L5: TORSION OF MEMBERS

Viorel Ungureneanu

Lecture: 19/02/2014

European Erasmus Mundus Master Course

Sustainable Constructions
under Natural Hazards and Catastrophic Events

520121-1-2011-1-CZ-ERA MUNDUS-EMMC

Universitatea Politehnica Timișoara

Theoretical background

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On the other hand, some of the **instability phenomena** that may occur in steel members (particularly lateral-torsional buckling of beams and flexural-torsional buckling of columns) depend on the behaviour in torsion.

TORSION FUNDAMENTALS

The shear center is the point through which the applied loads must pass to produce bending without twisting.

If a cross-section has a line of symmetry, the shear center will always lie on that line. For cross-sections with two lines of symmetry, the shear center is at the intersection of those lines (as is the centroid).

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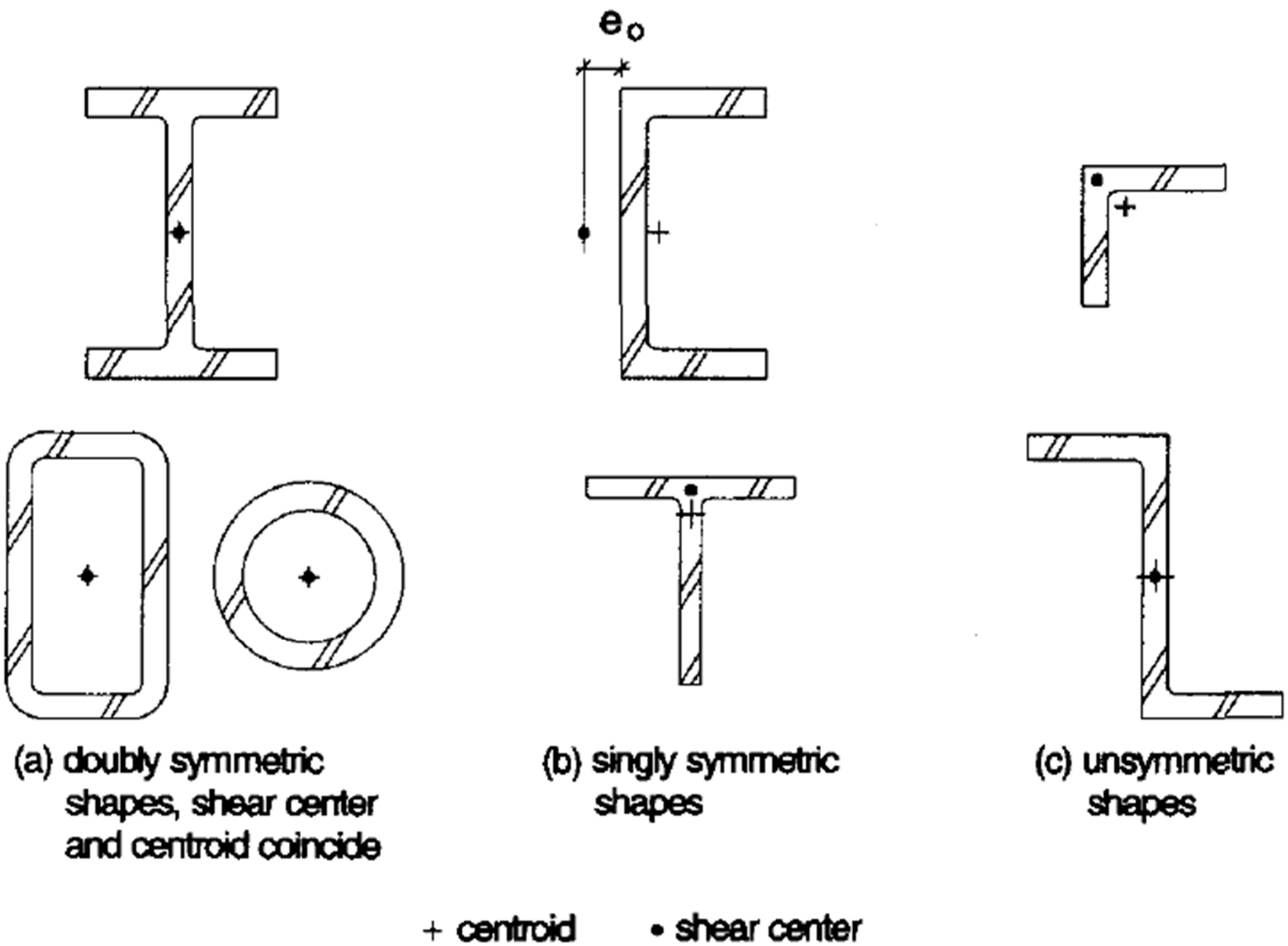

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When a member is subjected to a torsional moment *T*, the cross-sections rotate around the longitudinal axis of the member (<u>axis that is defined by the shear centre of the cross-sections</u>) and warp, i.e. they undergo differential longitudinal displacements, and plane sections no longer remain plane.

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If the torsional moment is variable or warping is restrained at any cross section (usually at the supports), the member is under ***non-uniform torsion.***

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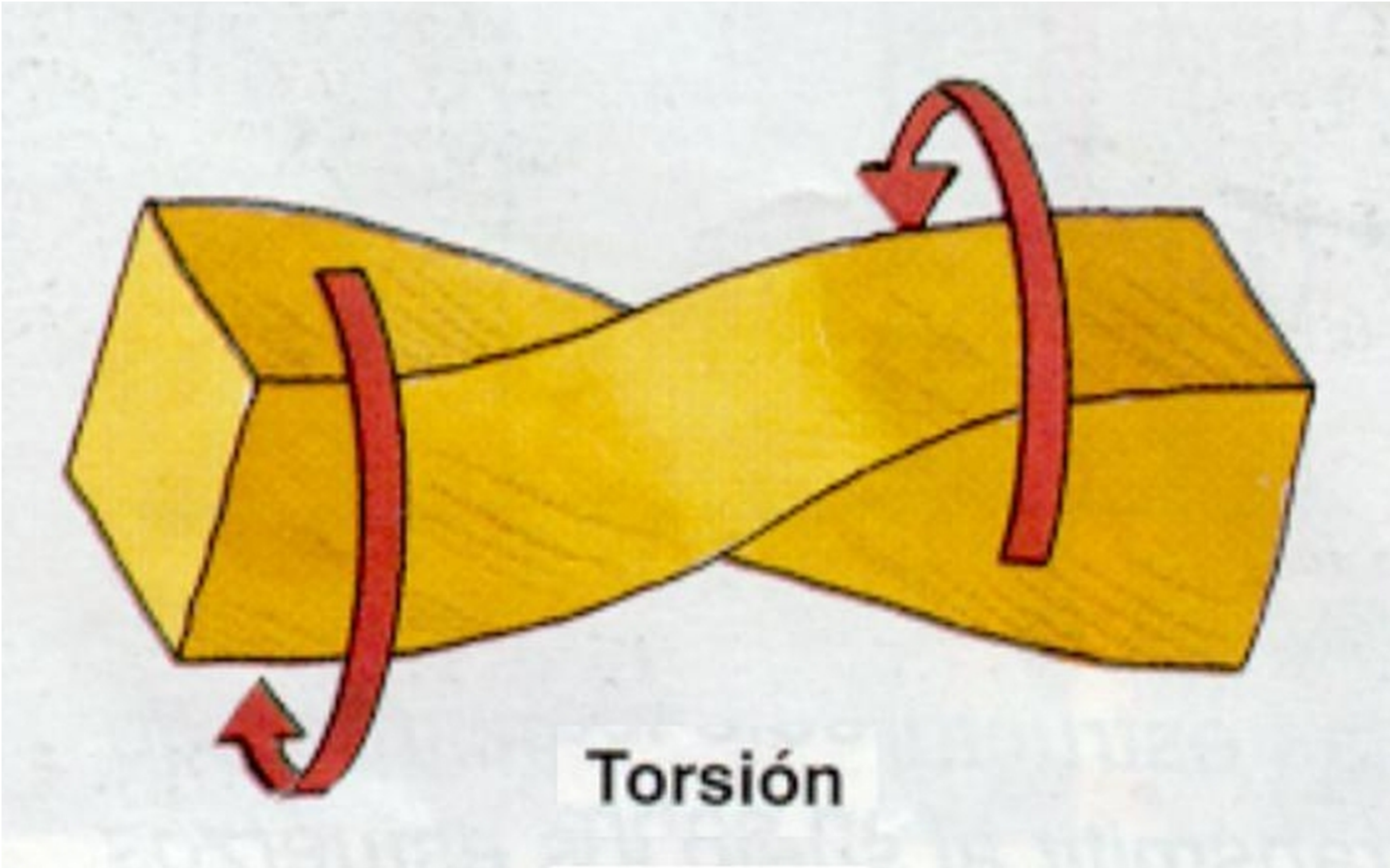

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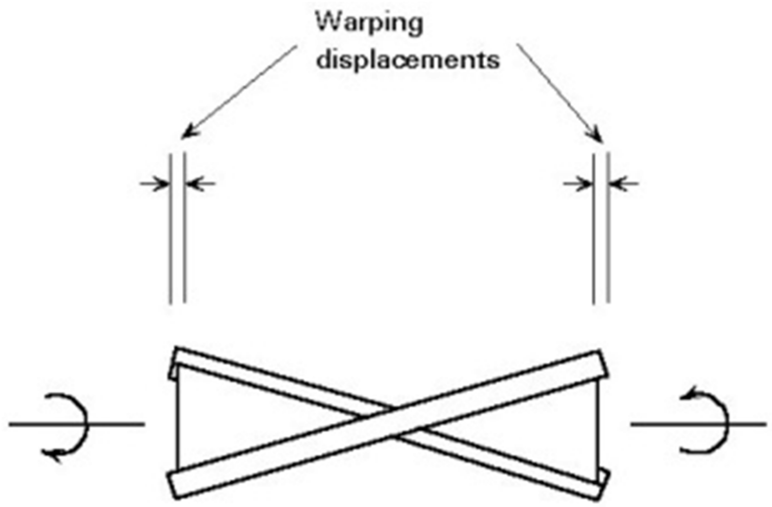

(a) Torsion with unrestrained warping

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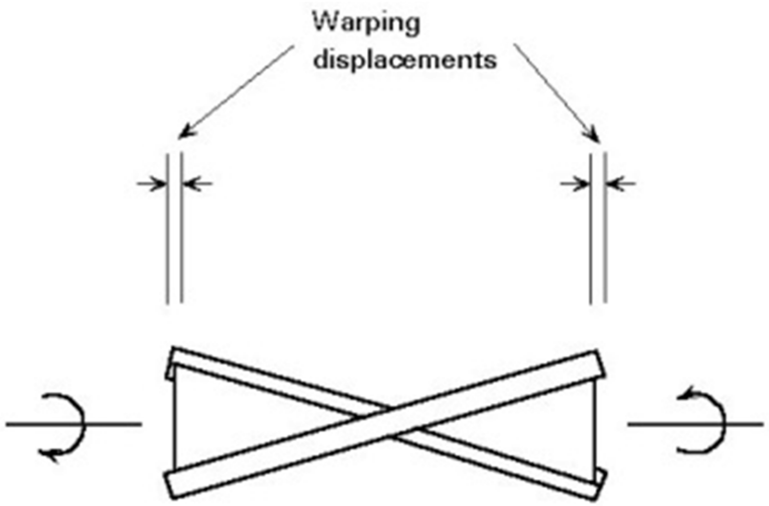
⇒ shear stresses appear to balance the applied torsional moment *T*;



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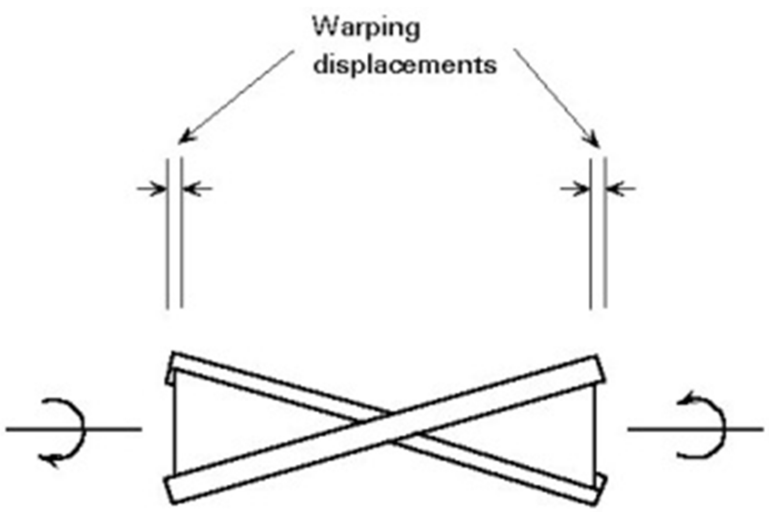

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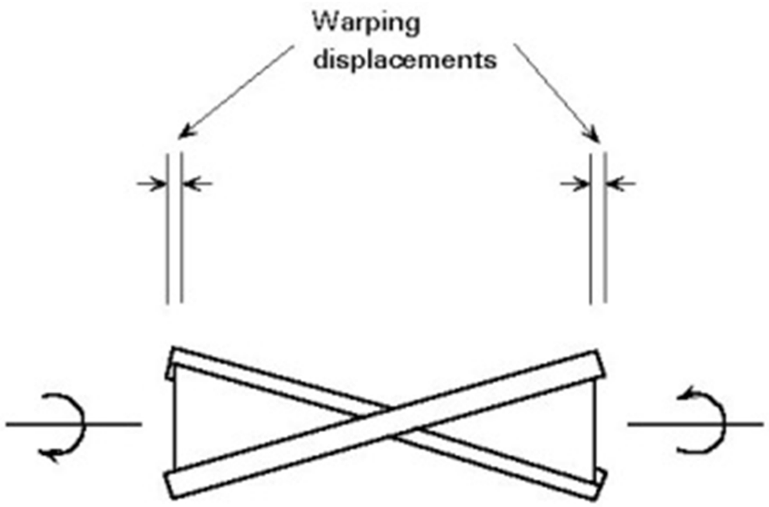

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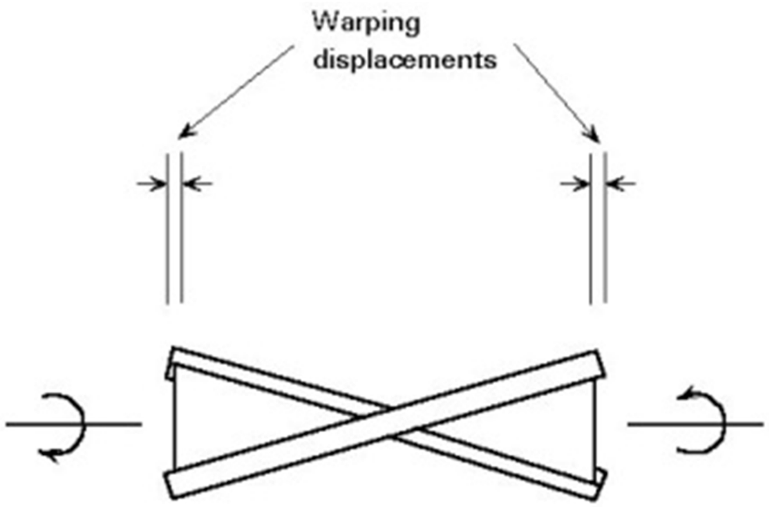
St. Venant Torsion

$$\boxed{T = T_t}$$



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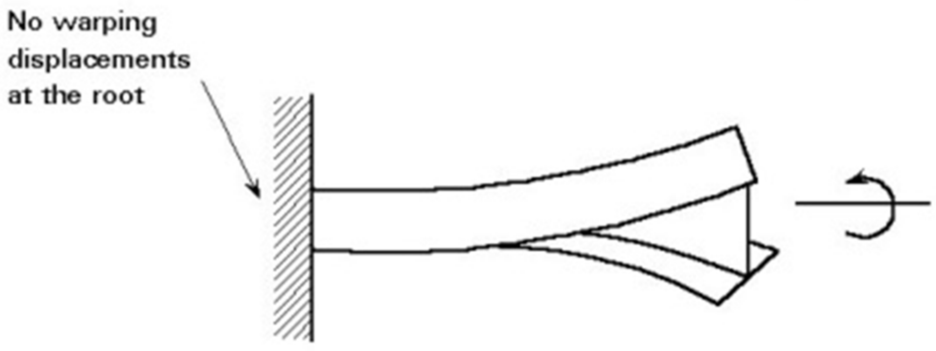

(b) Torsion with restrained warping

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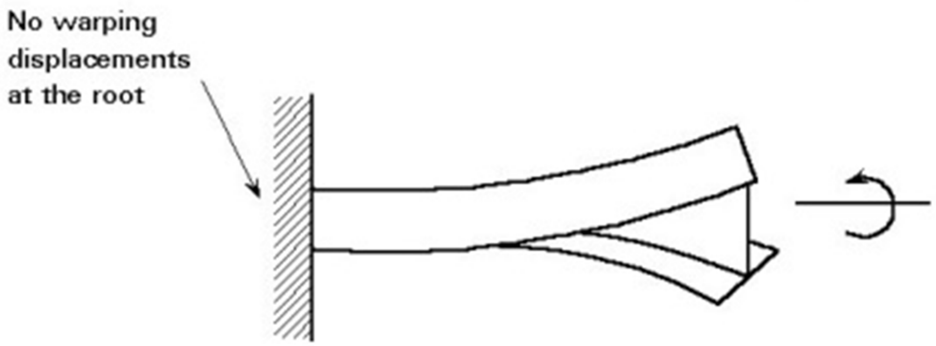
These longitudinal strains generate self-equilibrating normal stresses at the cross-sectional level that, depending on the level of restriction to warping, vary along the member.



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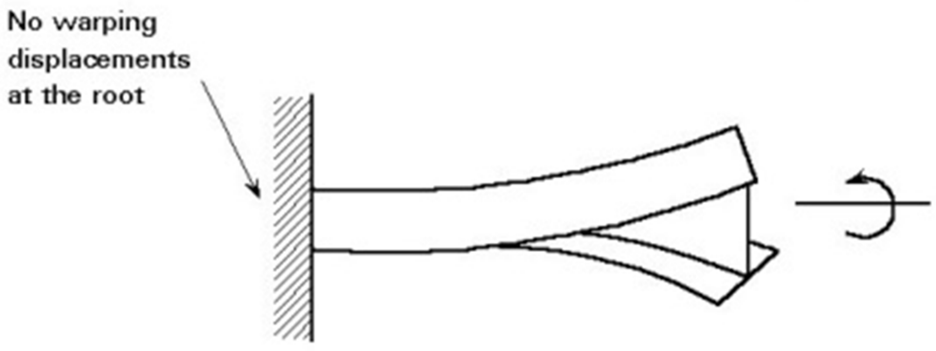
The existence of varying normal stresses implies (by equilibrium in the longitudinal direction) the existence of additional shear stresses that also resist to torsional moments, leading to:



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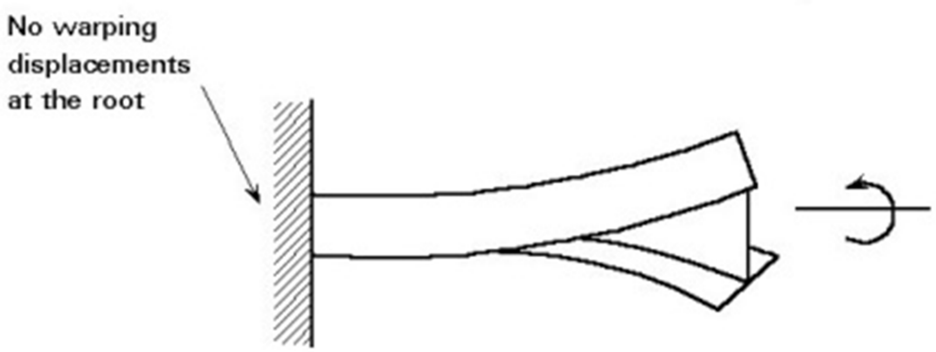
$$\boxed{T = T_t + T_w}$$



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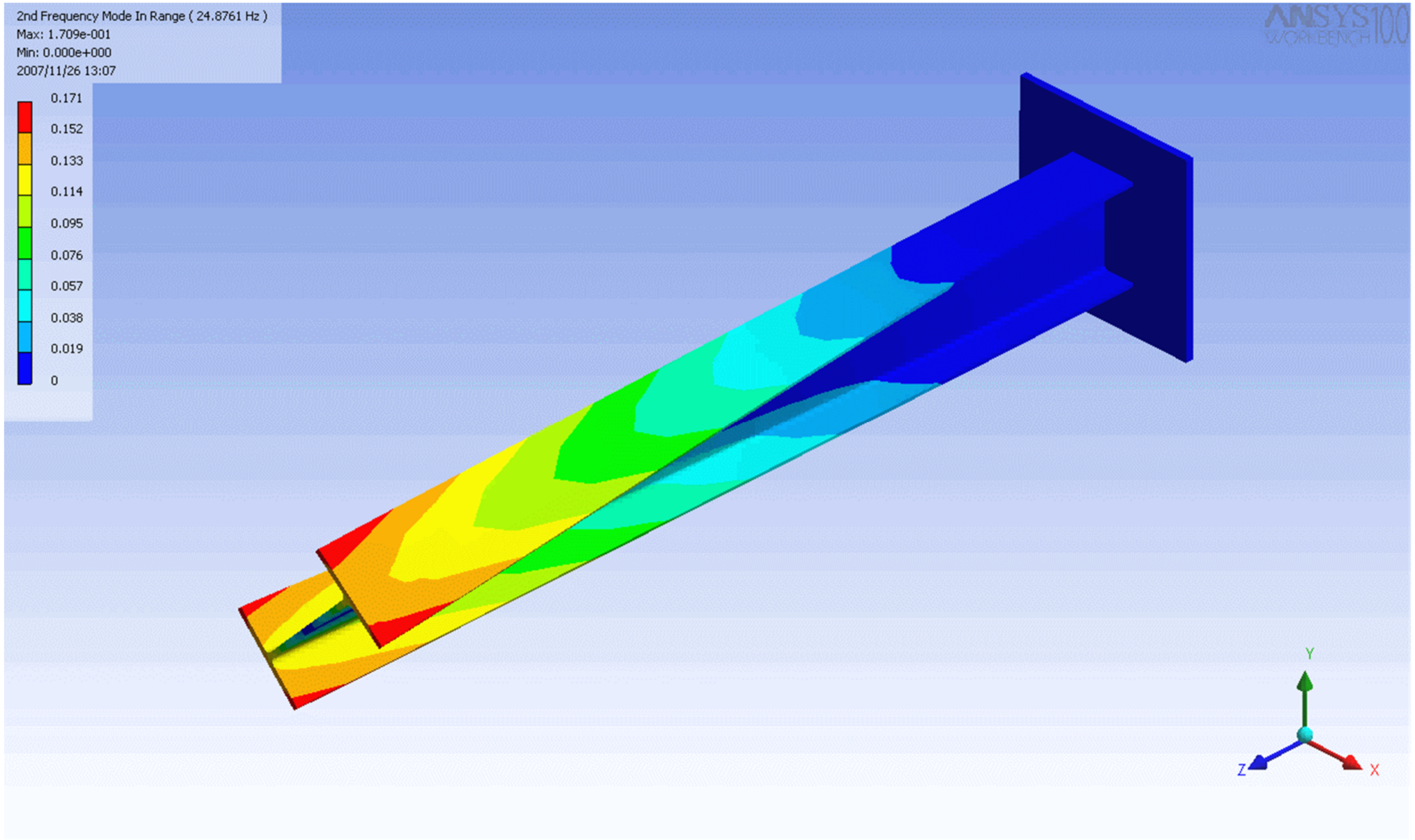
The applied torsional moment T is thus balanced by two terms:
-one due to the torsional rotation of the cross-section (T_t) and;
-the other caused by the restraint to warping, designated by warping torsion (T_w).



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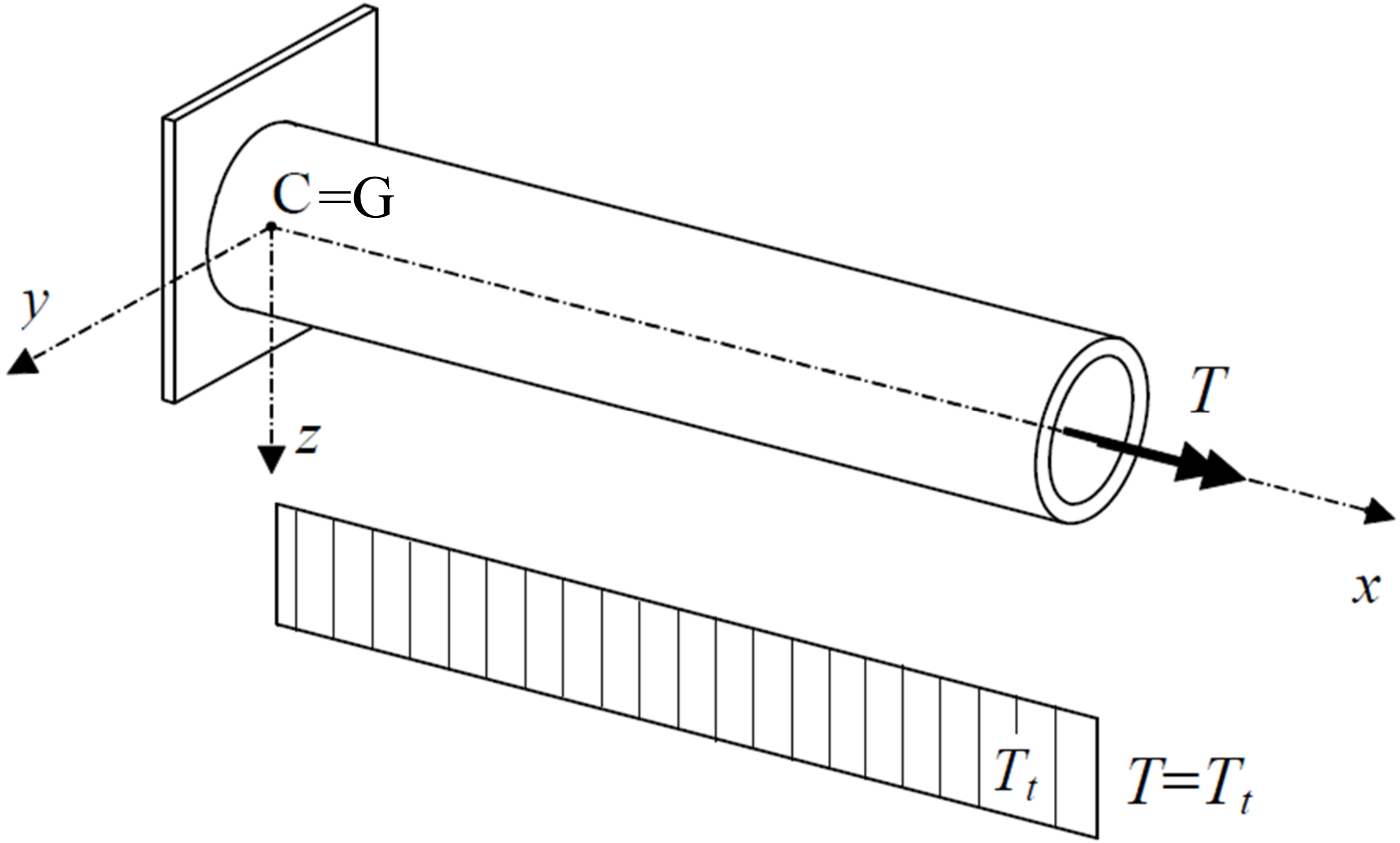

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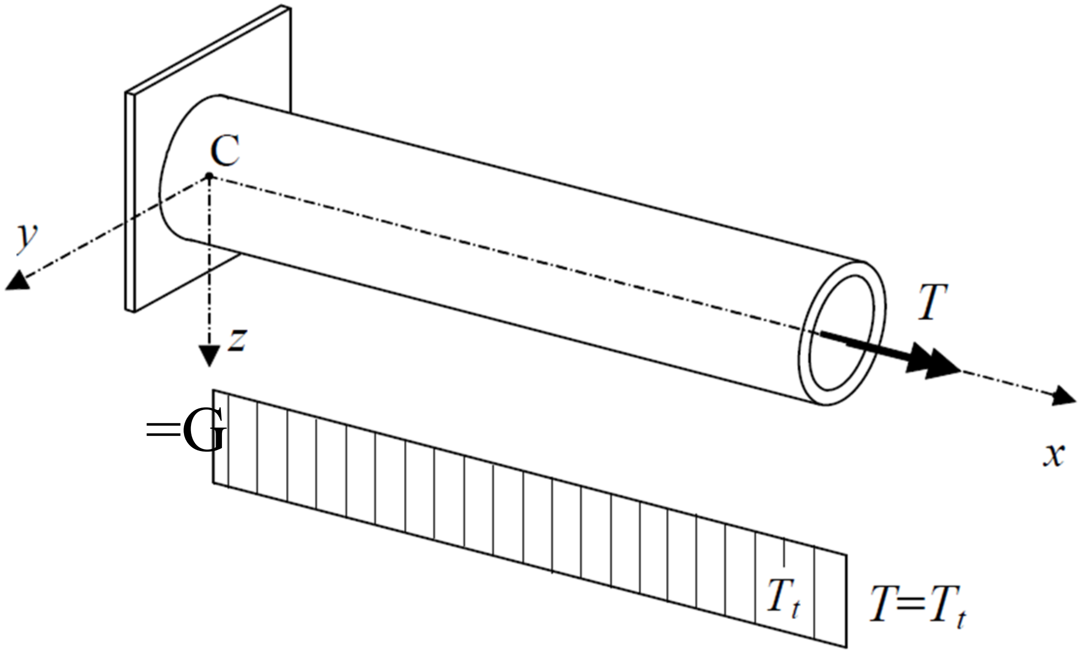

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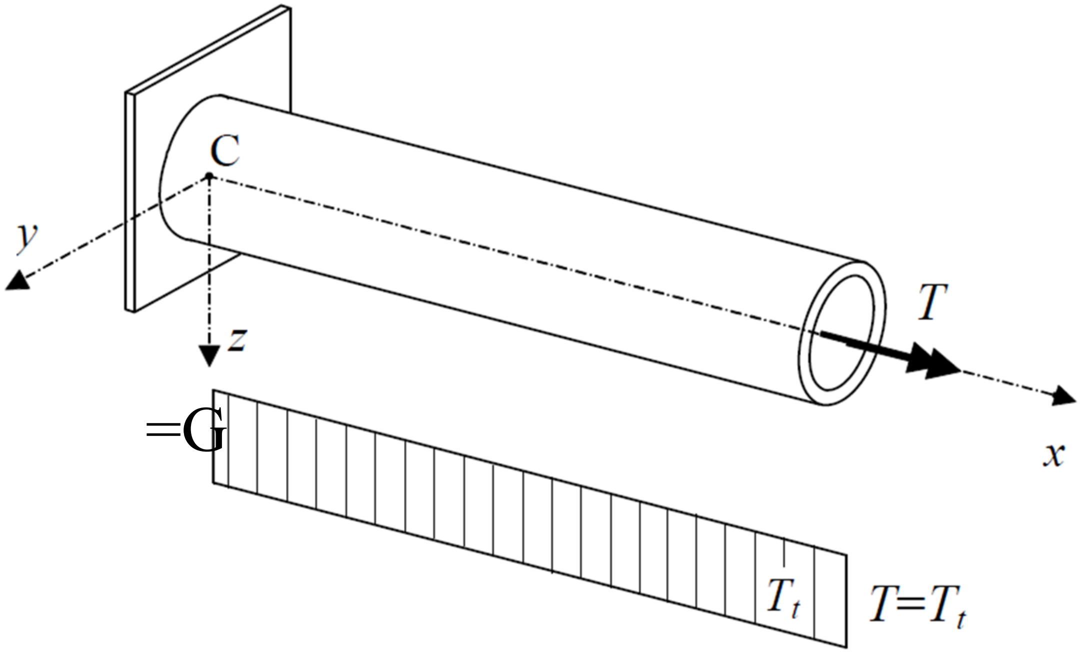

In thin-walled closed cross-sections (<u>the most appropriate to resist torsion</u>), uniform torsion is predominant.

⇒Therefore, in the analysis of thin-walled closed cross-sections subjected to torsion, the warping torsion (T_w) is normally neglected.

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In case of members with thin-walled open cross-sections (such as I or H sections), where only the uniform torsion component appears, it is necessary that the supports do not prevent warping and than the torsional moment is constant.

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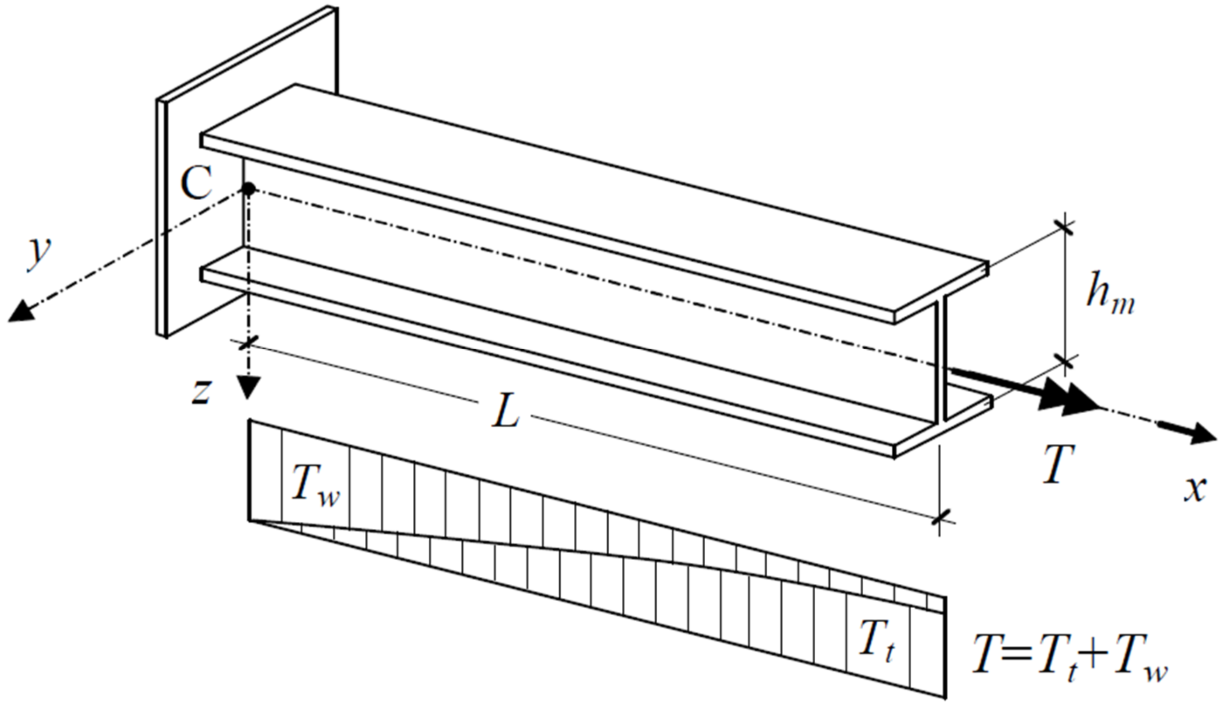

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•the warping moment is developed only if warping deformation is restrained.

Theoretical background

The figure compares section properties for four different shapes of equal area.

Section type	Flat	H-Sections (Typical)	I-Sections (Typical)	Hollow sections (Typical)
Section properties				
A Area	1	1	1	1
I_y (Vertical loading)	1	0,35	1	0,2
I_z (Horizontal loading)	0,2	3,5	1	3,5
I_t (Twisting)	1	1	1	100

Types of cross-section used as beams showing relative values of section properties

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The figure shows the high torsional stiffness in case of closed cross-sections.

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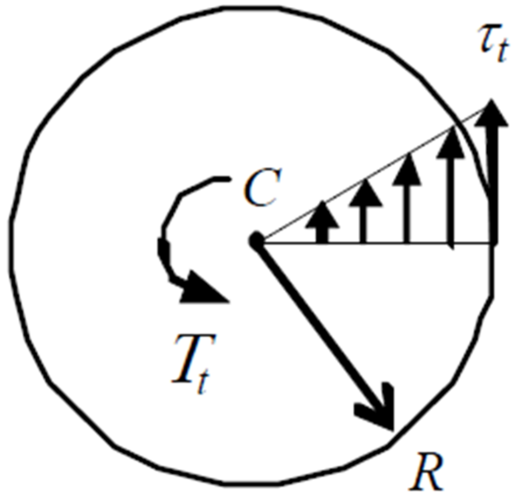
where
T_t = is the torsional moment;
I_t = is the torsion constant;
G = is the shear modulus;
φ = is the twist of the section;
x = is a variable with the direction of the longitudinal axis of the member.

Uniform torsion

The shear stresses due to uniform torsion are obtained according to different methodologies (some are exact and others approximate), depending on the shape of the cross-section.

•for <u>cross-sections with circular shape</u>, the shear stresses vary linearly with the distance to the shear centre.



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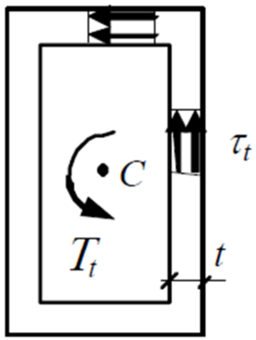

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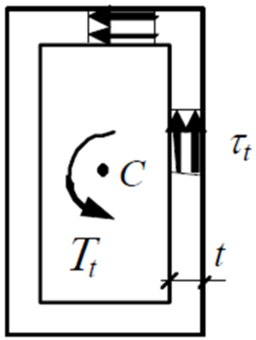

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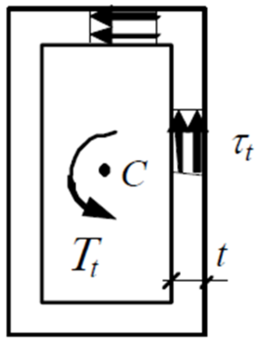

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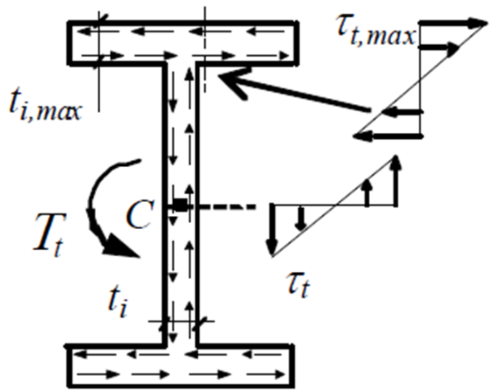

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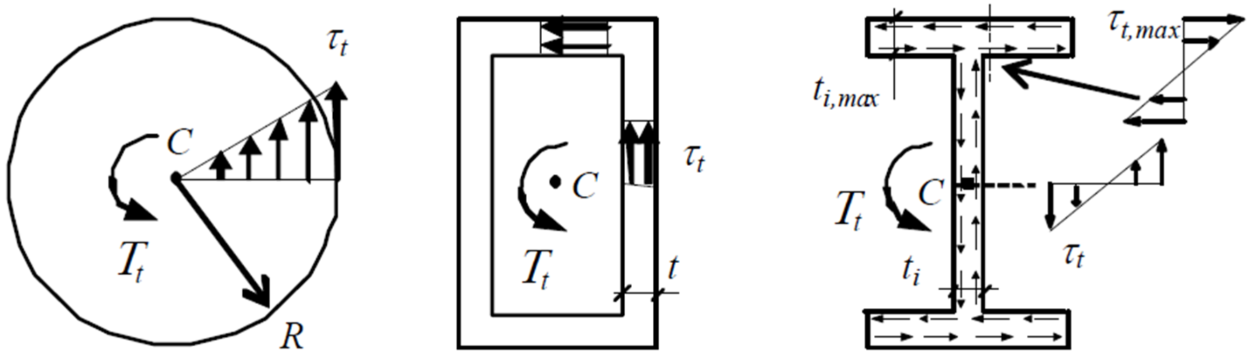
Shear stresses and torsion constant for typical steel cross-section shapes:



a) *Circular section* b) *Rectangular hollow section* c) I *section*

Section	Shear stress	Torsion constant
Circular (solid or hollow)	$\tau_t = \frac{T}{I_p} r$	$I_T = I_p$
Thin-walled closed	$\tau_t = \frac{T}{2 A_m t}$	$I_T = \frac{4 A_m^2}{\oint \frac{ds}{t}}$
Thin-walled open	$\tau_{t,\max} \approx \frac{T}{I_T} t_{i,máx}$	$I_T \approx \frac{1}{3} \sum_{i=1}^{n} h_i t_i^3$

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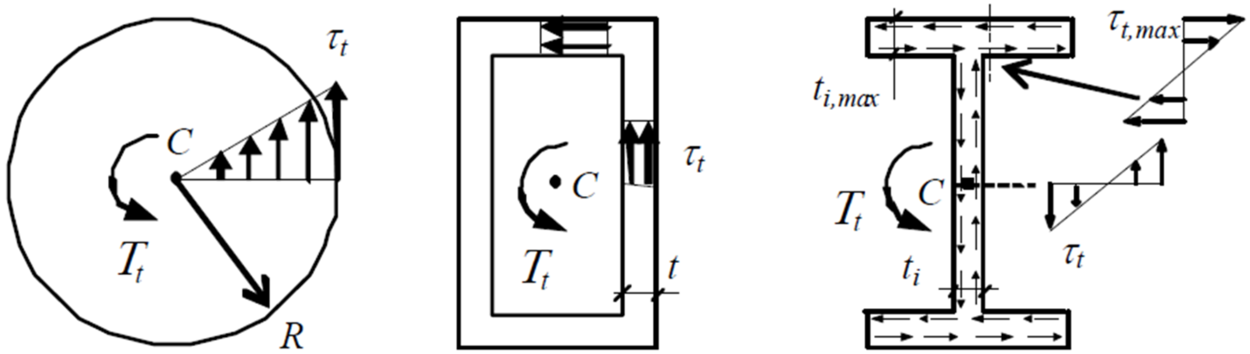

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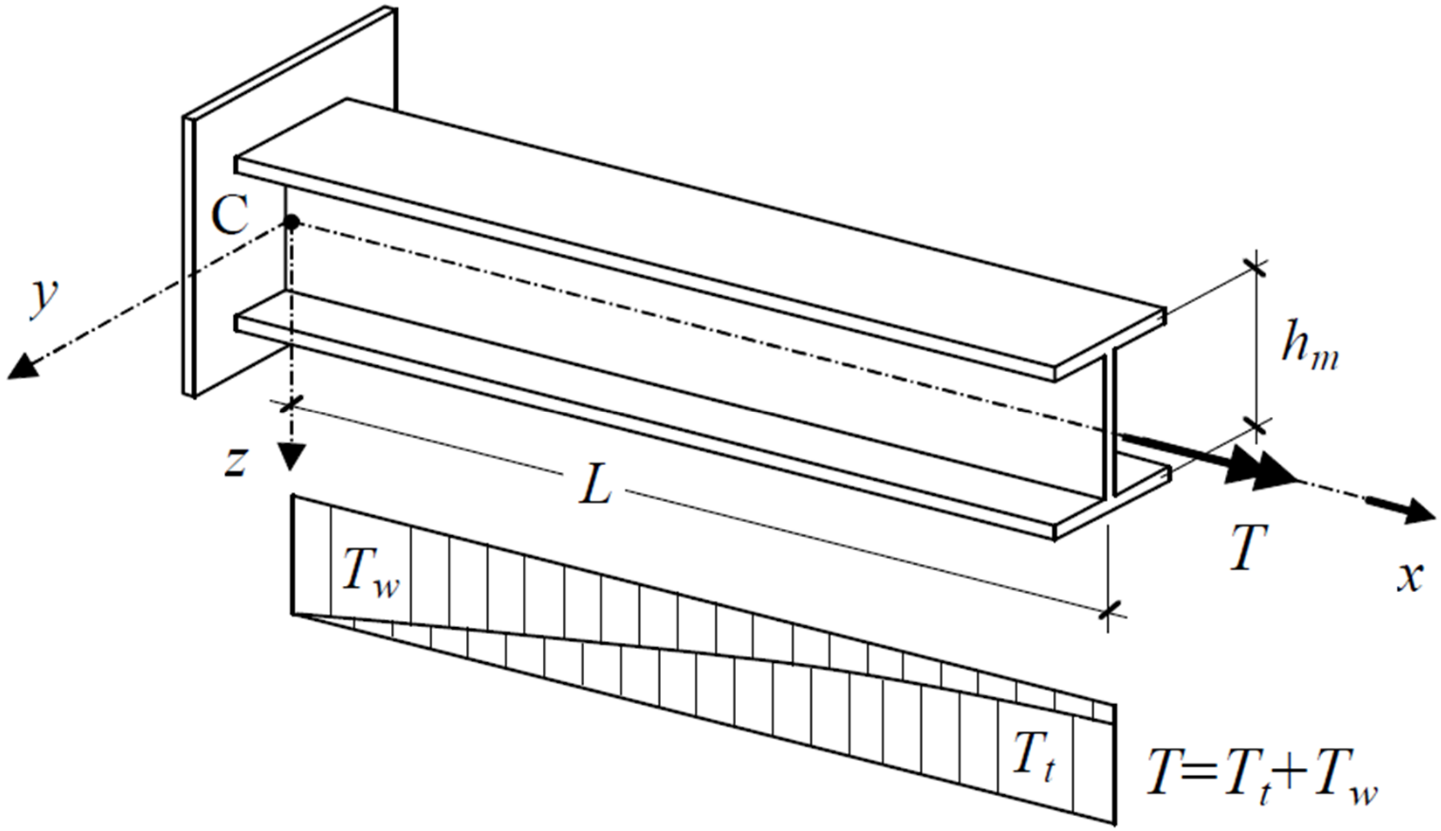

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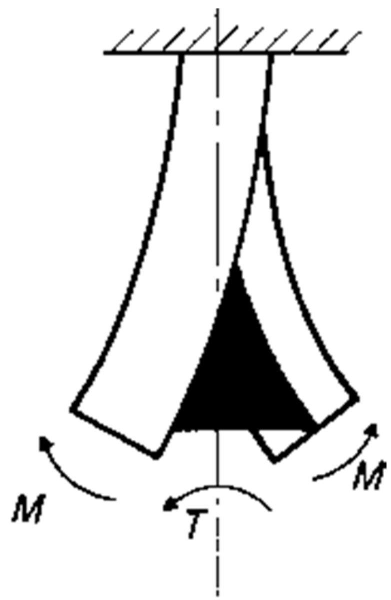

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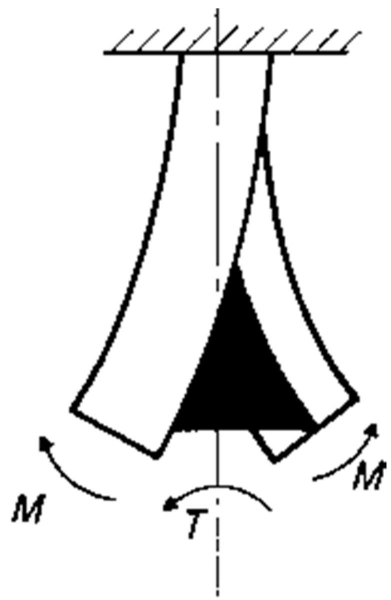
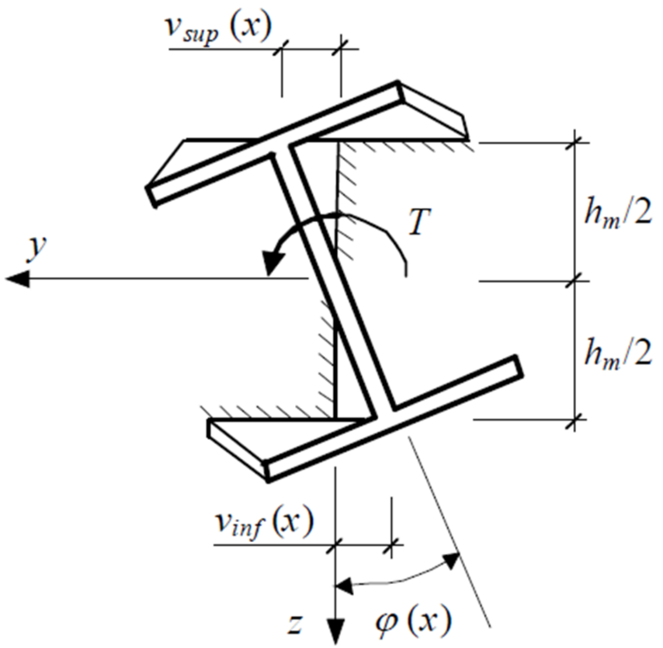
This type of behaviour was the subject of a classical investigation by Vlaslov who termed this force system induced in the flanges by warping restraint a ***bi-moment***.



Non-uniform torsion

⇒ a generic section at a distance x from the support is subjected to the following deformations:

- $\varphi(x)$ rotations around the axis of the member, due to uniform torsion T_t;
- transverse displacements of the upper flange ($v_{sup}(x)$) and lower flange ($v_{inf}(x)$) due to bending in its own plane (around z), due to the additional component T_w.



Non-uniform torsion

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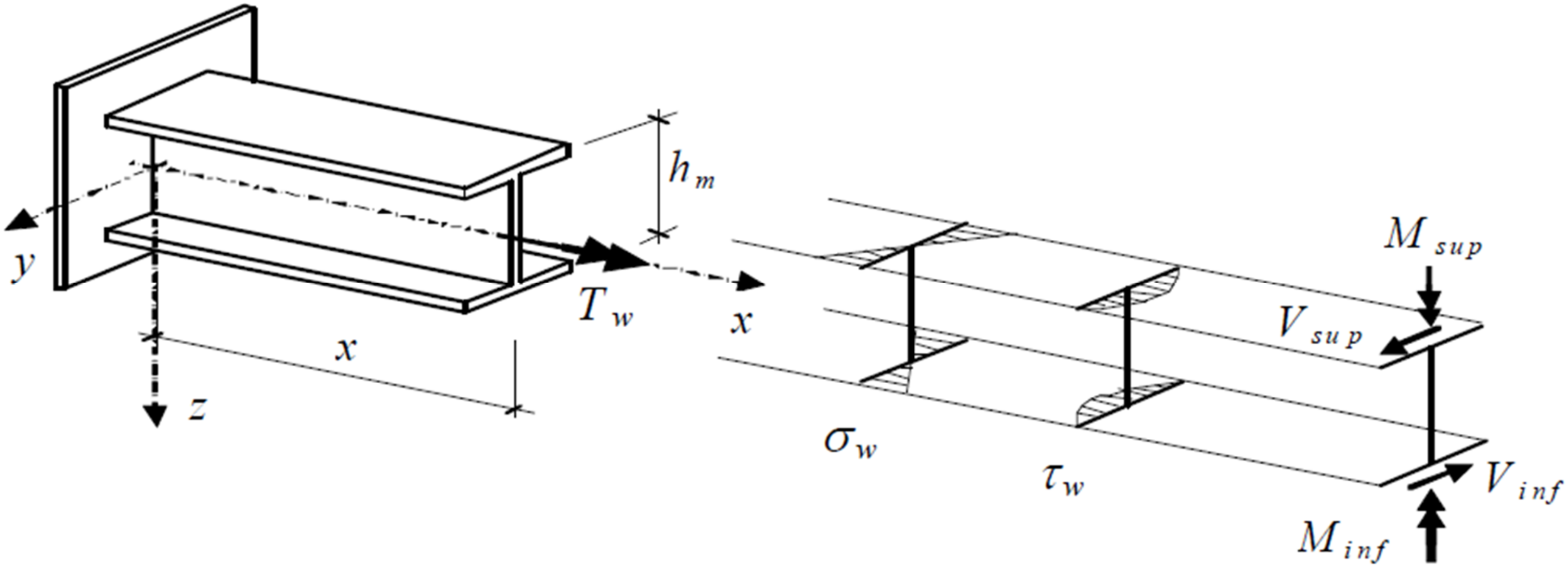


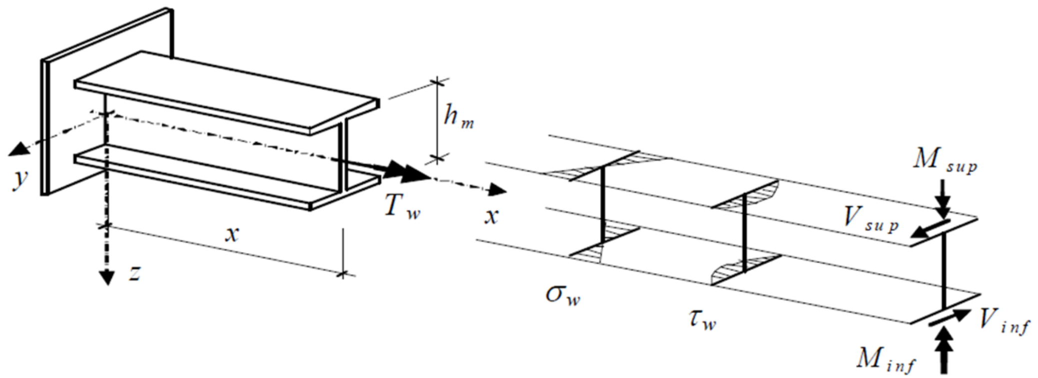
Non-uniform torsion

The normal stresses σ_w are calculated from the pair of moments M_{sup} or M_{inf}, based on the so-called bi-moment, i.e.

$$\boxed{B = M_{\sup} h_m (= M_{\inf} h_m)}$$

Shear stresses τ_w, which develop in the flanges, are due to the pair of shear forces V_{sup} and V_{inf}, statically equivalent to the warping torsion, T_w,

$$\boxed{T_w = V_{\sup} h_m (= V_{\inf} h_m)}$$



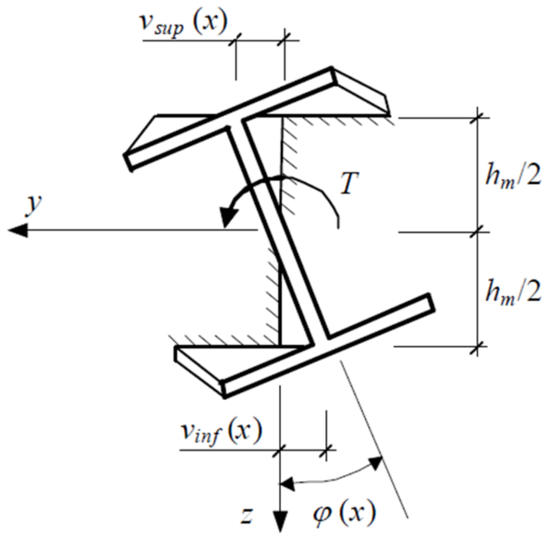
Non-uniform torsion

The differential equation of a member subject to non-torsion starts from:

$$\frac{d^2 v_{\sup}(x)}{dx^2} = -\frac{M_{\sup}}{E I_{fz}};$$

$$v_{\sup}(x) = \varphi(x)\frac{h_m}{2};$$

$$V_{\sup} = \frac{dM_{\sup}}{dx} = -\frac{d^3 v_{\sup}(x)}{dx^3} E I_{fz}$$



I_{fz} is the second moment of the flange area with respect to the z axis.

$$M_{\sup} = -\frac{d^2\varphi(x)}{dx^2} E I_{fz} \frac{h_m}{2};$$

$\Rightarrow$

$$V_{\sup} = -\frac{d^3 v_{\sup}(x)}{dx^3} E I_{fz} = -\frac{d^3\varphi(x)}{dx^3} E I_{fz} \frac{h_m}{2}$$

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where, for a thin-walled I-section $I_{fz} \cong I_z / 2$ and $I_w = \dfrac{I_{fz} h_m^{\ 2}}{2} \cong \dfrac{I_z h_m^{\ 2}}{4}$

is the warping constant and EI_w is the warping stiffness of the section.

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Finally, the differential equation of non-uniform torsion is:

$$\boxed{T = T_t(x) + T_w(x) = GI_t \frac{d\varphi(x)}{dx} - EI_w \frac{d^3\varphi(x)}{dx^3}}$$

Non-uniform torsion

The warping torsion, T_w *is:*

$$T_w = V_{\sup} h_m = -EI_{fz} \frac{h_m^{\ 2}}{2} \frac{d^3\varphi(x)}{dx^3} = -EI_w \frac{d^3\varphi(x)}{dx^3}$$

where, for a thin-walled I-section $I_{fz} \cong I_z/2$ and $I_w = \dfrac{I_{fz} h_m^{\ 2}}{2} \cong \dfrac{I_z h_m^{\ 2}}{4}$

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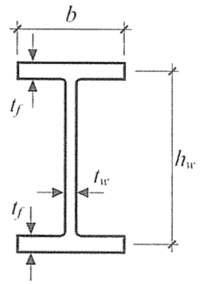
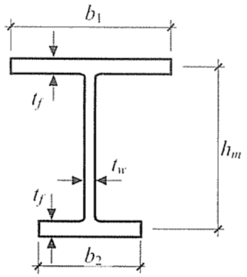
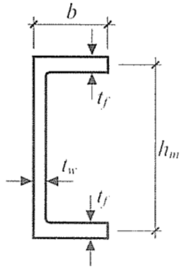
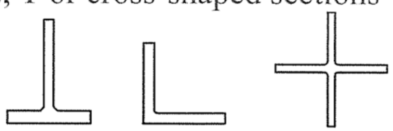
the general equation for the torsion of a non-circular section

Warping constant for typical cross-sections

Section	I_W
Circular (solid or hollow)	0
Thin-walled closed	≈ 0
I or H of equal flanges 	$\frac{t_f h_m^2 b^3}{24}$
I or H of unequal flanges 	$\frac{t_f h_m^2}{12} \frac{b_1^3 b_2^3}{b_1^3 + b_2^3}$
Channel 	$\frac{t_f b^3 h_m^2}{12} \frac{3 b t_f + 2 h_m t_w}{6 b t_f + h_m t_w}$
L, T or cross-shaped sections 	0

Non-uniform torsion

The evaluation of the longitudinal stresses and shear stresses due to an applied torque requires the solution of this equation with appropriate boundary conditions.

By differentiation $$\frac{dT(x)}{dx} = -EI_w \frac{d^4\varphi(x)}{dx^4} + GI_t \frac{d^2\varphi(x)}{dx^2} = -m(x)$$

or $$\frac{d^4\varphi(x)}{dx^4} - k^2 \frac{d^2\varphi(x)}{dx^2} = \frac{m(x)}{EI_w}$$

where

$$k = \sqrt{\frac{GI_t}{EI_w}}$$

m is the intensity of a distributed torsional moment *(m* = 0 for a concentrated torsional moment).

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C_1 to C_4 are constants;
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$\Rightarrow$ The particular solution is connected with load distribution

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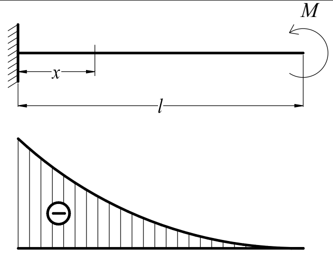
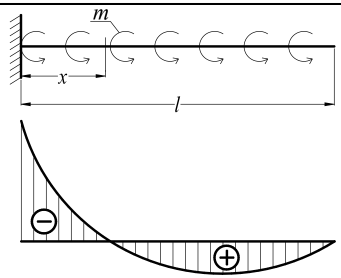
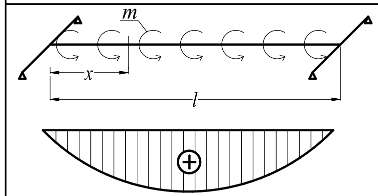
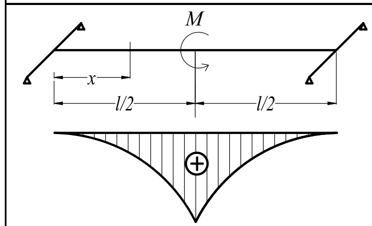
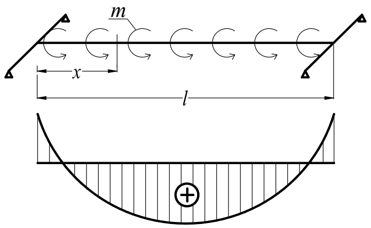
(1) Fixed end – one which is built-in and can neither twist nor warp, i.e.

$$\begin{cases} \varphi = 0 \\ \dfrac{d\varphi}{dx} = 0 \end{cases}$$

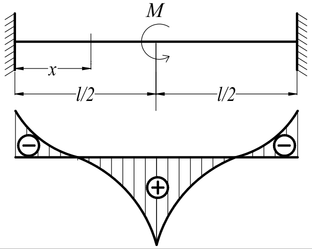
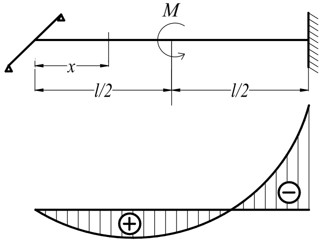
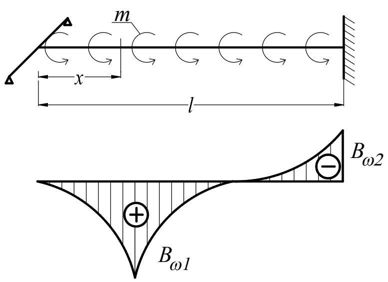
(2) Simply supported end – one which cannot twist but is free to warp and is therefore free of longitudinal stresses due to torsion

$$\begin{cases} \varphi = 0 \\ \dfrac{d^2\varphi}{dx^2} = 0 \quad \text{i.e.} \quad B = 0 \end{cases}$$

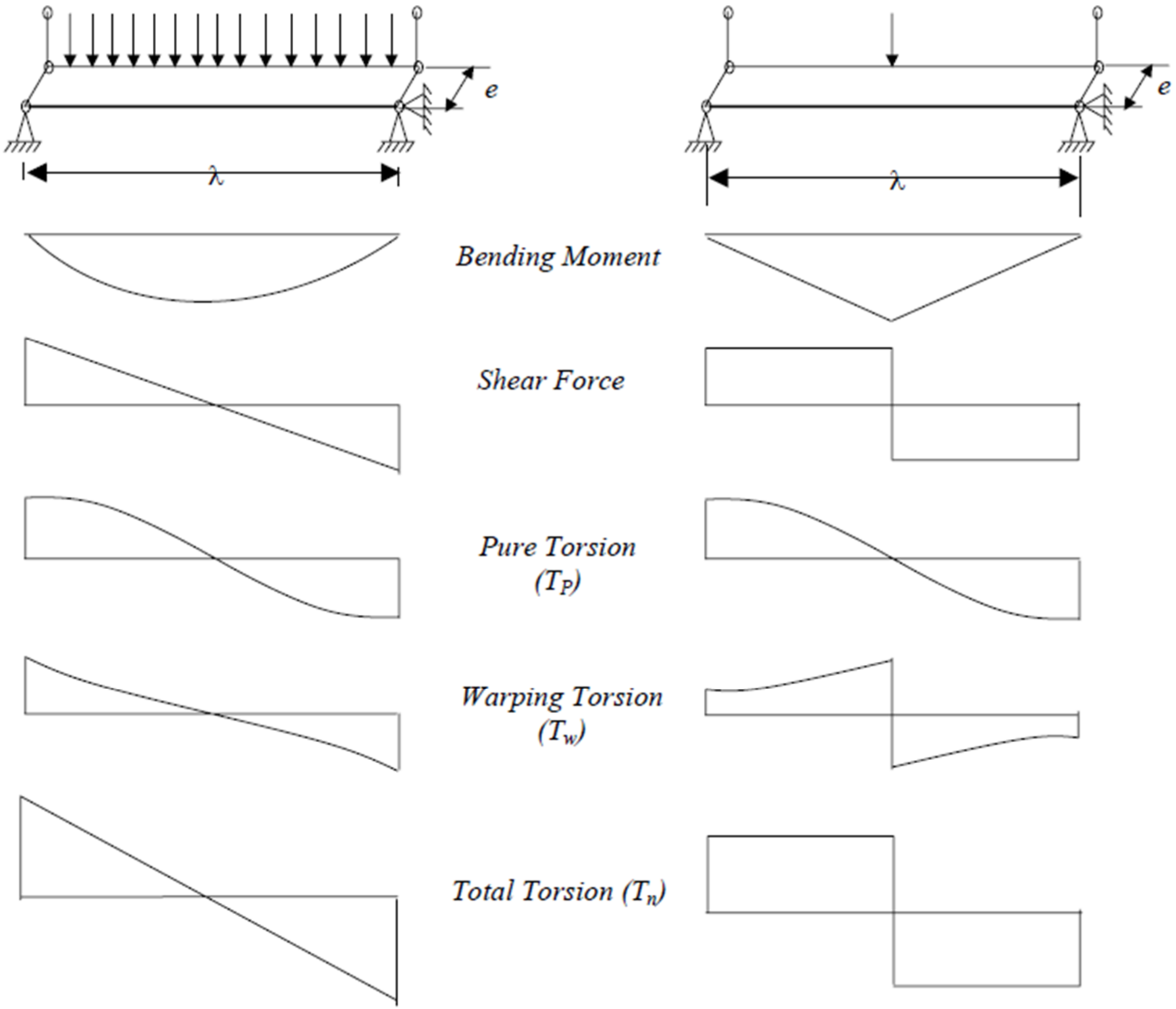
Bi-moment variation for common boundary conditions

Load condition	Bimoment equation	Maximum values	
	$B_\omega = -Ml\dfrac{\sinh k(l-x)}{kl \cdot \cosh kl}$	$x = 0$	$B_\omega = Mlb$
	$B_\omega = -\dfrac{m}{k^2 \cdot \cosh kl} \cdot [kl \cdot \sinh k\left(l-x\right) - \cosh kl + \cosh kx]$	$x = 0$	$B_\omega = ml^2 c$
	$B_\omega = \dfrac{m}{k^2} \cdot \left[1 - \dfrac{\cosh k\left(\frac{l}{2}-x\right)}{\cosh \frac{kl}{2}}\right]$	$x = \dfrac{l}{2}$	$B_\omega = ml^2 p$
	$B_\omega = \dfrac{M}{2k} \cdot \dfrac{\sinh kx}{\cosh \frac{kl}{2}}$	$x = \dfrac{l}{2}$	$B_\omega = \dfrac{Ml}{2} f$
	$B_\omega = \dfrac{m}{k^2} \cdot \left[1 - \dfrac{kl \cdot \cosh k\left(\frac{l}{2}-x\right)}{2 \cdot \sinh \frac{kl}{2}}\right]$	$\left.\begin{matrix} x = 0 \\ x = l \end{matrix}\right\}$ $x = \dfrac{l}{2}$	$B_\omega = ml^2 g$ $B_\omega = ml^2 j$

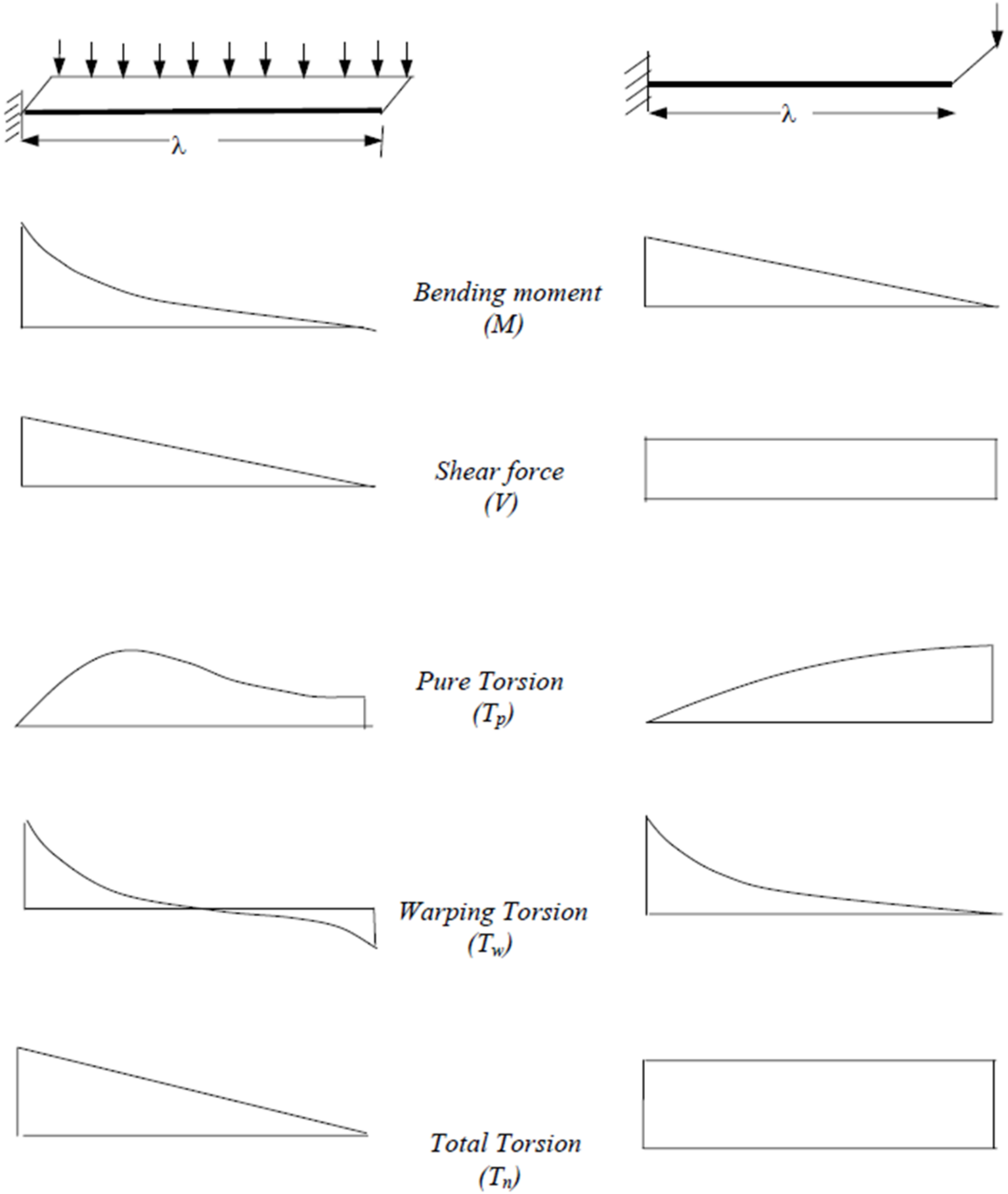
Bi-moment variation for common boundary conditions

Load condition	Bimoment equation	Maximum values	
	$B_\omega = \frac{M}{2k}\cdot\frac{\cosh kx - \cosh k\left(\frac{l}{2}-x\right)}{\sinh\frac{kl}{2}}$	$x=0$ $x=\frac{l}{2}$ $x=l$	$B_\omega = \frac{Ml}{2}n$
	$B_\omega = \frac{m}{k^2}\cdot[1-\cosh kx +$ $+\frac{1+kl\cdot\sinh kl-\cosh kl-\frac{k^2l^2}{2}}{kl\cdot\cosh kl-\sin h\; kl}\cdot\sinh kx$	$x=l$	$B_\omega = \frac{ml}{2}w$
	$B_{\omega 1} = \frac{M}{k}\cdot\frac{1}{kl\cdot\sinh kl-\sinh kl}\cdot$ $\cdot\left(kl\cdot\cosh\frac{kl}{2}-\sinh\frac{kl}{2}-\frac{kl}{2}\right)\cdot\sinh kx$ $B_{\omega 2} = \frac{M}{k}\cdot\left[\frac{\mathrm{sh}\cdot kx}{kl\cdot\mathrm{ch}\; kl-\mathrm{sh}\; kl}\cdot\right.$ $\left.\left(kl\cdot\mathrm{ch}\frac{kl}{2}-\mathrm{sh}\frac{kl}{2}-\frac{kl}{2}\right)-\mathrm{sh}\; k\left(x-\frac{l}{2}\right)\right]$	$x=\frac{l}{2}$ $x=l$	$B_\omega = \frac{Ml}{2}v$ $B_\omega = \frac{Ml}{2}u$
$b=\frac{\tanh kl}{kl}$; $c=\frac{kl\cdot\sinh kl-\cosh kl+1}{k^2l^2\cosh kl}$; $p=\frac{\cosh\frac{kl}{2}-1}{k^2l^2\cosh\frac{kl}{2}}$; $f=\frac{\cosh kl-1}{kl\sinh kl}$ $g=\frac{\frac{kl}{2}(\cosh kl+1)-\sinh kl}{k^2l^2\sinh kl}$; $j=\frac{\sinh kl-kl\cosh\frac{kl}{2}}{k^2l^2\sinh kl}$; $n=\frac{\sinh kl-2\sinh\frac{kl}{2}}{kl(\cosh kl-1)}$ $v=\frac{kl\sinh kl-\cosh kl+1-kl\sinh\frac{kl}{2}}{kl(kl\cosh kl-\sinh kl)}$; $w=\frac{kl\sinh kl-2\cosh kl+2}{kl(kl\cosh kl-\sinh kl)}$; $u=\frac{\sinh kl-2\sinh\frac{kl}{2}}{kl\cosh kl-\sinh kl}$			

Torsion in simply supported beam with free end warping



Torsion in Cantilevers



DESIGN FOR TORSION (clause 6.2.7, EN 1993-1-1)

For members subject to torsion for which distortional deformations may be disregarded the design value of the torsional moment T_{Ed} at each cross-section should satisfy:

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The total torsional moment T_{Ed} at any cross- section should be considered as the sum of two internal effects:

$$\boxed{T_{Ed} = T_{t,Ed} + T_{w,Ed}}$$

$T_{t,Ed}$ is the internal St. Venant torsion;
$T_{w,Ed}$ is the internal warping torsion.

The values of $T_{t,Ed}$ and $T_{w,Ed}$ at any cross-section may be determined from T_{Ed} by elastic analysis, taking account of the section properties of the cross-section of the member, the conditions of restraint at the supports and the distribution of the actions along the member.

The values of $T_{t,Ed}$ and $T_{w,Ed}$ at any cross-section may be determined from T_{Ed} by elastic analysis, taking account of the <u>section properties of the cross-section of the member</u>, the <u>conditions of restraint at the supports</u> and the <u>distribution of the actions along the member</u>.

The following stresses due to torsion should be taken into account:

– the shear stresses $\tau_{t,Ed}$ due to St. Venant torsion $T_{t,Ed}$

– the direct stresses $\sigma_{w,Ed}$ due to the bi-moment B_{Ed} and shear stresses $\tau_{w,Ed}$ due to warping torsion $T_{w,Ed}$.

In cross-sections subject to torsion, the following conditions should be satisfied:

$$\boxed{\sigma_{tot,Ed} \leq f_y / \gamma_{M0}}$$

$$\boxed{\tau_{tot,Ed} \leq \frac{f_y / \sqrt{3}}{\gamma_{M0}}}$$

$$\boxed{\sqrt{\sigma^2_{tot,Ed} + 3 \cdot \tau^2_{tot,Ed}} \leq 1.1 \frac{f_y}{\gamma_{M0}}}$$

$\sigma_{tot,Ed}$ is the total longitudinal stress, calculated on the relevant cross-section;

$\tau_{tot,Ed}$ is the total shear stress, calculated on the gross cross-section.

The total longitudinal stress $\sigma_{tot,Ed}$ and the total shear stress $\tau_{tot,Ed}$ should by obtained from:

$$\sigma_{tot,Ed} = \sigma_{N,Ed} + \sigma_{My,Ed} + \sigma_{Mz,Ed} + \sigma_{w,Ed}$$

$$\tau_{tot,Ed} = \tau_{Vy,Ed} + \tau_{Vz,Ed} + \tau_{t,Ed} + \tau_{w,Ed}$$

where:

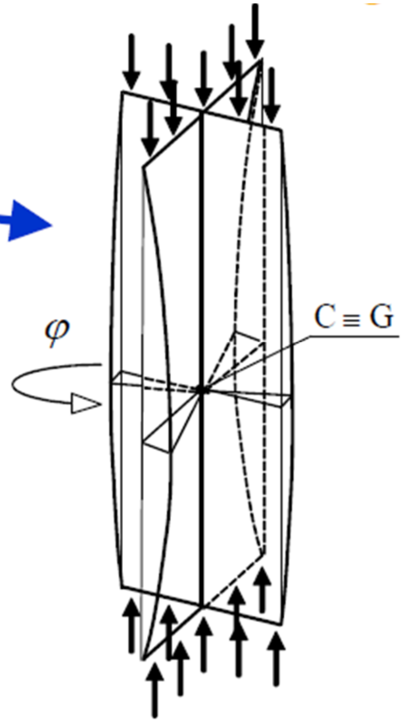
$\sigma_{My,Ed}$	is the direct stress due to the bending moment $M_{y,Sd}$;
$\sigma_{Mz,Ed}$	is the direct stress due to the bending moment $M_{z,Sd}$;
$\sigma_{N,Ed}$	is the direct stress due to the axial force N_{Sd};
$\sigma_{w,Ed}$	is the direct stress due to warping;
$\tau_{Vy,Ed}$	is the shear stress due to the transverse shear force $V_{y,Sd}$;
$\tau_{Vz,Ed}$	is the shear stress due to the transverse shear force $V_{z,Sd}$;
$\tau_{t,Ed}$	is the shear stress due to uniform (St. Venant) torsion;
$\tau_{w,Ed}$	is the shear stress due to warping.

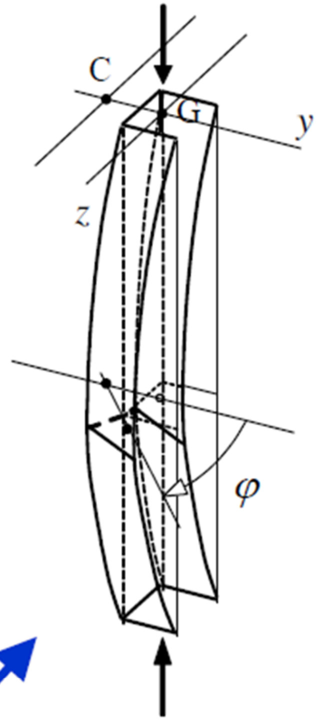
Instability phenomena that may occur in steel members depending on the behaviour in torsion

Torsional or Flexural-Torsional Buckling

$$N_{cr,T} = \frac{1}{i_C^{\,2}}\left(G\,I_T + \frac{\pi^2\,E\,I_W}{L_{ET}^{\,2}}\right)$$

$$i_C^{\,2} = y_C^{\,2} + \left(I_y + I_z\right)/A$$





$$N_{cr,TF} = \frac{1}{2\,\beta}\left[\left(N_{cr,y} + N_{cr,T}\right) - \sqrt{\left(N_{cr,y} + N_{cr,T}\right)^2 - 4\,\beta\,N_{cr,y}\,N_{cr,T}}\right]$$

$$\beta = 1 - \left(y_C / i_C\right)^2$$

$$\bar{\lambda} = \left[\beta_A \frac{A f_y}{N_{cr}} \right]^{0.5}$$

β_A = 1 for class 1-3 sections

$\beta_A = A_{eff}/A$ for class 4 sections

$$\varphi = 0.5[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2]$$

$$\chi = \frac{1}{\phi + [\phi^2 - \bar{\lambda}^2]^{0.5}} \leq 1$$

design buckling resistance ***$N_{b,Rd}$***

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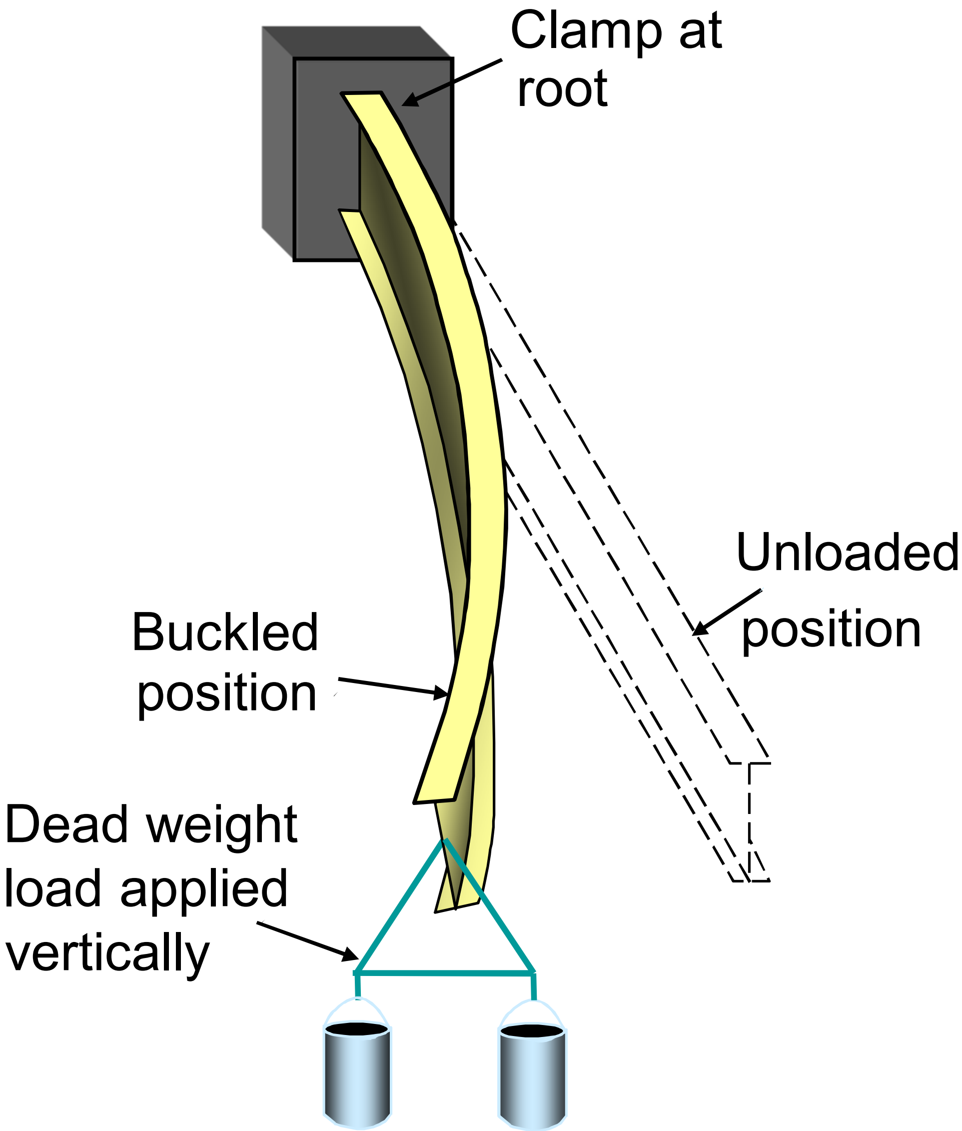
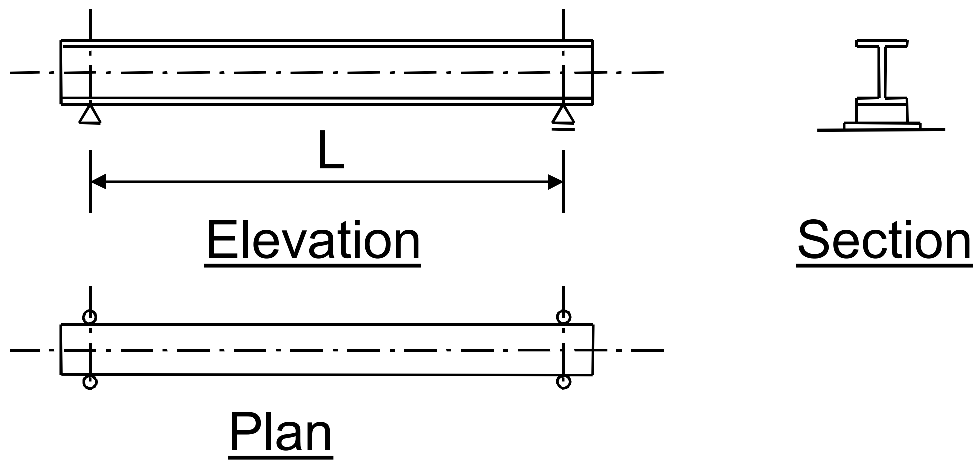


Instability phenomena and torsion



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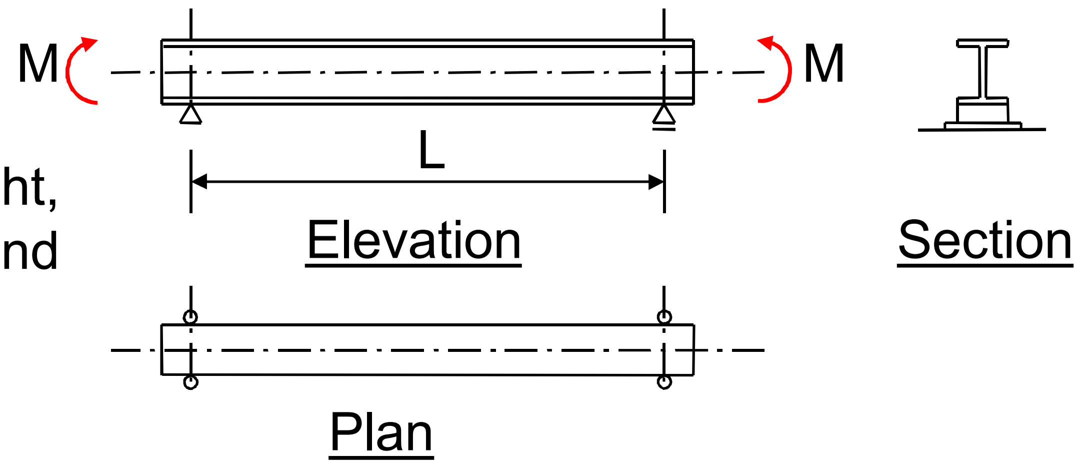
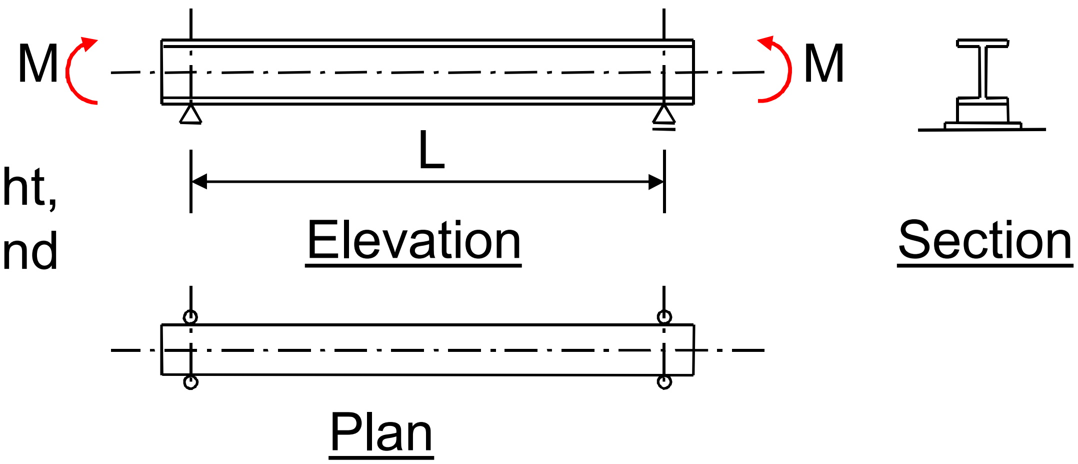


Perfectly elastic, initially straight, loaded by equal and opposite end moments about its major axis.

Instability phenomena and torsion



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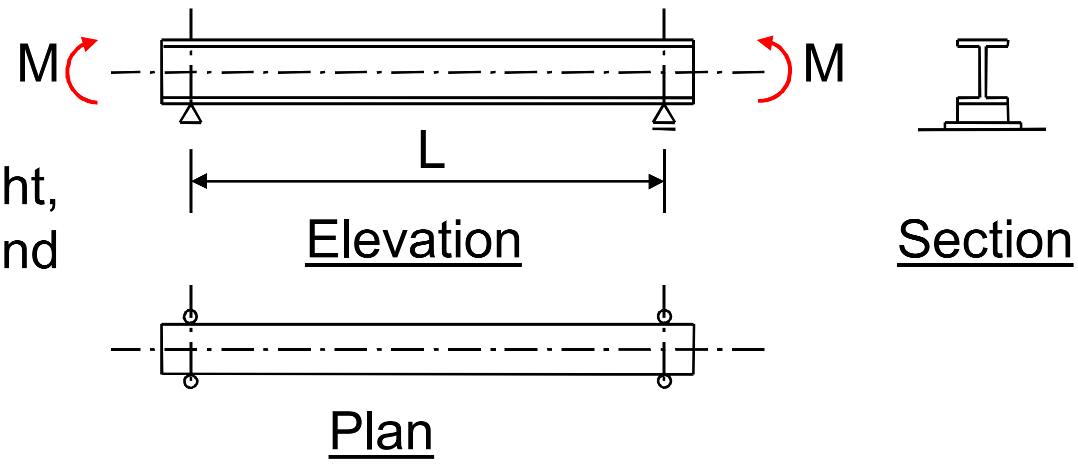

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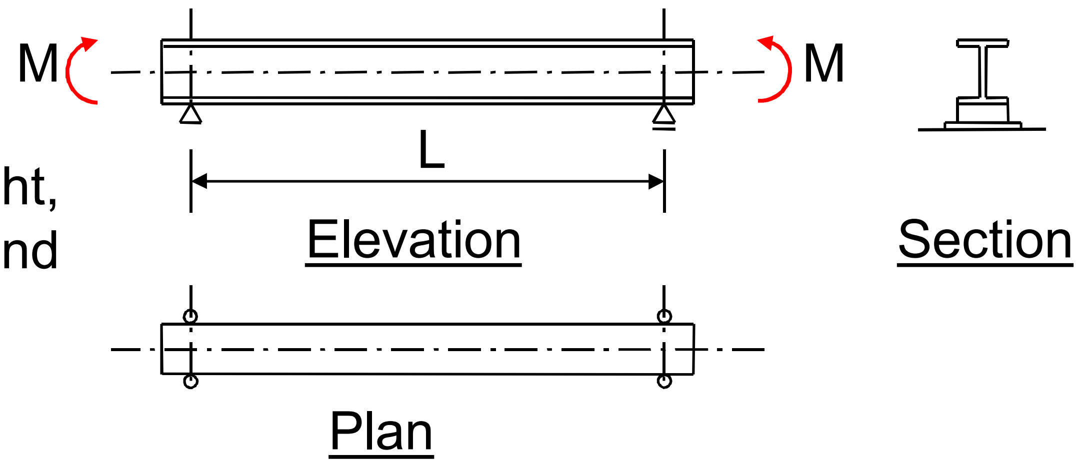
- End Supports …
  - Twisting and lateral deflection prevented.
  - Free to rotate both in the plane of the web and on plan.

Instability phenomena and torsion



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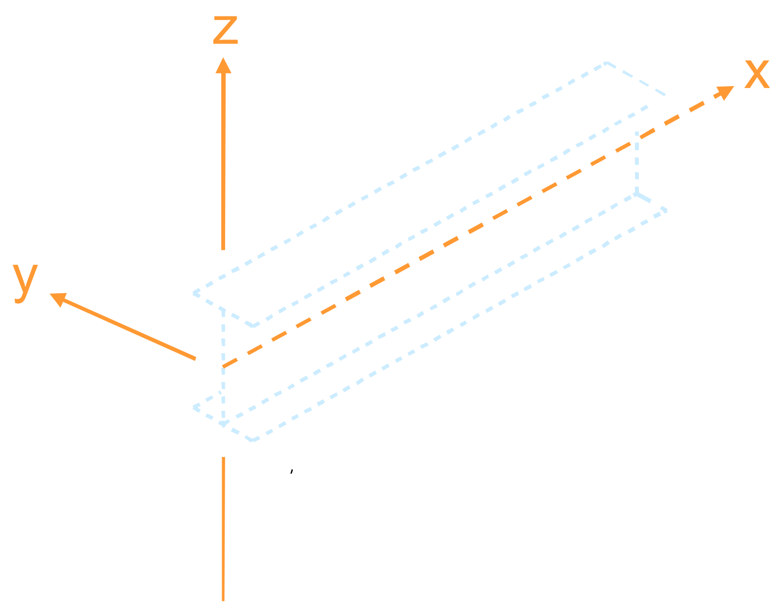

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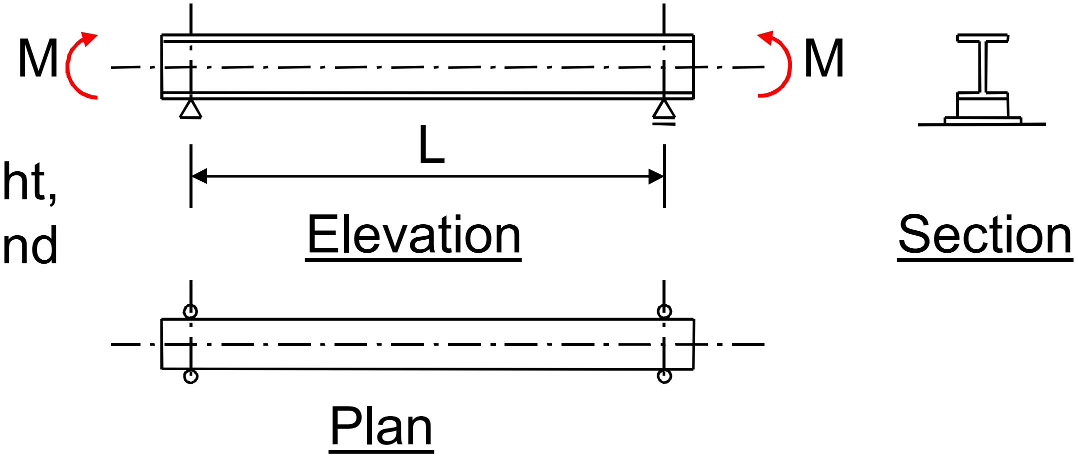

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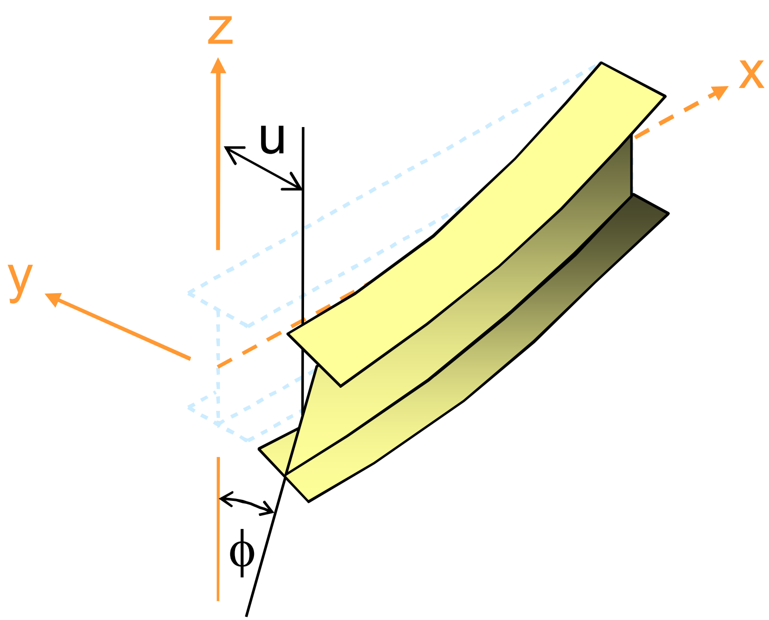

Instability phenomena and torsion

Critical Buckling Moment for uniform bending moment diagram is

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$$\boxed{M_{cr} = \frac{\pi^2 EI_z}{L^2}\sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}\right]}}$$

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Their relative importance depends on the type of cross-section used.

Instability phenomena and torsion

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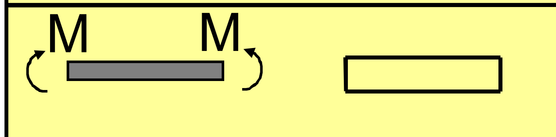
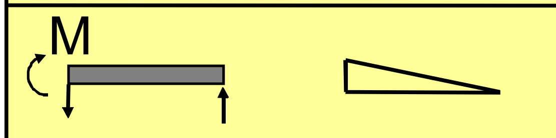
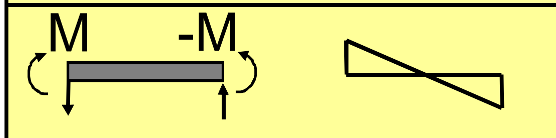
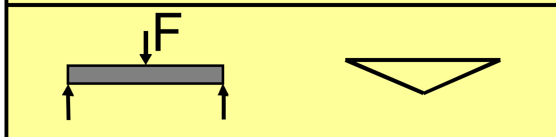
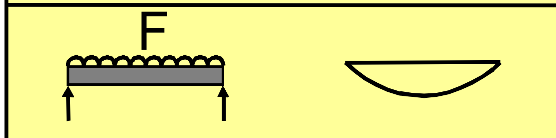
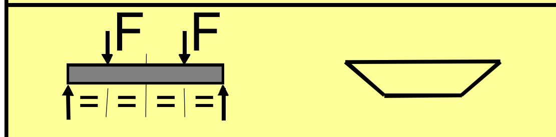
Loads	Bending moment	M_{max}	C_1
		M	1,00
		M	1,879
		M	2,752
		FL/4	1,365
		FL/8	1,132
		FL/4	1,046

Instability phenomena and torsion

$$\overline{\lambda}_{LT} = \sqrt{W_y f_y / M_{cr}}$$

$$\varphi_{LT} = 0.5[1 + \alpha(\overline{\lambda}_{LT} - 0.2) + \overline{\lambda}_{LT}^{\,2}]$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + [\phi_{LT}^{\,2} - \overline{\lambda}_{LT}^{\,2}]^{0.5}} \leq 1$$

design buckling resistance $M_{b,Rd}$

Avoiding, controlling and minimizing torsion

Designing for torsion in practice

- ***"Avoid Torsion - if you can!"***

Designing for torsion in practice

- ***"Avoid Torsion - if you can!"***

- The loads are usually applied in such a manner that their resultant passes through the centroid in the case of symmetrical sections and shear centre in the case of unsymmetrical sections.

Designing for torsion in practice

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- Arrange connections suitably.

Designing for torsion in practice

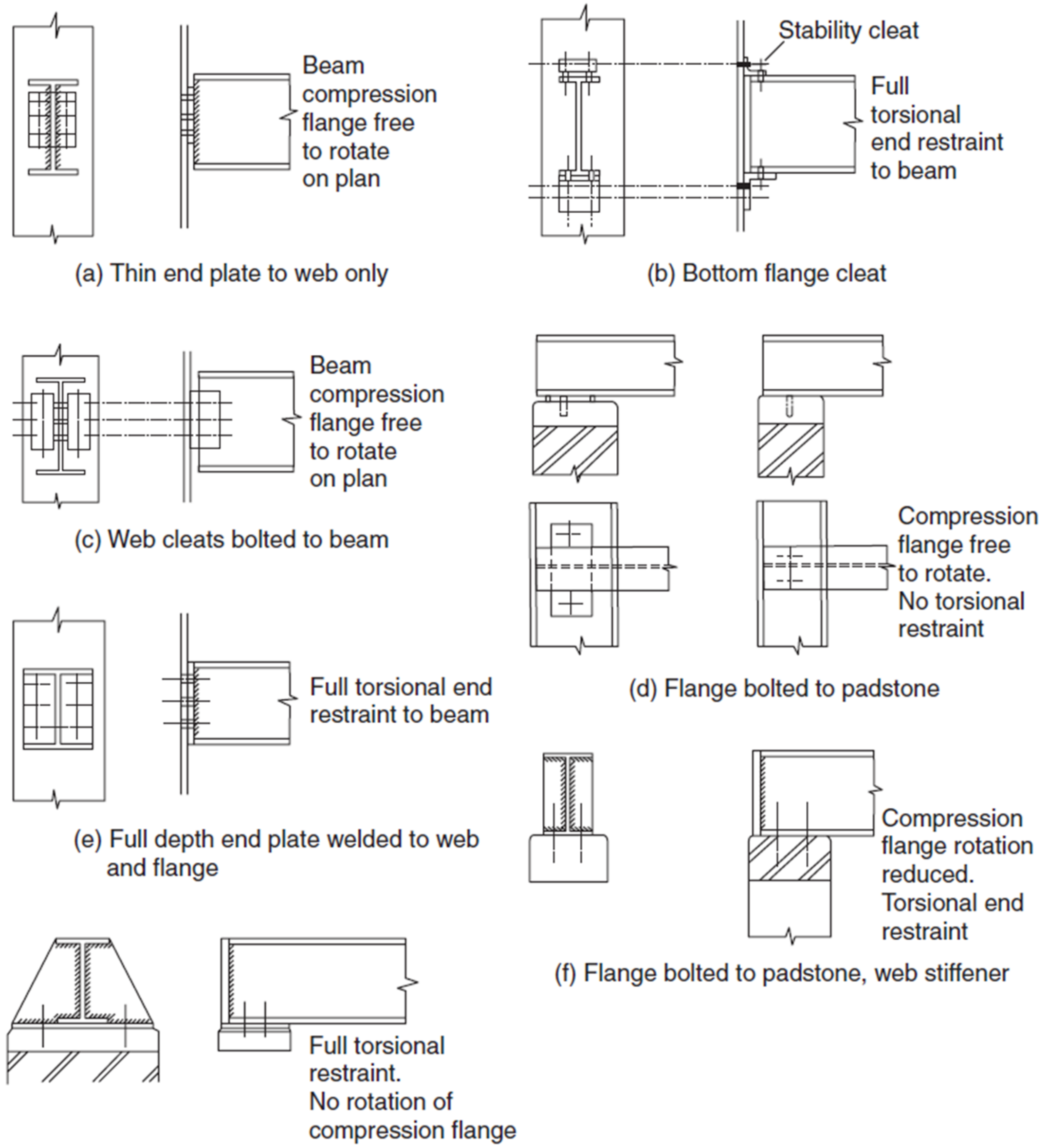
- ***"Avoid Torsion - if you can!"***

- The loads are usually applied in such a manner that their resultant passes through the centroid in the case of symmetrical sections and shear centre in the case of unsymmetrical sections.

- Arrange connections suitably.

- Where significant eccentricity of loading (which would cause torsion) is unavoidable, alternative methods of resisting torsion like design using box, tubular sections or lattice box girders should be investigated.

Typical connection and support detail



(a) Thin end plate to web only

(b) Bottom flange cleat

(c) Web cleats bolted to beam

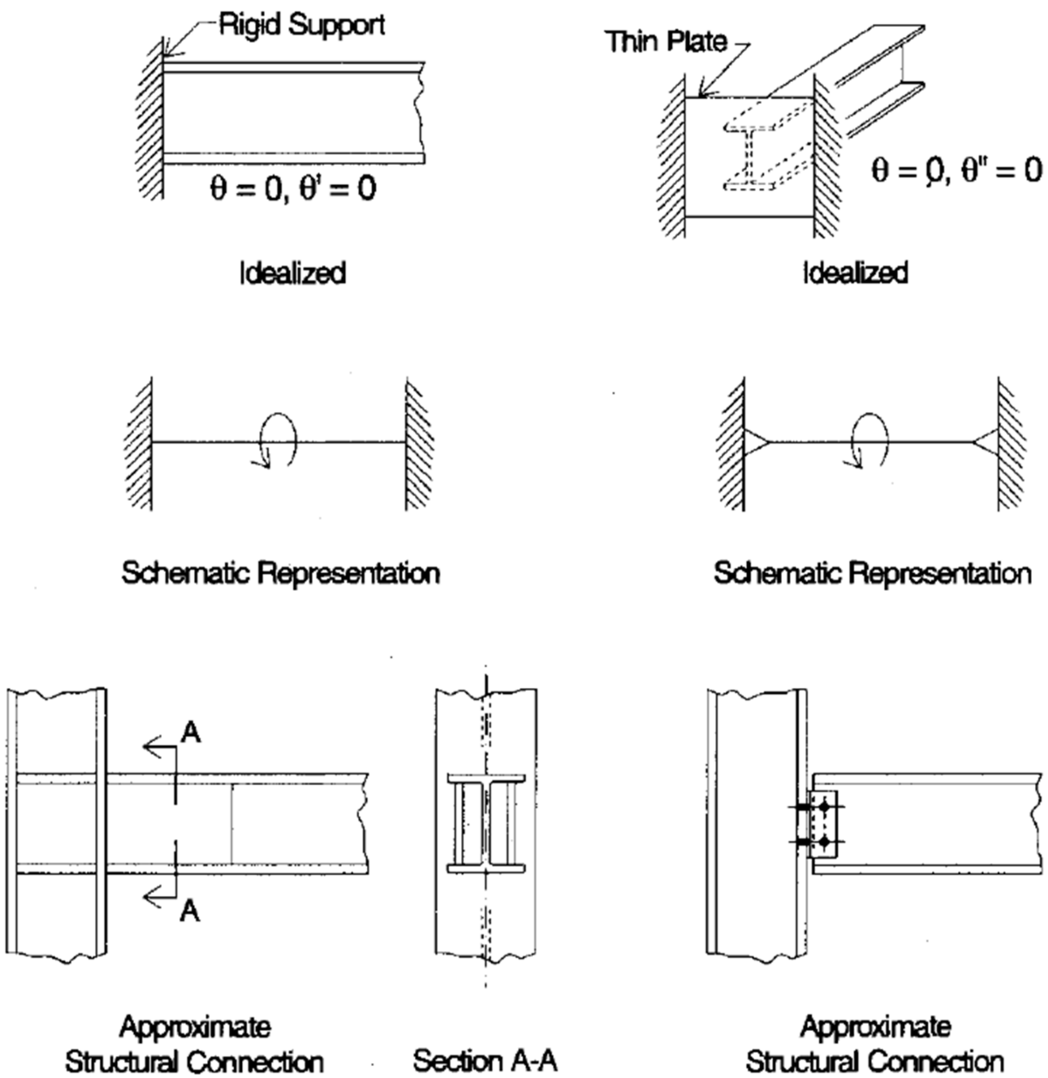
(d) Flange bolted to padstone

(e) Full depth end plate welded to web and flange

(f) Flange bolted to padstone, web stiffener

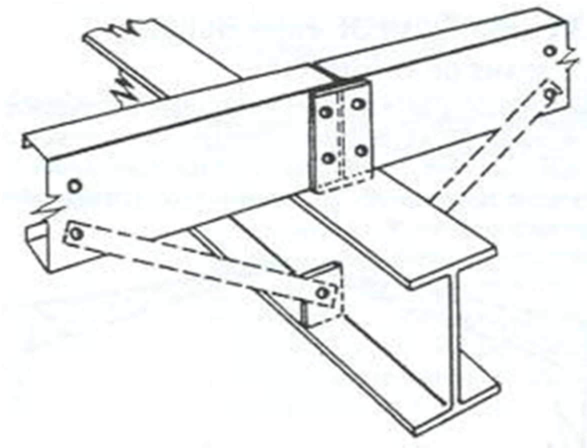
(g) Full end stiffener

Typical connection and support detail

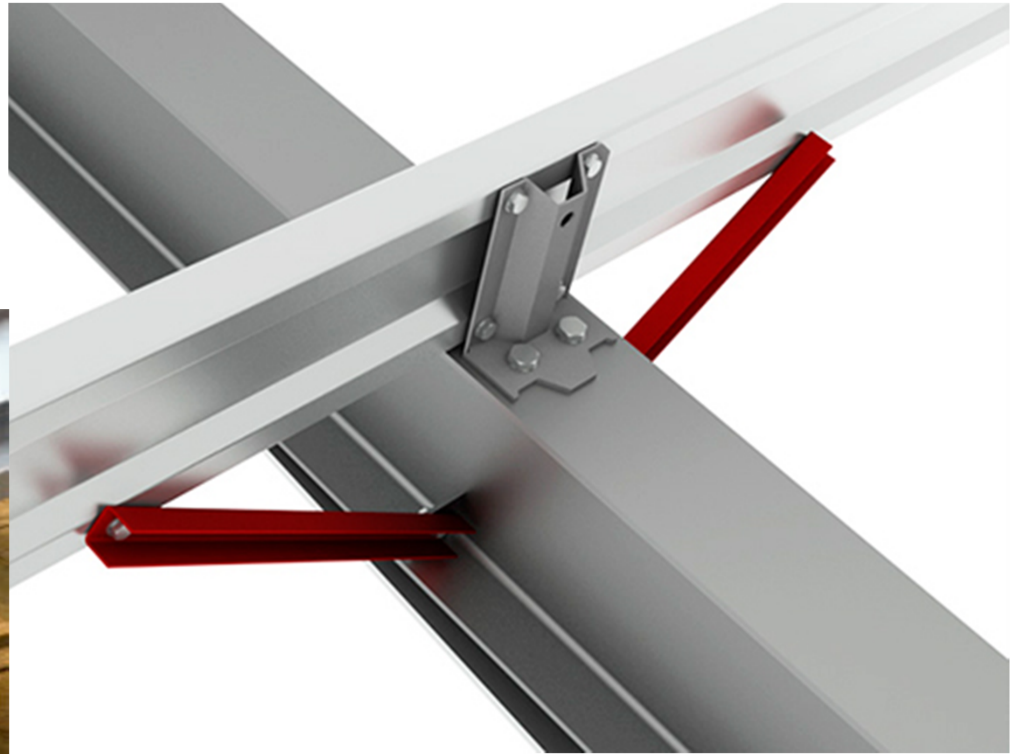


(a) Torsionally Fixed End

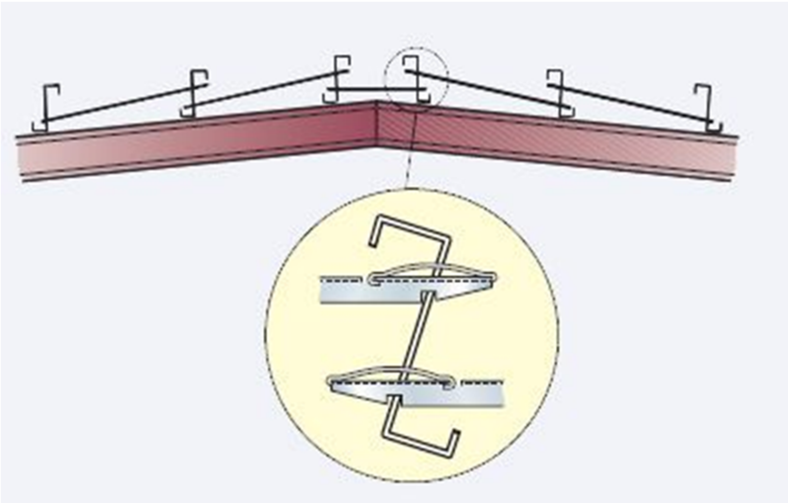
(b) Torsionally Pinned End

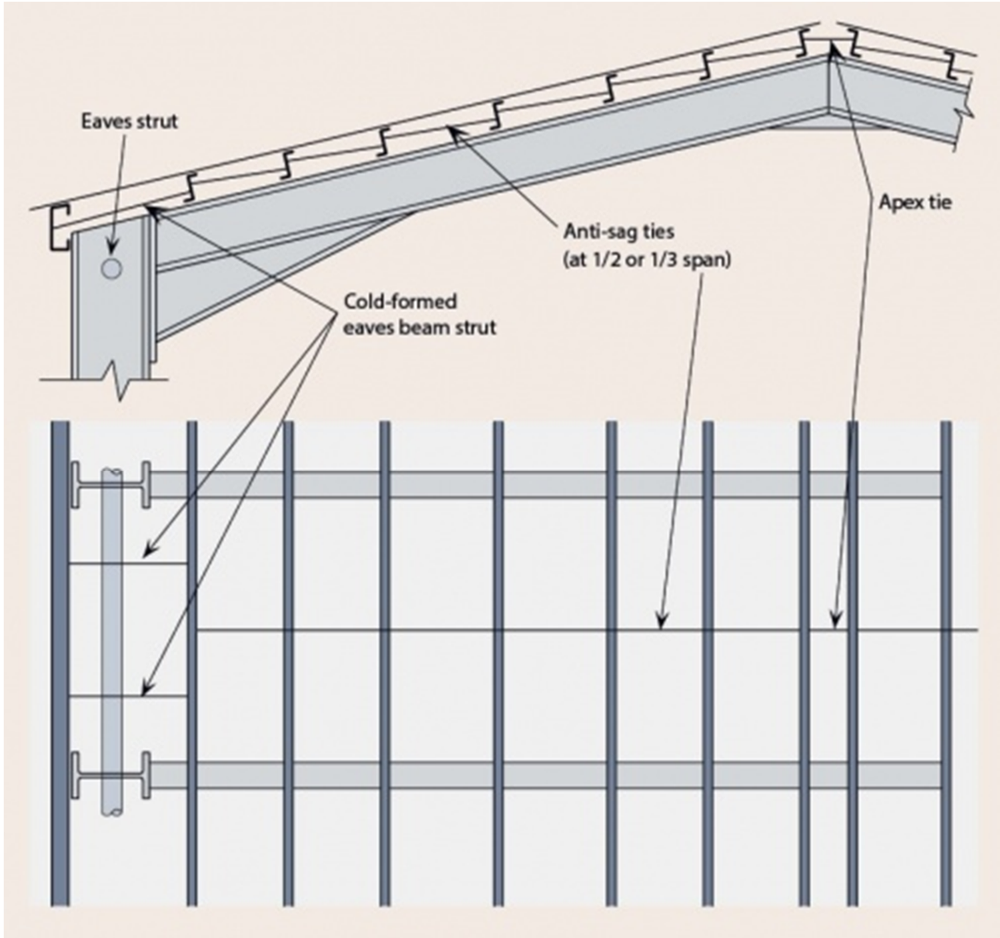


Use fly bracings for rafters and colums

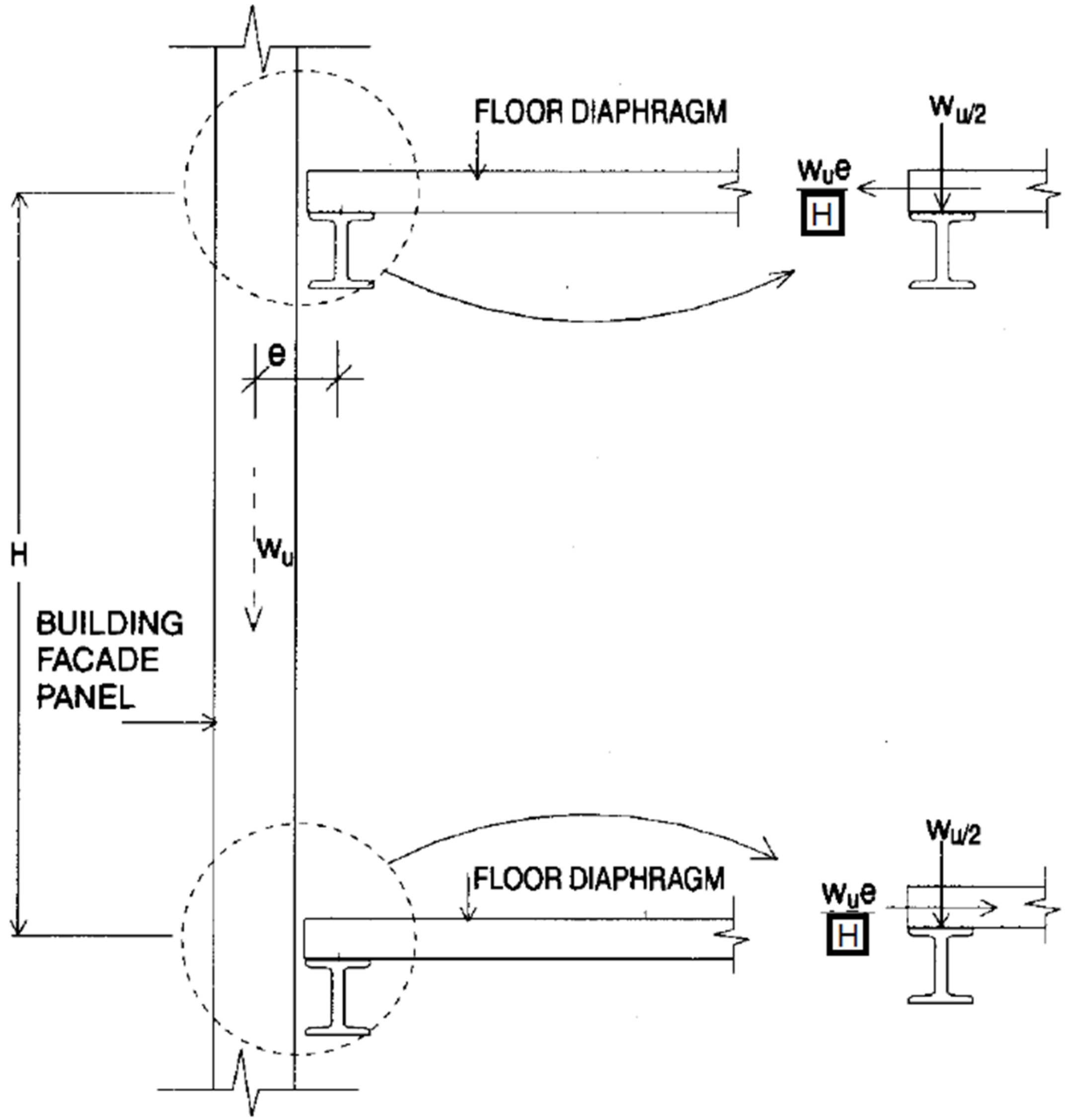


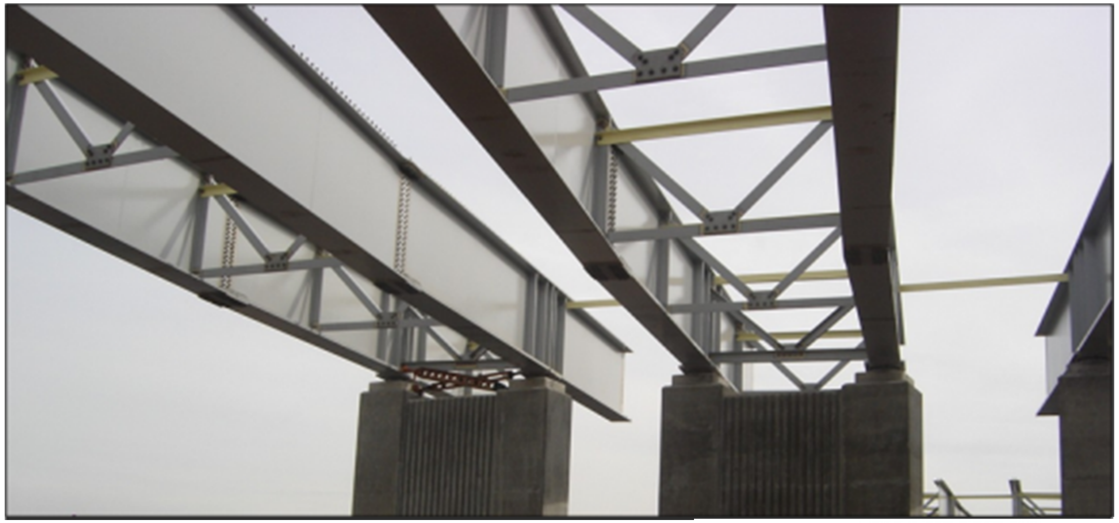
Use anti-sag ties for purlins



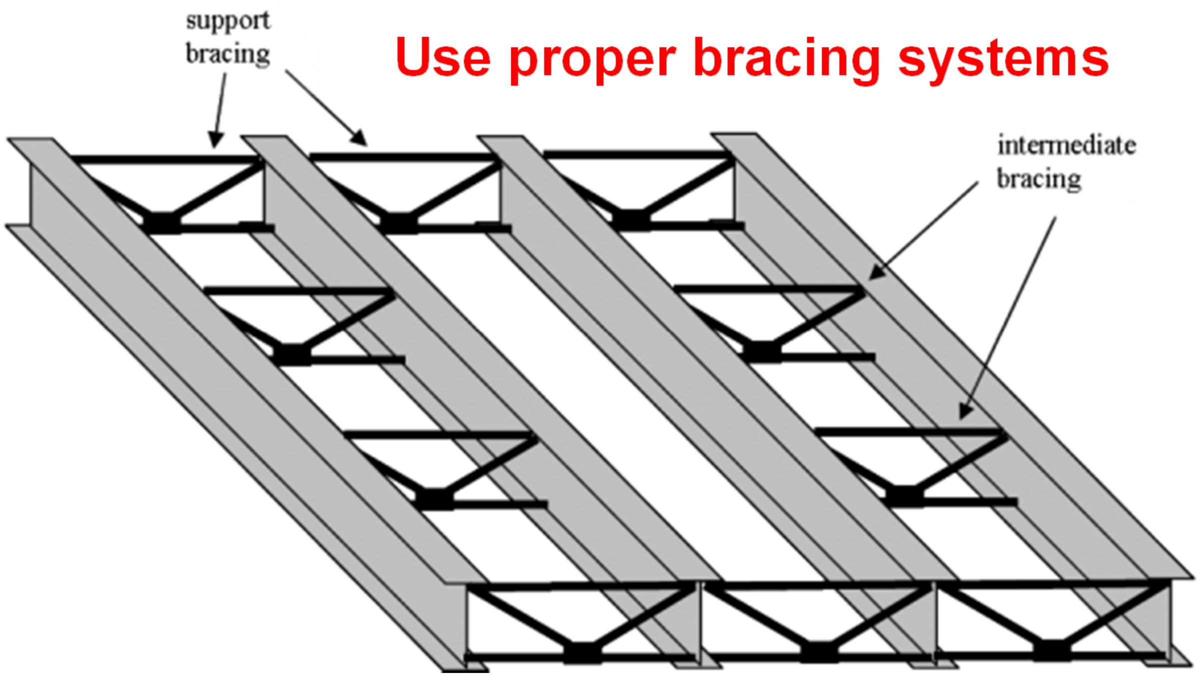


For example a rigid facade elements spanning between floors: the weight of which would otherwise induce torsional loading of the spandrel girder, may be designed to transfer lateral forces into the floor diaphragms and resist the eccentric effect.





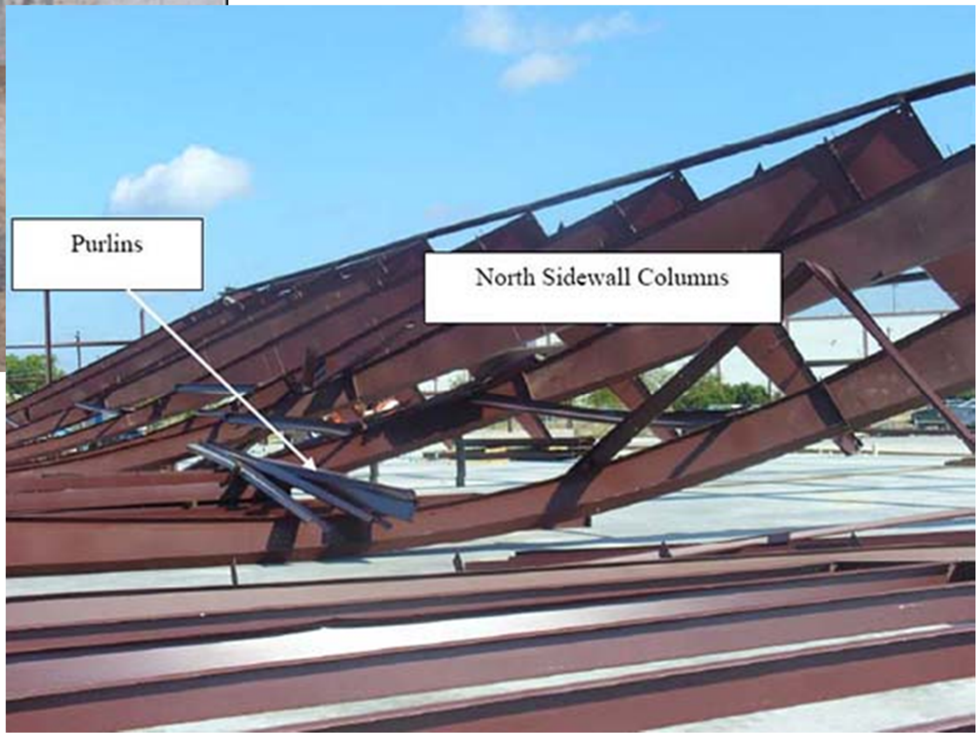
Use proper bracing systems



Otherwise …

Otherwise …

Otherwise …



Viorel.ungureanu@upt.ro

http://steel.fsv.cvut.cz/suscos

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