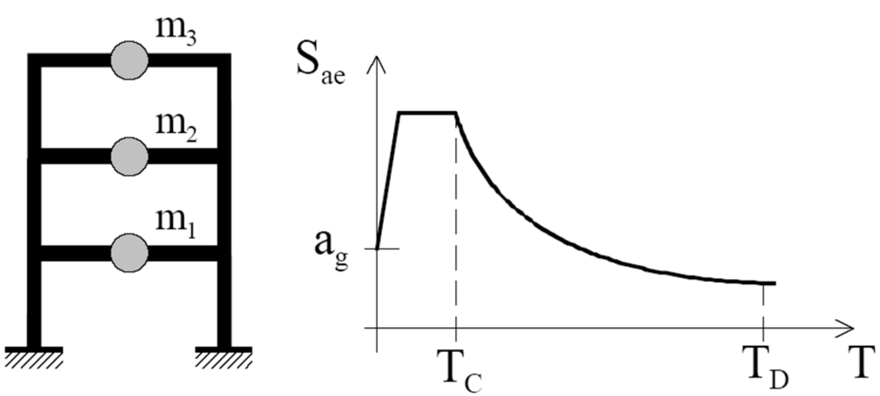


- Evaluation of displacement demand:
  - N2 Method (Fajfar, 2000)
  - EN 1998-1:2004, Annex B

N2: step 1 – initial data

- **Structural model:**
  - Plane (2D) structures
  - Modelling of nonlinear response of structural components

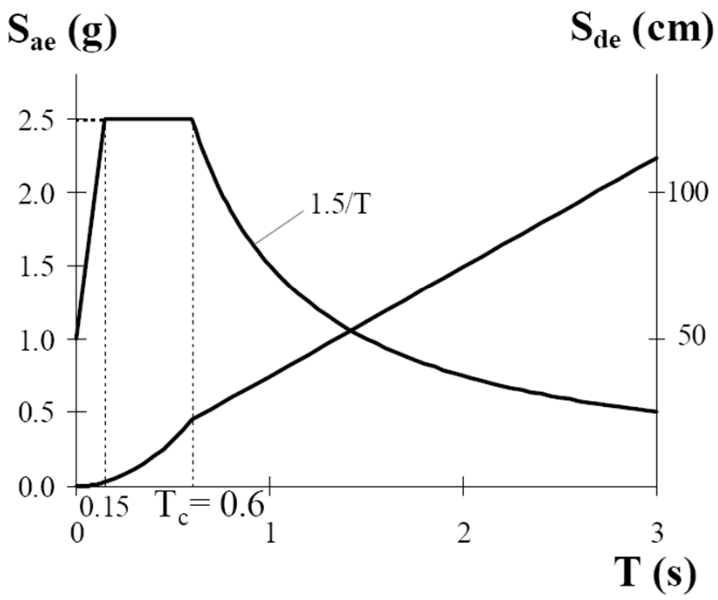
- **Seismic action: elastic (pseudo-) acceleration response spectrum**

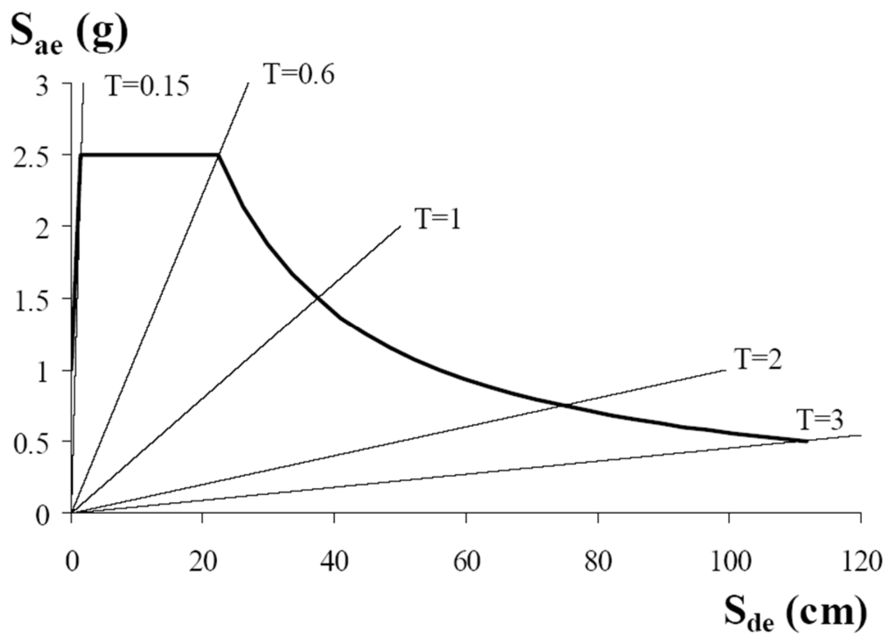


N2: step 2 – response spectrum in AD format

- **Elastic response spectrum in AD format: Acceleration – Displacement (S_{de}-S_{ae})**

$$S_{de} = \frac{S_{ae}}{\omega^2} = S_{ae}\frac{T^2}{4\cdot\pi^2}$$



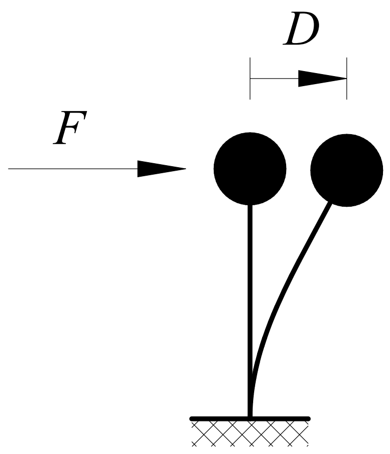


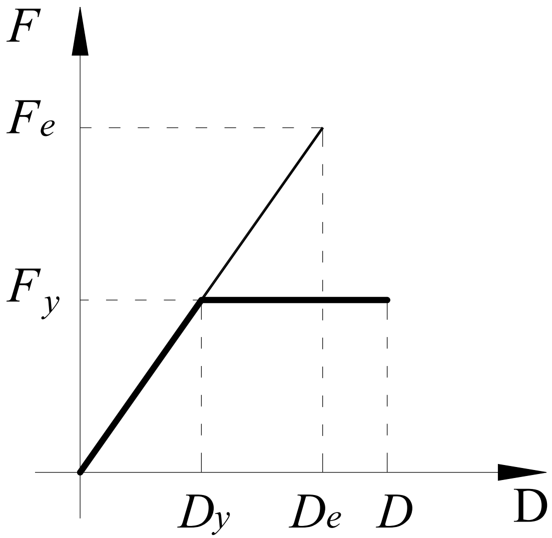
N2: step 2 – response spectrum in AD format

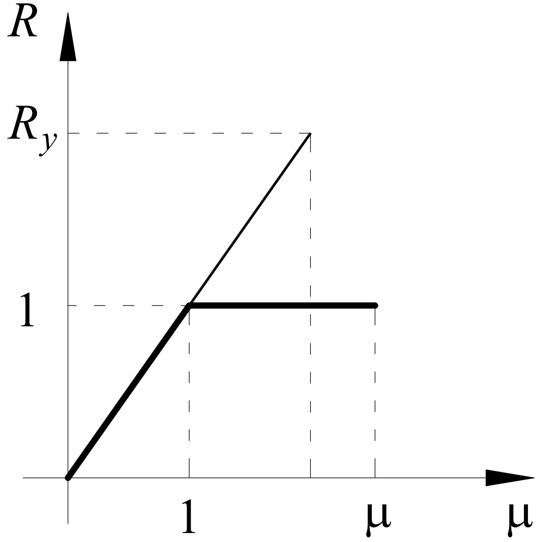
- **Inelastic response spectrum in AD format: (S_d-S_a)**
  - **Ductility** $\mu = D/D_y$
  - **Seismic force reduction factor:** $R_\mu = F_e/F_y$

$$S_a = \frac{S_{ae}}{R_\mu} \qquad D = \mu \cdot D_y = \frac{\mu}{R_\mu} \cdot D_e$$

$$S_d = \frac{\mu}{R_\mu} \cdot S_{de} = \frac{\mu}{R_\mu} \cdot \frac{T^2}{4 \cdot \pi^2} \cdot S_{ae} = \mu \cdot \frac{T^2}{4 \cdot \pi^2} \cdot S_a$$





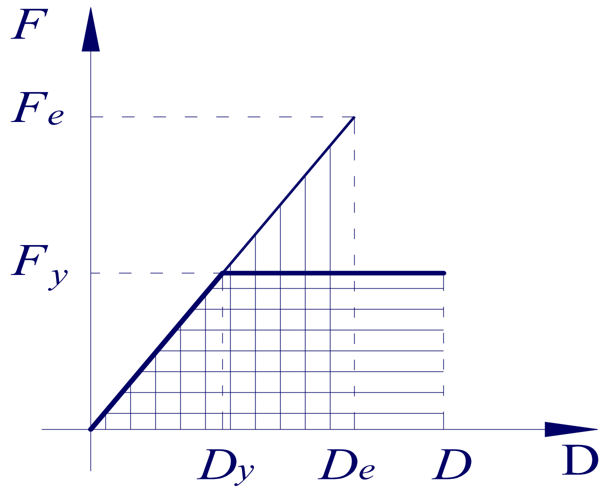


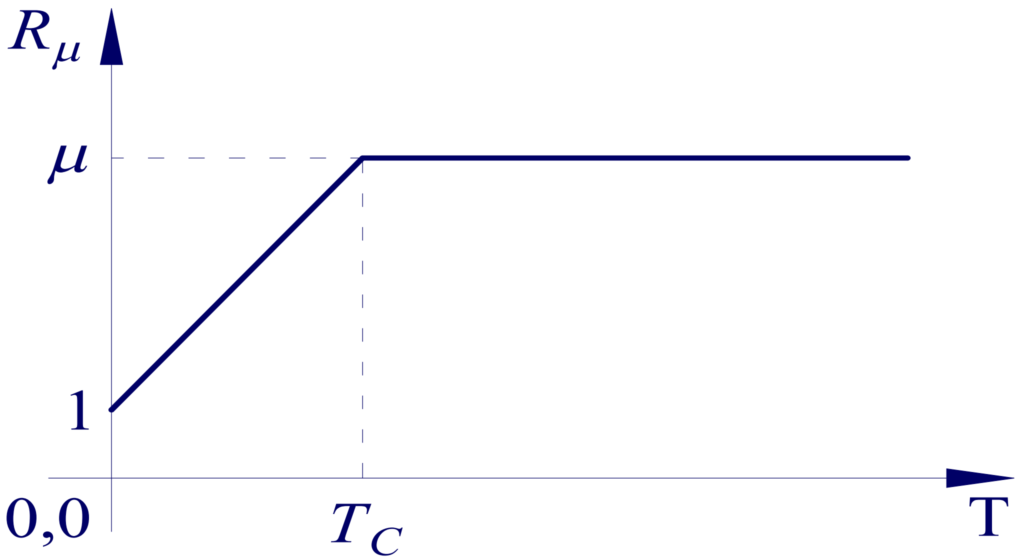
N2: step 2 – response spectrum in AD format

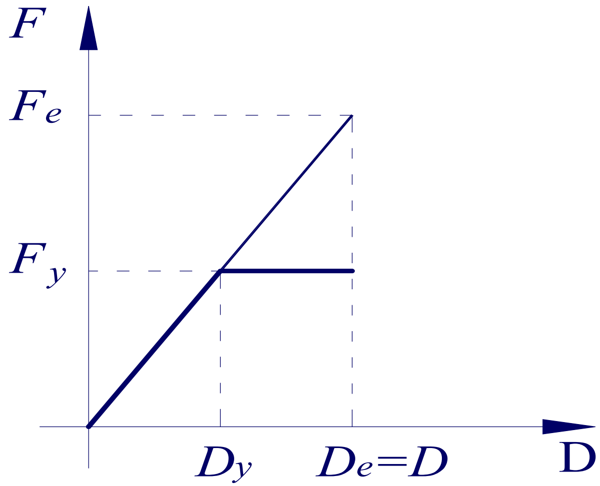
- **Inelastic response spectrum in AD format: (S_d-S_a)**
  - **μ-R_μ relationship**

$$R_\mu = (\mu - 1)\frac{T}{T_C} + 1 \qquad T < T_C$$

$$R_\mu = \mu \qquad T \geq T_C$$

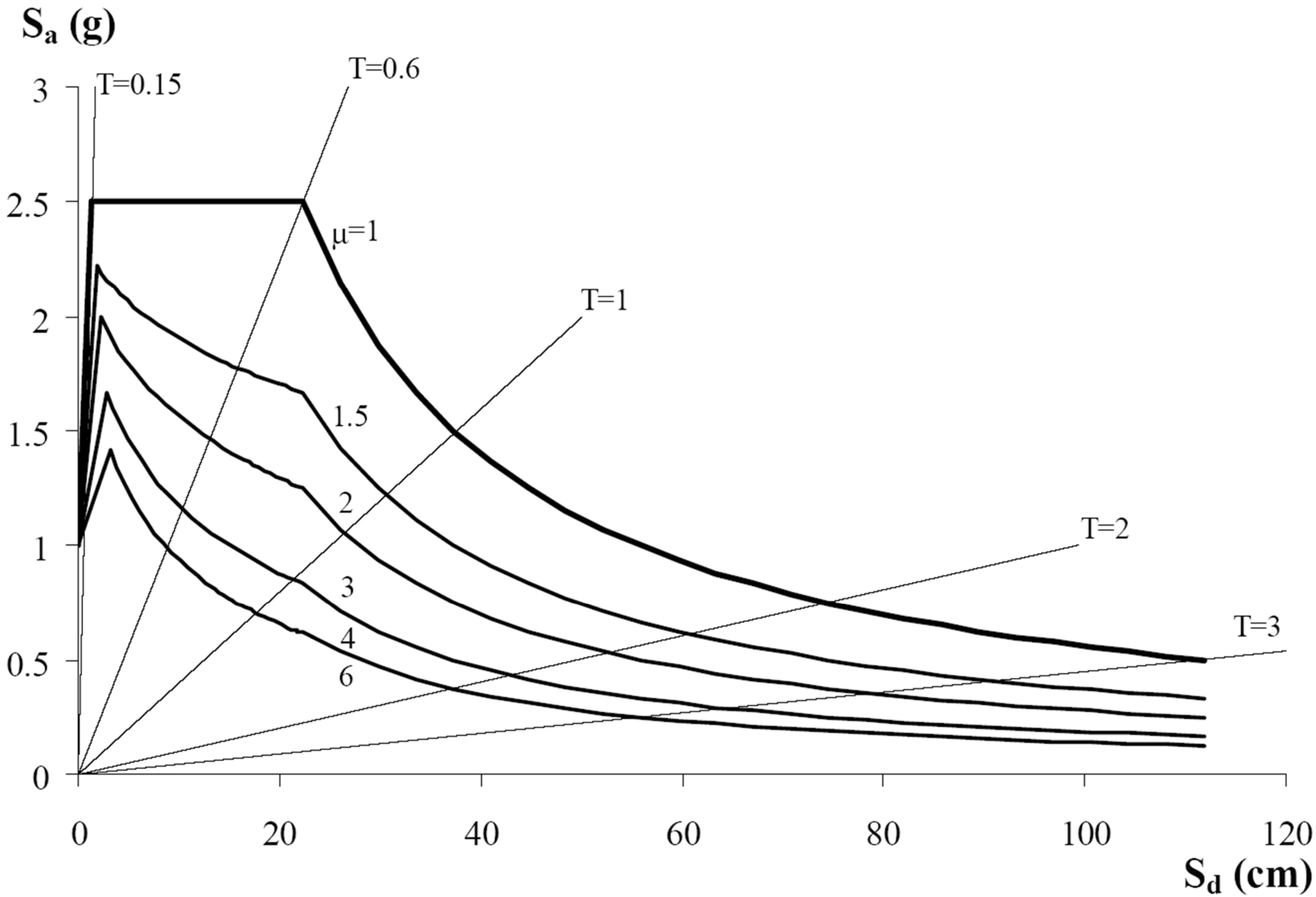






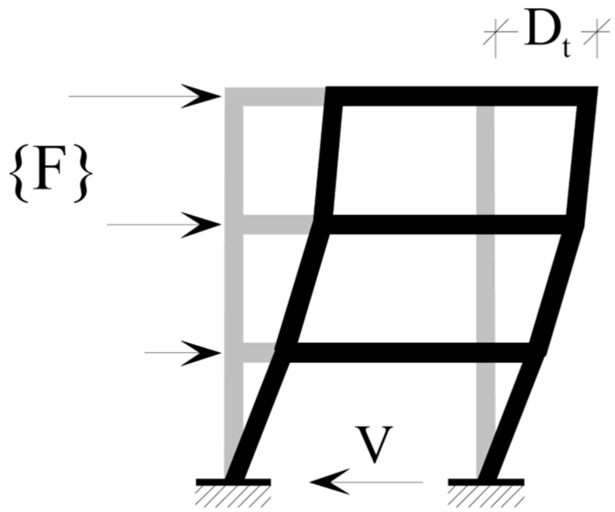
N2: step 2 – response spectrum in AD format

- **Inelastic response spectrum in AD format: (S_d-S_a)**



N2: step 3 – nonlinear static analysis

- **Nonlinear static analysis ⇒ base shear – top displacement relationship of the MDOF system is obtained**
- **MDOF – multi degree of freedom system**
- **Natural modes of vibration $\{\phi\}$:**
  - **Fundamental mode shape**
  - **Uniform distribution**
- **Distribution of lateral forces: $\{F\}=f\cdot[m]\cdot\{\phi\}$**
  - ***f* – magnitude of lateral forces**
  - **[*m*] – mass matrix**
  - **$\{\phi\}$ – assumed deformed shape**
- **Lateral forces at storey *i*: $F_i=f\cdot m_i\cdot\phi_i$**
  - **m_i – mass of storey *i***
  - **ϕ_i – modal displacement at storey *i***



N2: step 4 – equivalent SDOF system

- **Determination of the displacement demand in an equivalent SDOF system**
- **SDOF – single degree of freedom system**
- **Equation of motion of MDOF system:**

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+[k]\{u\}=-[m]\{1\}a_g$$

as damping is included in the response spectrum, this can be expressed as

$$[m]\{\ddot{u}\}+\{R\}=-[m]\{1\}a_g$$

  – $\{u\}$ – **vector of displacements**
  – $\{R\}$ – **vector of internal forces**
  – $\{1\}$ – **unit vector**
  – a_g - **ground acceleration**

N2: step 4 – equivalent SDOF system

- **Vector of displacements can be expressed as:**

$$\{u\}=\{\phi\}D_t$$

  - D_t – **time–dependent displacement at the top of the structure**
  - $\{\phi\}$ – **deformed shape in a natural mode of vibration, normalized in such a way that the component at the top is equal to 1**

N2: step 4 – equivalent SDOF system

- **From the equilibrium condition:** $\{F\}=\{R\}$

- **Replacing**

  $\{F\}=f\cdot[m]\cdot\{\phi\}$ $\{u\}=\{\phi\}D_t$ $\{F\}=\{R\}$

  in

  $$[m]\{\ddot{u}\}+\{R\}=-[m]\{1\}a_g$$

  it is obtained

  $$[m]\{\phi\}\ddot{D}_t+f[m]\{\phi\}=-[m]\{1\}a_g$$

  multiplying to the left with $\{\phi\}^T \Rightarrow$

  $$\{\phi\}^T[m]\{\phi\}\ddot{D}_t+\{\phi\}^T[m]\{\phi\}f=-\{\phi\}^T[m]\{1\}a_g$$

N2: step 4 – equivalent SDOF system

- **And multiplying and dividing**

$$\{\phi\}^T[m]\{\phi\}\ddot{D}_t + \{\phi\}^T[m]\{\phi\}f = -\{\phi\}^T[m]\{1\}a_g$$

by

$$\{\phi\}^T[m]\{1\}$$

the following is obtained

$$\frac{\{\phi\}^T[m]\{\phi\}}{\{\phi\}^T[m]\{1\}}\{\phi\}^T[m]\{1\}\ddot{D}_t + \frac{\{\phi\}^T[m]\{\phi\}}{\{\phi\}^T[m]\{1\}}\{\phi\}^T[m]\{1\}f = -\{\phi\}^T[m]\{1\}a_g$$

N2: step 4 – equivalent SDOF system

$$\underbrace{\frac{\{\phi\}^T[m]\{\phi\}}{\{\phi\}^T[m]\{1\}}}_{\frac{1}{\Gamma}}\underbrace{\{\phi\}^T[m]\{1\}}_{m^*}\ddot{D}_t+\underbrace{\frac{\{\phi\}^T[m]\{\phi\}}{\{\phi\}^T[m]\{1\}}}_{\frac{1}{\Gamma}}\underbrace{\{\phi\}^T[m]\{1\}}_{m^*}f=-\underbrace{\{\phi\}^T[m]\{1\}}_{m^*}a_g$$

$$m^*\frac{\ddot{D}_t}{\Gamma}+m^*\frac{f}{\Gamma}=-m^*a_g$$

$$m^*\ddot{D}^*+F^*=-m^*a_g$$

N2: step 4 – equivalent SDOF system

- ***m**** **- the equivalent mass of the SDOF system:**
  $m^*=\{\phi\}[m]\{1\}=\sum m_i \cdot \phi_i$
- ***D**** **- the displacement of the equivalent SDOF system:**
  $D^*=D_t/\Gamma$
- ***F**** **- the force of the equivalent SDOF system:**
  $F^*=V/\Gamma$
- ***V*** **– base shear force**
  $V= \sum F_i=\{\phi\}[m]\{1\}f=f\sum m_i \cdot \phi_i=f \cdot m^*$

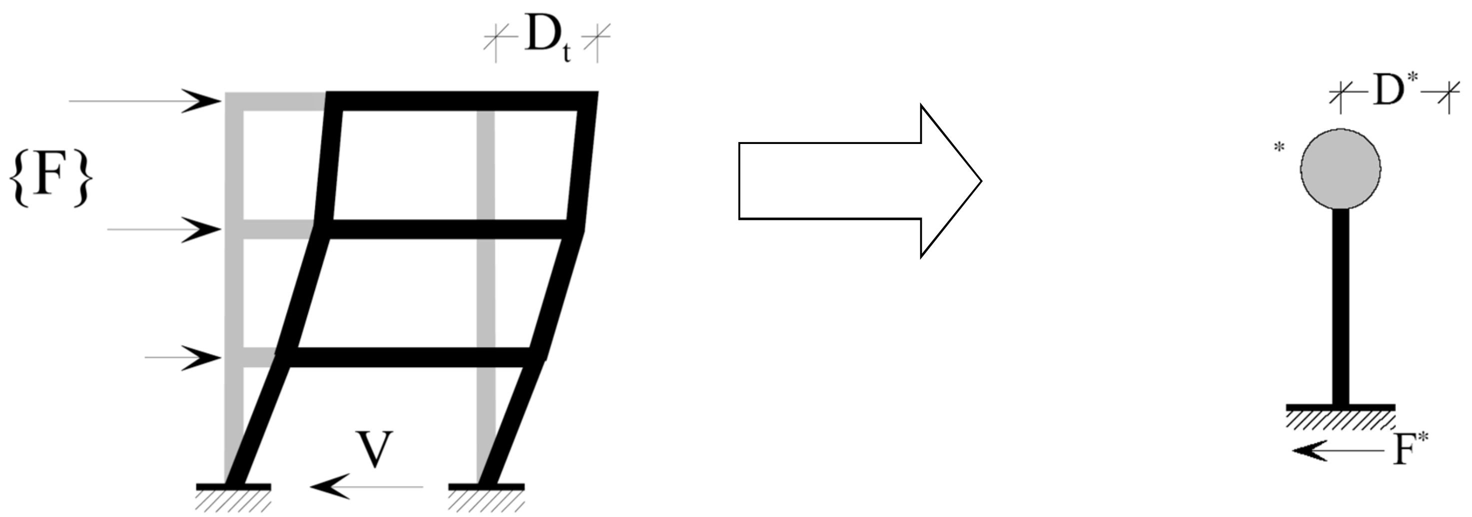
- **Γ - modal participation factor, used to transform the MGLD system into an equivalent SDOF system**

$$\Gamma = \frac{\{\phi\}^T [m]\{1\}}{\{\phi\}^T [m]\{\phi\}} = \frac{\sum m_i \phi_i}{\sum m_i \phi_i^2} = \frac{m^*}{\sum m_i \phi_i^2}$$

N2: step 4 – equivalent SDOF system

$$[m]\{\ddot{u}\}+\{R\}=-[m]\{1\}a_g$$

$$m^*\ddot{D}^*+F^*=-m^*a_g$$

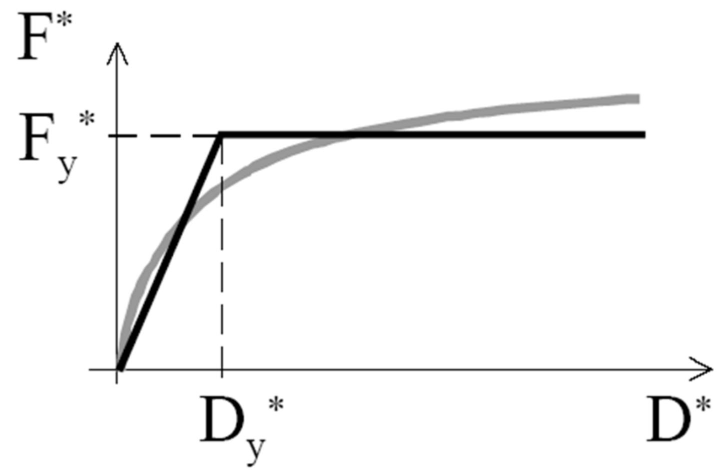


V-D$_t$

F-D**

N2: step 4 – equivalent SDOF system

- V-$D_t \Leftrightarrow F^*$-$D^* \Rightarrow$ **bilinear relationship (elastic – perfectly plastic)**
- **Equivalent bilinear** F^*-D^* **curve:**
  - **Can be obtained by equating the areas below the two curves up to the displacement at the formation of plastic mechanism (**D_m^***)**
  - **If there is a large difference between the displacement demand** D^* **(obtained through the N2 method) and displacement at which the plastic mechanism is formed (**D_m^***) it is recommended to use** D^* **at the determination of the bilinear relationship (iterative procedure)**

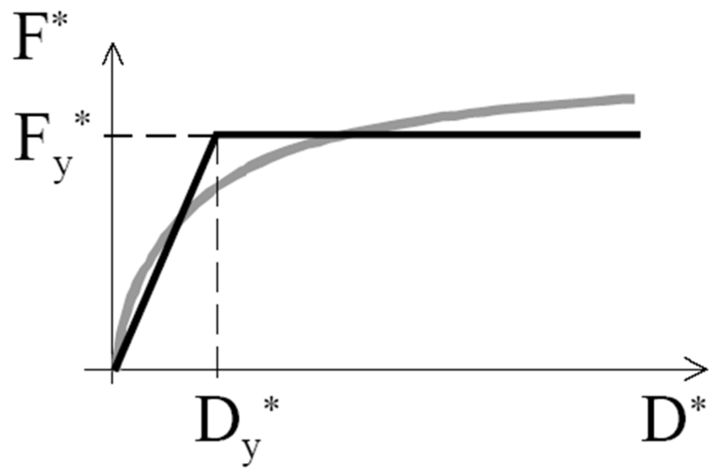


Example of application:

- $D_y^* = 2(D_m^* - E_m / F_y^*)$
- F_y^***: Force at the plastic mechanism**
- E_m**: area under the** F^*-D^* **curve corresponding to the formation of plastic mechanism**

N2: step 4 – equivalent SDOF system

- $V\text{-}D_t \Leftrightarrow F^*\text{-}D^* \Rightarrow$
  bilinear relationship
  (elastic – perfectly plastic)



- **Natural period of vibration of the SDOF system**

$$T^* = \frac{2\pi}{\omega^*} = \frac{2\pi}{\sqrt{k^*/m^*}} = 2\pi\sqrt{\frac{m^* \cdot D_y^*}{F_y^*}}$$

- $F^*\text{-}D^* \Leftrightarrow S_a\text{-}S_d$ **(capacity curve in AD format)**

$$S_a = \frac{F^*}{m^*}$$

N2: step 5 – displ. demand in the SDOF system

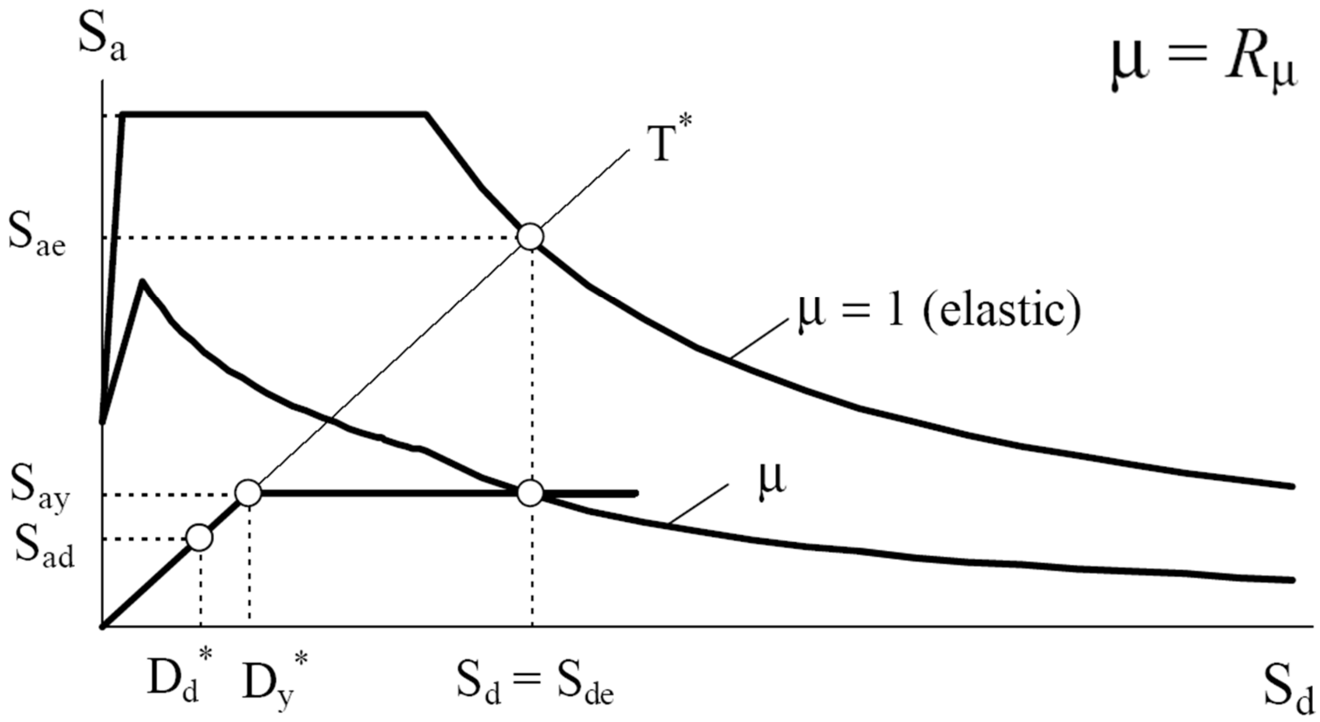
- **Reduction factor R_μ**

$$R_\mu = \frac{S_{ae}\left(T^*\right)}{S_{ay}}$$

- ***$T^* \geq T_C$* case:**

$$S_d = S_{de}(T^*) \qquad T^* \geq T_C$$

$$\mu = R_\mu$$



N2: step 5 – displ. demand in the SDOF system

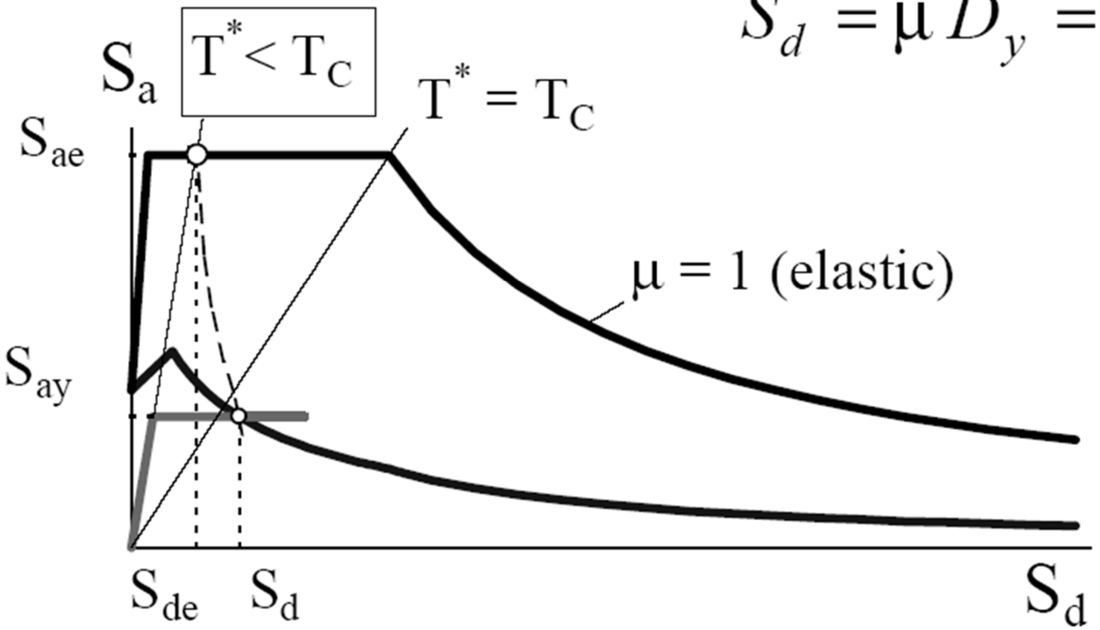
- **Reduction factor R_μ**

$$R_\mu = \frac{S_{ae}\left(T^*\right)}{S_{ay}}$$

- ***T^*<T_C* case:**

$$\mu = \left(R_\mu - 1\right)\frac{T_C}{T^*} + 1 \qquad T^* < T_C$$

$$S_d = \mu\, D_y^* = \frac{S_{de}}{R_\mu}\left(1 + \left(R_\mu - 1\right)\frac{T_C}{T^*}\right)$$



N2: step 6 – response of the MDOF system

- **Displacement demand of the SDOF system**

  $D^* = S_d$

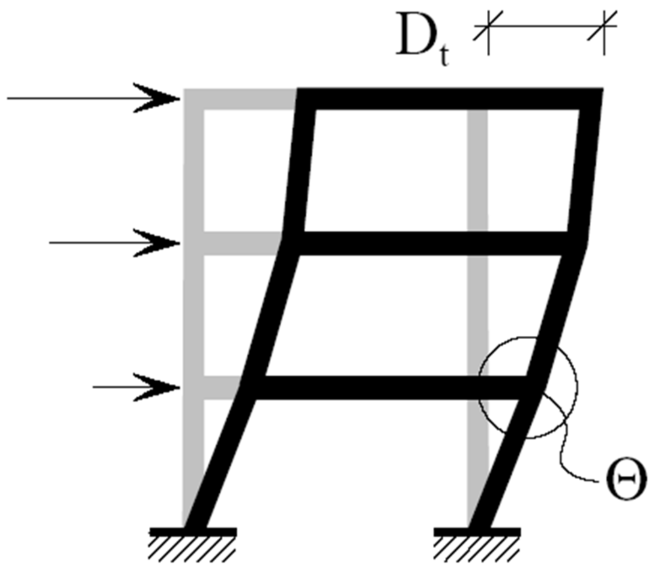
  ⇩

- **Displacement demand of the MDOF system**

  $D_t = \Gamma \cdot D^*$

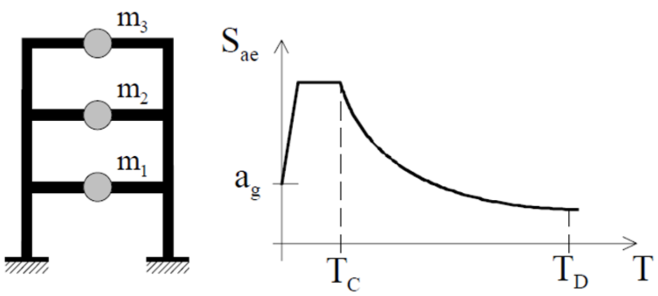
  ⇩

- **Evaluation of performance of the structure at the target displacement** D_t

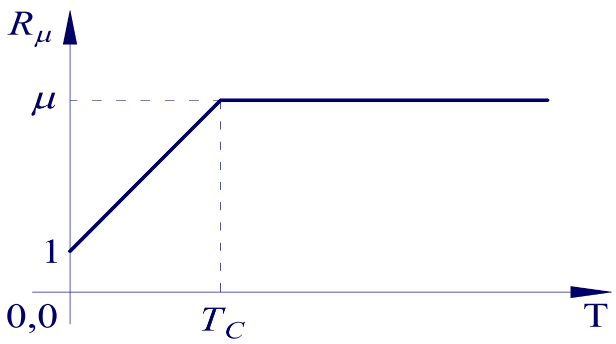


N2: performance evaluation procedure

1. **Initial data**
   - **Properties of the structure**
   - **Elastic pseudo-acceleration response spectrum S_{ae}**

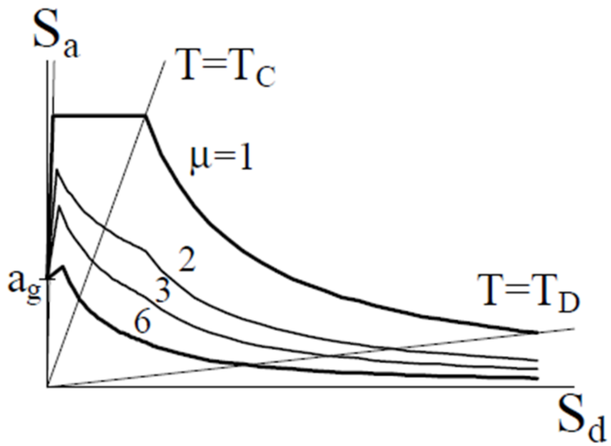


2. **Determination of spectra in AD format for constant values of ductility, e.g. μ=1, 2, 4, 6, etc. (only if graphical representation of the method is needed)**



$$R_\mu = (\mu - 1)\frac{T}{T_C} + 1 \qquad T < T_C$$

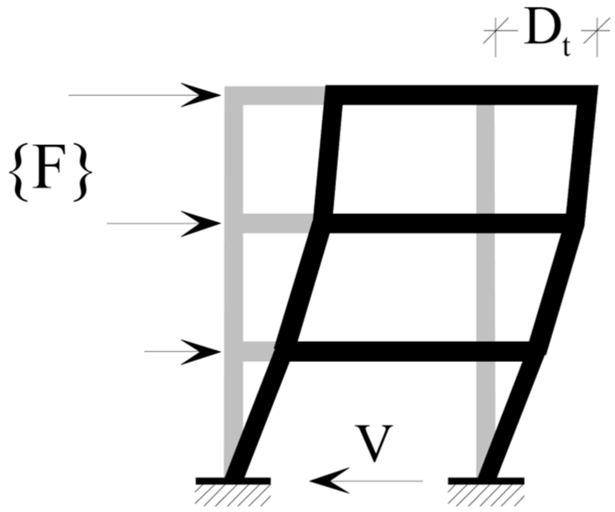
$$R_\mu = \mu \qquad T \geq T_C$$

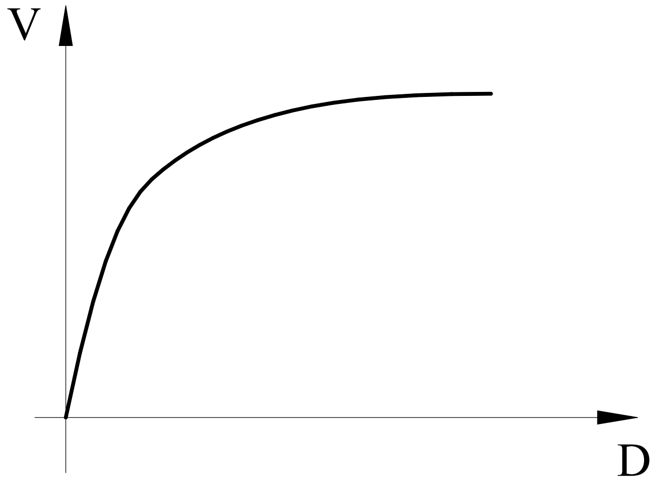


N2: performance evaluation procedure

3. Nonlinear static analysis

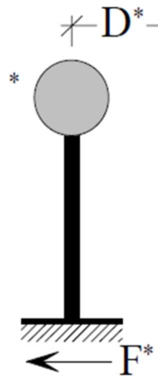
- **Assume displacement shape {ϕ}**
  Note: normalized in such a way that the component at the top is equal to 1
- **Determine vertical distribution of lateral forces**
  $F_i=[m]\{\phi\}=m_i \cdot \phi_i$
- **Determine base shear (V)–top displacement (Dt) relationship by performing the nonlinear static analysis**





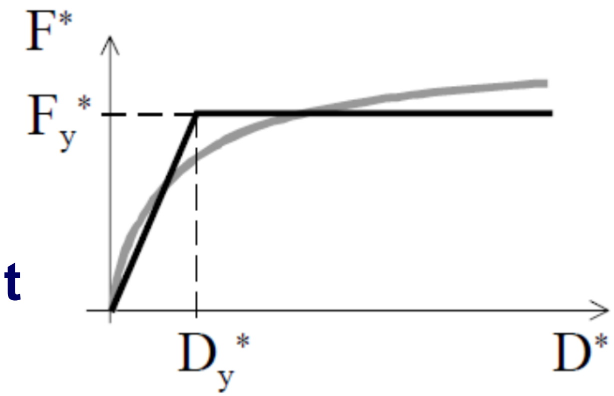
N2: performance evaluation procedure

4. Equivalent SDOF system



- **Determine mass *m****
  $m^* = \sum m_i \cdot \phi_i$
- **Transform MDOF quantities (Q) to SDOF quantities (Q*)**
  $Q^* = Q / \Gamma$

$$\Gamma = \frac{m^*}{\sum m_i \phi_i^2}$$



- **Determine an approximate elasto-plastic force – displacement relationship *F*-D****
- **Determine strength F_y*, yield displacement D_y*, and period *T****

$$T^* = 2\pi \sqrt{\frac{m^* \cdot D_y^*}{F_y^*}}$$

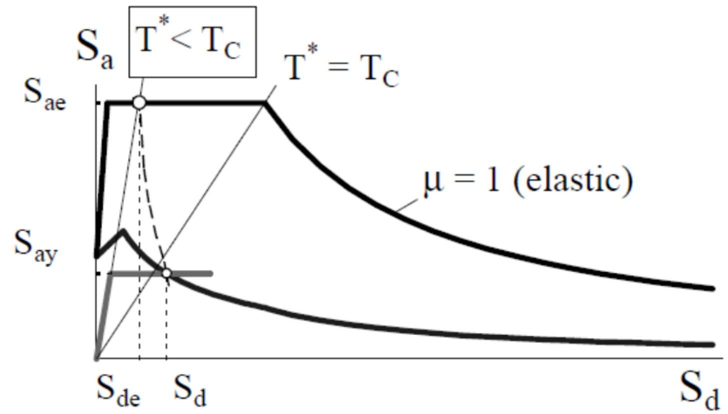
- **Determine capacity diagram S_a-S_d (only if graphical representation of the method is needed)**

$$S_a = \frac{F^*}{m^*} \qquad S_d = D^*$$

N2: performance evaluation procedure

5. **Seismic demand for SDOF system**
   - **Determine reduction factor R_μ**

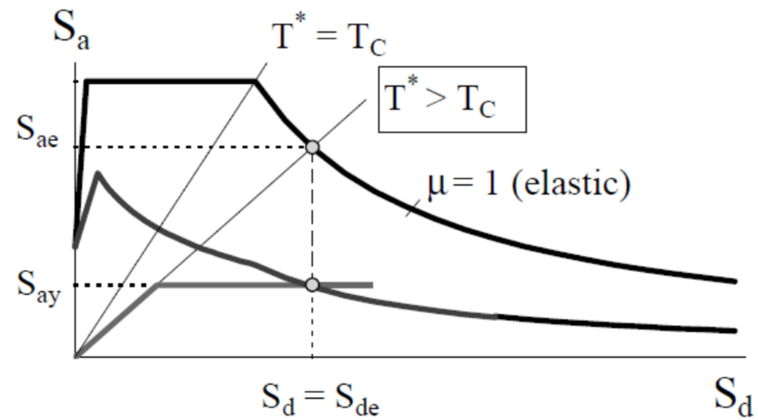
$$R_\mu = \frac{S_{ae}\left(T^*\right)}{S_{ay}}$$



   - **Determine displacement demand** $S_d = D^*$

$$S_d = \frac{S_{de}}{R_\mu}\left(1 + \left(R_\mu - 1\right)\frac{T_C}{T^*}\right) \qquad T^* < T_C$$

$$S_d = S_{de} \qquad T^* \geq T_C$$



Nonlinear static analysis: torsional effects

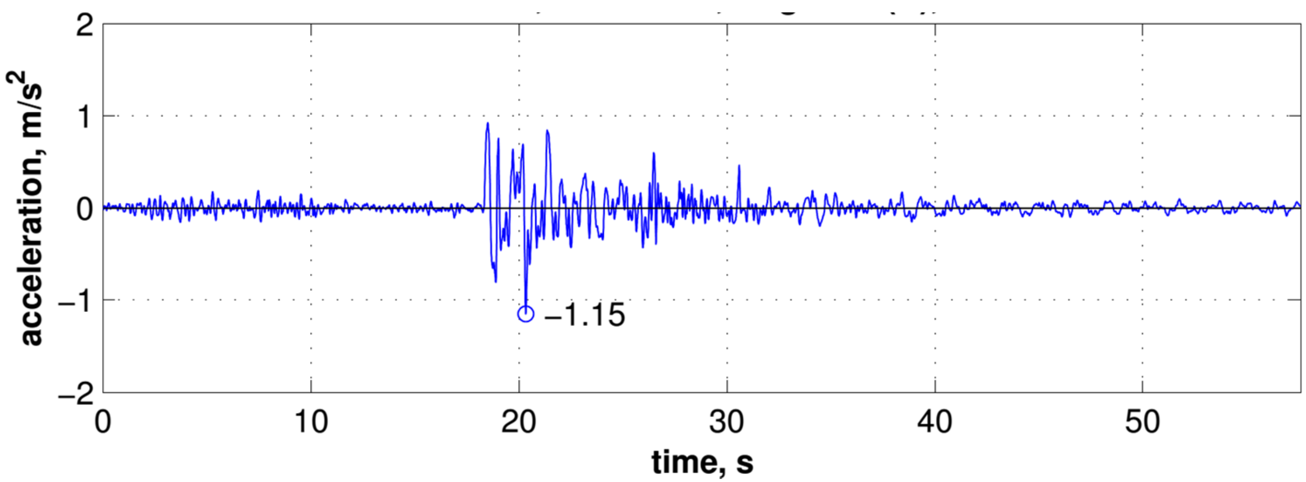
- **Buildings <u>irregular in plan</u> shall be analysed using a <u>spatial model</u>. Two independent analyses with lateral loads applied in one direction only may be performed.**
- **For spatial models(3D), torsional effects may be accounted for by amplifying the displacements of the stiff/strong side based on the results of an elastic modal analysis of the spatial model.**

- **For buildings <u>regular in plan</u> can be analysed using two <u>planar models</u>, one for each main horizontal direction.**
- **For planar models (2D): the accidental torsional effects may be accounted for by amplifying the target displacement resulting from analysis by a factor δ**

$$\delta = 1 + 1.2\frac{x}{L_e}$$

Nonlinear dynamic analysis

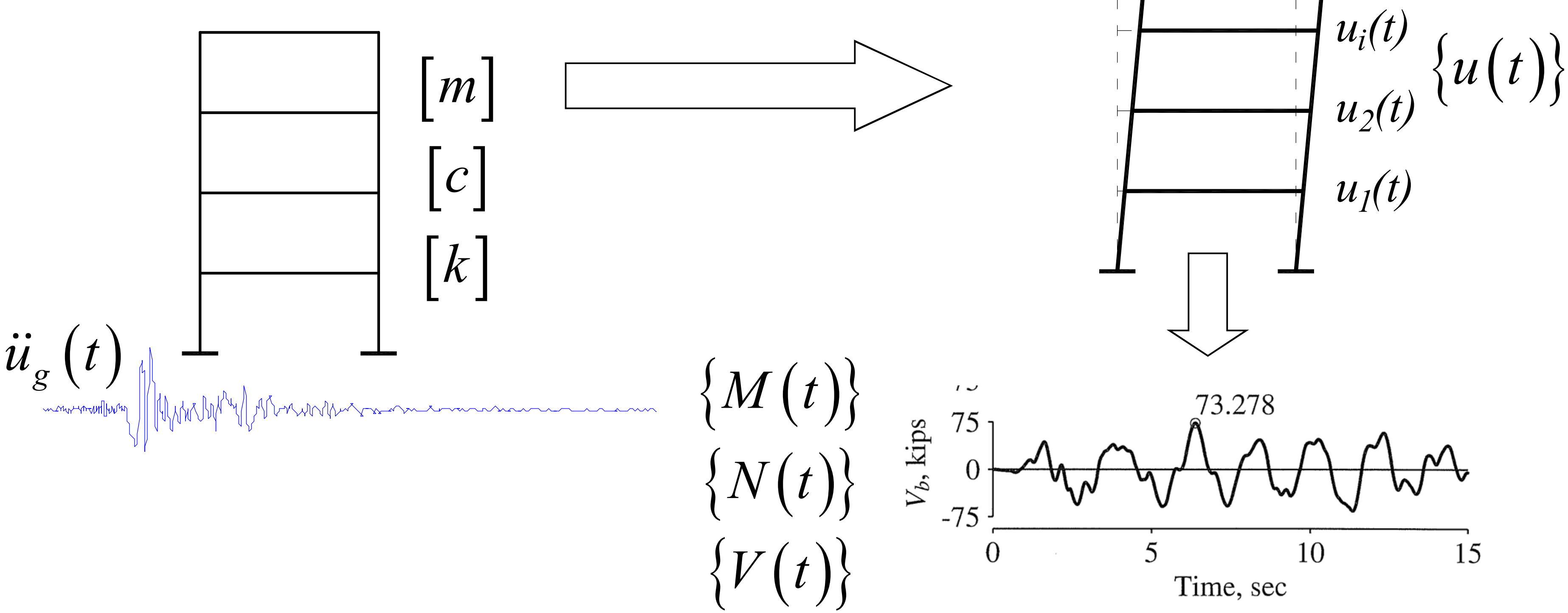
- **Nonlinear response at the material, cross-section and member levels**
- **Geometrical nonlinearity:**
  - **First order analysis: equilibrium is formulated on undeformed structure**
  - **Second order analysis: equilibrium is formulated on deformed structure**
- **Modelling of seismic action: accelerograms digitized at time steps of 0.005 – 0.02 sec**



Nonlinear dynamic analysis

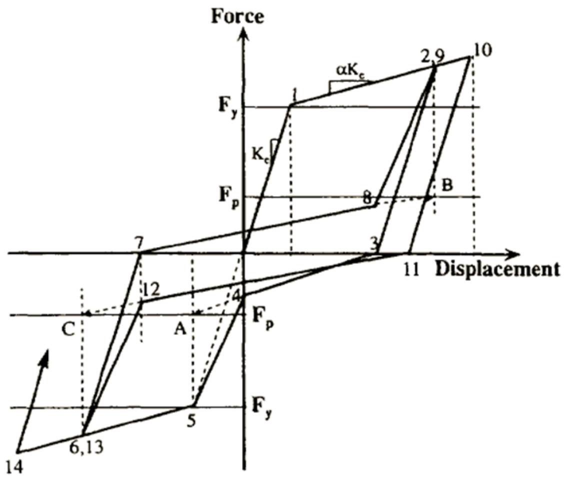
- **Time history response is obtained through direct integration (numerical methods) of the equations of motion**
- **For a system with *N* degrees of freedom, there are *N* coupled differential equations to be solved numerically**

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+\{f_s(\mathrm{u})\}=-[m]\{1\}\ddot{u}_g(t)$$



Nonlinear dynamic analysis

- Superposition of effects CANNOT be applied
- Structural model should include cyclic response of members, and, eventually strength and stiffness degradation
- The principal aim of nonlinear response history analysis is to determine if the computed deformations of the structure are within appropriate limits. Strength requirements for the designated lateral load-resisting elements do not apply because element strengths are established prior to the analysis.



Nonlinear dynamic analysis

- Nonlinear dynamic analysis is not used as part of the normal design process for typical structures. In some cases, however, nonlinear analysis is recommended, and in certain cases required, to obtain a more realistic assessment of structural response and verify the results of simpler methods of analysis.
- Such is the case for systems with highly irregular force-deformation relationships.
- The principal aim of nonlinear response history analysis is to determine if the computed deformations of the structure are within appropriate limits. Strength requirements for the designated lateral load-resisting elements do not apply because element strengths are established prior to the analysis. These initial strengths typically are determined from a preliminary design using linear analysis.

Nonlinear dynamic analysis

- **Advantages:**
  - The most "realistic" modelling of a structure under seismic action
  - Direct assessment of seismic performance at member and structure levels
  - Ability to model a wide variety of nonlinear material behaviors, geometric nonlinearities (including large displacement effects), gap opening and contact behavior, and nonclassical damping, and to identify the likely spatial and temporal distributions of inelasticity
- **Disadvantages:**
  - Increased effort to develop the analytical model
  - Modelling of the structure requires specialised knowledge
  - Analysis is resource-intensives
  - Large amount of results $\Rightarrow$ a time-consuming post-processing of results
  - Sensitivity of computed response to system parameter

Nonlinear dynamic analysis

- Allows evaluation of inealstic deformations – the most relevant response quantity in the case of inelastic response

- "Local" checks:
  - Interstorey drifts
  - Strength demands in non-dissipative components
  - Ductility of dissipative components

- "Global" checks – failure at the structure level
  - Failure to resist further gravity loading
  - Failure of the first vertical element essential for stability of the structure

Nonlinear dynamic analysis: modelling of damping

- In the context of the nonlinear dynamic procedure, equivalent viscous damping is associated with the reduction in vibrations through energy dissipation other than that which is calculated directly by the nonlinear hysteresis in the modeled elements.
- This so-called inherent damping occurs principally in
  - structural components that are treated as elastic but where small inelastic cracking or yielding occurs,
  - the architectural cladding, partitions, and finishes, and
  - the foundation and soil (if these are not modeled otherwise).
- Special energy dissipation components (e.g., viscous, friction, or hysteretic devices) should be modeled explicitly in the analysis, rather than as inherent damping.
- The inherent damping is usually modelled using Rayleigh damping $[c] = a_0 [m] + a_1 [k]$

Nonlinear dynamic analysis: modelling of damping

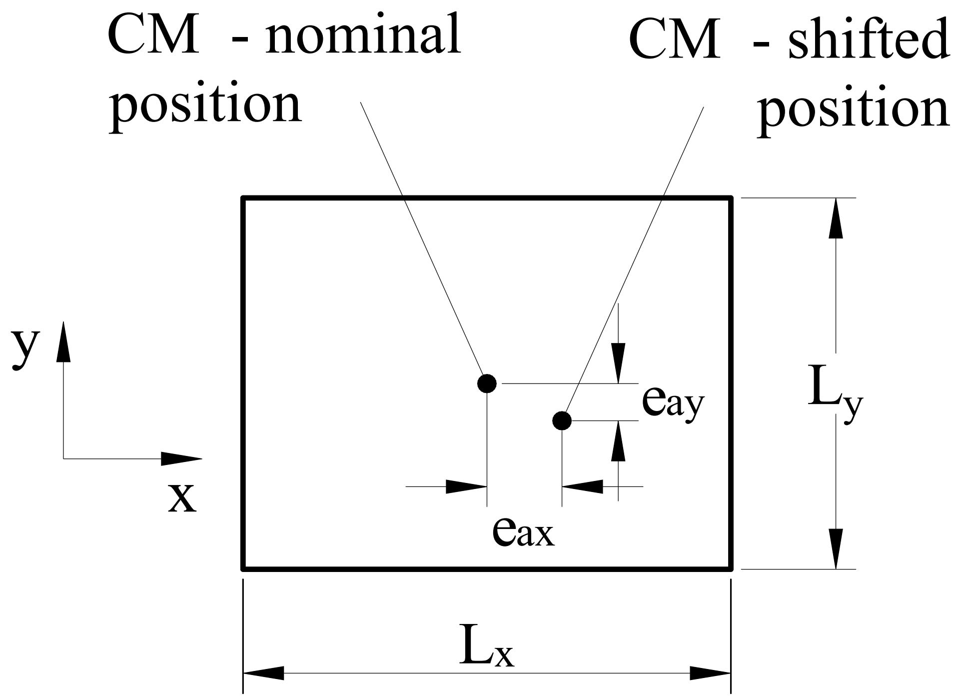
- **It is suggested to specify equivalent viscous damping in the range of 1% to 5% of critical damping over the range of elastic periods from 0.2T to 1.5T (where T is the fundamental period of vibration).**
- **If the damping matrix is based on the initial stiffness of the system, artificial damping may be generated by system yielding. In some cases, the artificial damping can completely skew the computed response.**
- **One method to counter this occurrence is to base the damping matrix on the mass and the instantaneous tangent stiffness.**

Modelling of mass and seismic action

- **Modelling of mass distribution → see linear dynamic analysis**
- **Selection, scaling and number of accelerograms → see linear dynamic analysis**
- **Components of seismic action → see linear dynamic analysis**

Linear dynamic analysis: accidental eccentricity

- **For spatial models (3D): the accidental torsional effects accounted for by shifting the centre of mass from its nominal location with the value of the eccentricity in each of the two horizontal directions**
- **Accidental eccentricity $e_{ai} = \pm 0.05\, L_i$ (EN 1998-1)**

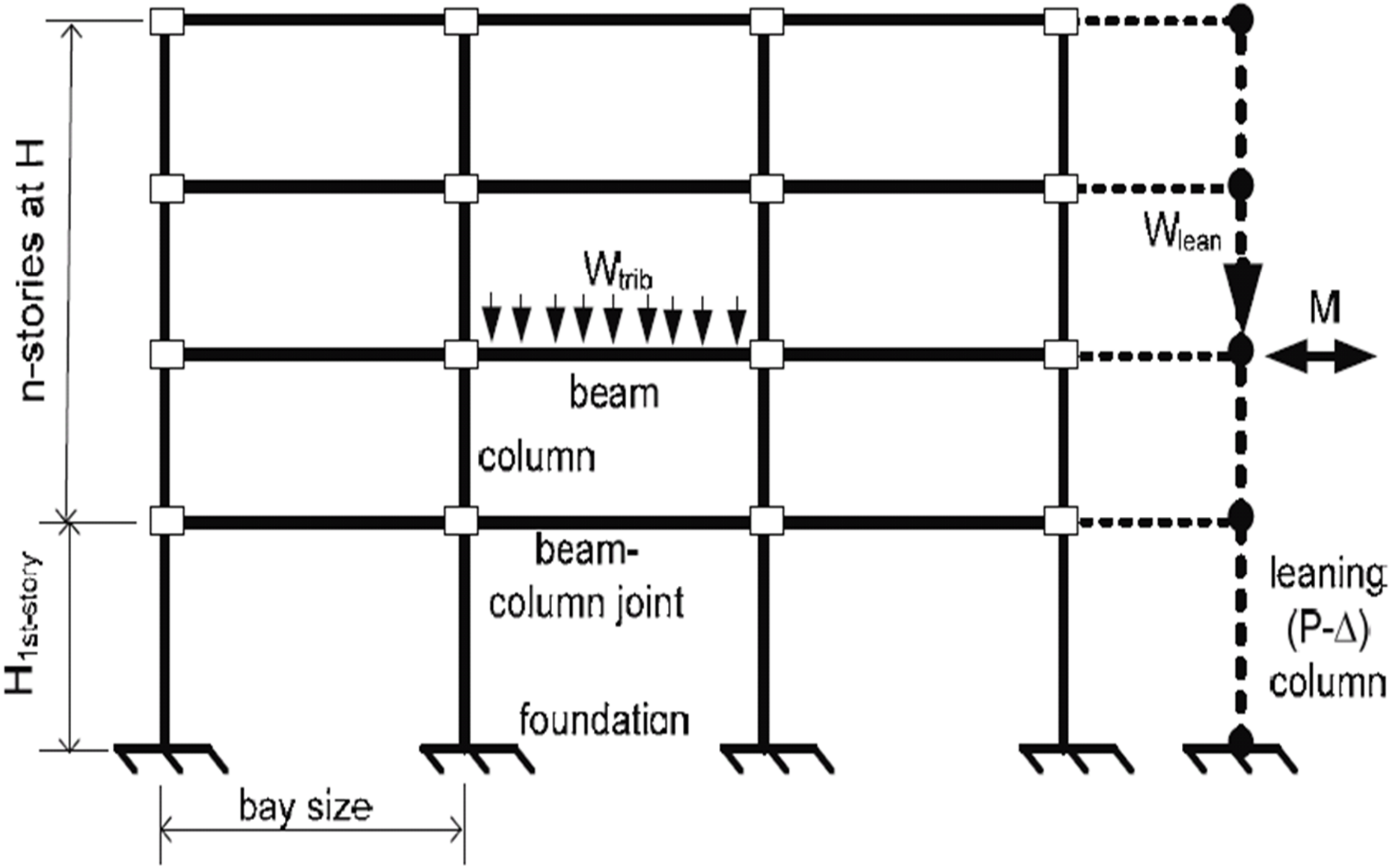


Modelling of 2nd order effects in planar models

- **Most structures are composed of both gravity and lateral force resisting systems**
- **Gravity actions applied on gravity frames produce second order effects which are resisted by lateral force resisting frames**
- **If a plan model of the structure is adopted, second order effects produced by gravity frames should be applied to the lateral force resisting ones**

Modelling of 2nd order effects in planar models

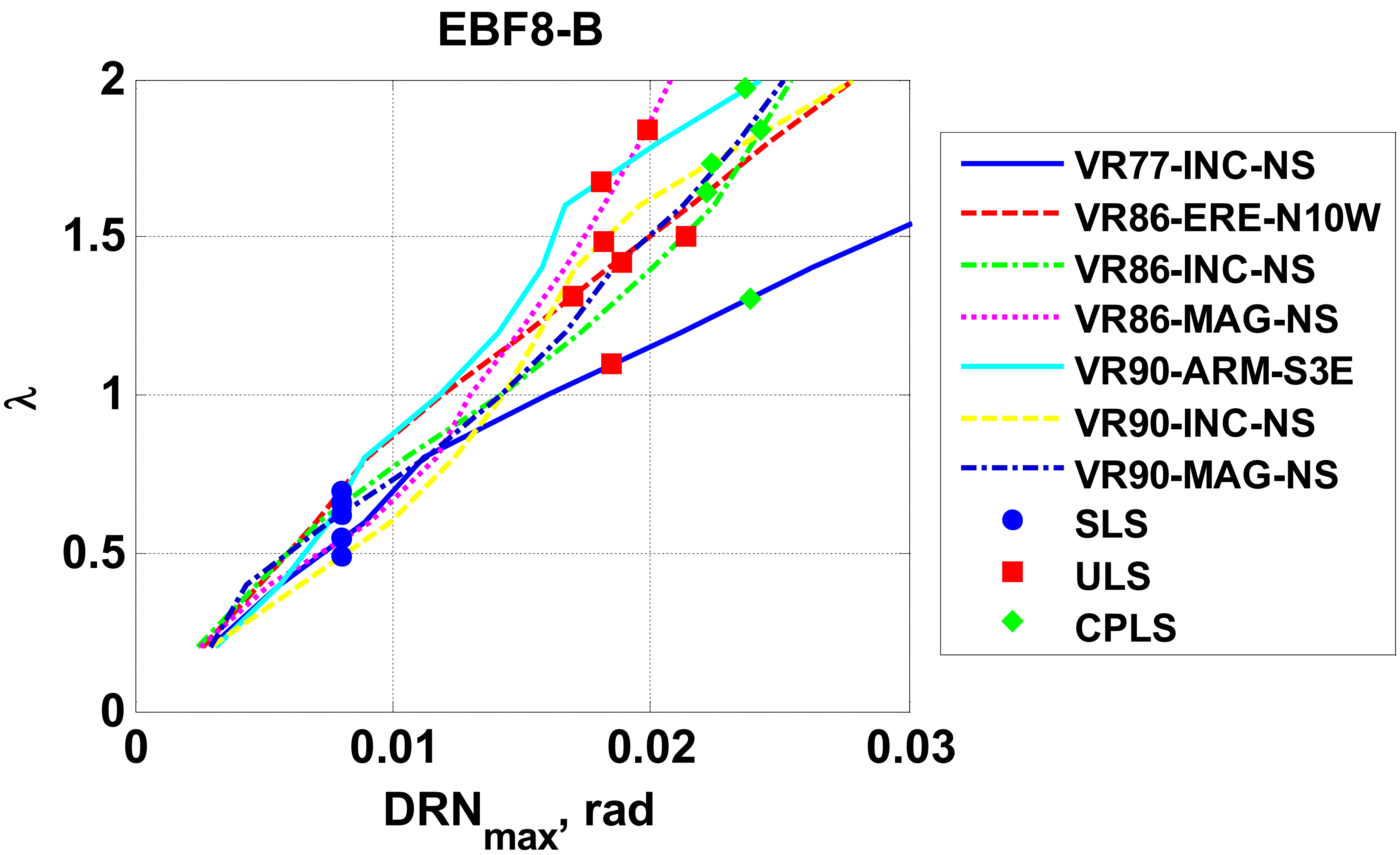
- **Example of considering gravity forces on gravity frames ("leaning column – *P-Δ*") in second-order analysis of lateral force resisting frames**



Incremental Dynamic Analysis (IDA)

- Incremental Dynamic Analysis (IDA) is a parametric analysis method that involves subjecting a structural model to one (or more) ground motion records, each scaled to multiple levels of intensity, thus producing one (or more) curves of response parameterized versus intensity level. It allows:
  - thorough understanding of the range of response or ”demands” versus the range of potential levels of a ground motion record
  - better understanding of the structural implications of rarer / more severe ground motion levels
  - better understanding of the changes in the nature of the structural response as the intensity of ground motion increases
  - producing estimates of the dynamic capacity of the global structural system
  - finally, given a multi-record IDA study, how stable (or variable) all these items are from one ground motion record to another.

Incremental Dynamic Analysis (IDA)



References / additional reading

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- Gregory G. Deierlein, Andrei M. Reinhorn, Michael R. Willford (2010). "Nonlinear Structural Analysis for Seismic Design. A Guide for Practicing Engineers". EHRP Seismic Design Technical Brief No. 4. NIST GCR 10-917-5.
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