Training Course on
Design of Cold-formed Steel Structures

Eurocode 3: Design of Steel Structures
Part 1-3 – Design of Cold-formed Steel Structures
Resistance of Members

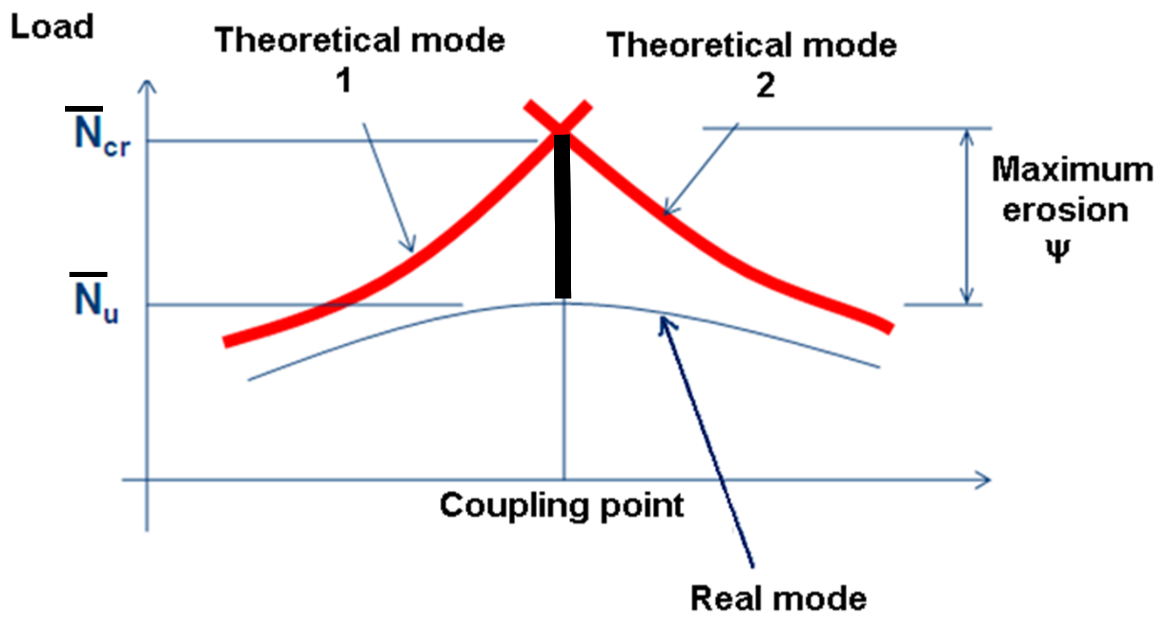
Professor Dan Dubina

Behaviour and Design Resistance of Bar Members

- General
- Compression members
- Buckling strength of bending members
- Buckling of members in bending and axial compression
- Beams restrained by sheeting
- Design of beams at serviceability limit states

Behaviour and Design Resistance of Bar Members

General



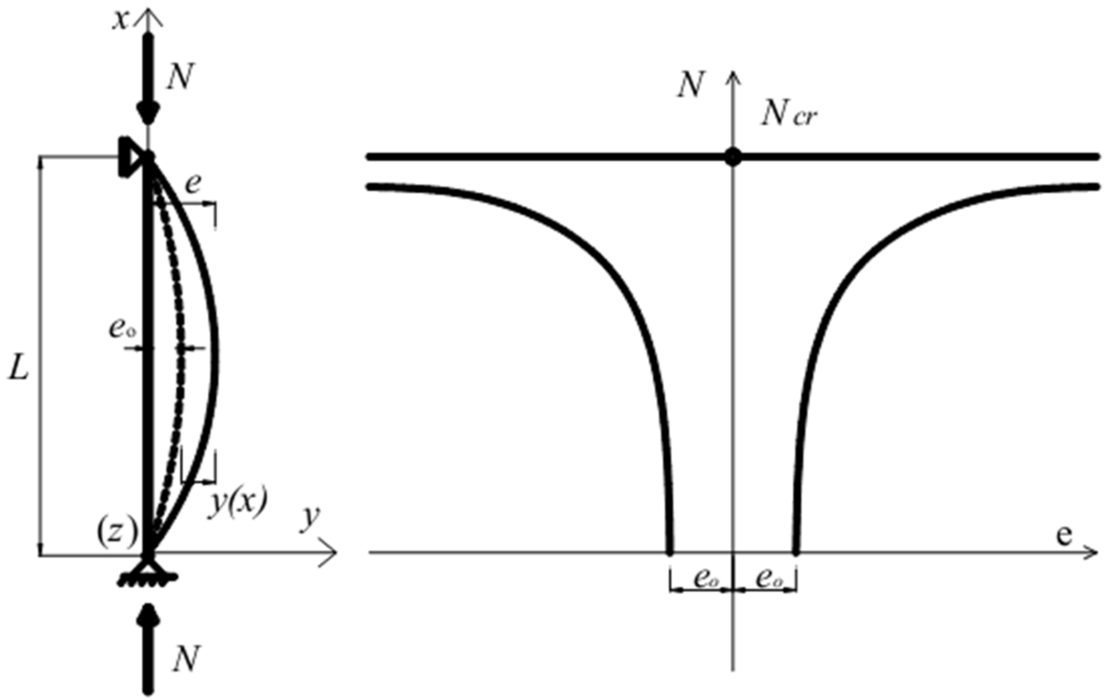
The *erosion factor* ψ was introduced as a *measure* of erosion of critical load. Gioncu (1994) has classified the interaction types by means of this erosion factor, as follows:

class I: weak interaction (W), $\psi \leq 0.1$;
class II: moderate interaction (M), $0.1 < \psi \leq 0.3$;
class III: strong interaction (S), $0.3 < \psi \leq 0.5$;
class IV: very strong interaction (VS), $\psi > 0.5$.

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background –** ***Theoretical background***

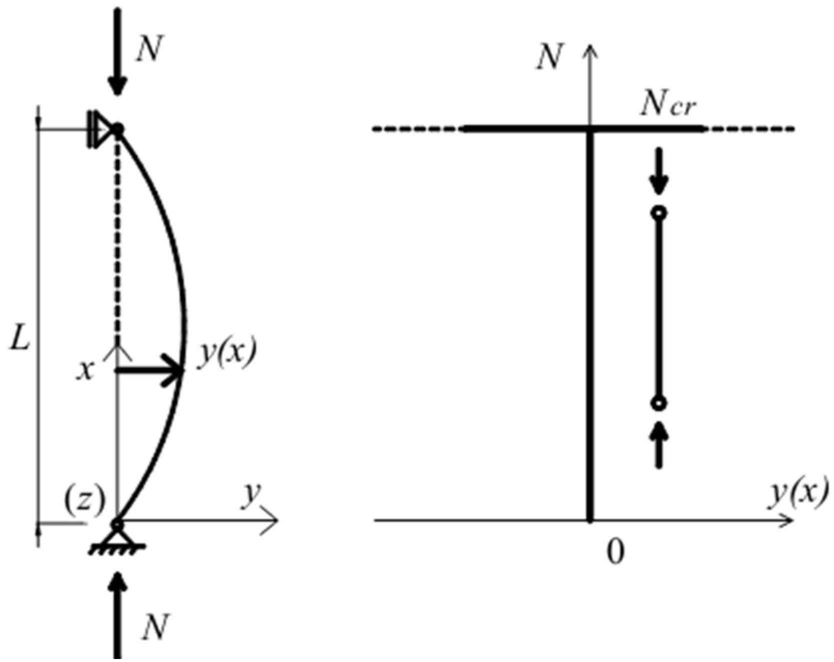


Imperfect bar member in compression   load – deflection path

$$(\Sigma M)_z = EI\frac{d^2y}{dx^2} + N \cdot y = 0$$

- **Theoretical background – *Theoretical background***

Model of ideal pinned bar member under pure compression (Euler's column)



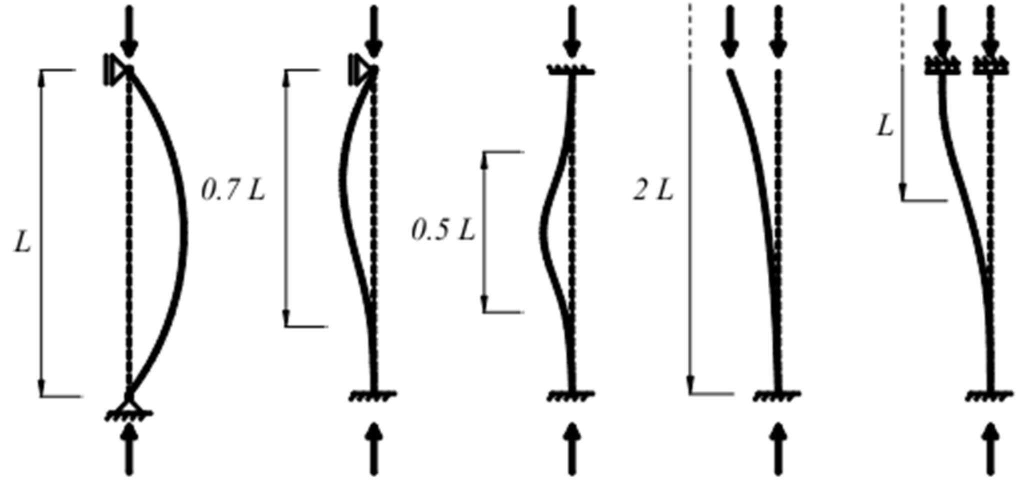
The critical load is obtained from:

$$kL = n\pi \;\Rightarrow\; k^2 = \frac{n^2\pi^2}{L^2} = \frac{N}{EI}$$

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Theoretical background***



Buckling length L_{cr} as a function of the real length L of the column

$$L_{cr} = \mu L \qquad N_{cr} = \frac{\pi^2 EI}{L_{cr}^2}$$

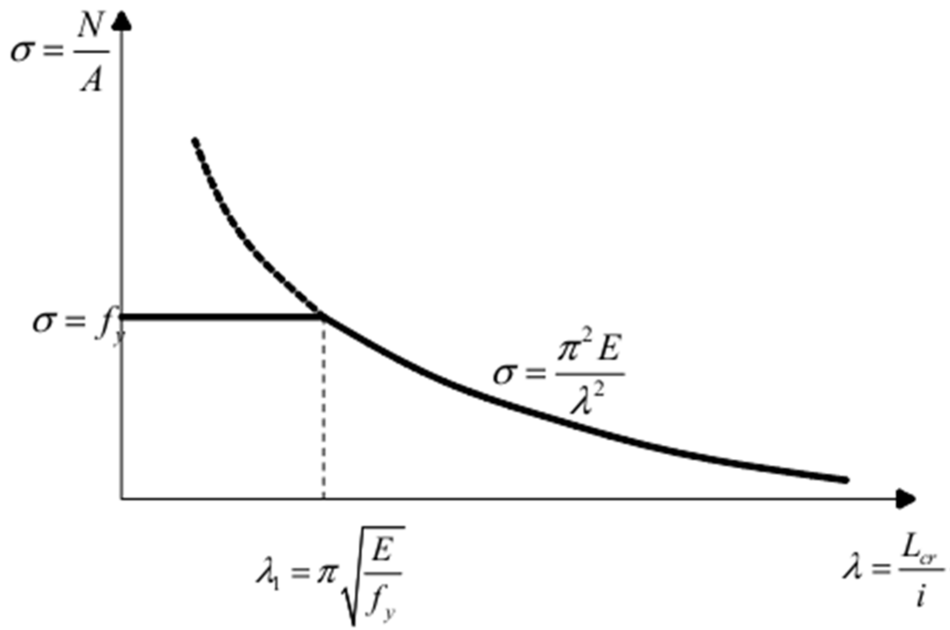
$$\sigma_{cr} = \frac{\pi^2 EI}{A\, L_{cr}^2} = \frac{\pi^2 E}{\lambda^2}$$

where, $\lambda = L_{cr} / i$ is the slenderness coefficient

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Theoretical background***

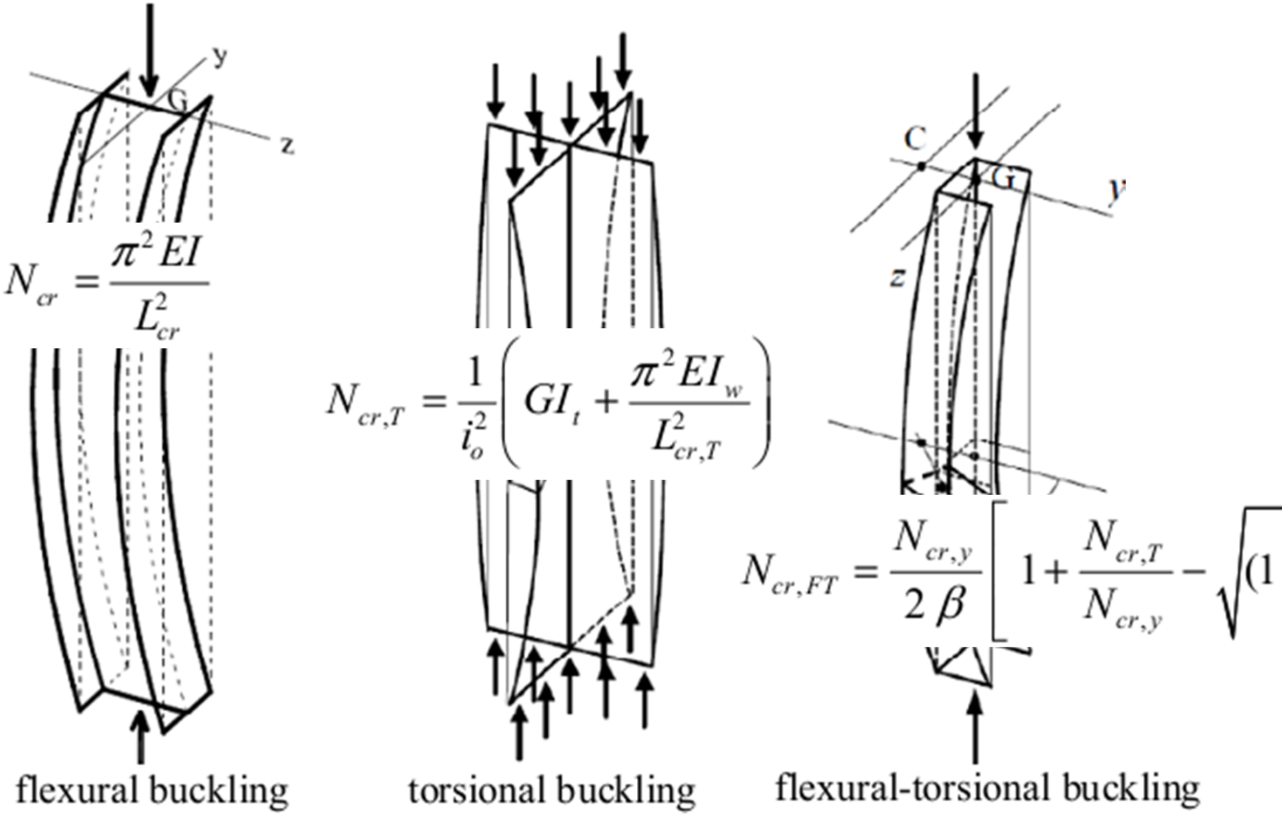


$\sigma - \lambda$ relationship of a compressed ideal member

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background** – ***Theoretical background***



$$N_{cr} = \frac{\pi^2 EI}{L_{cr}^2}$$

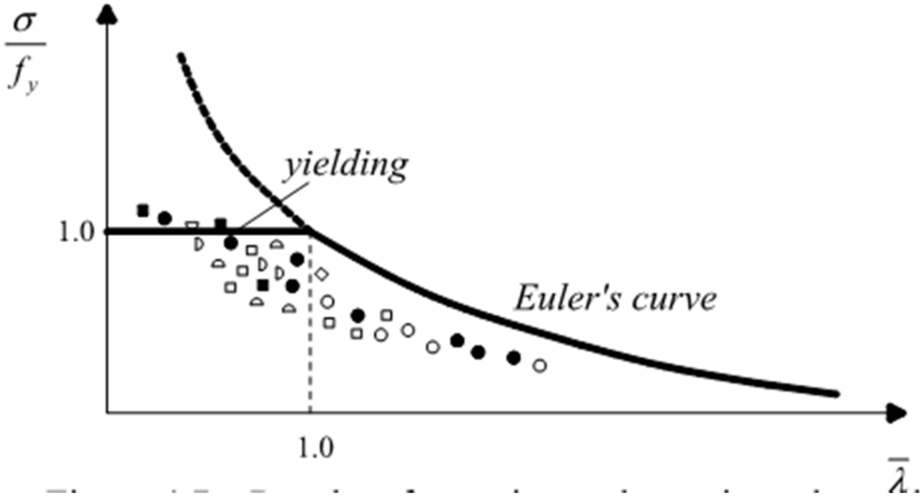
$$N_{cr,T} = \frac{1}{i_o^2}\left(GI_t + \frac{\pi^2 EI_w}{L_{cr,T}^2}\right)$$

$$N_{cr,FT} = \frac{N_{cr,y}}{2\beta}\left[1 + \frac{N_{cr,T}}{N_{cr,y}} - \sqrt{\left(1 + \frac{N_{cr,T}}{N_{cr,y}}\right)^2 - 4\beta\frac{N_{cr,T}}{N_{cr,y}}}\right]$$

Behaviour and Design Resistance of Bar Members

Compression members

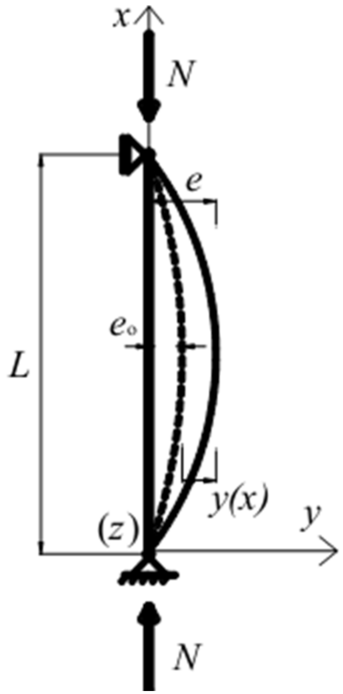
- **Theoretical background** – ***Imperfect member***



Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Imperfect member***



$$\frac{N}{A} + \frac{N\,e}{W_{el}} = f_y$$

$$e = \frac{e_o}{1 - \dfrac{N}{N_{cr}}}$$

$$\frac{N}{N_{pl}} + \frac{N\,e_o\,A}{W_{el}\left(1 - \dfrac{N}{N_{pl}}\dfrac{N_{pl}}{N_{cr}}\right)N_{pl}} = 1$$

Defining $\chi = N / N_{pl} = \overline{N}$ yields:

$$\chi + \frac{\chi}{\left(1 - \chi\,\overline{\lambda}^2\right)}\frac{e_o\,A}{W_{el}} = 1$$

Behaviour and Design Resistance of Bar Members

Compression members

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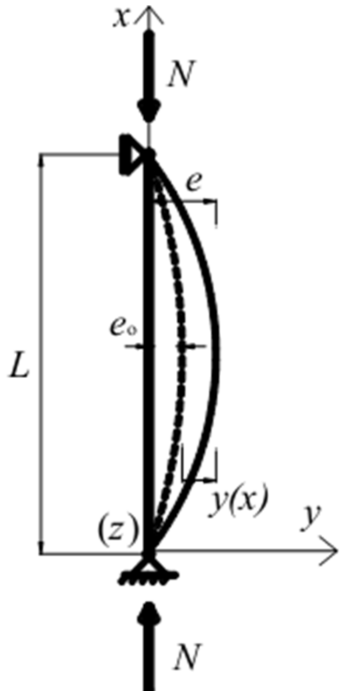


Defining $\chi = N / N_{pl} = \overline{N}$ yields:

$$\chi + \frac{\chi}{\left(1 - \chi\,\overline{\lambda}^2\right)} \frac{e_o\,A}{W_{el}} = 1$$

Ayrton-Perry equation

$$(1-\chi)\left(1-\chi\,\overline{\lambda}^2\right) = \frac{e_o\,A}{W_{el}}\chi = \eta\chi$$

Maquoi & Rondal, 1978

$$\eta = \alpha(\overline{\lambda} - 0.2)$$

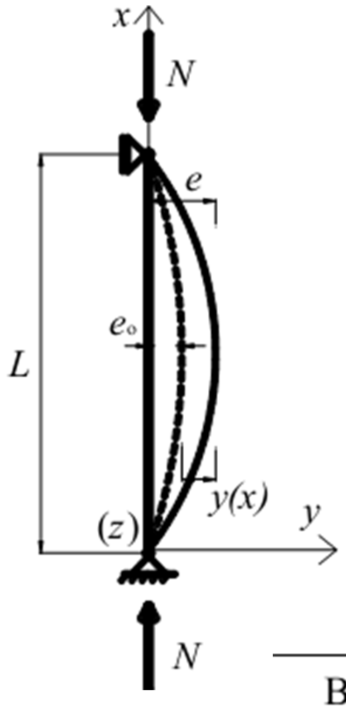
the imperfection factor α depends on the shape of the cross section.

$$\left(1-\chi\,\overline{\lambda}^2\right)(1-\chi) = \eta\chi = \alpha\chi(\overline{\lambda}-0.2) \qquad (1-\overline{N})\left(1-\overline{N}\,\overline{\lambda}^2\right) = \alpha\overline{N}(\overline{\lambda}-0.2)$$

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Imperfect member***



$$\chi = \frac{\phi - \sqrt{\phi^2 - \bar{\lambda}^2}}{\bar{\lambda}^2}$$

$$\phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right]$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$

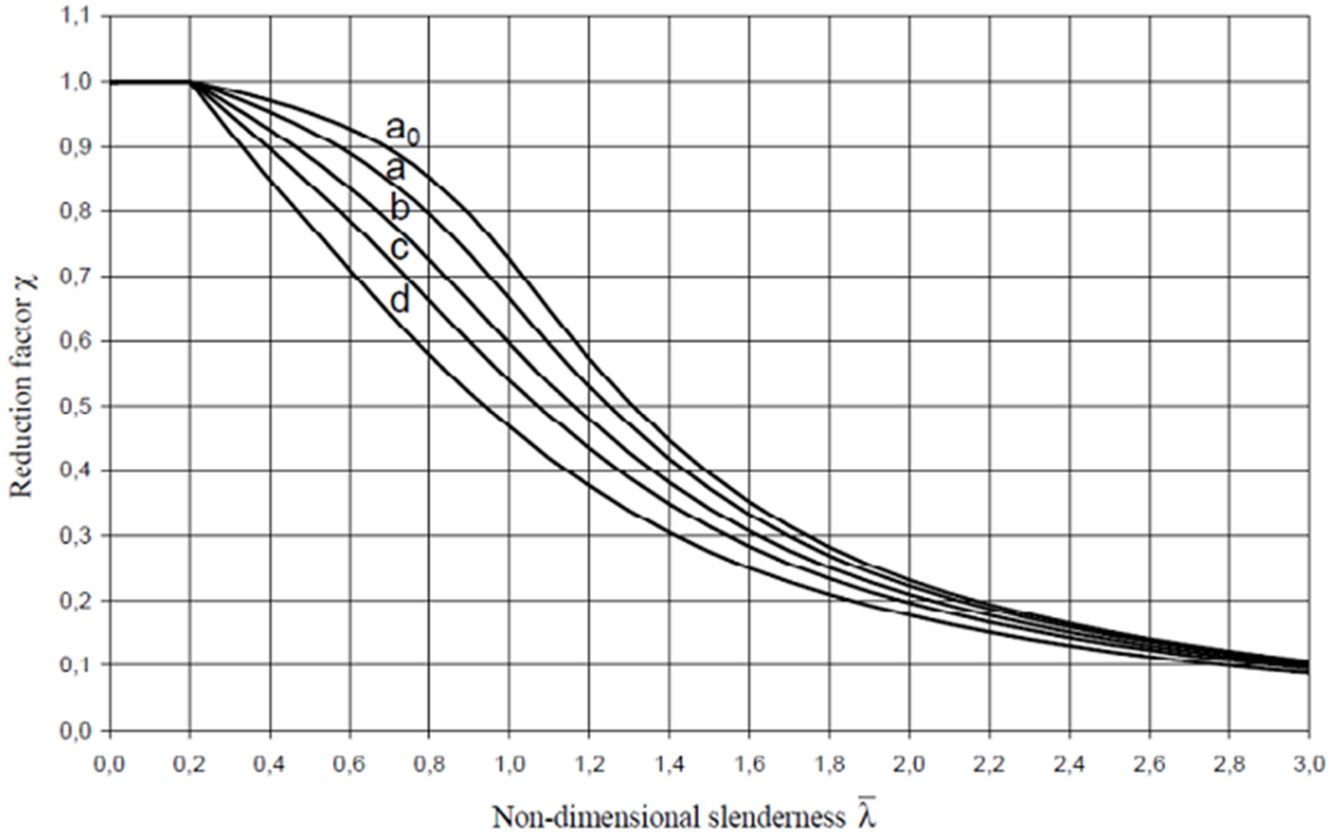
Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0.13	0.21	0.34	0.49	0.76

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Imperfect member***

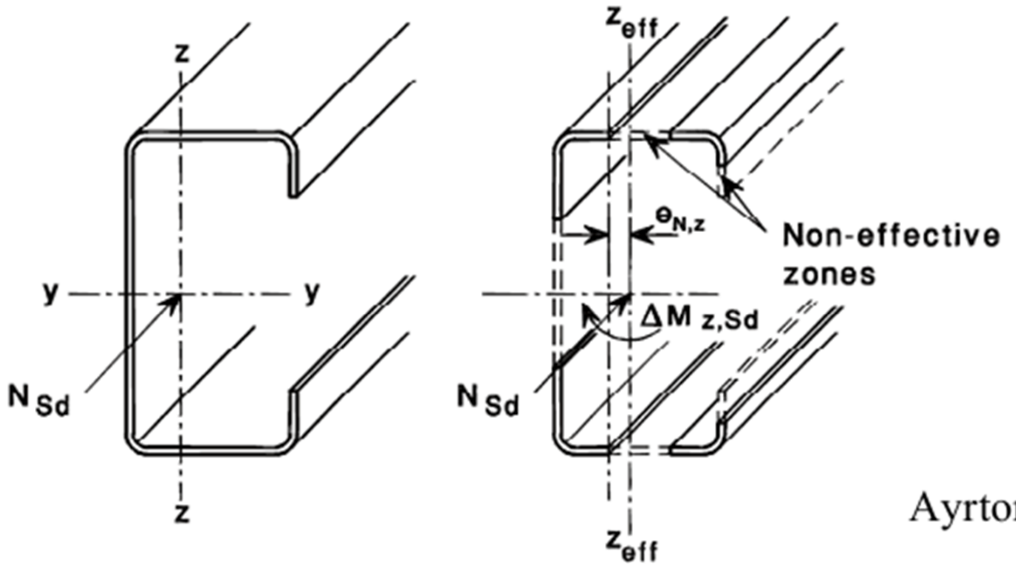


European design buckling curves

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background –** ***Class 4 sections: local-global interactive buckling***



$$N = A_{eff} f_y$$

$$A_{eff} = QA$$

$$N = A_{eff} f_y = Q\,A\,f_y$$

Ayrton-Perry formula *Interactive Buckling*

$$(Q - \overline{N})(1 - \overline{\lambda}^2 \overline{N}) = \alpha(\overline{\lambda} - 0.2)\overline{N}$$

$$\overline{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} = \frac{\lambda}{\lambda_1}\sqrt{Q}$$

- **Theoretical background – *Class 4 sections: local-global interactive buckling – Direct Strength Method*** Schafer & Peköz (1998)

The nominal axial strength, P_{ne}, for *flexural, torsional, or flexural-torsional buckling* is:

$$\lambda_c \le 1.5 \qquad P_{ne} = \left(0.658^{\lambda_c^2}\right) P_y$$

$$\lambda_c > 1.5 \qquad P_{ne} = \left(\frac{0.877}{\lambda_c^2}\right) P_y$$

where

$$\lambda_c = \sqrt{P_y / P_{cre}}$$

$$P_y = A f_y$$

P_{cre} = minimum of the critical elastic column buckling load for flexural, torsional or flexural-torsional buckling.

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Class 4 sections: local-global interactive buckling – Direct Strength Method*** Schafer & Peköz (1998)

The nominal axial strength, P_{nl}, for *local buckling* is:

$$\lambda_l \leq 0.776 \qquad P_{nl} = P_{ne}$$

$$\lambda_l > 0.776 \qquad P_{nl} = \left[1 - 0.15\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} P_{ne}$$

where

$$\lambda_l = \sqrt{P_{ne} / P_{crl}}$$

P_{crl} = critical elastic local buckling load

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Class 4 sections: local-global interactive buckling – Direct Strength Method*** Schafer & Peköz (1998)

The nominal axial strength, P_{nd}, for *distortional buckling* is:

$$\lambda_d \leq 0.561 \qquad P_{nd} = P_y$$

$$\lambda_d > 0.561 \qquad P_{nd} = \left[1 - 0.25\left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6} P_y$$

where

$$\lambda_d = \sqrt{P_y / P_{crd}}$$

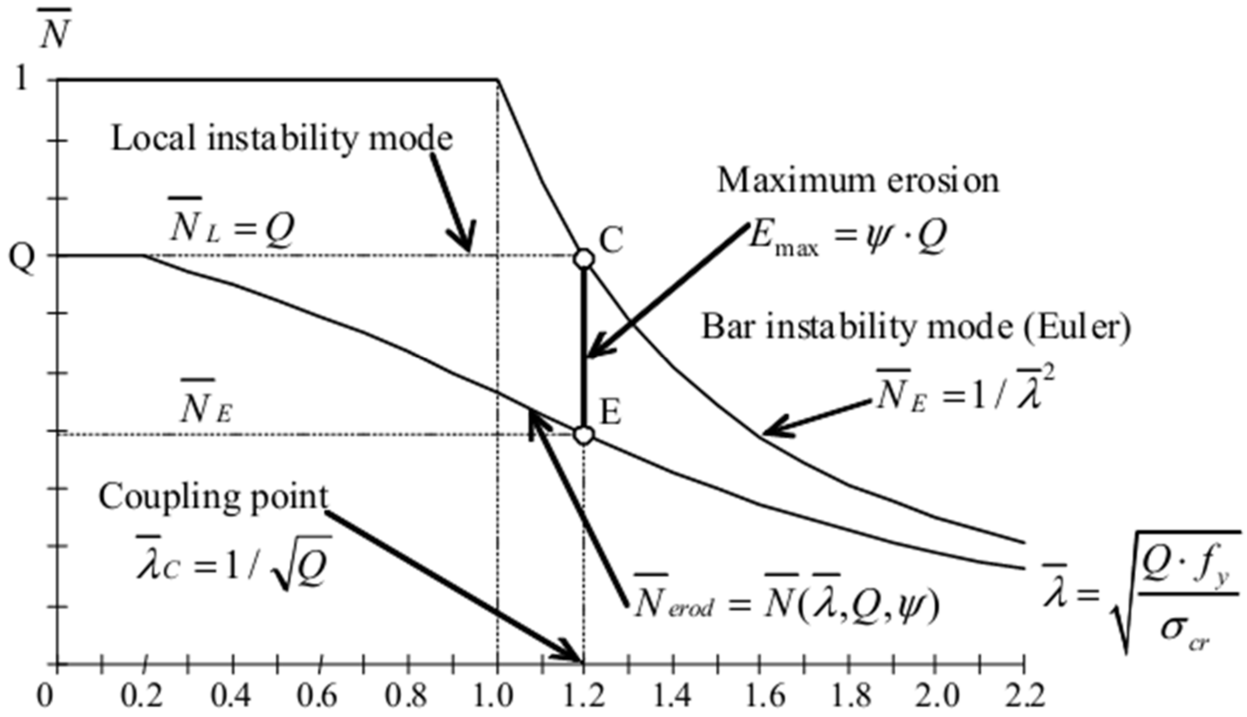
$$P_y = A f_y$$

P_{crd} = critical elastic distortional buckling load.

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background –** ***Class 4 sections: local-global interactive buckling – ECBL approach*** (Dubina, 2001)



The interactive buckling model based on the ECBL theory

Behaviour and Design Resistance of Bar Members

Compression members

- **Theoretical background – *Class 4 sections: local-global interactive buckling – ECBL approach*** (Dubina, 2001)

$$\overline{N}(\overline{\lambda}=1,\alpha)=\frac{1}{2}[2+0.8\alpha-\sqrt{(2+0.8\alpha)^2-4}]=1-\psi$$

$$\alpha=\frac{\psi^2}{0.8(1-\psi)}$$

$$\overline{N}=\frac{1+\alpha\left(\overline{\lambda}-0.2\right)+Q\overline{\lambda}^2}{2\overline{\lambda}^2}-$$

$$-\frac{1}{2\overline{\lambda}^2}\sqrt{\left[1+\alpha\left(\overline{\lambda}-0.2\right)+Q\overline{\lambda}^2\right]^2-4Q\overline{\lambda}^2}=(1-\psi)Q$$

which leads to

$$\alpha=\frac{\psi^2}{1-\psi}\cdot\frac{\sqrt{Q}}{1-0.2\sqrt{Q}}$$

Behaviour and Design Resistance of Bar Members

Compression members

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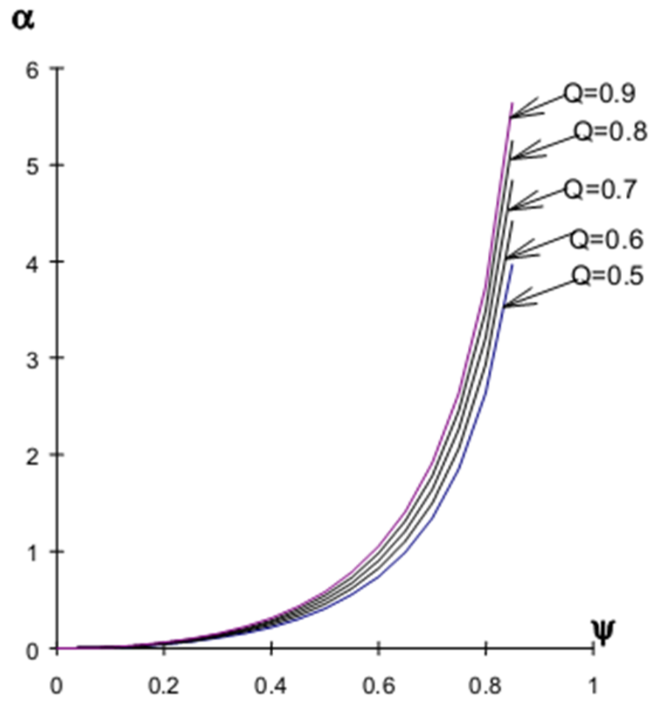

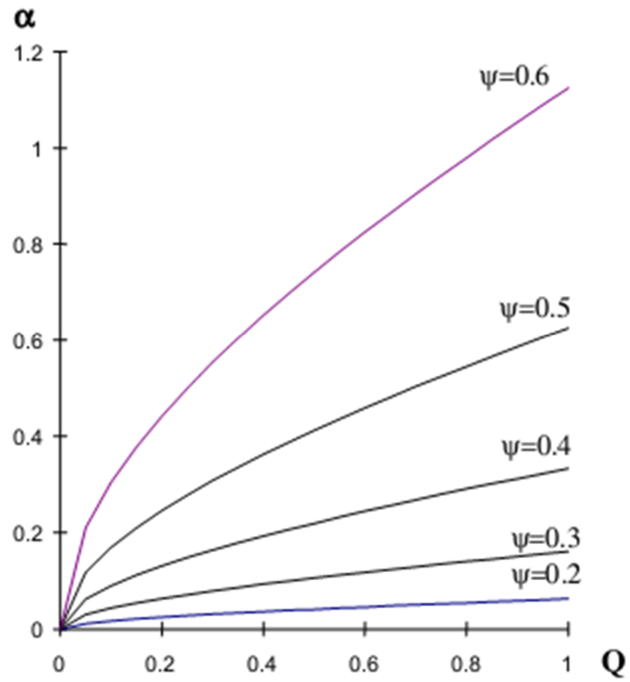

- **Buckling resistance of uniform members in compression. Design according to EN1993-1-3**

$$\frac{N_{Ed}}{N_{b.Rd}} \leq 1$$

where

N_{Ed} is the design value of the compression force;

$N_{b.Rd}$ is the design buckling resistance of the compression member.

The design buckling resistance of a compression member with Class 4 cross section should be taken as:

$$N_{b.Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} \qquad (4.46)$$

where χ is the reduction factor for the relevant buckling mode.

EN1993-1-1 + §§6.2.1 of EN1993-1-3

Behaviour and Design Resistance of Bar Members

Compression members

- **Buckling resistance of uniform members in compression. Design according to EN1993-1-3**

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$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad \text{but} \quad \chi \leq 1 \qquad (4.46)$$

γ_{M1}

where χ is the reduction factor $\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$.

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}}$$ for class 4 cross sections.

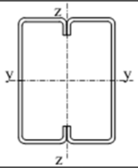
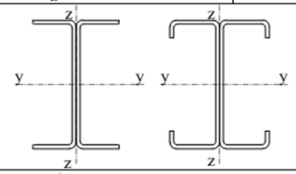
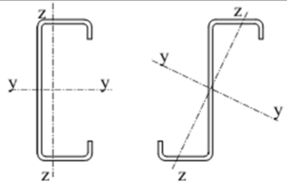
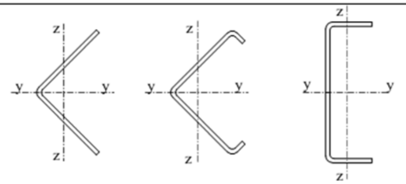
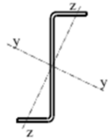
Behaviour and Design Resistance of Bar Members

Compression members

- **Buckling resistance of uniform members in compression. Design according to EN1993-1-3**

Flexural buckling

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} = \frac{L_{cr}}{i} \frac{\sqrt{\frac{A_{eff}}{A}}}{\lambda_1}$$

Type of cross section		Buckling about axis	Buckling curve
	if f_{yb} is used	any	b
	if f_{ya} is used *)	any	c
		$y-y$	a
		$z-z$	b
		any	b
 	or other cross section	any	c

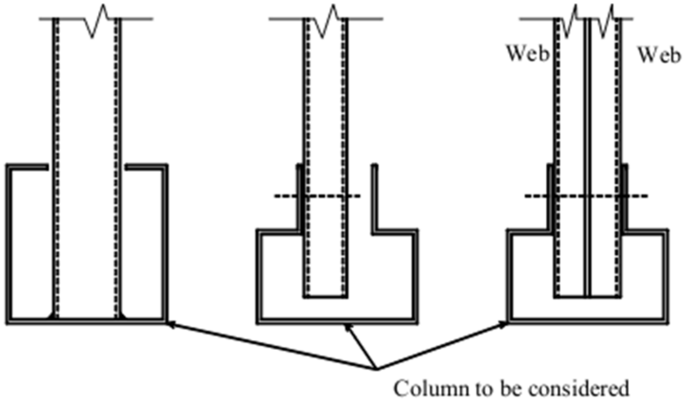
*) The average yield strength f_{ya} should not be used unless $A_{eff} = A_g$

Behaviour and Design Resistance of Bar Members

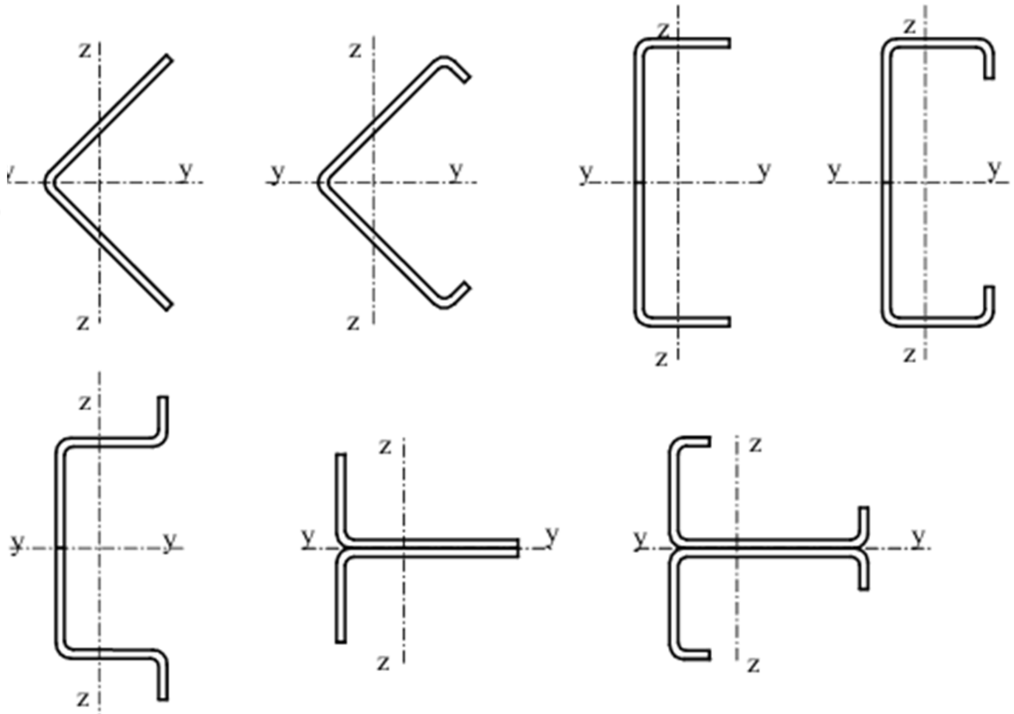
Compression members

- **Buckling resistance of uniform members in compression. Design according to EN1993-1-3**

Torsional and Flexural-Torsional buckling



Connections capable of giving partial torsional and warping restraint



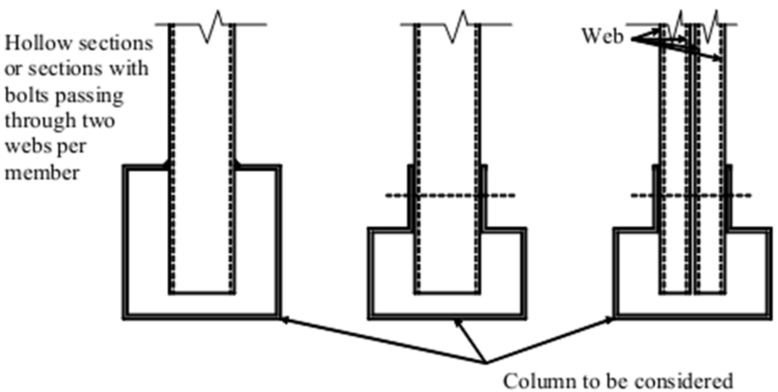
Mono-symmetric cross sections susceptible to torsional-flexural buckling

Behaviour and Design Resistance of Bar Members

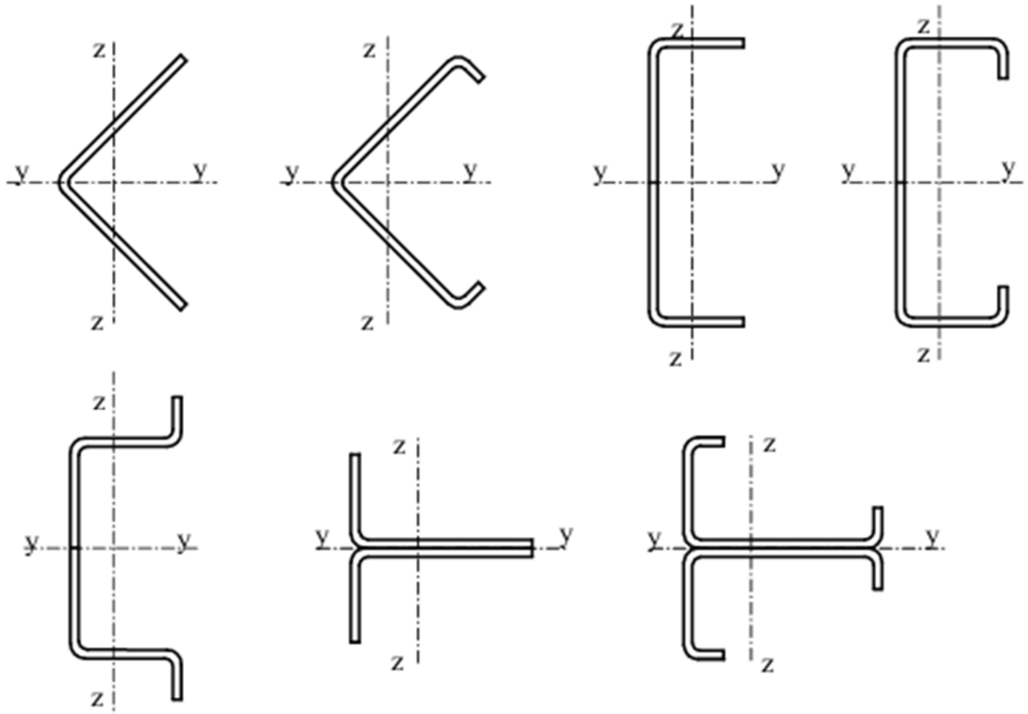
Compression members

- **Buckling resistance of uniform members in compression. Design according to EN1993-1-3**

Torsional and Flexural-Torsional buckling



Connections capable of giving significant torsional and warping restraint



Mono-symmetric cross sections susceptible to torsional-flexural buckling

Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

Basic Data

Height of column $H = 3.00\ \text{m}$

Span of floor $L = 6.00\ \text{m}$

Spacing between floor joists $S = 0.6\ \text{m}$

Distributed loads applied to the floor:

- dead load – lightweight slab: $1.5\ \text{kN/m}^2$

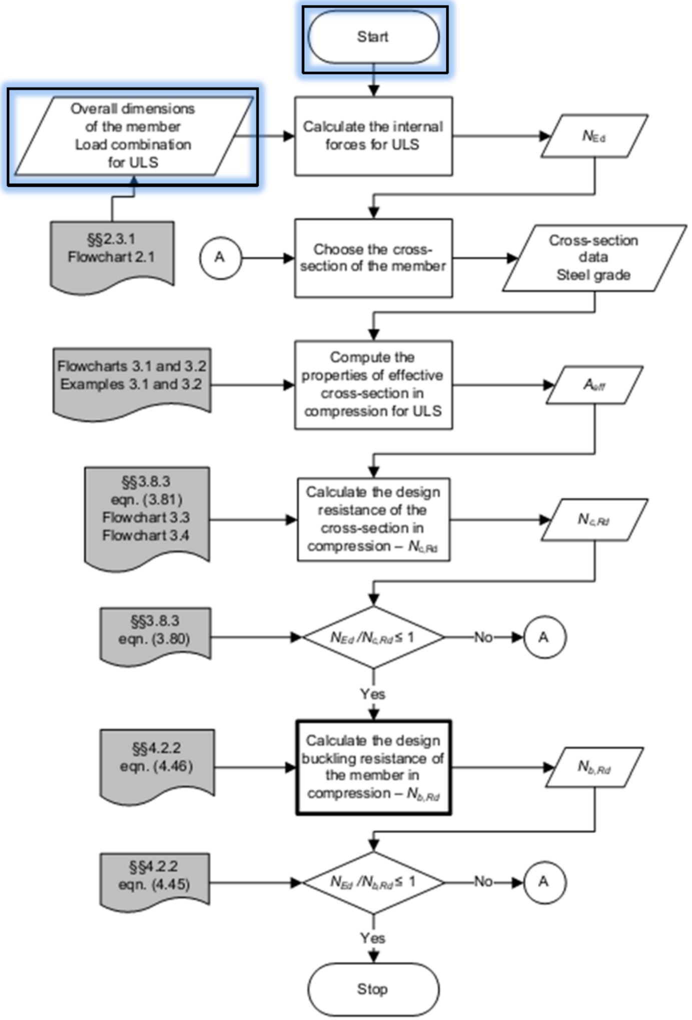
$$q_G = 1.5 \times 0.6 = 0.9\ \text{kN/m}$$

- imposed load: $3.00\ \text{kN/m}^2$

$$q_Q = 3.00 \times 0.6 = 1.80\ \text{kN/m}$$

Ultimate Limit State concentrated load

$$Q = 7.0\ \text{kN}$$



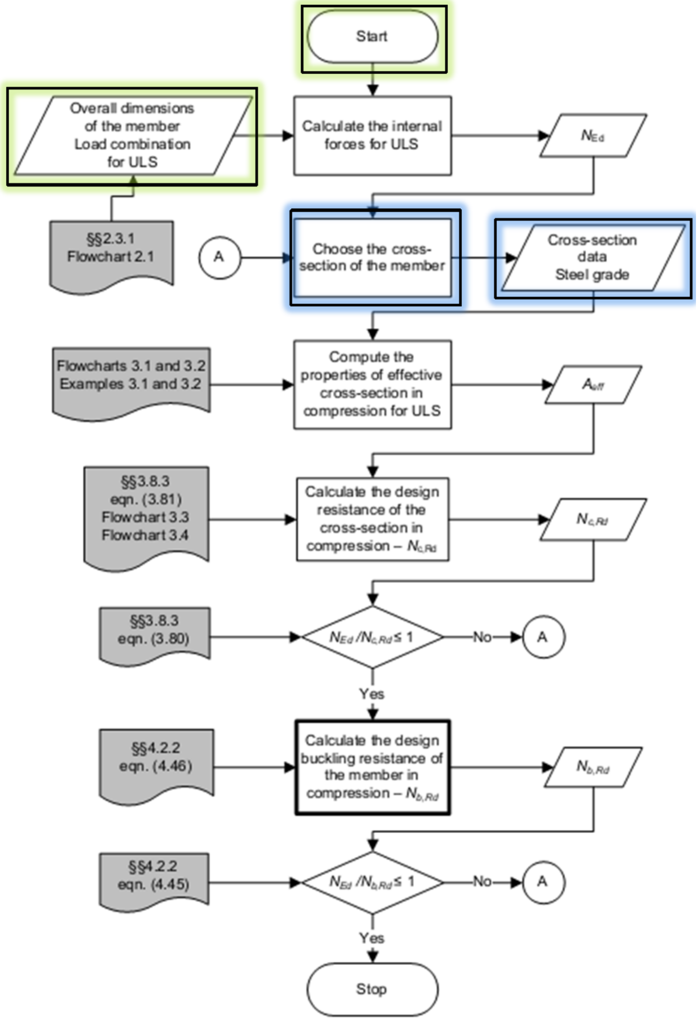
Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

Basic Data

Total height	$h = 150$ mm
Total width of flange	$b = 40$ mm
Total width of edge fold	$c = 15$ mm
Internal radius	$r = 3$ mm
Nominal thickness	$t_{nom} = 1.2$ mm
Steel core thickness (§§2.4.2.3)	$t = 1.16$ mm
Steel grade	S350GD+Z
Basic yield strength	$f_{yb} = 350\ \mathrm{N/mm^2}$
Modulus of elasticity	$E = 210000\ \mathrm{N/mm^2}$
Poisson's ratio	$\nu = 0.3$
Shear modulus	$G = 81000\ \mathrm{N/mm^2}$
Partial factors	$\gamma_{M0} = 1.0$
	$\gamma_{M1} = 1.0$
	$\gamma_G = 1.35$
	$\gamma_Q = 1.50$

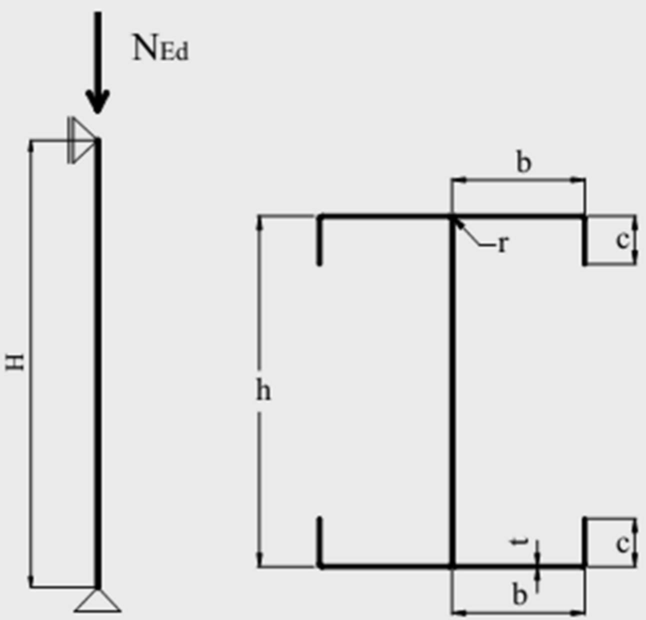


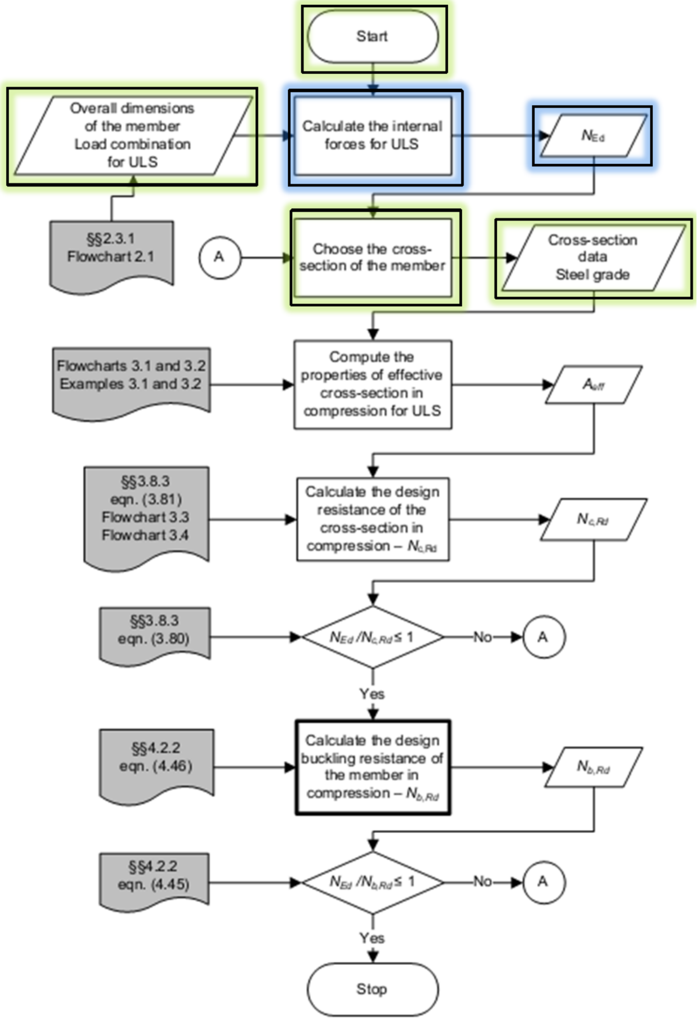
Compression members

- Example – *Design of an internal wall stud in compression*

Basic Data

$$N_{Ed} = (\gamma_G q_G + \gamma_Q q_Q) L/2 + Q = 16.79\text{kN}$$





Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

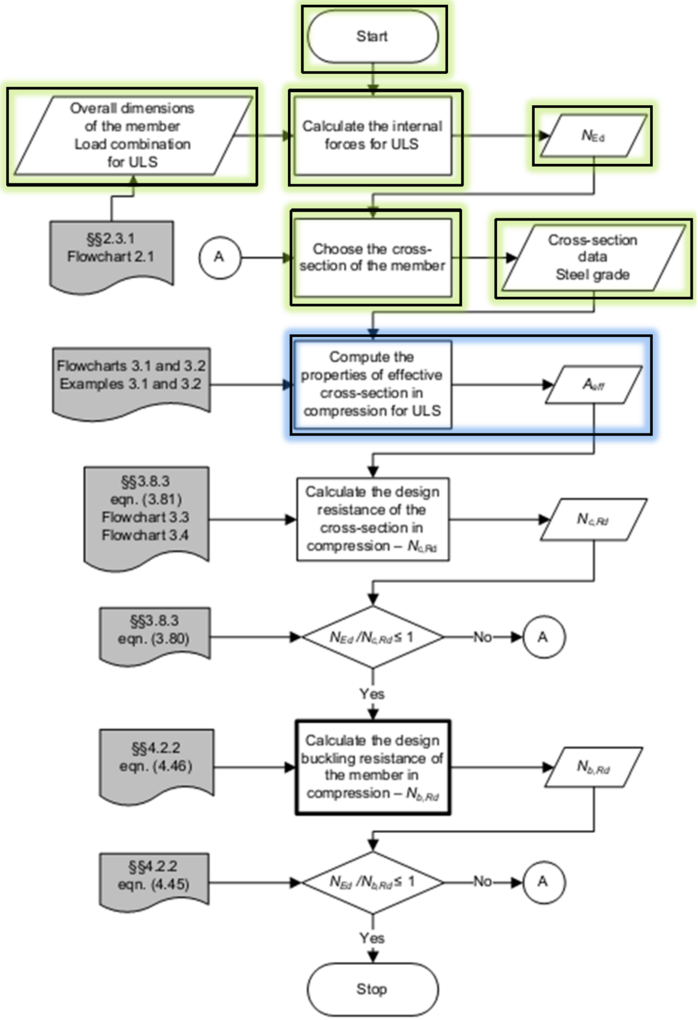
Properties of the gross cross section

Area of gross cross section:	$A = 592\ \text{mm}^2$
Radii of gyration:	$i_y = 57.2\ \text{mm}$; $i_z = 18\ \text{mm}$
Second moment of area about *y-y*:	$I_y = 1.936 \times 10^6\ \text{mm}^4$
Second moment of area about *z-z*:	$I_z = 19.13 \times 10^4\ \text{mm}^4$
Warping constant:	$I_w = 4.931 \times 10^8\ \text{mm}^6$
Torsion constant:	$I_t = 266\ \text{mm}^4$

Effective section properties of the cross section

Effective area of the cross section when subjected to compression only:

$$A_{eff} = 322\ \text{mm}^2$$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example – *Design of an internal wall stud in compression***

Resistance check of the cross section

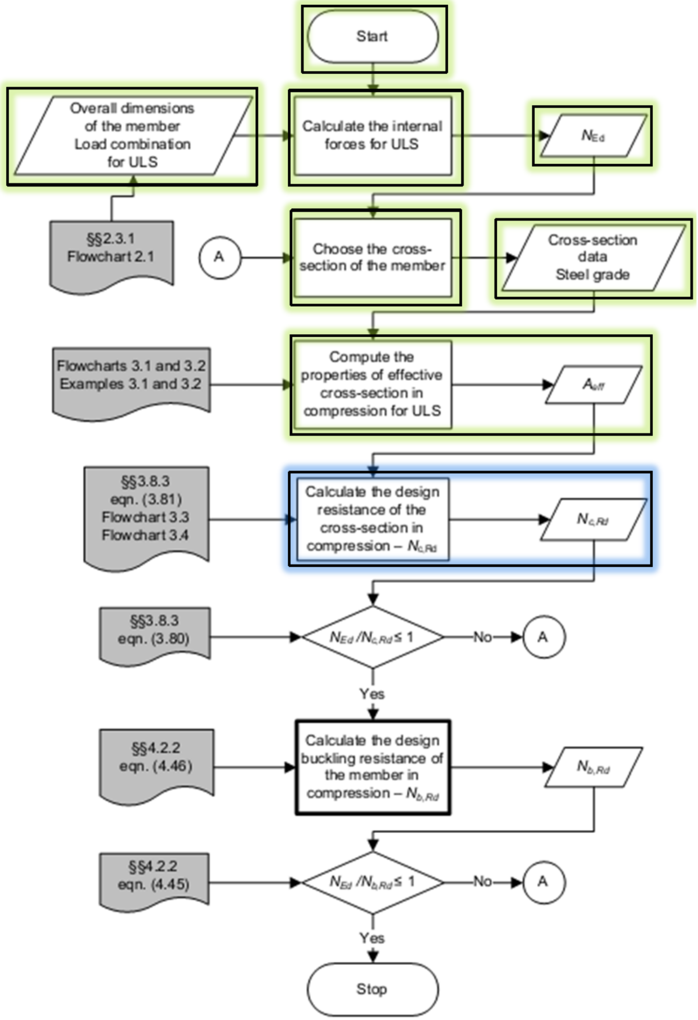
The following criterion should be met

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1$$

where

$$N_{c,Rd} = A_{eff} f_{yb} / \gamma_{M0}$$

The cross section is doubly symmetric and so the shift of the centroidal y–y axis is $e_{Ny} = 0$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

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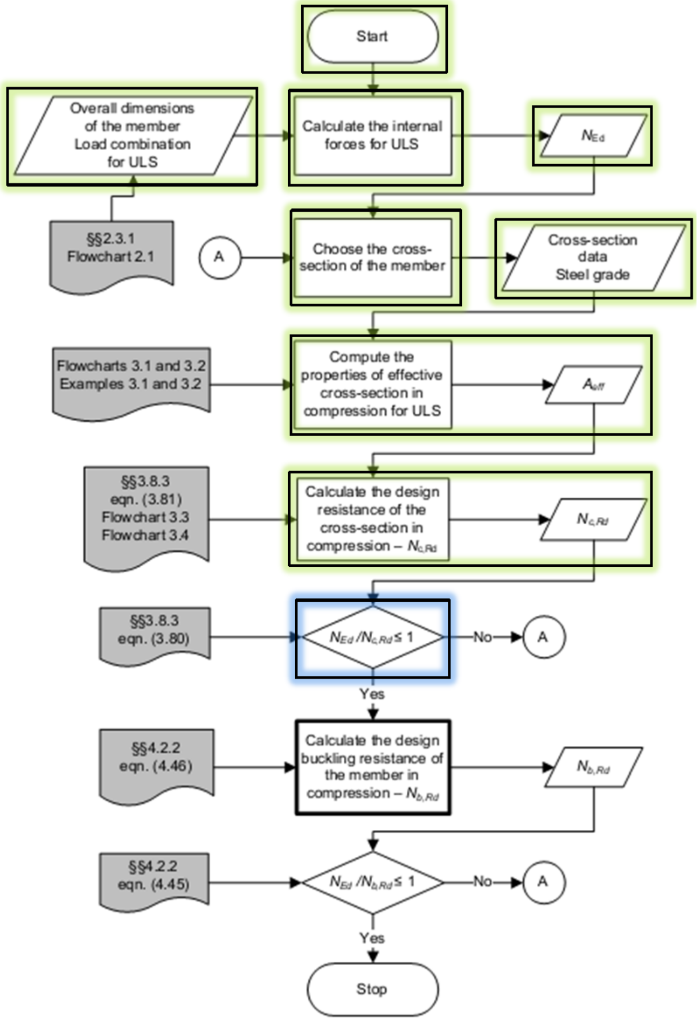
The resistance check is:

$$\frac{16.79 \times 10^3}{322 \times 350/1.0} = 0.149 < 1 - \text{OK}$$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

Buckling resistance check

Members which are subjected to axial compression should satisfy

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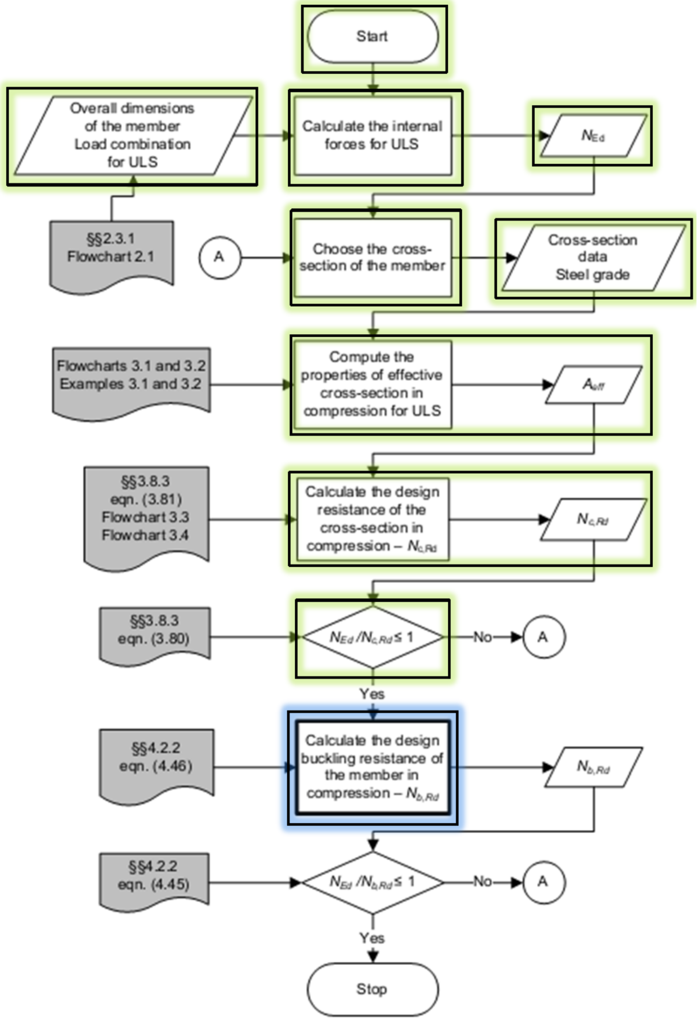
$$N_{b.Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}}$$, where χ is the reduction factor for the relevant buckling mode.

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad \text{but} \quad \chi \leq 1.0$$

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

α – imperfection factor

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}}$$



Behaviour and Design Resistance of Bar Members

Compression members

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$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1$$

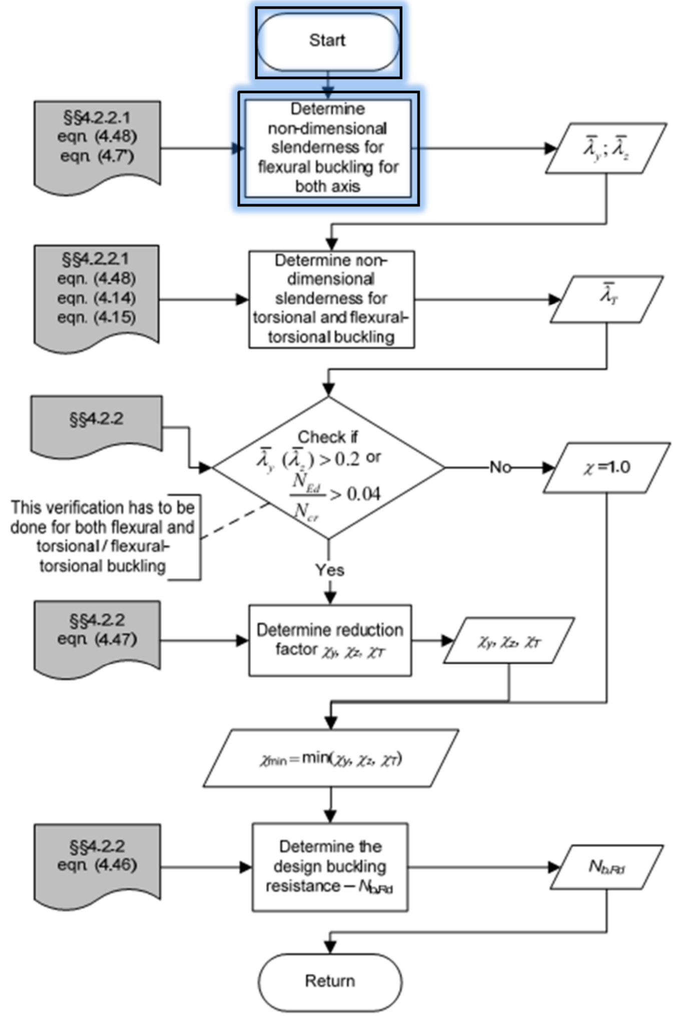
$$N_{b.Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}},$$ where χ is the reduction factor for the relevant buckling mode.

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad \text{but} \quad \chi \leq 1.0$$

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

α – imperfection factor

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}}$$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

Flexural buckling

$$\bar{\lambda}_F = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}} = \frac{L_{cr}}{i} \frac{\sqrt{A_{eff}/A}}{\lambda_1}$$

The buckling length:

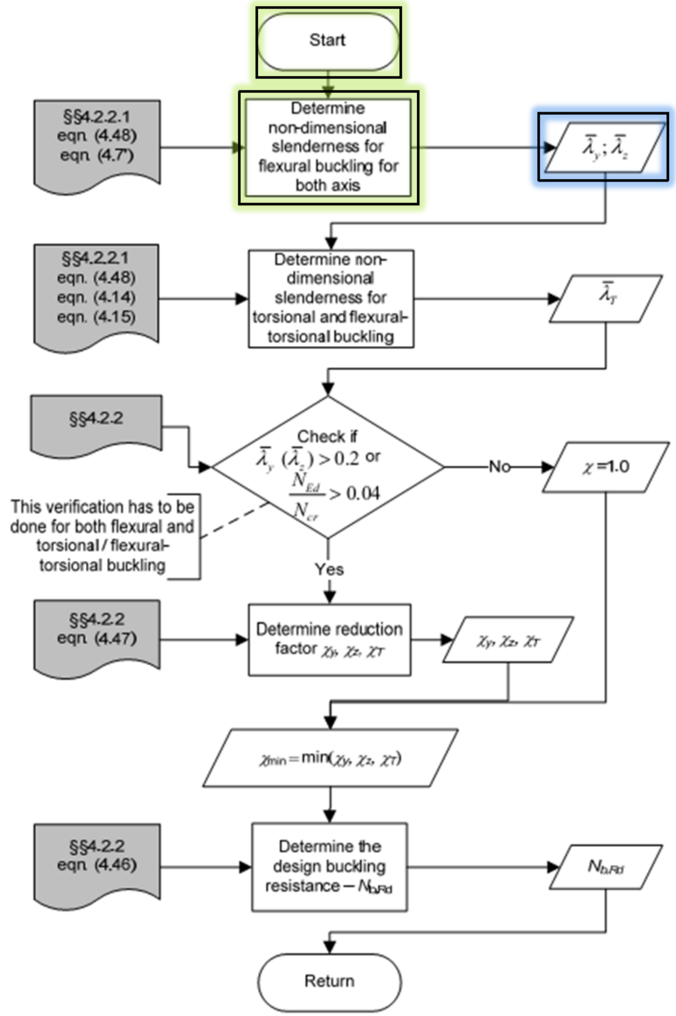
$$L_{cr,y} = L_{cr,z} = H = 3000 \text{ mm}$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_{yb}}} = \pi \times \sqrt{\frac{210000}{350}} = 76.95$$

Buckling about y–y axis

$$\bar{\lambda}_y = \frac{L_{cr,y}}{i_y} \frac{\sqrt{A_{eff}/A}}{\lambda_1} = \frac{3000}{57.2} \times \frac{\sqrt{322/592}}{76.95} = 0.503$$

$$\alpha_y = 0.21$$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example** – ***Design of an internal wall stud in compression***

Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

Flexural buckling

$$\bar{\lambda}_F = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}} = \frac{L_{cr}}{i} \frac{\sqrt{A_{eff}/A}}{\lambda_1}$$

The buckling length:

$$L_{cr} = L = H = 3000 \text{ mm}$$

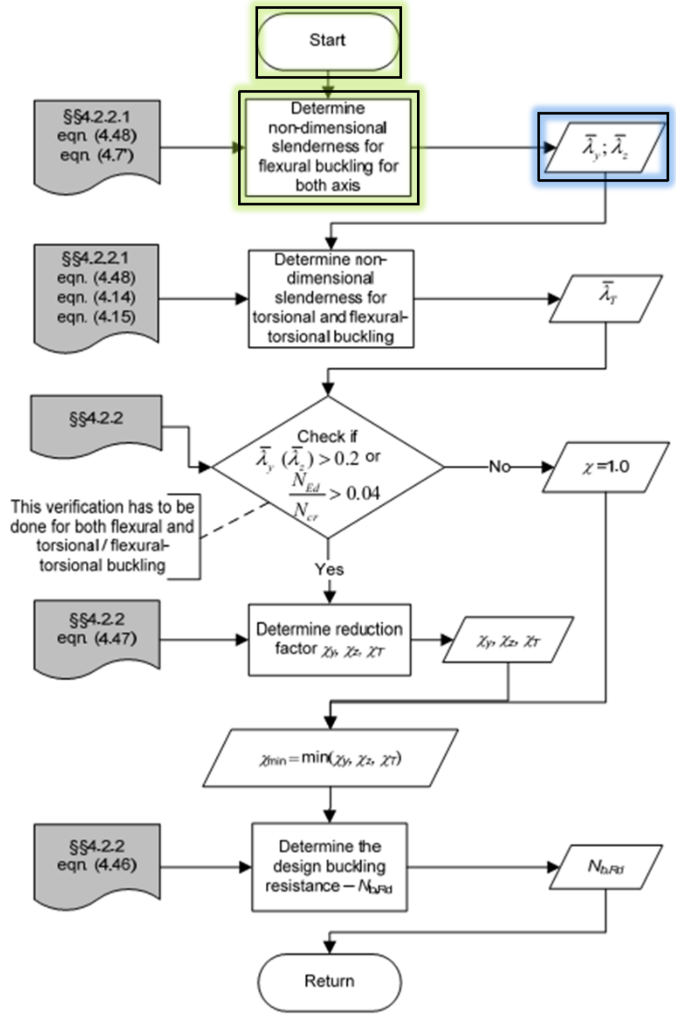
$$\phi_y = 0.5\left[1 + \alpha_y\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^{\,2}\right] =$$

$$= 0.5 \times \left[1 + 0.21 \times (0.503 - 0.2) + 0.503^2\right] = 0.658$$

$$\chi_y = \frac{1}{\phi_y + \sqrt{\phi_y^{\,2} - \bar{\lambda}_y^{\,2}}} = \frac{1}{0.658 + \sqrt{0.658^2 - 0.503^2}} = 0.924$$

$$\bar{\lambda}_y = \frac{L_{cr,y}}{i_y} \frac{\sqrt{A_{eff}/A}}{\lambda_1} = \frac{3000}{57.2} \times \frac{\sqrt{322/592}}{76.95} = 0.503$$

$$\alpha_y = 0.21$$



- **Example** – ***Design of an internal wall stud in compression***

Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

Buckling about z–z axis

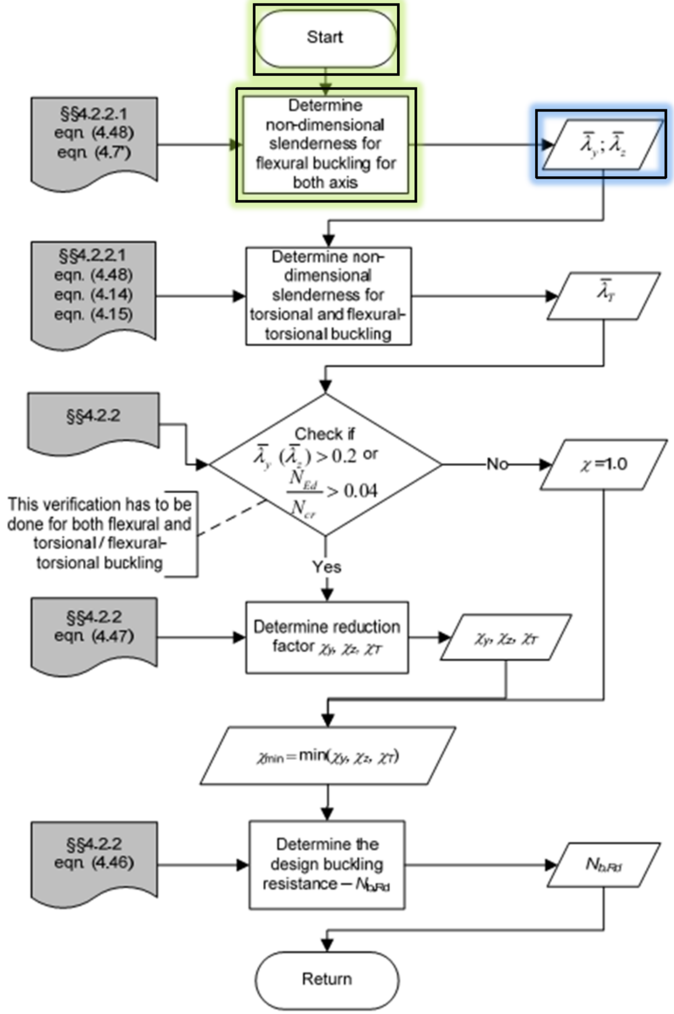
$$\bar{\lambda}_z = \frac{L_{cr,z}}{i_z}\frac{\sqrt{A_{eff}/A}}{\lambda_1} = \frac{3000}{18}\times\frac{\sqrt{322/592}}{76.95} = 1.597$$

$$\alpha_z = 0.34$$

$$\phi_z = 0.5\left[1+\alpha_z\left(\bar{\lambda}_z - 0.2\right)+\bar{\lambda}_z^{\,2}\right] =$$

$$= 0.5\times\left[1+0.34\times(1.597-0.2)+1.597^2\right] = 2.013$$

$$\chi_z = \frac{1}{\phi_z+\sqrt{\phi_z^{\,2}-\bar{\lambda}_z^{\,2}}} = \frac{1}{2.013+\sqrt{2.013^2-1.597^2}} = 0.309$$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

Torsional buckling

$$N_{cr,T} = \frac{1}{i_o^2}\left(GI_t + \frac{\pi^2 EI_w}{l_T^2}\right)$$

where

$$i_o^2 = i_y^2 + i_z^2 + y_o^2 + z_o^2$$

$$y_o = z_o = 0$$

$$i_o^2 = 57.2^2 + 18^2 + 0 + 0 = 3594 \text{ mm}^2$$

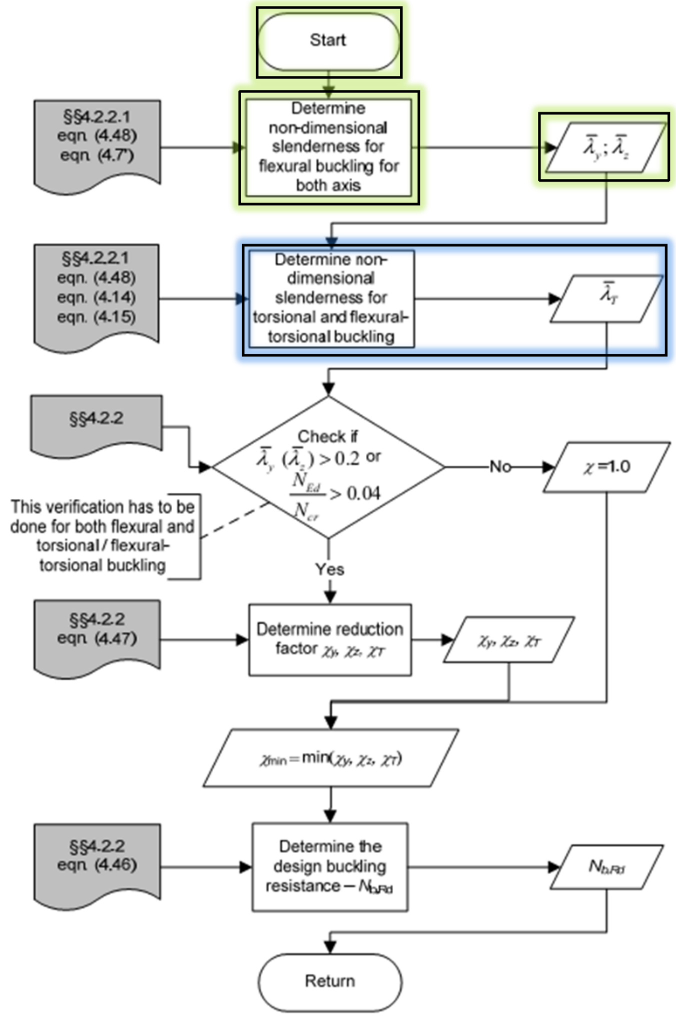
$$l_T = H = 3000 \text{ mm}$$

The elastic critical force for torsional buckling is:

$$N_{cr,T} = \frac{1}{3594} \times \left(81000 \times 266 + \frac{\pi^2 \times 210000 \times 4.931 \times 10^8}{3000^2}\right) =$$

$$= 37.59 \times 10^3 \text{ N}$$

The elastic critical force will be: $N_{cr} = N_{cr,T} = 37.59 \text{ kN}$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

Torsional buckling

The non-dimensional slenderness is:

$$\bar{\lambda}_T = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}} = \sqrt{\frac{322 \times 350}{37.59 \times 10^3}} = 1.731$$

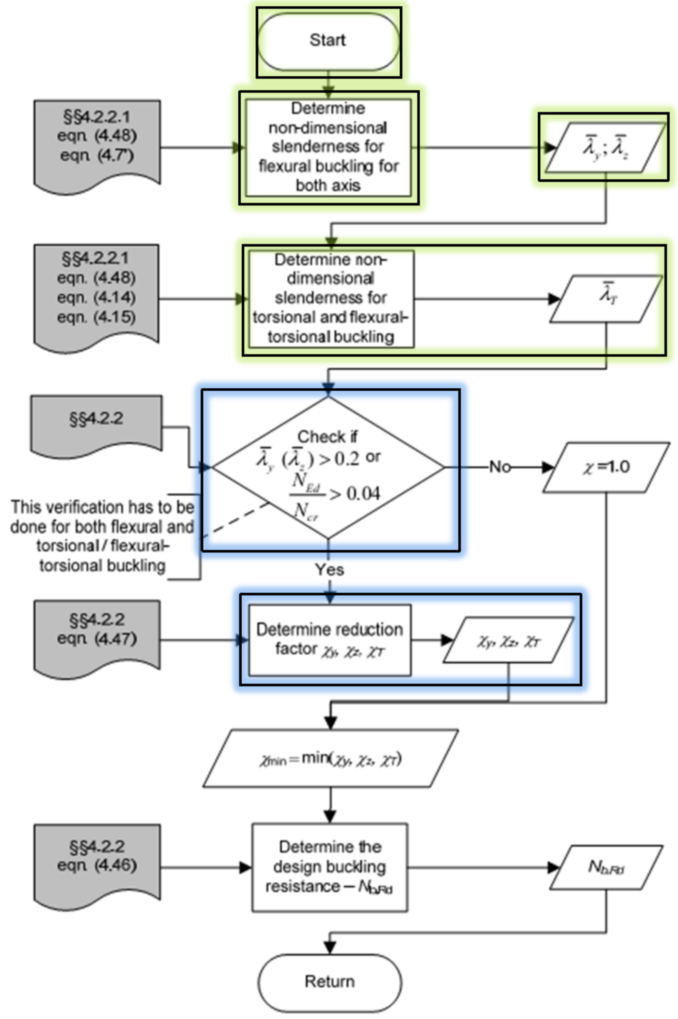
$$\alpha_T = 0.34 \text{ – buckling curve } b$$

$$\phi_T = 0.5\left[1 + \alpha_T\left(\bar{\lambda}_T - 0.2\right) + \bar{\lambda}_T^{\,2}\right] =$$

$$= 0.5 \times \left[1 + 0.34 \times \left(1.731 - 0.2\right) + 1.731^2\right] = 2.258$$

The reduction factor for torsional buckling is:

$$\chi_T = \frac{1}{\phi_T + \sqrt{\phi_T^{\,2} - \bar{\lambda}_T^{\,2}}} = \frac{1}{2.258 + \sqrt{2.258^2 - 1.731^2}} = 0.270$$



Behaviour and Design Resistance of Bar Members

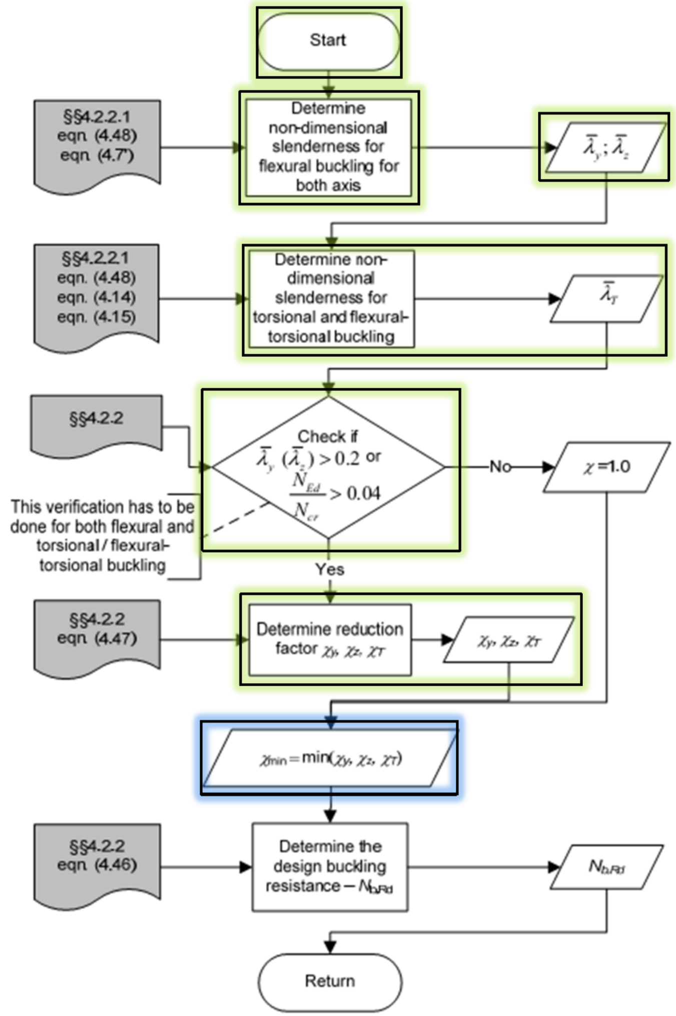
Compression members

- **Example –** ***Design of an internal wall stud in compression***

Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

$$\chi = \min(\chi_y, \chi_z, \chi_T) = \min(0.924, 0.309, 0.270) = 0.270$$



Behaviour and Design Resistance of Bar Members

Compression members

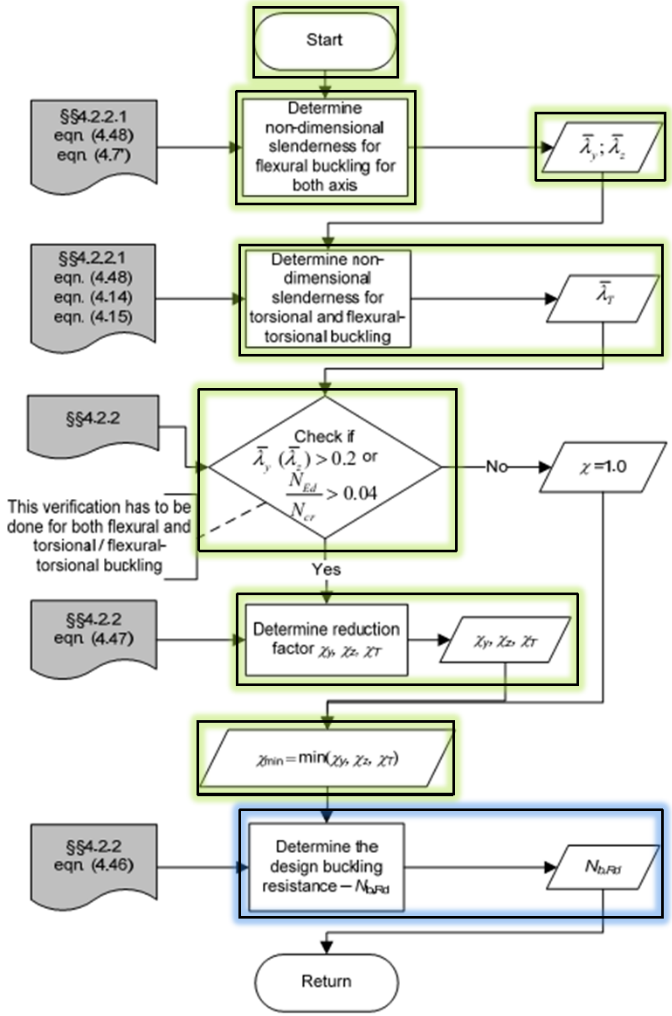
- **Example –** ***Design of an internal wall stud in compression***

Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

$$\chi = \min(\chi_y, \chi_z, \chi_T) = \min(0.924, 0.309, 0.270) = 0.270$$

$$N_{b.Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} = \frac{0.270 \times 322 \times 350}{1.00} = 30429\,\text{N} = 30.429\,\text{kN}$$



Behaviour and Design Resistance of Bar Members

Compression members

- **Example –** ***Design of an internal wall stud in compression***

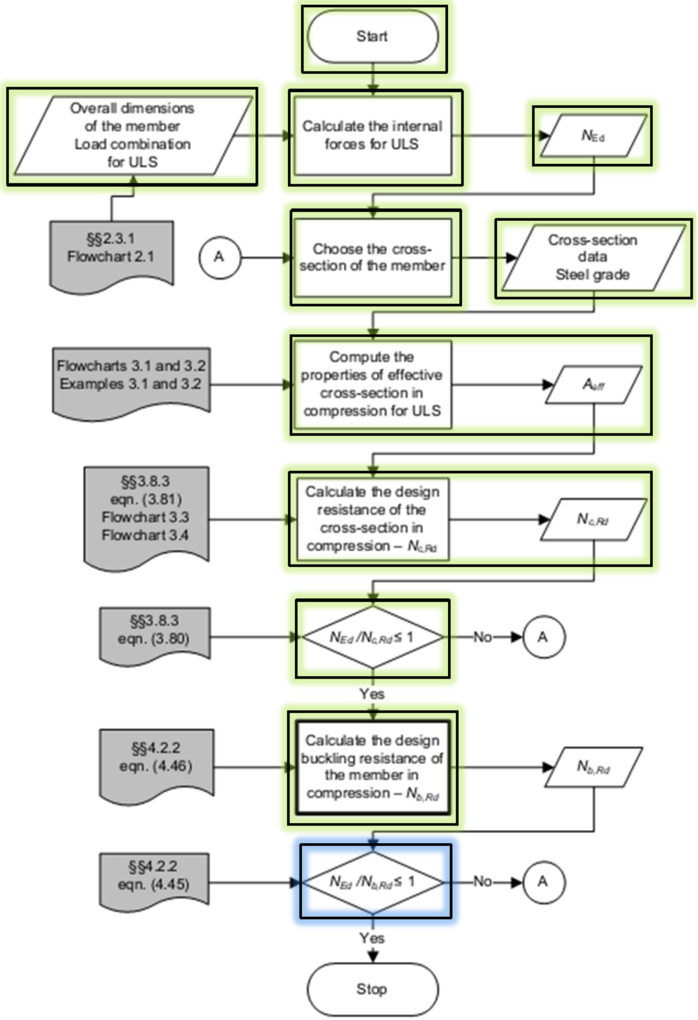
Buckling resistance check

Determination of the reduction factors χ_y, χ_z, χ_T

$$\chi = \min(\chi_y, \chi_z, \chi_T) = \min(0.924, 0.309, 0.270) = 0.270$$

$$N_{b.Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} = \frac{0.270 \times 322 \times 350}{1.00} = 30429\,\mathrm{N} = 30.429\mathrm{kN}$$

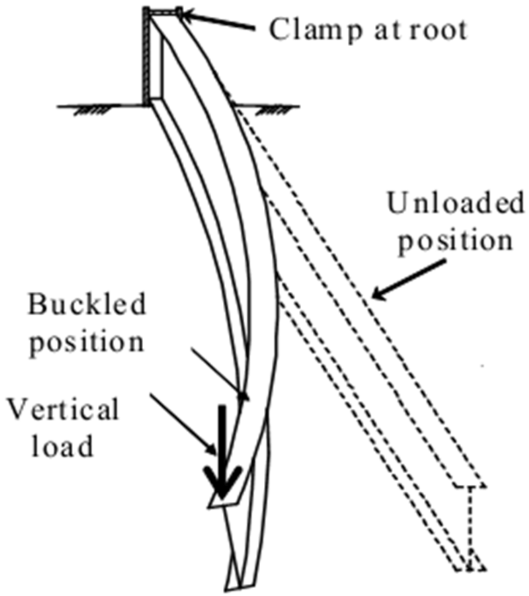
$$\frac{N_{Ed}}{N_{b,Rd}} = \frac{16.79}{30.429} = 0.552 \le 1 \text{ – OK}$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Theoretical background**



- bending about minor axis, z–z

$$EI_z \frac{d^2 v(x)}{dx^2} + \varphi(x) M_y = 0$$

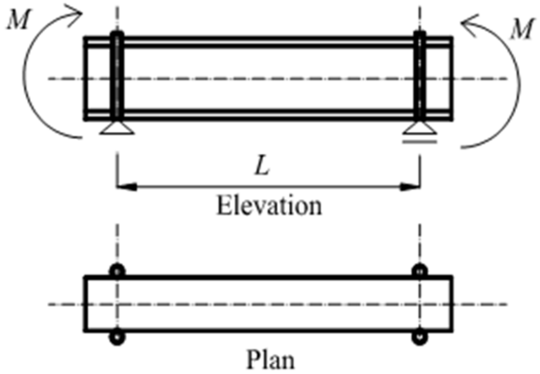
- torsion around x–x axis

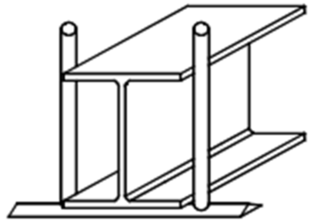
$$EI_w \frac{d^3 \varphi(x)}{dx^3} - GI_T \frac{d\varphi(x)}{dx} + M_y \frac{dv(x)}{dx} = 0$$

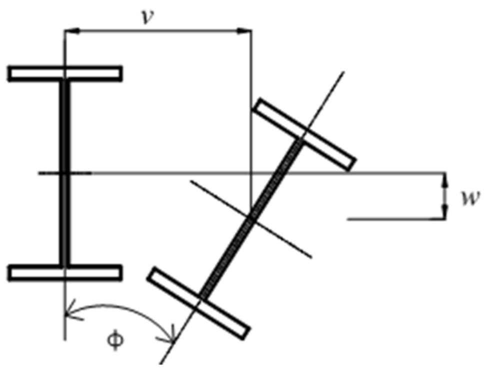
Behaviour and Design Resistance of Bar Members

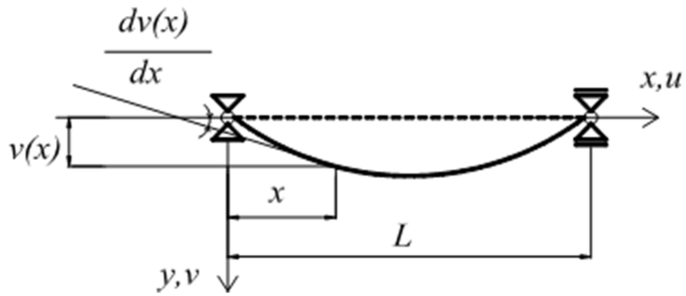
Buckling strength of bending members

- **Theoretical background**







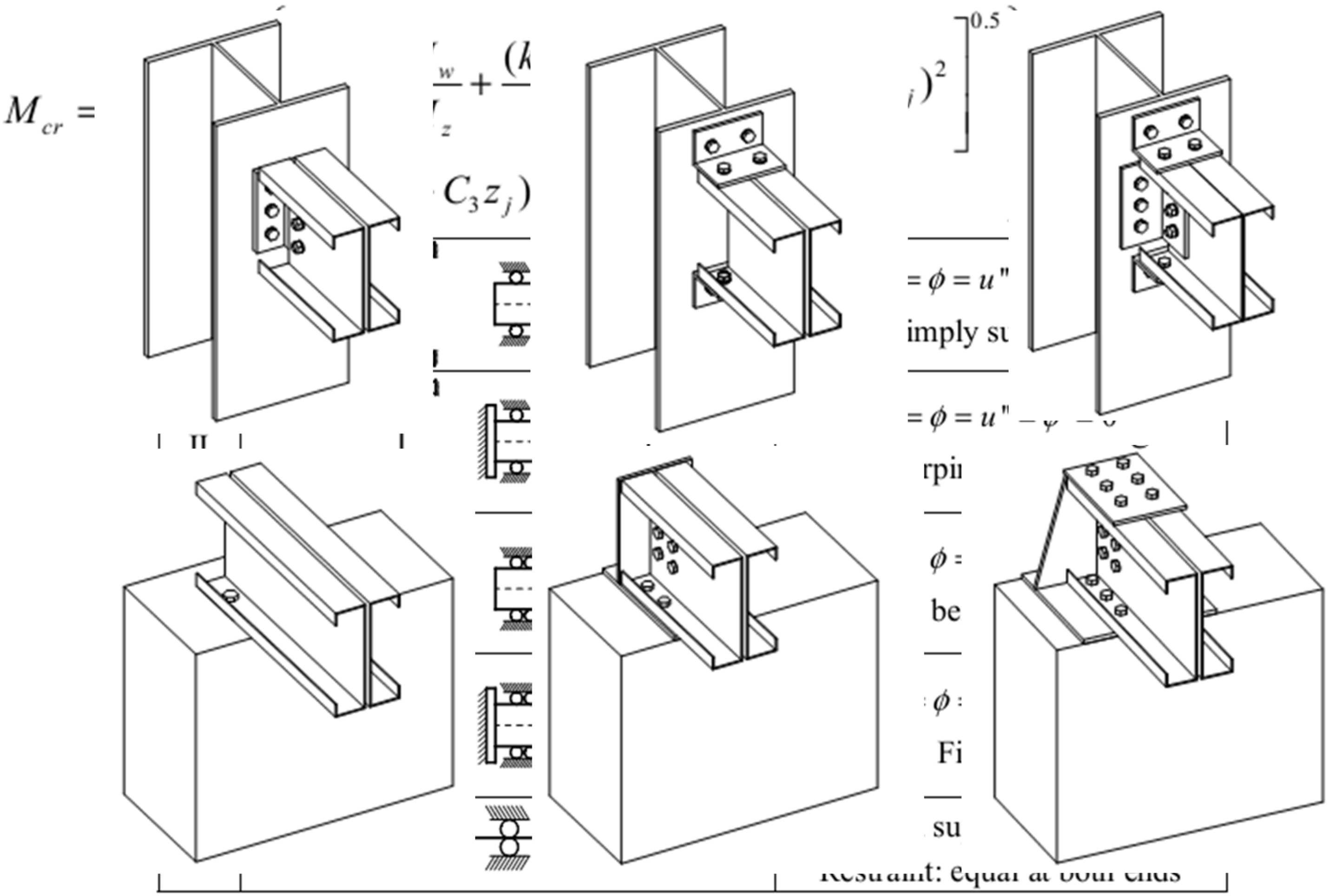


$$EI_w \frac{d^4\varphi(x)}{dx^4} - GI_T \frac{d^2\varphi(x)}{dx^2} - \frac{M_y{}^2}{EI_z}\varphi(x) = 0 \qquad \varphi = \varphi_0 \sin\frac{\pi x}{L} \qquad M_{cr} = \frac{\pi\sqrt{EI_z GI_T}}{L}\sqrt{1 + \frac{\pi^2 EI_w}{L^2 GI_T}}$$

Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

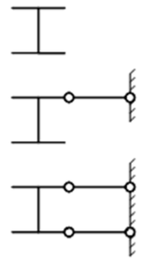
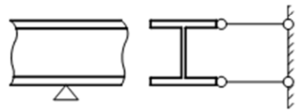
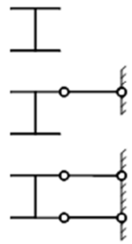
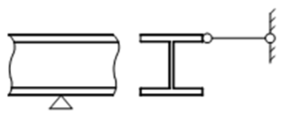
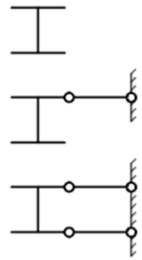
- **Theoretical background**



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Theoretical background**

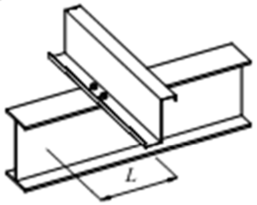
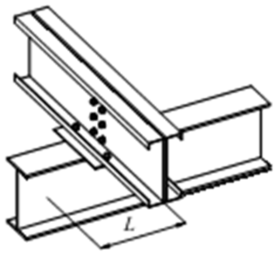
Restraint conditions		Effective length	
At root	At tip	Top flange loading	All other cases
		1.4L 1.4L 1.6L	0.8L 0.7L 0.6L
		2.5L 2.5L 1.5L	1.0L 0.9L 0.8L
		7.5L 7.5L 4.5L	3.0L 2.7L 2.4L

Idealised restraint conditions and effective length factors for cantilevers (Galambos, 1998)

Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Theoretical background**

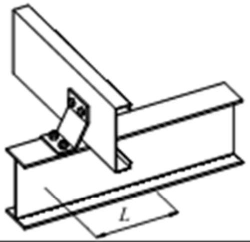
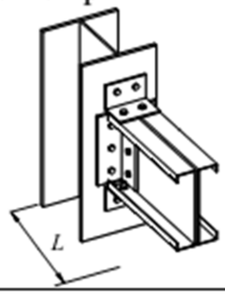
Restraint conditions		Loading conditions	
At support	At tip	Normal	Top flange (destabilizing)
(a) Continuous, with lateral restraint to top flange 	(1) Free (2) Lateral restraint to top flange (3) Torsional restraint (4) Lateral and torsional restraint	3.0L 2.7L 2.4L 2.1L	7.5L 7.5L 4.5L 3.6L
(b) Continuous, with partial torsional restraint 	(1) Free (2) Lateral restraint to top flange (3) Torsional restraint (4) Lateral and torsional restraint	2.0L 1.8L 1.6L 1.4L	5.0L 5.0L 3.0L 2.4L

Examples of practical details and corresponding effective lengths for cantilever beams without intermediate restraints

Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Theoretical background**

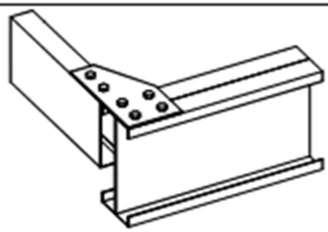
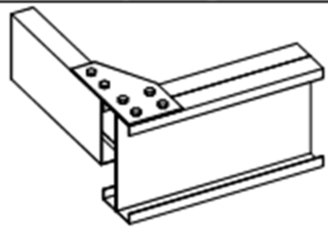
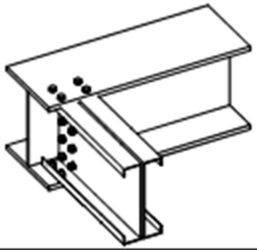
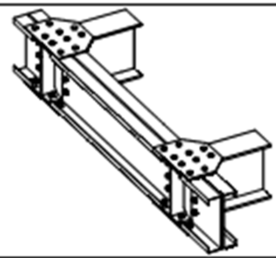
Restraint conditions		Loading conditions	
At support	At tip	Normal	Top flange (destabilizing)
(c) Continuous, with lateral and torsional restraint 	(1) Free	1.0L	2.5L
	(2) Lateral restraint to top flange	0.9L	2.5L
	(3) Torsional restraint	0.8L	1.5L
	(4) Lateral and torsional restraint	0.7L	1.2L
(d) Restrained laterally, torsionally and against rotation on plan 	(1) Free	0.8L	1.4L
	(2) Lateral restraint to top flange	0.7L	1.4L
	(3) Torsional restraint	0.6L	0.6L
	(4) Lateral and torsional restraint	0.5L	0.5L

Examples of practical details and corresponding effective lengths for cantilever beams without intermediate restraints

Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Theoretical background**

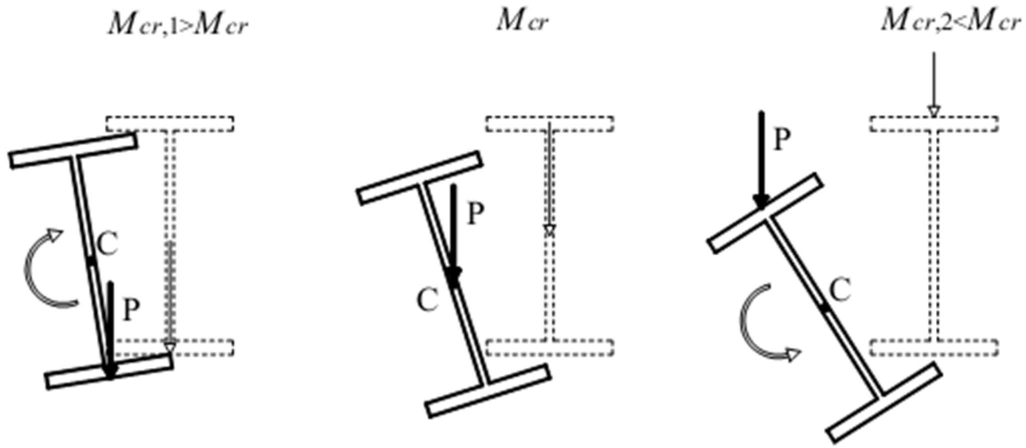
Tip restraint conditions			
(1) Free	(2) Lateral restraint to top flange	(3) Torsional restraint	(4) Lateral and torsional restraint
			
(not braced on plan)	(braced on plan in at least one bay)	(not braced on plan)	(braced on plan in at least one bay)

Examples of practical details and corresponding effective lengths for cantilever beams without intermediate restraints

Behaviour and Design Resistance of Bar Members

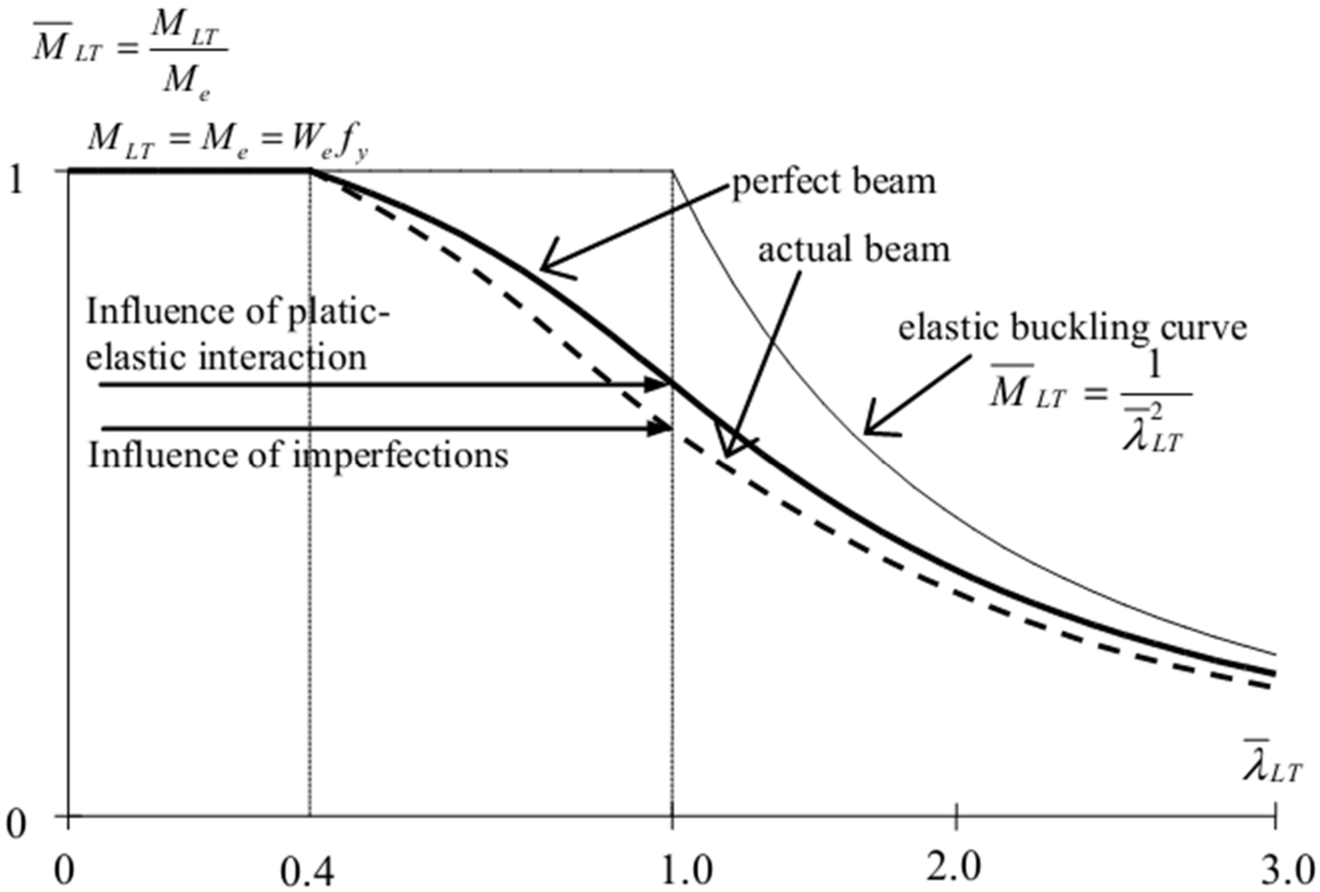
Buckling strength of bending members

- **Theoretical background**



Influence of the location of load application point

- **Theoretical background**

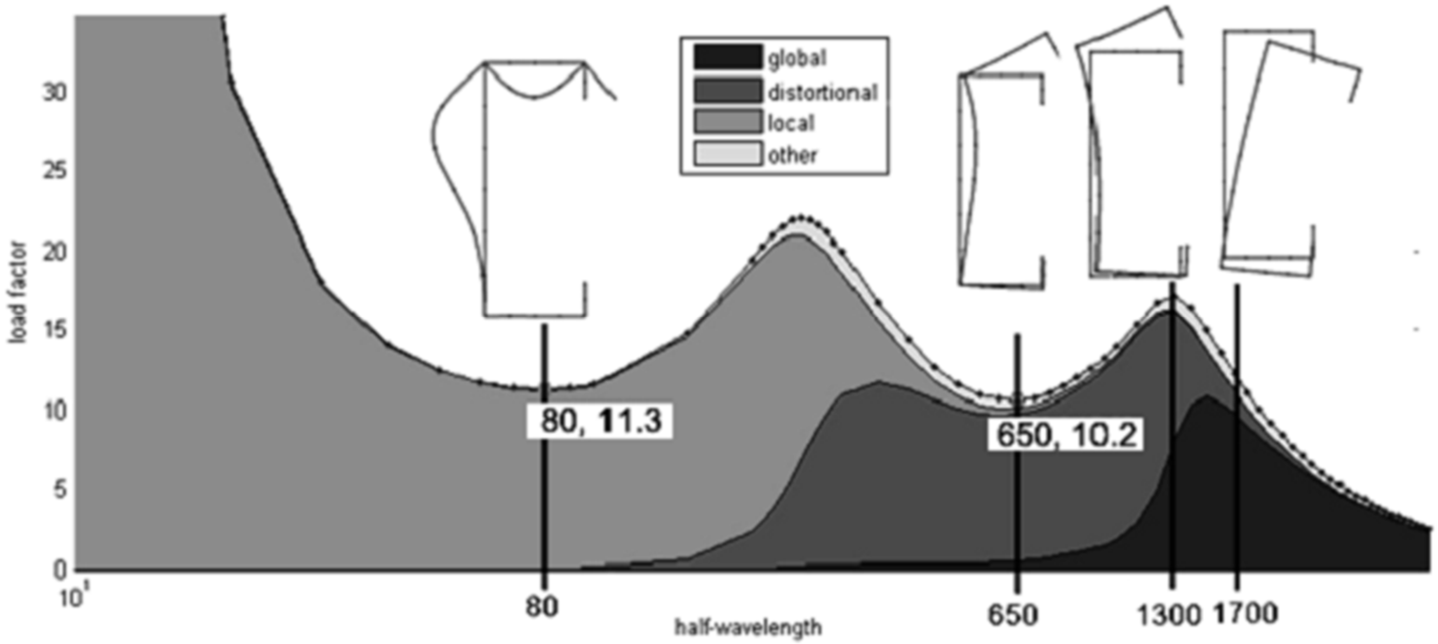


Lateral-torsional buckling curve of an unrestrained beam

Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Theoretical background**



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Design according to EN1993-1-3**

Lateral-torsional buckling of members subject to bending

A laterally unrestrained member subject to major axis bending should be verified against lateral-torsional buckling as follows:

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0 \qquad (4.55)$$

where

M_{Ed} is the design value of the moment;

$M_{b.Rd}$ is the design buckling resistance moment.

The design buckling resistance moment of a laterally unrestrained beam should be taken as:

$$M_{b,Rd} = \chi_{LT} W_y f_y / \gamma_{M1} \qquad (4.56)$$

where

W_y is the appropriate section modulus as follows:

$W_y = W_{el,y}$ is for class 3 cross section;

$W_y = W_{eff,y}$ is for class 4 cross section;

Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- **Design according to EN1993-1-3**

Lateral-torsional buckling of members subject to bending

In determining W_y, holes for fasteners at the beam ends need not to be taken into account.

χ_{LT} is the reduction factor for lateral-torsional buckling,

$$\chi_{LT} = \frac{1}{\phi_{LT} + \left(\phi_{LT}^2 - \bar{\lambda}_{LT}^2\right)^{0.5}} \text{, but } \chi_{LT} \leq 1$$

with: $\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$;

α_{LT} is the imperfection factor corresponding to buckling curve b, $\alpha_{LT} = 0.34$;

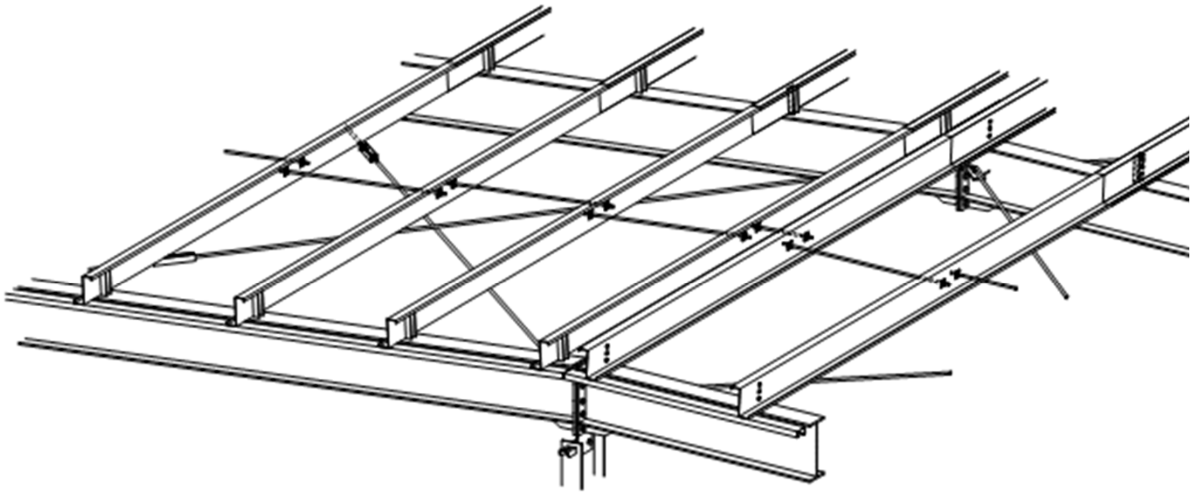
$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}};$$

Behaviour and Design Resistance of Bar Members

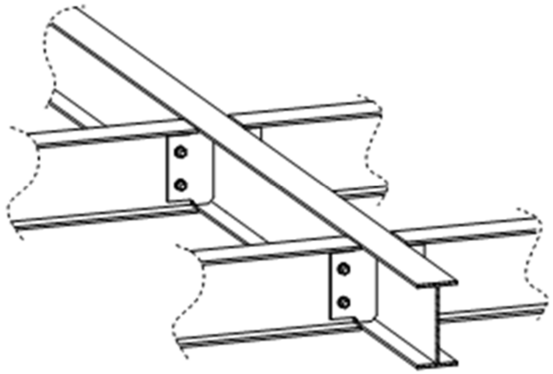
Buckling strength of bending members

- **Design according to EN1993-1-3**

Simplified assessment methods for beams with restraints in building



anti-sag bars for Z-purlins and Z-purlins for roof beams



secondary beams

Members with discrete lateral restraint

Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

Basic Data

Span of beam $L = 4.5\ \text{m}$

Spacing between beams $S = 3.0\ \text{m}$

Distributed loads applied to the joist:

self-weight of the beam $q_{G,beam} = 0.14\ \text{kN/m}$

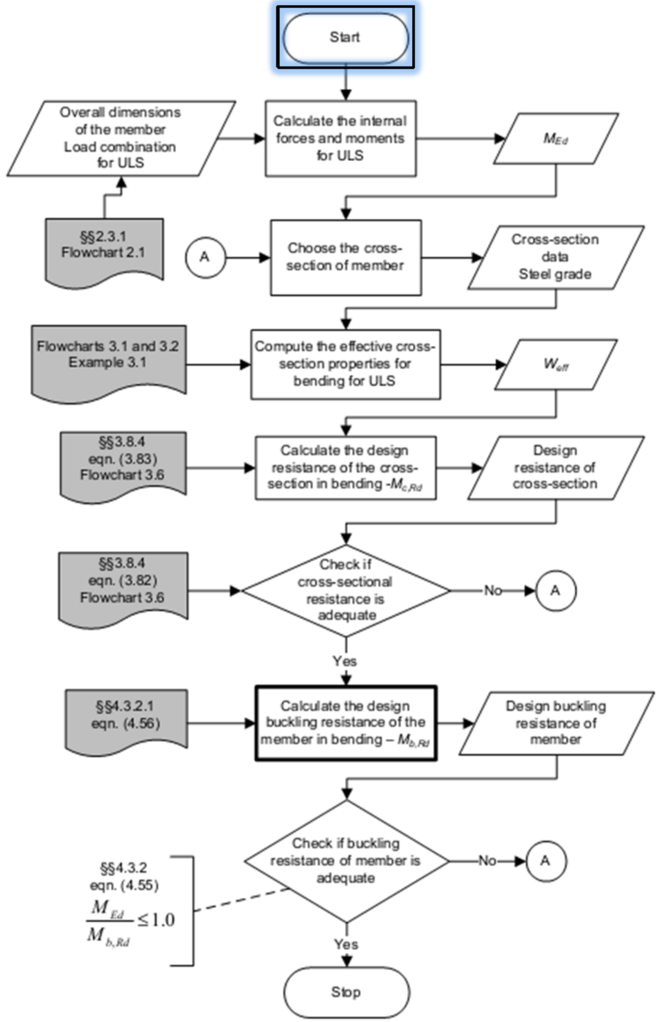
weight of the floor and $0.6\ \text{kN/m}^2$

$q_{G,slab} = 0.55 \times 3.0 = 1.65\ \text{kN/m}$

total dead load $q_G = q_{G,beam} + q_{G,slab} = 1.79\ \text{kN/m}$

imposed load $1.50\ \text{kN/m}^2$

$q_Q = 1.50 \times 3.0 = 4.50\ \text{kN/m}$

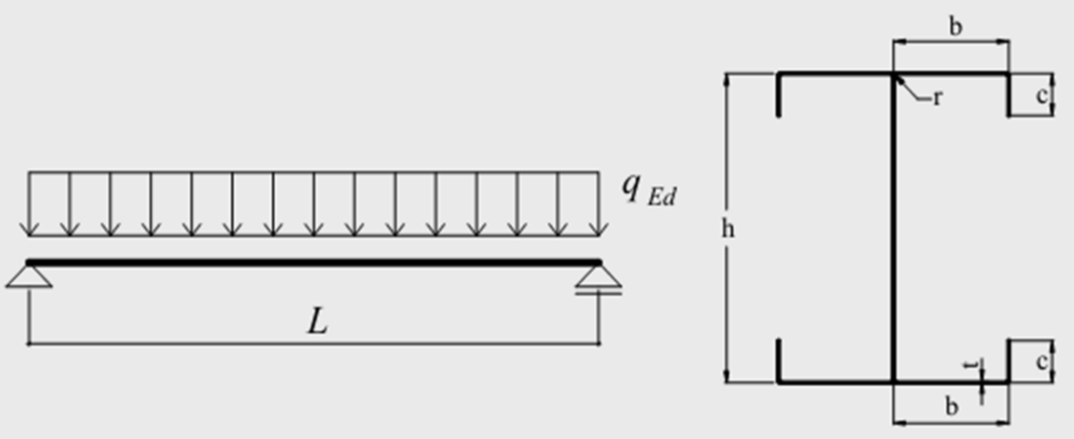


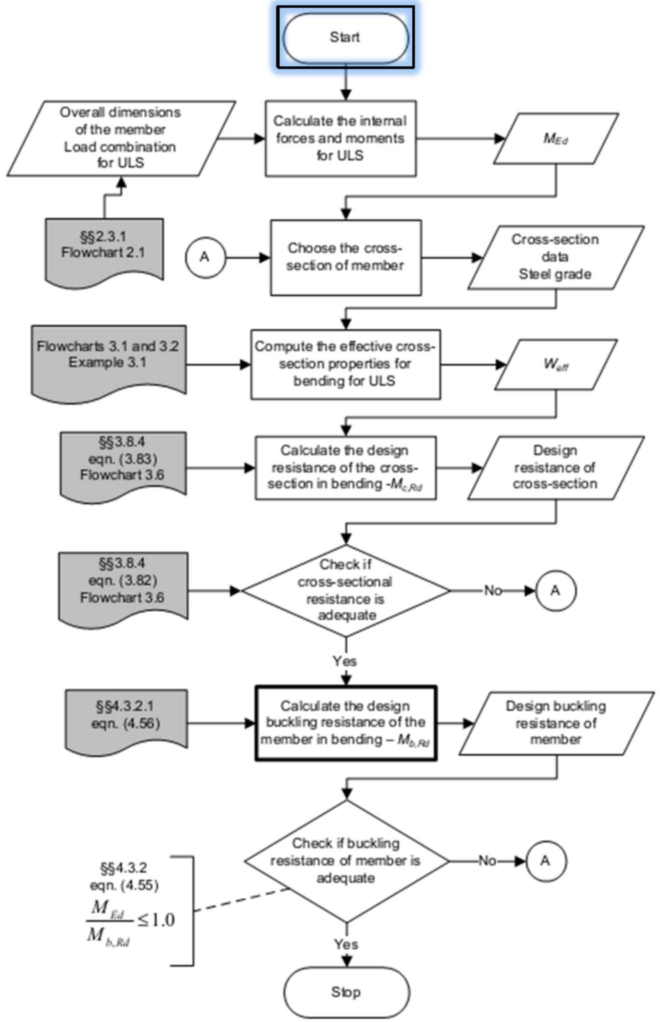
Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

Basic Data





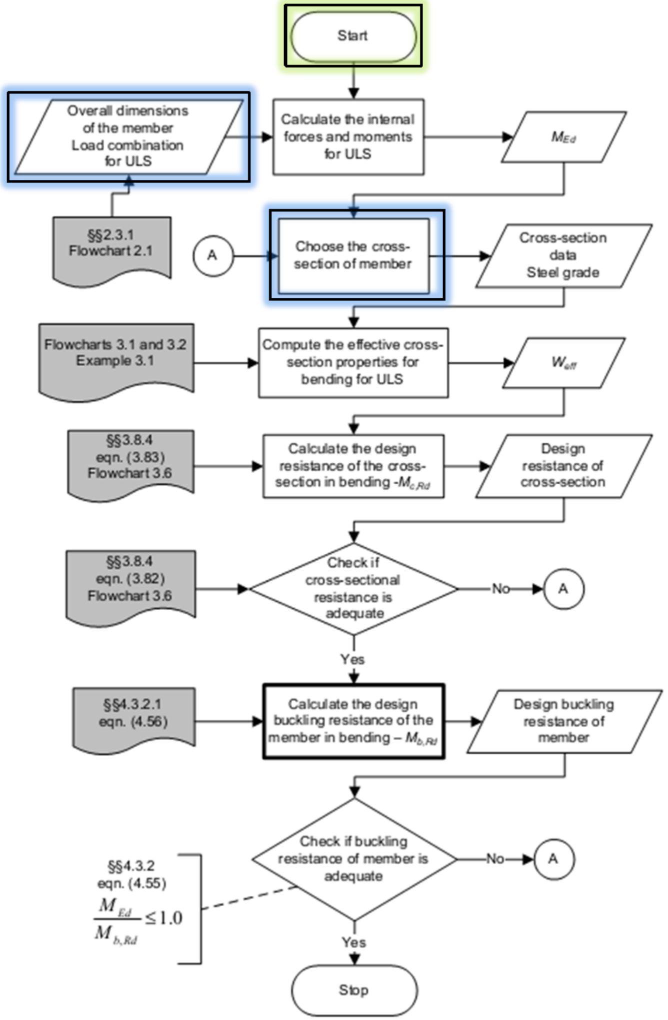
Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- ***Example – Design of an cold-formed steel beam in bending***

The dimensions of the cross section and the material properties are:

Total height	$h = 250$ mm
Total width of flanges	$b = 70$ mm
Total width of edge fold	$c = 25$ mm
Internal radius	$r = 3$ mm
Nominal thickness	$t_{nom} = 3.0$ mm
Steel core thickness (§§2.4.2.3)	$t = 2.96$ mm
Steel grade	S350GD+Z
Basic yield strength	$f_{yb} = 350\ \mathrm{N/mm^2}$
Modulus of elasticity	$E = 210000\ \mathrm{N/mm^2}$
Poisson's ratio	$\nu = 0.3$
Partial factors	$\gamma_{M0} = 1.0$
	$\gamma_{M1} = 1.0$
	$\gamma_G = 1.35$
	$\gamma_Q = 1.50$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- ***Example – Design of an cold-formed steel beam in bending***

Properties of the gross cross section

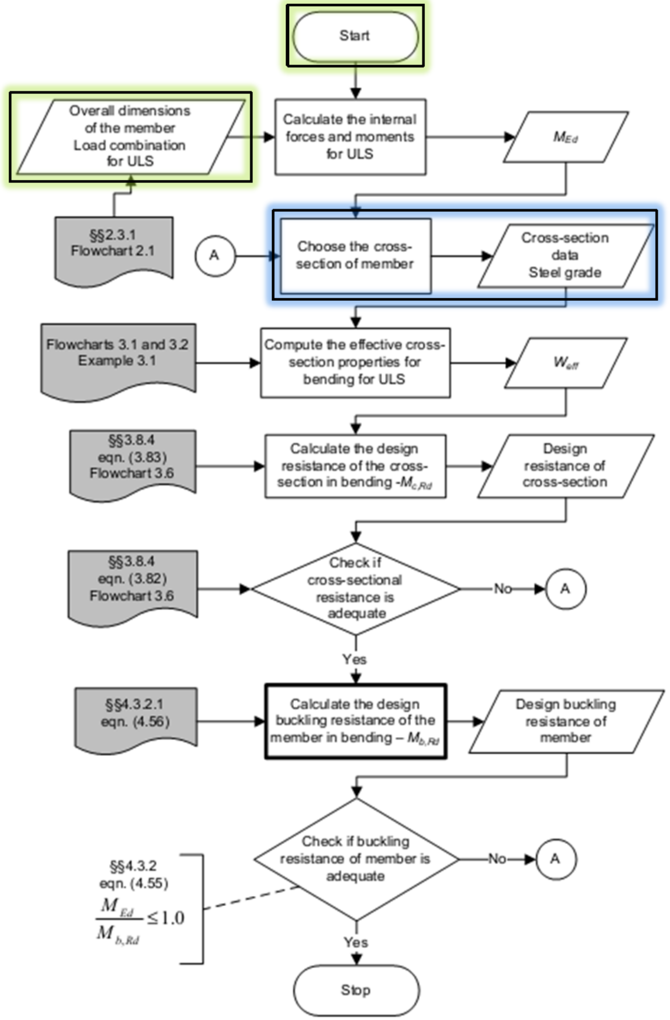
Second moment of area about y–y: $I_y = 2302.15 \times 10^4 \text{ mm}^4$

Second moment of area about z–z: $I_z = 244.24 \times 10^4 \text{ mm}^4$

Radii of gyration: $i_y = 95.3 \text{ mm}$; $i_z = 31 \text{ mm}$

Warping constant: $I_w = 17692.78 \times 10^6 \text{ mm}^6$

Torsion constant: $I_t = 7400 \text{ mm}^4$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

Effective section properties at the ultimate limit state

Second moment of area of cold-formed lipped channel section subjected to bending about its major axis:

$$I_{eff,y} = 22688890 \text{ mm}^4$$

Position of the neutral axis:

- from the flange in compression: $z_c = 124.6$ mm

- from the flange in tension: $z_t = 122.4$ mm

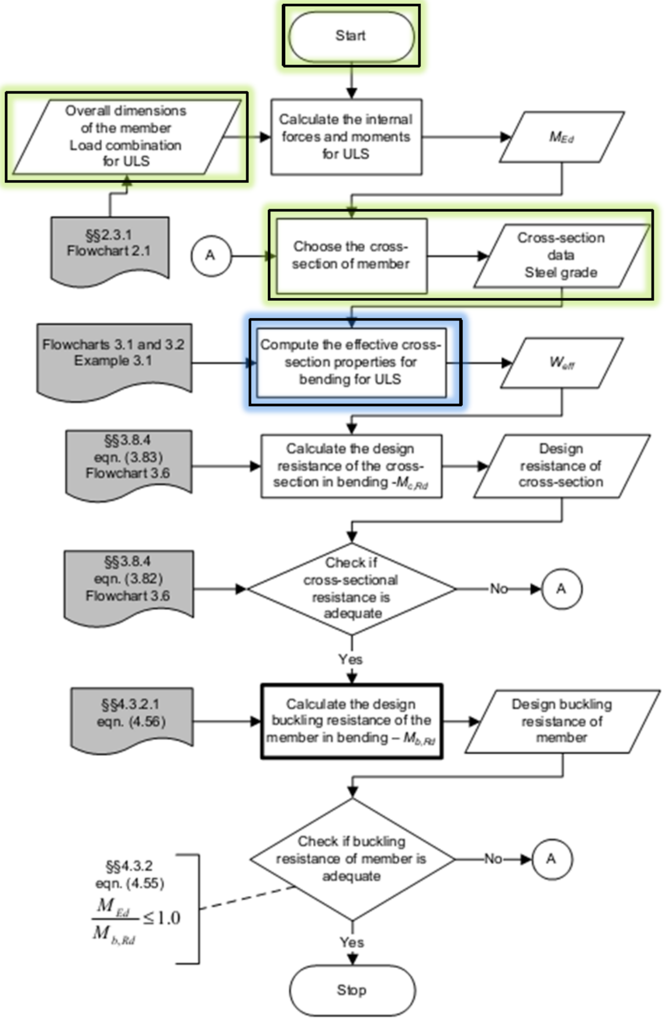
Effective section modulus:

- with respect to the flange in compression:

$$W_{eff,y,c} = \frac{I_{eff,y}}{z_c} = \frac{22688890}{124.6} = 182094 \text{ mm}^3$$

- with respect to the flange in tension:

$$W_{eff,y,t} = \frac{I_{eff,y}}{z_t} = \frac{22688890}{122.4} = 185367 \text{ mm}^3$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- ***Example – Design of an cold-formed steel beam in bending***

Effective section properties at the ultimate limit state

Second moment of area of cold-formed lipped channel section subjected to bending about its major axis:

$$I_{eff,y} = 22688890 \text{ mm}^4$$

Position of the neutral axis:

- from the flange in compression: $z_c = 124.6 \text{ mm}$
- from the flange in tension: $z_t = 122.4 \text{ mm}$

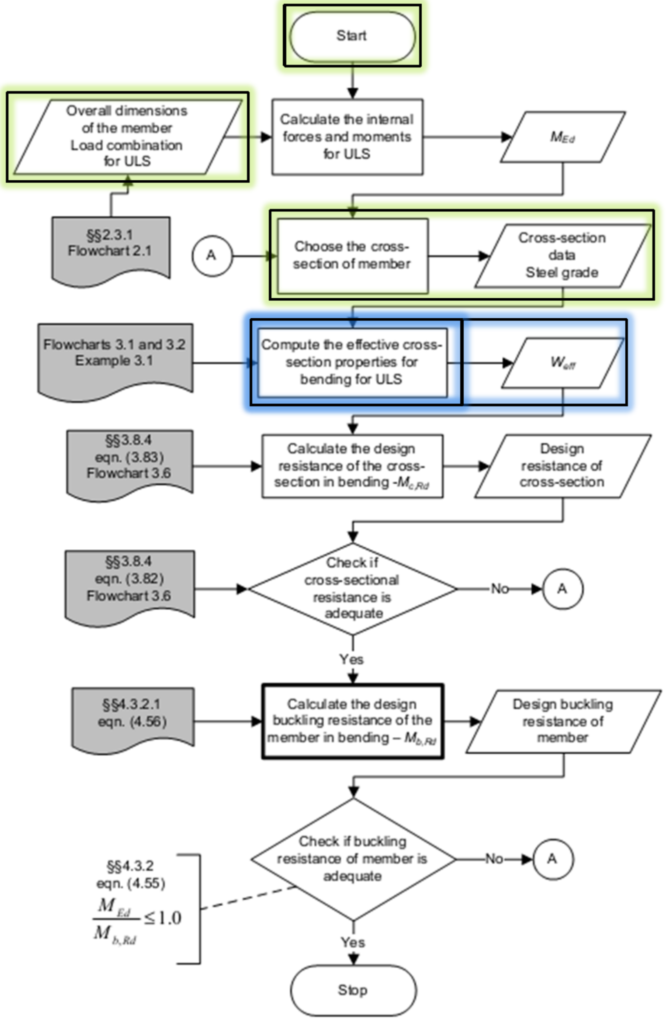
Effective section modulus:

- with respect to the flange in compression:

$$W_{eff,y} = \min\left(W_{eff,y,c}, W_{eff,y,t}\right) = 182094 \text{ mm}^3$$

- with respect to the flange in tension:

$$W_{eff,y,t} = \frac{I_{eff,y}}{z_t} = \frac{22688890}{122.4} = 185367 \text{ mm}^3$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

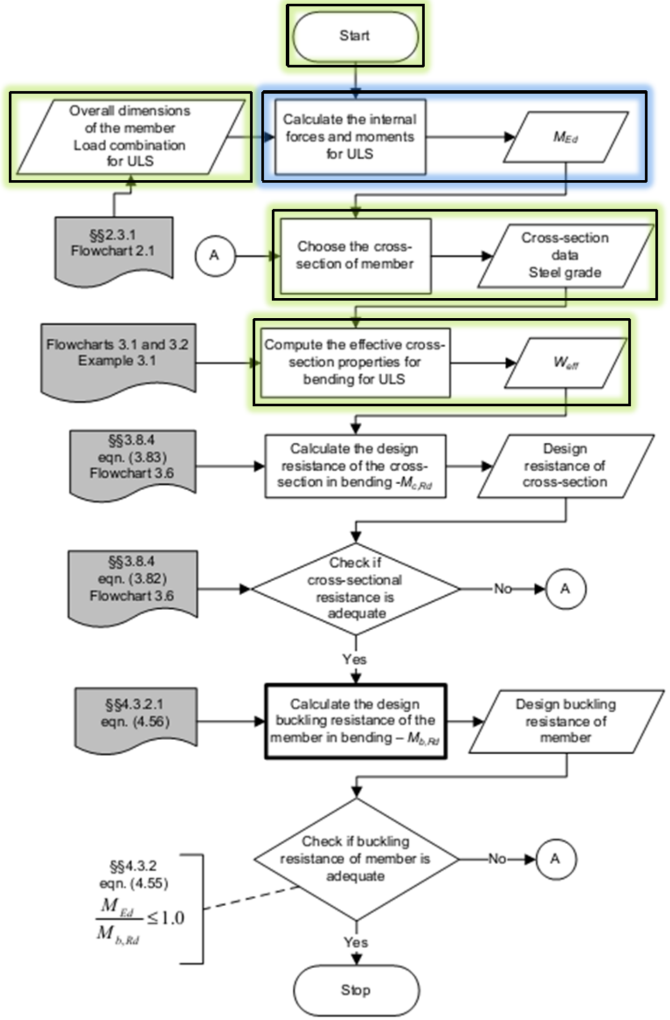
- ***Example – Design of an cold-formed steel beam in bending***

Applied loading on the beam at ULS

$$q_d = \gamma_G q_G + \gamma_Q q_Q = 1.35 \times 1.79 + 1.50 \times 4.5 = 9.17\,\mathrm{kN/m}$$

Maximum applied bending moment about the major axis *y-y*:

$$M_{Ed} = q_d L^2/8 = 9.17 \times 4.5^2/8 = 23.21\,\mathrm{kNm}$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

Applied loading on the beam at ULS

$$q_d = \gamma_G q_G + \gamma_Q q_Q = 1.35 \times 1.79 + 1.50 \times 4.5 = 9.17\,\text{kN/m}$$

Maximum applied bending moment about the major axis *y-y*:

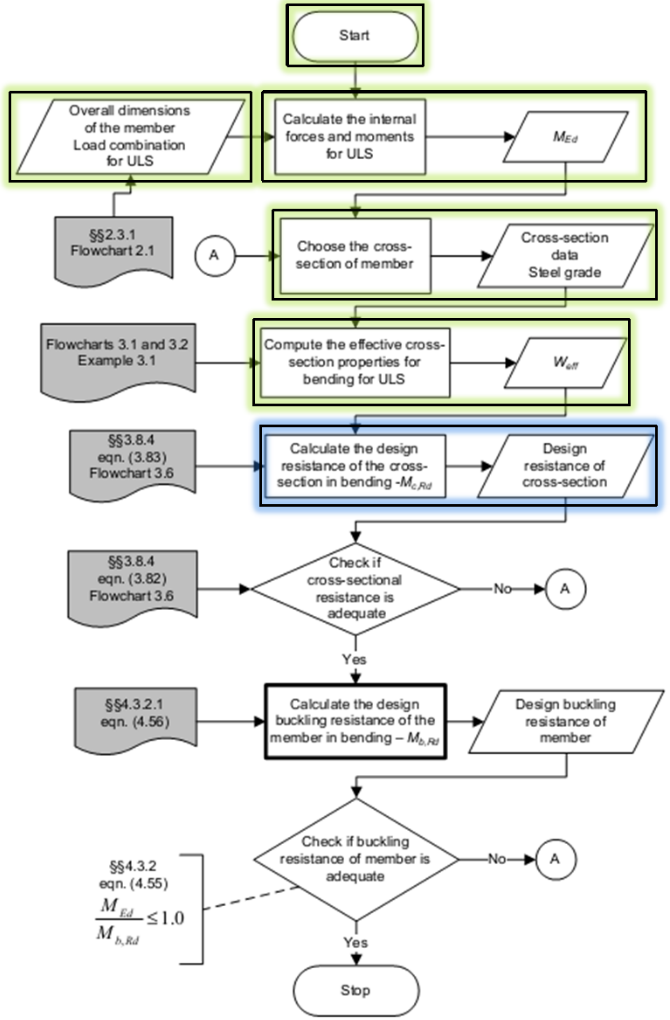
$$M_{Ed} = q_d L^2/8 = 9.17 \times 4.5^2/8 = 23.21\,\text{kNm}$$

Check of bending resistance at ULS

Design moment resistance of the cross section for bending

$$M_{c,Rd} = W_{eff,y} f_{yb}/\gamma_{M0} = 182094 \times 10^{-9} \times 350 \times 10^3/1.0 =$$

$$= 63.73\,\text{kNm}$$



- *Example – Design of an cold-formed steel beam in bending*

Applied loading on the beam at ULS

$$q_d = \gamma_G q_G + \gamma_Q q_Q = 1.35 \times 1.79 + 1.50 \times 4.5 = 9.17\,\text{kN/m}$$

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Check of bending resistance at ULS

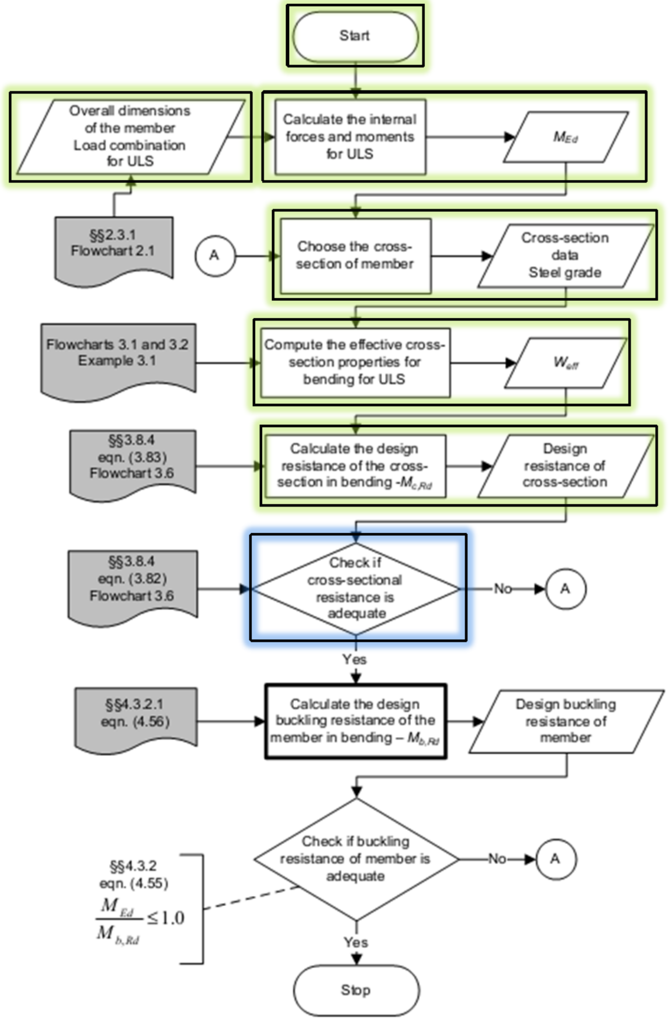
Design moment resistance of the cross section for bending

$$M_{c,Rd} = W_{eff,y} f_{yb}/\gamma_{M0} = 182094 \times 10^{-9} \times 350 \times 10^3/1.0 =$$

$$= 63.73\,\text{kNm}$$

<u>Verification of bending resistance</u>

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{23.21}{63.73} = 0.364 < 1 \;\; - \text{OK}$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

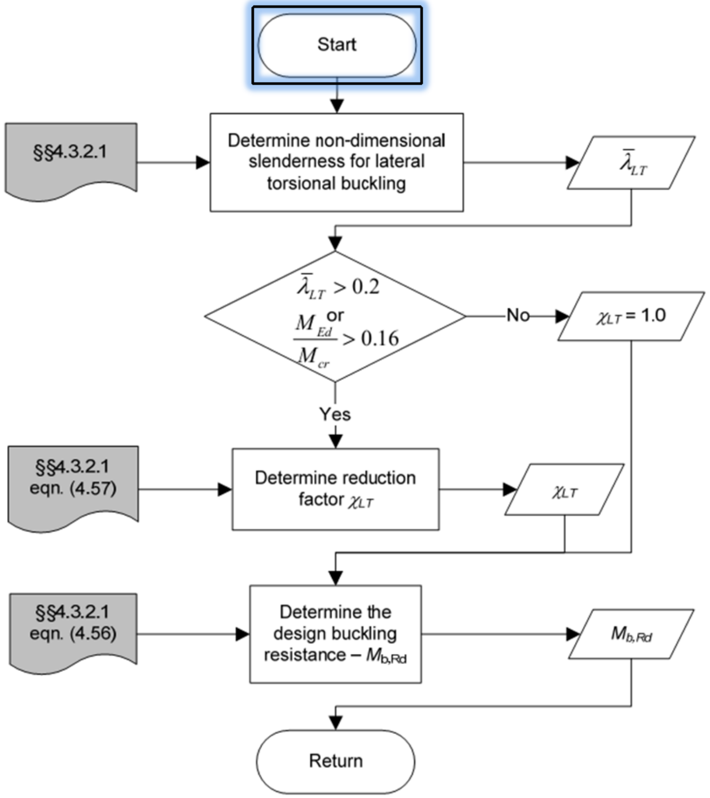
Determination of the reduction factor χ_{LT}

Lateral-torsional buckling

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \leq 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

$$\alpha_{LT} = 0.34 \quad \text{– buckling curve } b$$



- *Example – Design of an cold-formed steel beam in bending*

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Lateral-torsional buckling

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \le 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

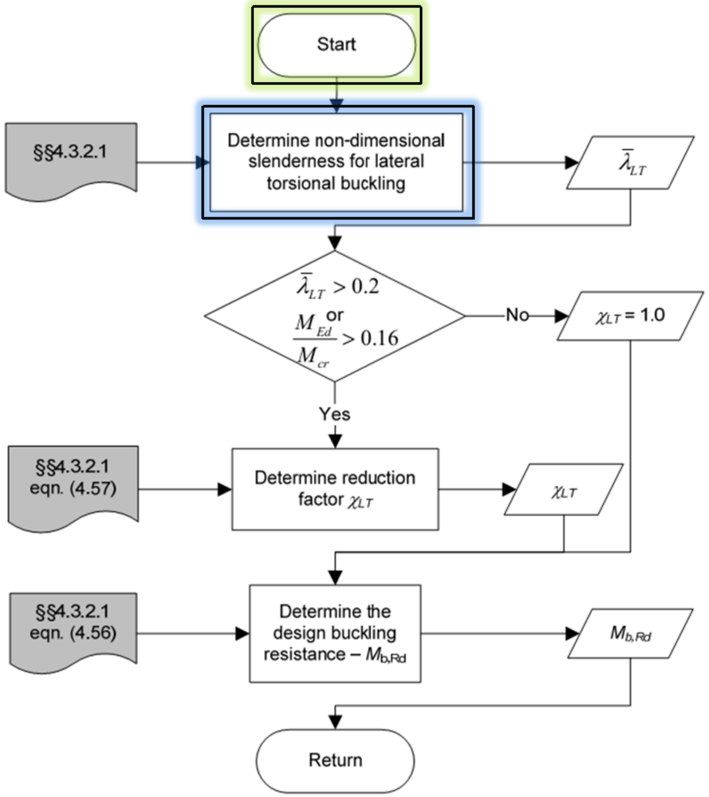
$$\alpha_{LT} = 0.34 \quad \text{– buckling curve } b$$

The non-dimensional slenderness is

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y,min} f_{yb}}{M_{cr}}}$$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z}}$$

$C_1 = 1.127$ for a simply supported beam under uniform loading



- *Example – Design of an cold-formed steel beam in bending*

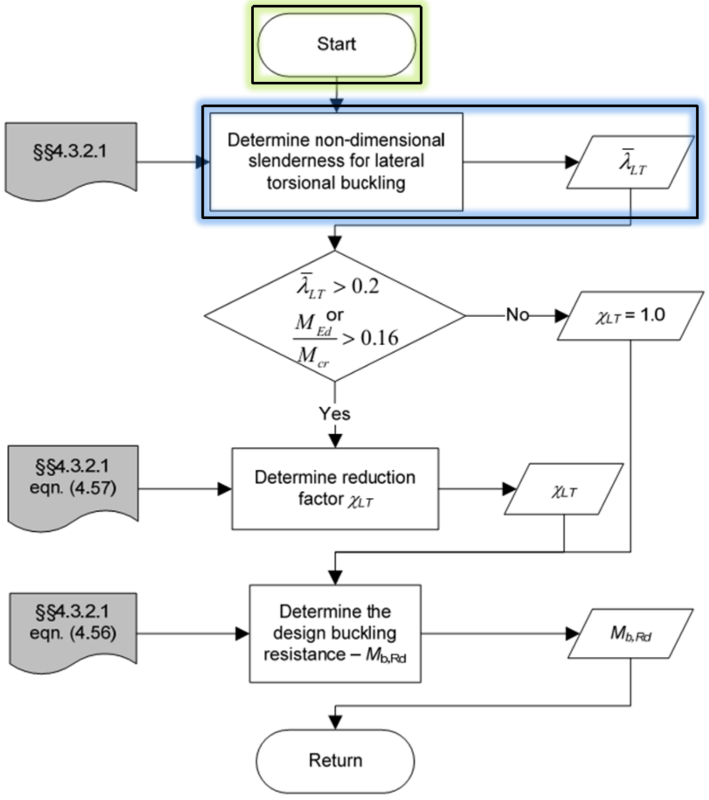
Determination of the reduction factor χ_{LT}

$$M_{cr} = 1.127 \times \frac{\pi^2 \times 210000 \times 244.24 \times 10^4}{4500^2} \times$$

$$\times \sqrt{\frac{17692.78 \times 10^6}{244.24 \times 10^4} + \frac{4500^2 \times 81000 \times 7400}{\pi^2 \times 210000 \times 244.24 \times 10^4}}$$

$$M_{cr} = 27.66 \text{ kNm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y,min} f_{yb}}{M_{cr}}} = \sqrt{\frac{182094 \times 350}{27.66 \times 10^6}} = 1.518$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

Determination of the reduction factor χ_{LT}

$$M_{cr} = 1.127 \times \frac{\pi^2 \times 210000 \times 244.24 \times 10^4}{4500^2} \times$$

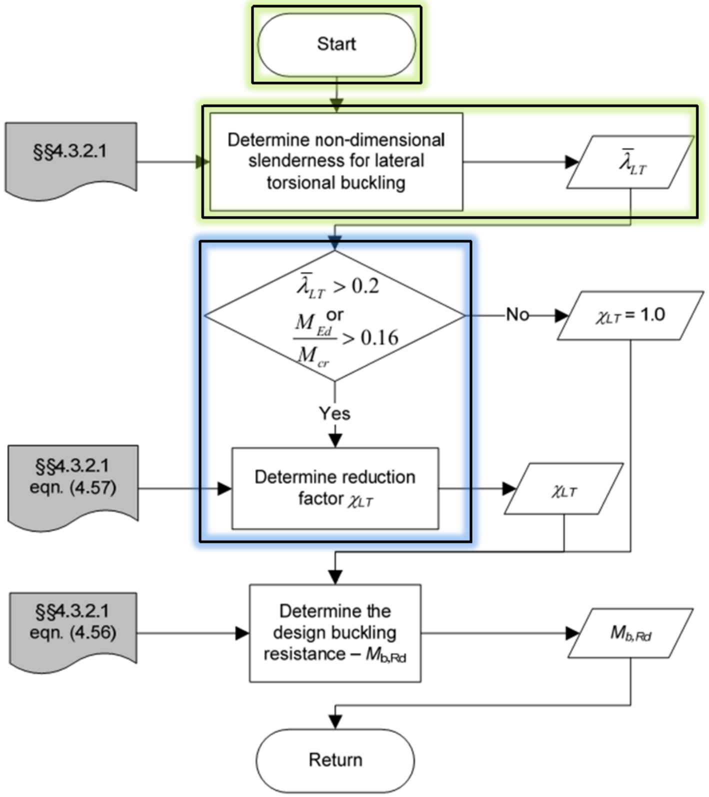
$$\times \sqrt{\frac{17692.78 \times 10^6}{244.24 \times 10^4} + \frac{4500^2 \times 81000 \times 7400}{\pi^2 \times 210000 \times 244.24 \times 10^4}}$$

$$M_{cr} = 27.66 \text{ kNm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y,min} f_{yb}}{M_{cr}}} = \sqrt{\frac{182094 \times 350}{27.66 \times 10^6}} = 1.518$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^{\,2}\right] =$$

$$= 0.5 \times \left[1 + 0.34 \times (1.437 - 0.2) + 1.437^2\right] = 1.743$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

Determination of the reduction factor χ_{LT}

$$M_{cr} = 1.127 \times \frac{\pi^2 \times 210000 \times 244.24 \times 10^4}{4500^2} \times$$

$$\times \sqrt{\frac{17692.78 \times 10^6}{244.24 \times 10^4} + \frac{4500^2 \times 81000 \times 7400}{\pi^2 \times 210000 \times 244.24 \times 10^4}}$$

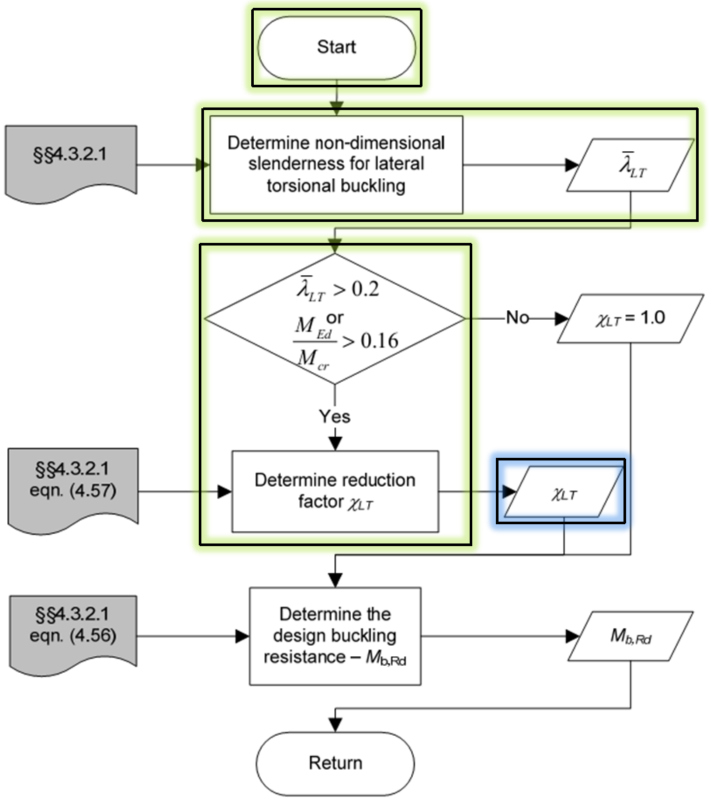
$$M_{cr} = 27.66 \text{ kNm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y,min} f_{yb}}{M_{cr}}} = \sqrt{\frac{182094 \times 350}{27.66 \times 10^6}} = 1.518$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^{\ 2}\right] =$$

$$= 0.5 \times \left[1 + 0.34 \times (1.437 - 0.2) + 1.437^2\right] = 1.743$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^{\ 2} - \bar{\lambda}_{LT}^{\ 2}}} = \frac{1}{1.743 + \sqrt{1.734^2 - 1.437^2}} = 0.369$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

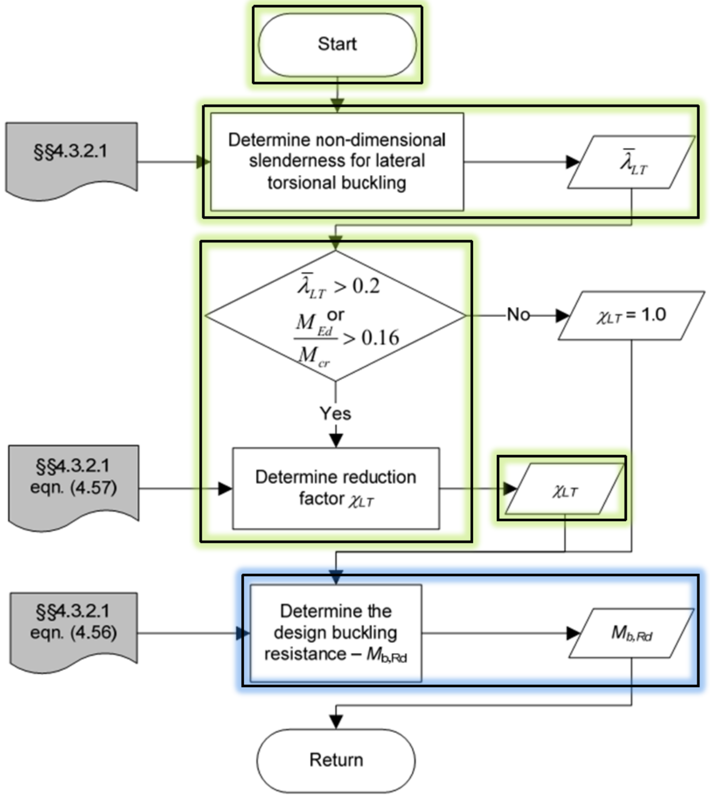
Check of buckling resistance at ULS

Design moment resistance of the cross section for bending

$$M_{b,Rd} = \chi_{LT} W_{eff,y} f_{yb} / \gamma_{M1} =$$

$$= 0.369 \times 182091 \times 10^{-9} \times 350 \times 10^{3} / 1.0 =$$

$$= 23.52 \text{ kNm}$$



- *Example – Design of an cold-formed steel beam in bending*

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Design moment resistance of the cross section for bending

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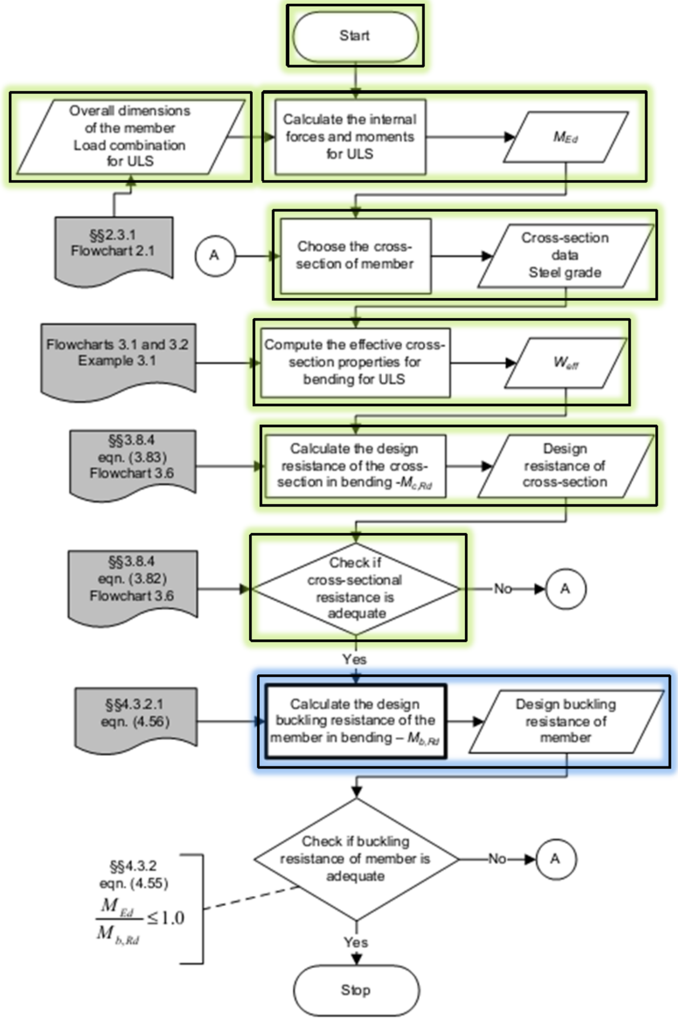
$$= 23.52 \text{ kNm}$$



Behaviour and Design Resistance of Bar Members

Buckling strength of bending members

- *Example – Design of an cold-formed steel beam in bending*

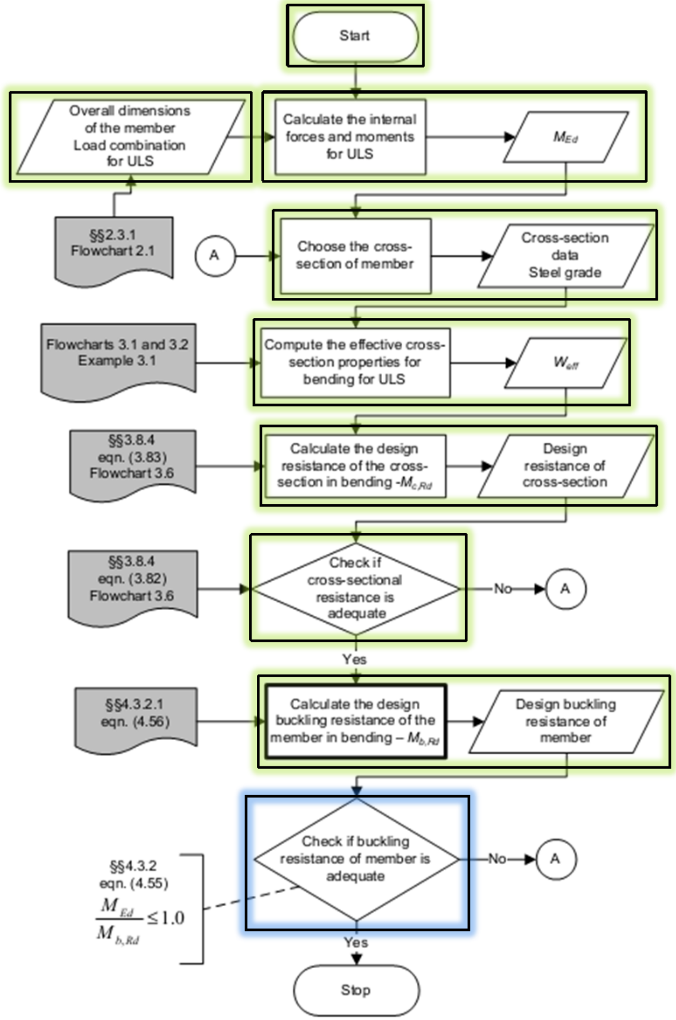
Check of buckling resistance at ULS

Design moment resistance of the cross section for bending

$$M_{b,Rd} = \chi_{LT} W_{eff,y} f_{yb} / \gamma_{M1} =$$
$$= 0.369 \times 182091 \times 10^{-9} \times 350 \times 10^{3} / 1.0 =$$
$$= 23.52 \text{ kNm}$$

Verification of buckling resistance

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{23.21}{23.52} = 0.987 < 1 \quad - \text{OK}$$



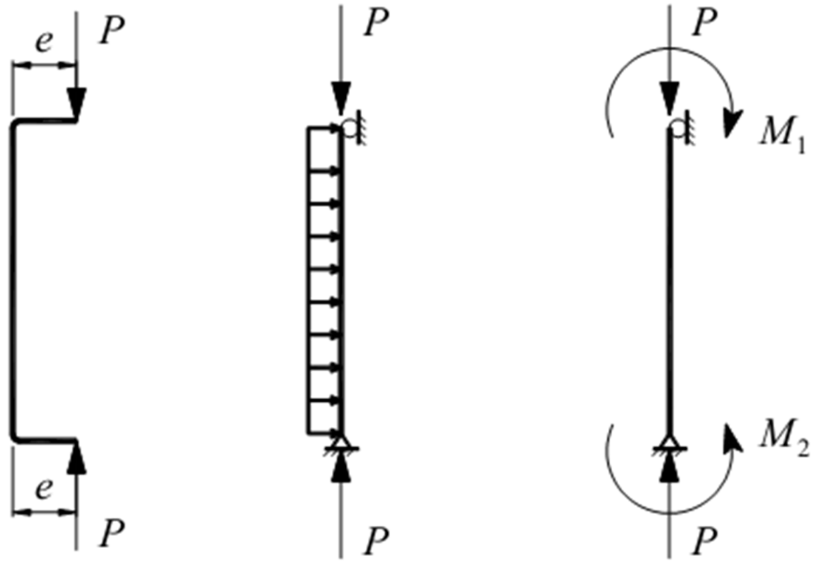
- **Buckling of members in bending and axial compression**

Theoretical background

Bending generally results from three sources.

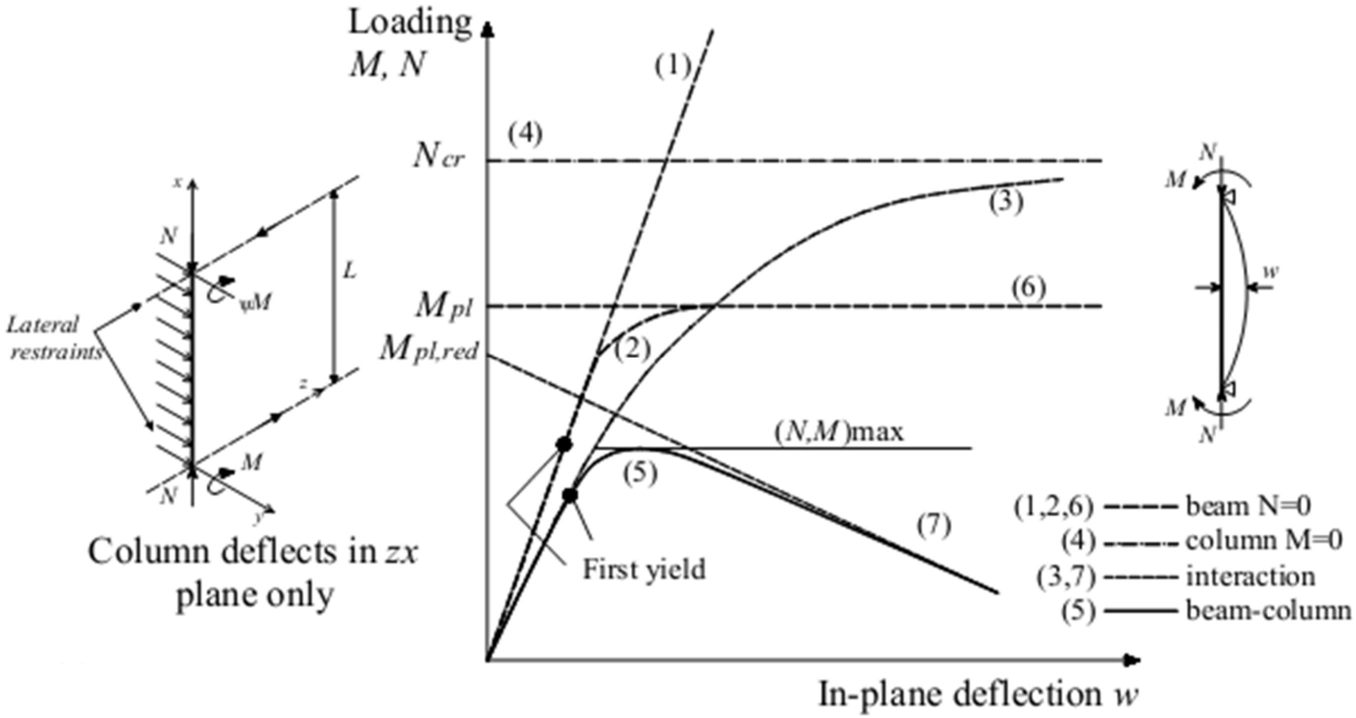
1. Eccentric axial load. A typical case is the pinned connection of the beam to the column when the connection is centred at the face of column (e.g. on the flange);

- Theoretical background



Buckling of members in bending and axial compression

- **Theoretical background**



In-plane behaviour of beam-columns

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Two different formats of the interaction formulae

Method 1 (Annex A of EN 1993–1–1) contains a set of formulae that favours transparency and provides a wide range of applicability together with a high level of accuracy and consistency.

Method 2 (Annex B of EN 1993–1–1) is based on the concept of global factors, in which simplicity prevails against transparency. This approach appears to be the more straightforward in terms of a general format.

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Members which are subjected to combined bending and axial compression should satisfy:

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \le 1.0$$

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \le 1.0$$

The interaction factors k_{yy}, k_{yz}, k_{zy}, k_{zz} depend on the method which is chosen, being derived from two alternative approaches: (1) Alternative method 1 – see Tables 4.7 and 4.8 (Annex A of EN1993-1-1) and (2) Alternative method 2 – see Tables 4.9, 4.10 and 4.11 (Annex B of EN1993-1-1).

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Interaction factors	Elastic cross sectional properties – class 3, class 4
k_{yy}	$C_{my}C_{mLT}\dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$
k_{yz}	$C_{mz}\dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$
k_{zy}	$C_{my}C_{mLT}\dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$
k_{zz}	$C_{mz}\dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Auxiliary terms	
	$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{W_{el,y}}{W_{pl,y}}$
$\mu_y = \dfrac{1 - \frac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \frac{N_{Ed}}{N_{cr,y}}}$	with $b_{LT} = 0.5 a_{LT} \bar{\lambda}_0^2 \frac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{M_{pl,z,Rd}}$
	$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq 0.6 \sqrt{\frac{w_z}{w_y}} \frac{W_{el,z}}{W_{pl,z}}$
$\mu_z = \dfrac{1 - \frac{N_{Ed}}{N_{cr,z}}}{1 - \chi_z \frac{N_{Ed}}{N_{cr,z}}}$	with $c_{LT} = 10 a_{LT} \frac{\bar{\lambda}_0^2}{5 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$
$w_y = \frac{W_{pl,y}}{W_{el,y}} \leq 1.5$	$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0.6 \sqrt{\frac{w_y}{w_z}} \frac{W_{el,y}}{W_{pl,y}}$
$w_z = \frac{W_{pl,z}}{W_{el,z}} \leq 1.5$	with $d_{LT} = 2 a_{LT} \frac{\bar{\lambda}_0}{0.1 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{C_{mz} M_{pl,z,Rd}}$
$n_{pl} = \frac{N_{Ed}}{[illegible]}$	$C_{zz} = 1 + ([illegible])\left[\left(2 - \frac{1.6}{[illegible]} C^2 \bar{\lambda} - \frac{1.6}{[illegible]} C^2 \bar{\lambda}^2\right) [illegible]\right] \geq \frac{W_{el,z}}{[illegible]}$

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

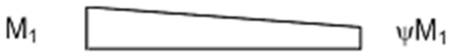
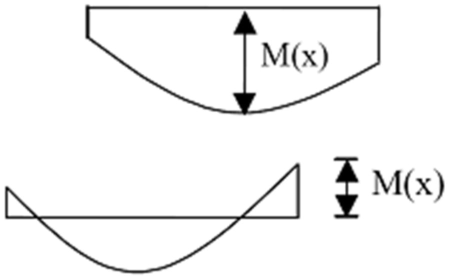
Auxiliary terms	
	$\bar{\lambda}_{LT}$ = non-dimensional slenderness for lateral-torsional buckling. If $\bar{\lambda}_0 \leq 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)} : C_{my} = C_{my,0}; C_{mz} = C_{mz,0}; C_{mLT} = 1.0$ If $\bar{\lambda}_0 > 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)} : C_{my} = C_{my,0} + \left(1-C_{my,0}\right)\frac{\sqrt{\varepsilon_y}a_{LT}}{1+\sqrt{\varepsilon_y}a_{LT}}$ $C_{mz} = C_{mz,0}; C_{mLT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}} \geq 1$; $C_{mi,0}$ see Table 4.8 $\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A}{W_{el,y}}$ for class 3; $\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A_{eff}}{W_{eff,y}}$ for class 4; $N_{cr,y}$ = elastic flexural buckling force about the y–y axis; $N_{cr,z}$ = elastic flexural buckling force about the z–z axis; $N_{cr,T}$ = elastic torsional buckling force;

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Table 4.8 – Equivalent uniform moment factors $C_{mi,0}$

Moment diagram	$C_{mi,0}$
 $-1 \le \psi \le 1$	$C_{mi,0} = 0.79 + 0.21\psi_i + 0.36(\psi_i - 0.33)\frac{N_{Ed}}{N_{cr,i}}$
	$C_{mi,0} = 1 + \left(\frac{\pi^2 EI_i \lvert\delta_x\rvert}{L^2 \lvert M_{i,Ed}(x)\rvert} - 1\right)\frac{N_{Ed}}{N_{cr,i}}$ $M_{i,Ed}(x)$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ according to the first order analyses $\lvert\delta_x\rvert$ is the maximum member deflection along the member δ_z (due to $M_{y,Ed}$) or δ_y (due to $M_{z,Ed}$)
	$C_{mi,0} = 1 - 0.18\frac{N_{Ed}}{N_{cr,i}}$ $C_{mi,0} = 1 - 0.03\frac{N_{Ed}}{N_{cr,i}}$

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Table 4.9 – Method 2 – Interaction factors k_{ij} for members not susceptible to torsional deformations

Interaction factors	Type of sections	Elastic cross sectional properties class 3, class 4
k_{yy}	I-sections RHS-sections	$C_{my}\left(1+0.6\bar{\lambda}_y\dfrac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)\leq$ $\leq C_{my}\left(1+0.6\dfrac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I-sections RHS-sections	k_{zz}
k_{zy}	I-sections RHS-sections	$0.8k_{yy}$
	I-sections	$C_{mz}\left(1+0.6\bar{\lambda}_z\dfrac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)\leq$

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Table 4.10 – Method 2 – Interaction factors k_{ij} for members susceptible to torsional deformations

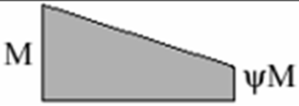
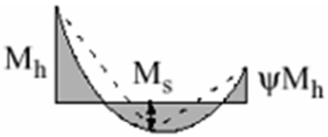
Interaction factors	elastic cross sectional properties class 3, class 4
k_{yy}	k_{yy} from Table 4.9
k_{yz}	k_{yz} from Table 4.9
k_{zy}	$\left[1-\frac{0.05\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] \geq \left[1-\frac{0.05}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$
k_{zz}	k_{zz} from Table 4.9

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

Table 4.11 – Equivalent uniform moment factors C_{mi} in Tables 4.9 and 4.10

Moment diagram	Range		C_{my}, C_{mz} and C_{mLT} Uniform loading	C_{my}, C_{mz} and C_{mLT} Concentrated load
	$-1 \le \psi \le 1$		$0.6+0.4\psi \ge 0.4$	
 $\alpha_h = M_S / M_h$	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0.2+0.8\alpha_s \ge 0.4$	$0.2+0.8\alpha_s \ge 0.4$
	$-1 \le \alpha_s < 1$	$0 \le \psi \le 1$	$0.1-0.8\alpha_s \ge 0.4$	$-0.8\alpha_s \ge 0.4$
		$-1 \le \psi < 0$	$0.1(1-\psi)-$ $-0.8\alpha_s \ge 0.4$	$0.2(-\psi)-$ $-0.8\alpha_s \ge 0.4$
 $\alpha_h = M_h / M_s$	$0 \le \alpha_h \le 1$	$-1 \le \psi \le 1$	$0.95+0.05\alpha_h$	$0.90+0.10\alpha_h$
	$-1 \le \alpha_h < 1$	$0 \le \psi \le 1$	$0.95+0.05\alpha_h$	$0.90+0.10\alpha_h$
		$-1 \le \psi < 0$	$0.95+$ $+0.05\alpha_h(1+2\psi)$	$0.90+$ $+0.10\alpha_h(1+2\psi)$

In the calculation of α_h and α_h, a hogging moment should be taken as negative

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

As an alternative, the interaction formula may be used

$$\left(\frac{N_{Ed}}{N_{b,Rd}}\right)^{0.8} + \left(\frac{M_{Ed}}{M_{b,Rd}}\right)^{0.8} \leq 1.0$$

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

General method for lateral and lateral-torsional buckling of structural components

- ***In-plane analysis of the structural component.*** The objective is to determine the design effects in the structural component under the design loading and then to assess the magnitude of the effects in relation to the characteristic resistance at the most critical cross section, considering only the in-plane behaviour. The ratio between characteristic resistance and design effects is expressed as the load amplifier, $\alpha_{ult.k}$;
- ***Out-of-plane buckling analysis of the structural component.*** The objective is to determine the magnitude of the loading, as a multiple of the design loading, at which the structural component fails by out-of-plane elastic buckling. The magnitude is expressed as the amplifier $\alpha_{cr.op}$;

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

General method for lateral and lateral-torsional buckling of structural components

- ***Check of the overall resistance of the structural component.*** The objective is to verify the adequacy of the structural component, considering the interaction between the in-plane behaviour of the structural component and the out-of-plane behaviour.

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

General method for lateral and lateral-torsional buckling of structural components

The overall resistance to out-of-plane buckling for any structural component conforming to the scope presented above can be verified by ensuring that:

$$\frac{\chi_{op}\alpha_{ult,k}}{\gamma_{M1}} \geq 1.0$$

where

$\alpha_{ult.k}$ is the minimum load amplifier of the design loads to reach the characteristic resistance of the most critical cross section of the structural component considering its in-plane behaviour without taking lateral or lateral-torsional buckling into account however accounting for all effects due to in plane geometrical deformation and imperfections, global and local, where relevant;

χ_{op} is the reduction factor for the non-dimensional slenderness $\bar{\lambda}_{op}$, to take account of lateral and lateral-torsional buckling;

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

General method for lateral and lateral-torsional buckling of structural components

The global non-dimensional slenderness $\bar{\lambda}_{op}$ for the structural component should be determined from:

$$\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}}$$

where

$\alpha_{cr.op}$ is the minimum amplifier for the in-plane design loads to reach the elastic critical load of the structural component with regards to lateral or lateral-torsional buckling without accounting for in plane flexural buckling. In determining $\alpha_{cr.op}$ and $\alpha_{ult.k}$ Finite Element analysis may be used.

Behaviour and Design Resistance of Bar Members

Buckling of members in bending and axial compression

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

General method for lateral and lateral-torsional buckling of structural components

The reduction factor χ_{op} may be determined from either of the following methods:

a) the minimum value of

χ_z for lateral buckling according to §§4.2.2;

χ_{LT} for lateral torsional-buckling according to §§4.3.2;

each calculated for the global non-dimensional slenderness $\bar{\lambda}_{op}$.

b) a value interpolated between the values χ_z and χ_{LT} as determined in a) by using the formula for $\alpha_{ult,k}$ corresponding to the critical cross section.

$$\alpha_{ult,k} = \frac{1}{\dfrac{N_{Ed}}{N_{Rk}} + \dfrac{M_{y,Ed}}{M_{y,Rk}}} \qquad \frac{\chi_{op}\alpha_{ult,k}}{\gamma_{M1}} = \frac{\chi_{op}}{\gamma_{M1}\left(\dfrac{N_{Ed}}{N_{Rk}} + \dfrac{M_{y,Ed}}{M_{y,Rk}}\right)} \geq 1.0 \qquad \frac{N_{Ed}}{N_{Rk}/\gamma_{M1}} + \frac{M_{y,Ed}}{M_{y,Rk}/\gamma_{M1}} \leq \chi_{op}$$

- **Design of beam-columns according to EN1993-1-1 and EN1993-1-3**

General method for lateral and lateral-torsional buckling of structural components

$$\chi_{op} = \frac{\left(\dfrac{N_{Ed}}{N_{Rk}} + \dfrac{M_{y,Ed}}{M_{y,Rk}}\right)}{\left(\dfrac{N_{Ed}}{\chi_z N_{Rk}} + \dfrac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}}\right)} \geq 1.0$$

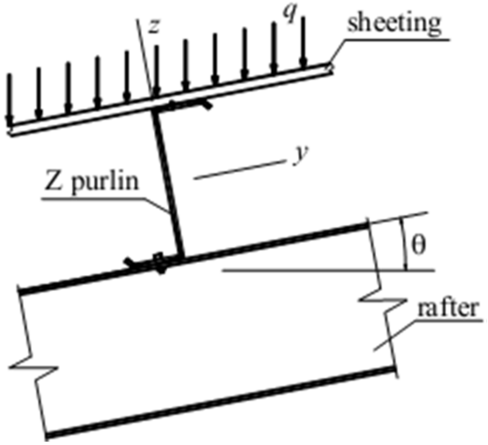
$$\frac{\chi_{op}\alpha_{ult,k}}{\gamma_{M1}} = \frac{\left(\dfrac{N_{Ed}}{N_{Rk}} + \dfrac{M_{y,Ed}}{M_{y,Rk}}\right)}{\left(\dfrac{N_{Ed}}{\chi_z N_{Rk}} + \dfrac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}}\right)} \frac{1}{\gamma_{M1}\left(\dfrac{N_{Ed}}{N_{Rk}} + \dfrac{M_{y,Ed}}{M_{y,Rk}}\right)} \geq 1.0$$

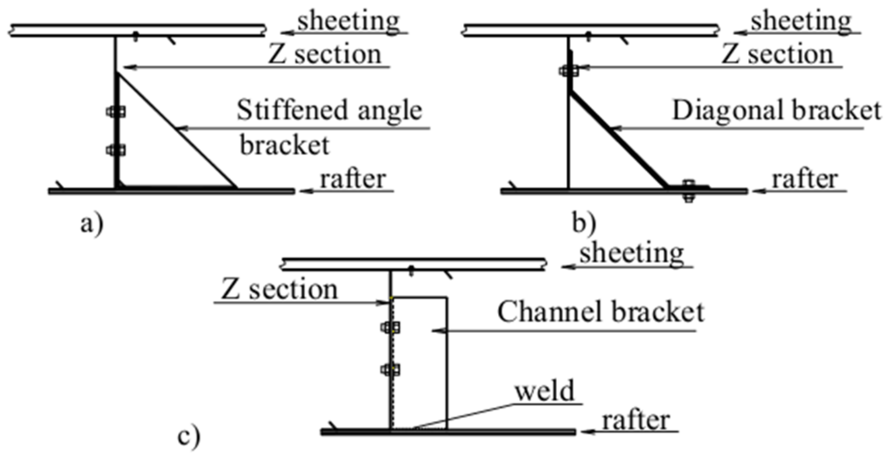
$$\frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} + \frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk} / \gamma_{M1}} \leq 1$$

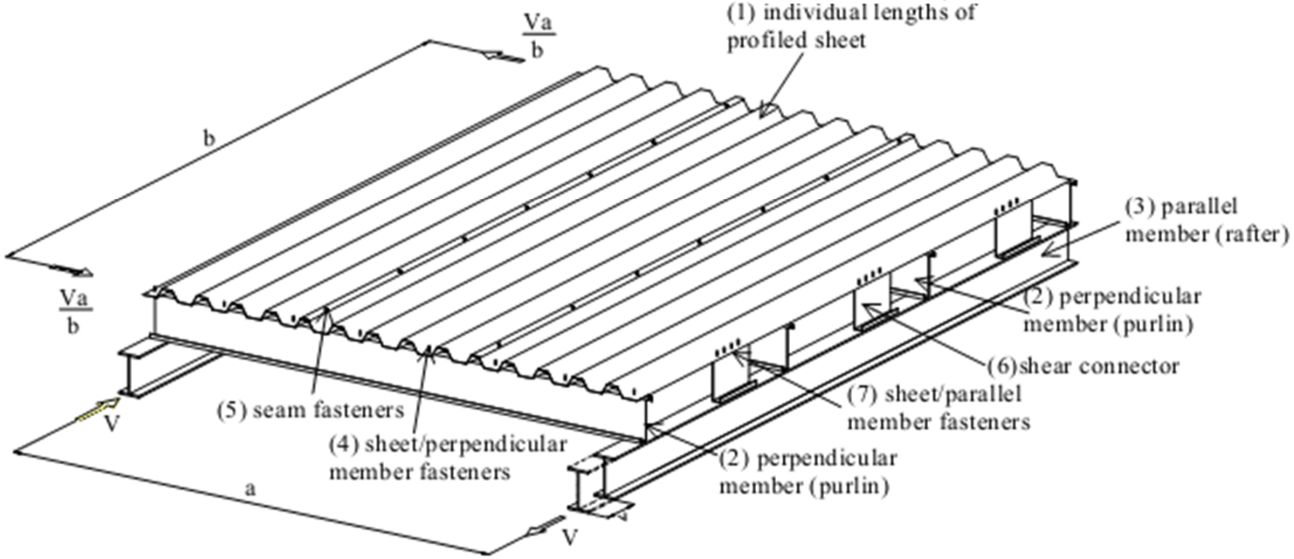
Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **General. Constructional detailing and static system**



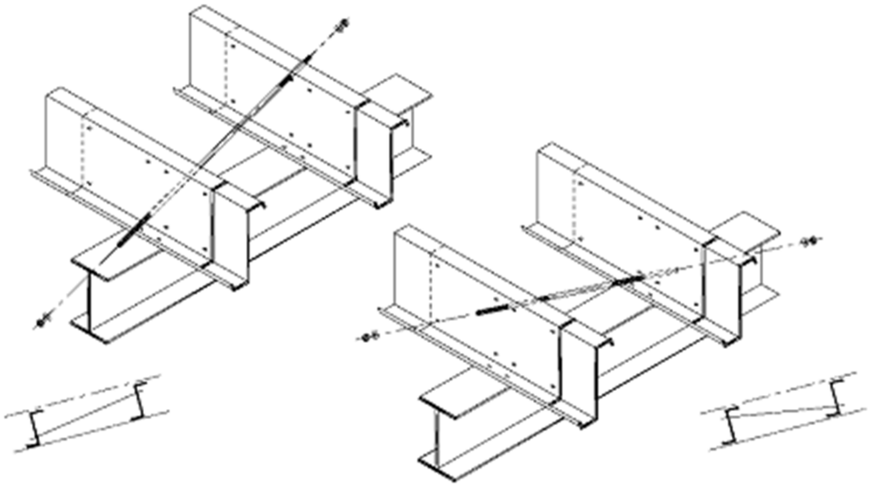




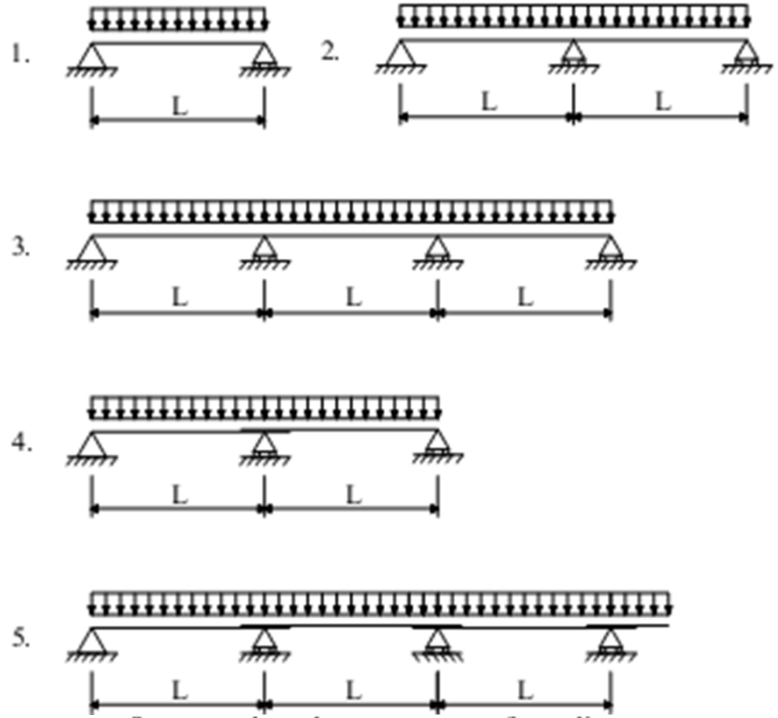
Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **General. Constructional detailing and static system**



Purlin directly fastened onto the top of primary beam (Lindab Systemline)

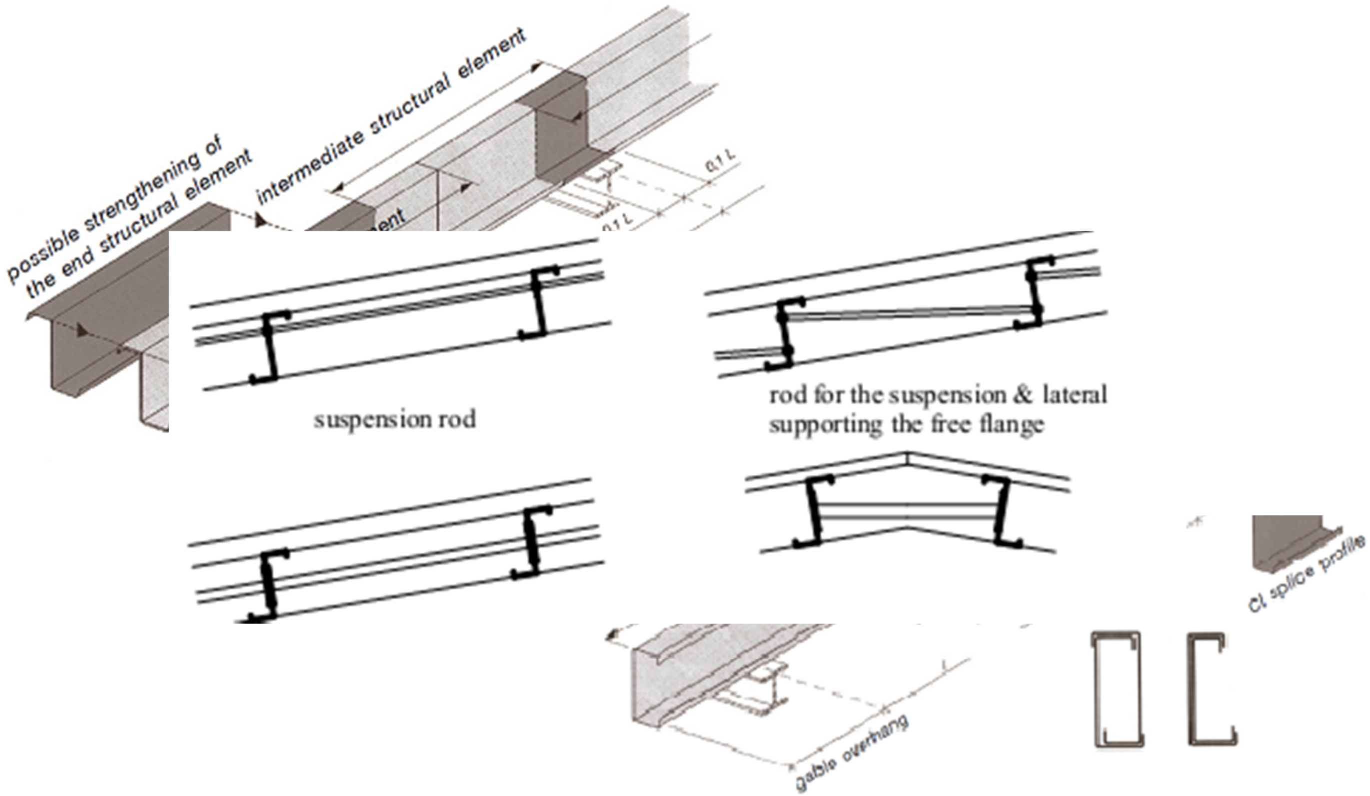


Support and overlap arrangements for purlins

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **General. Constructional detailing and static system**



Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Modeling of beam-sheeting interaction**

The restraining effect of sheeting has to be considered for an effective design of purlin-sheeting systems. It is generally assumed that the sheeting can provide the necessary in-plane stiffness and capacity to carry the component of the load in the plane of the sheeting

the purlin resists to normal component. In fact, the sheeting may provide not only lateral restraint but can also partially restrain the twisting of purlins, taking account of the flexural stiffness of the sheeting if substantial (e.g. as for trapezoidal sheeting) and properly connected to the purlin.

According to EN1993-1-3, the purlin is laterally restrained

$$S \geq \left(EI_w \frac{\pi^2}{L^2} + GI_t + EI_z \frac{\pi^2}{L^2} 0.25 h^2 \right) \frac{70}{h^2}$$

where

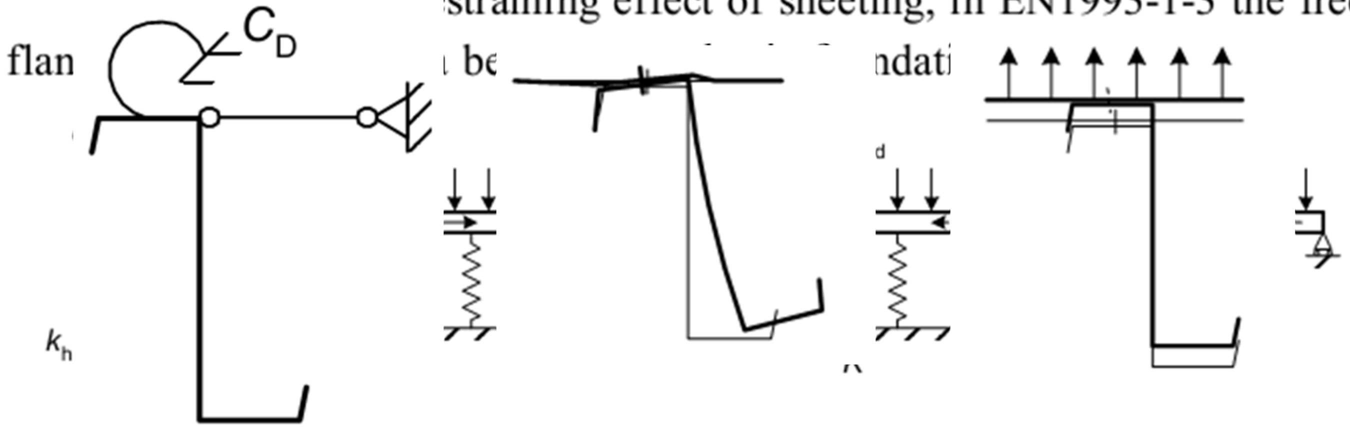
S is the portion of the shear stiffness provided by the sheeting for the examined member connected to the sheeting at each rib. If the sheeting is connected to a purlin every second rib only, then S should be replaced by $0.20\,S$;

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Modeling of beam-sheeting interaction**

straining effect of sheeting, in EN1993-1-3 the free



The equivalent lateral spring stiffness for the strength and stability check is obtained by a combination of:

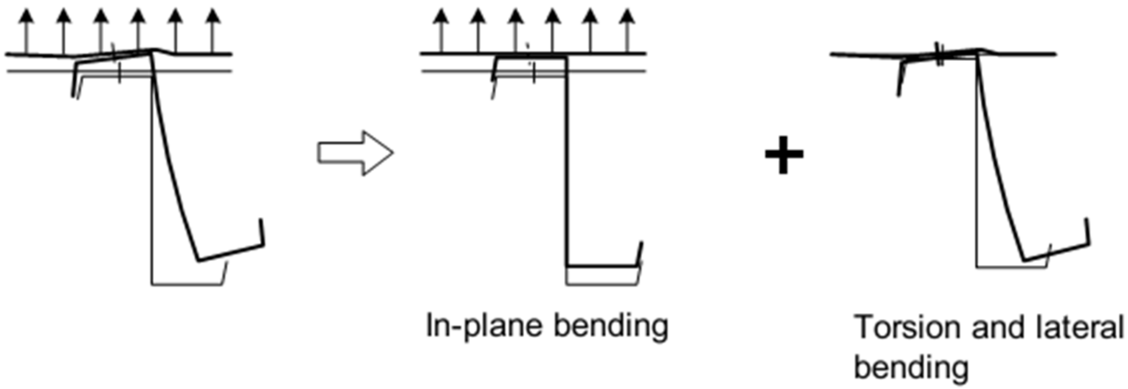
1. Rotational stiffness of the connection between the sheeting and the purlin C_D,
2. Distortion of the cross section of the purlin, K_B,
3. Bending stiffness of the sheeting, $C_{D,C}$, perpendicular to the span of the purlin (see Figure 4.49).

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Modeling of beam-sheeting interaction**

The composed distortion-torsion-bending effect



- **Modeling of beam-sheeting interaction**

According to EN1993-1-3, the partial torsional restraint may be represented by a rotational spring with a spring stiffness C_D, which can be calculated based on the stiffness of the sheeting and the connection between the sheeting and the purlin, as follows,

$$\frac{1}{C_D} = \frac{1}{C_{D,A}} + \frac{1}{C_{D,C}}$$

where

$C_{D,A}$ is the rotational stiffness of the connection between the sheeting and the purlin;

$C_{D,C}$ is the rotational stiffness corresponding to the flexural stiffness of the sheeting.

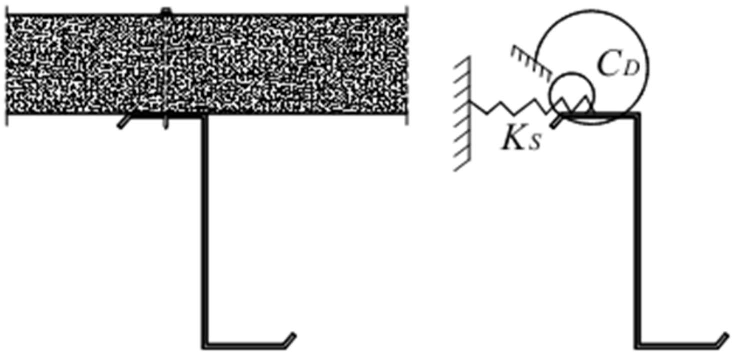
Both $C_{D,A}$ and $C_{D,C}$ are specified in Section 10.1.5 of EN1993-1-3.

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Modeling of beam-sheeting interaction**

Alternatively, the $C_{D,A}$ stiffness and $C_{D,C}$ stiffness values can be obtained experimentally applying the recommendation in Annex A5 of EN1993-1-3.



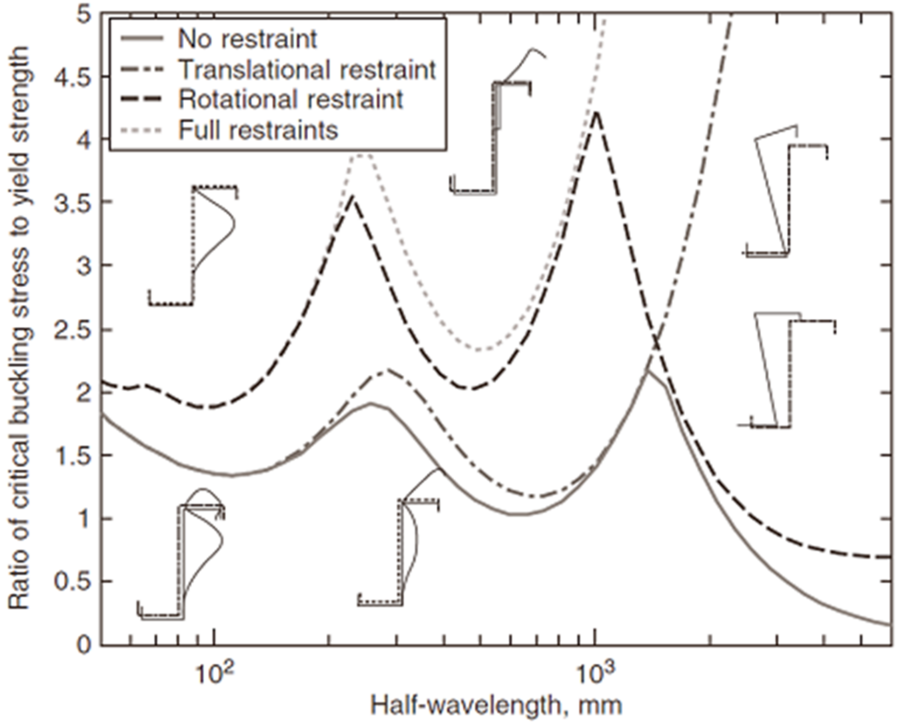
Model adopted for sandwich panel – purlin interaction (Davies, 2001)

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Modeling of beam-sheeting interaction**

The restraints of the sheeting to the purlin have important influence on the buckling behaviour of the purlin.



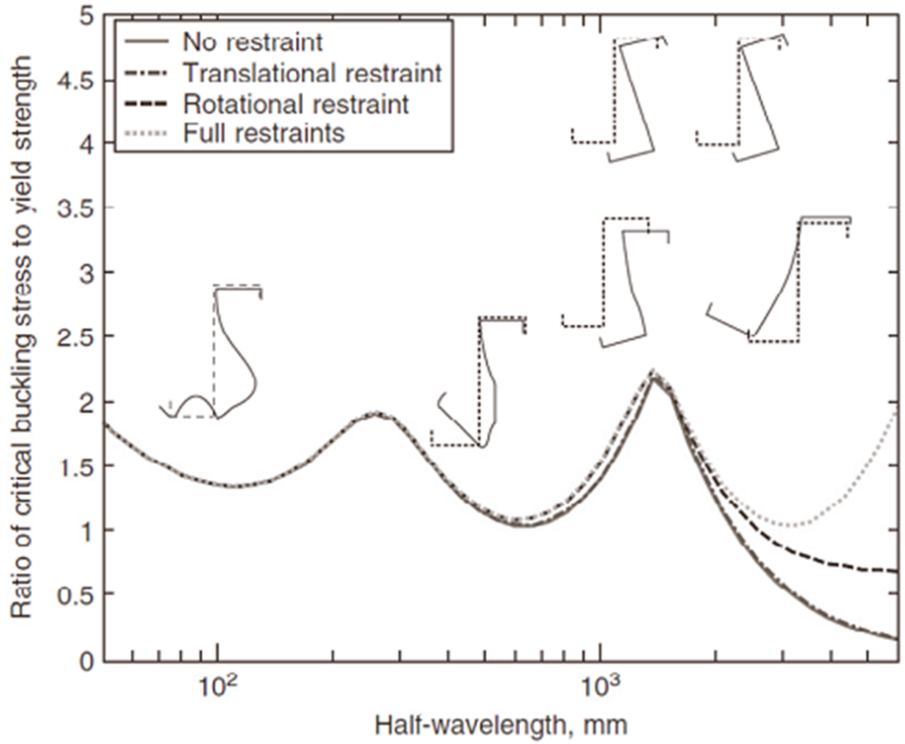
Buckling curves of a simply supported zed section beam with different restraint applied at the junction between web and compression flange subjected to pure bending (h=202 mm, b=75 mm, c=20 mm, t=2.3mm) (Martin & Purkiss, 2008)

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Modeling of beam-sheeting interaction**

The restraints of the sheeting to the purlin have important influence on the buckling behaviour of the purlin.



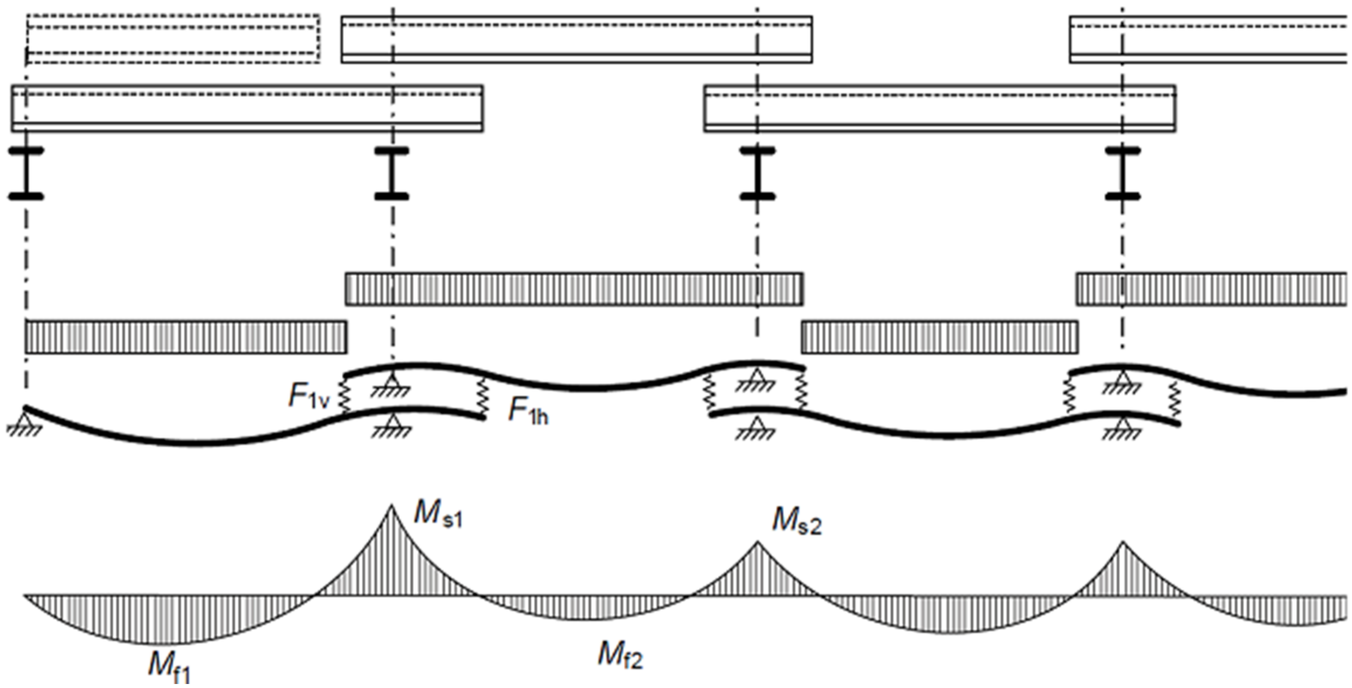
Buckling curves of a simply supported zed section beam with different restraint applied at the junction between web and tension flange subjected to pure bending (h=202 mm, b=75 mm, c=20 mm, t =2.3mm) (Martin & Purkiss, 2008)

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design criteria



Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design criteria

For gravity loading, the purlin should satisfy the following criteria:

- at internal supports, the resistance to combined support reaction and moment;
- near supports, the resistance to combined shear force and bending moment;
- in the spans, the criteria for cross section resistance;
- if the purlin is subject to axial compression, the criteria for stability of the free flange.

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design criteria

For uplift loading, the purlin should satisfy the following criteria:

- at internal supports, the resistance to combined support reaction and moment, taking into account the fact that the support reaction is a tensile force in this case;
- near supports, the resistance to combined shear force and bending moment;
- in the spans, the criteria for stability of the free flange;
- if the purlin is subjected to axial compression, the criteria for stability of the free flange.

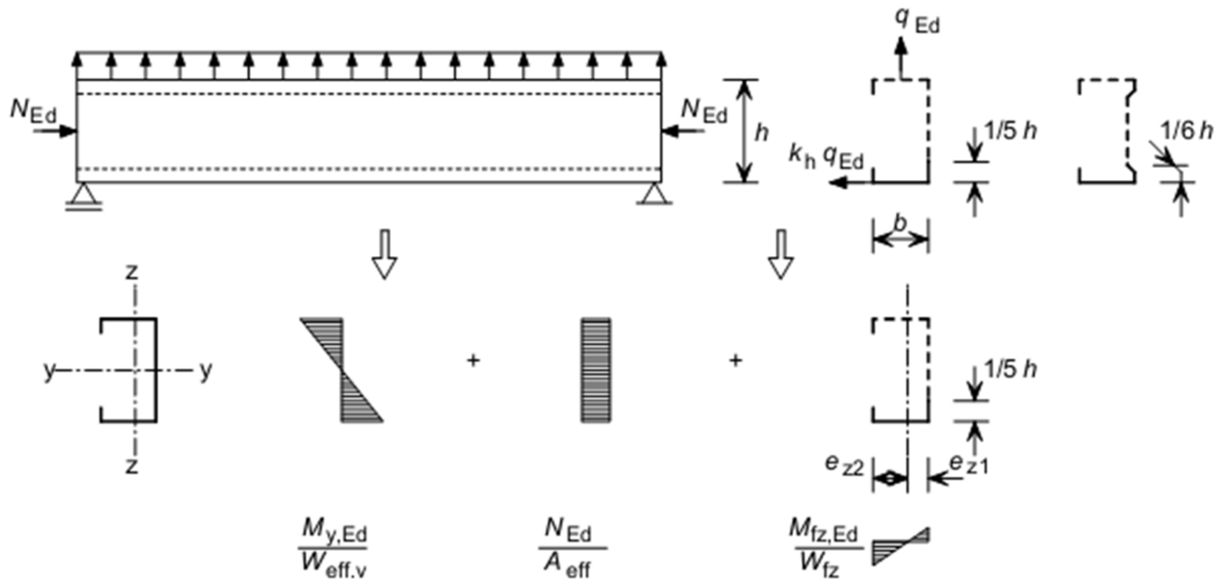
The serviceability criteria relevant to purlins should also be satisfied.

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – resistance of cross-section



The maximum stresses in the cross section should satisfy the following:

- restrained flange: $\sigma_{max,Ed} = \dfrac{M_{y,Ed}}{W_{eff,y}} + \dfrac{N_{Ed}}{A_{eff}} \leq f_y / \gamma_M$

Behaviour and Design Resistance of Bar Members

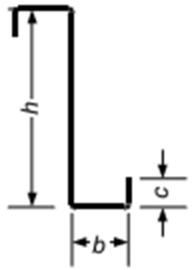
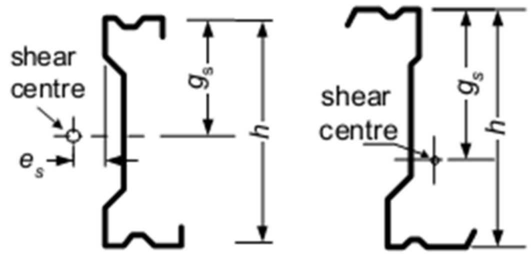
Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – resistance of cross-section

The equivalent lateral load $q_{h,Ed}$ acting on the free flange, due to torsion and lateral bending, should be obtained from:

$$q_{h,Ed} = k_h \, q_{Ed}$$

 $k_{h0} = \dfrac{ht(b^2 + 2cb - 2c^2 b/h)}{4I_y}$ Simple symmetrical Z section	 $k_{h0} = \dfrac{I_{yz}}{I_y}\dfrac{g_s}{h}$ Z, C or Σ sections

a) k_{h0} factor for lateral load on free bottom flange (k_{h0} corresponds to loading at the shear centre)

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Resistance of cross-section

The lateral bending moment $M_{fz,Ed}$ may be determined

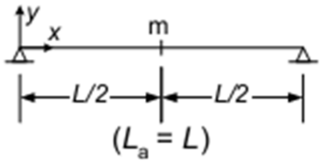
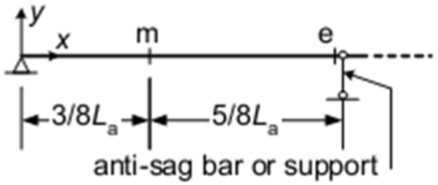
$$M_{fz,Ed} = \kappa_R \; M_{0,fz,Ed}$$

where

$M_{0,fz,Ed}$ is the initial lateral bending moment in the free flange w any spring support;

κ_R is a correction factor for the effective spring support.

The initial lateral bending moment in the free flange $M_{0,fz,Ed}$

System	Location	$M_{0,fz,Ed}$	κ_R
	m	$\frac{1}{8} q_{h,Ed} L_a^2$	$\kappa_R = \frac{1-0.0225R}{1+1.013R}$
	m	$\frac{9}{128} q_{h,Ed} L_a^2$	$\kappa_R = \frac{1-0.0141R}{1+0.416R}$
	e	$-\frac{1}{8} q_{h,Ed} L_a^2$	$\kappa_R = \frac{1+0.0314R}{1+0.396R}$

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Resistance of cross-section

The validity of the Table is limited to the range R ≤ 40

$$R = \frac{K L_a^4}{\pi^4 E I_{fz}} \qquad (4.78)$$

where

I_{fz} is the second moment of area of the gross cross section of the free flange plus the contributing part of the web for bending about the z–z axis (see Figure 4.55);

K is the lateral spring stiffness per unit length;

L_a is the distance between anti-sag bars, or if none are present, the span L of the purlin.

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Buckling resistance of free flange

If the free flange is in compression, its buckling resistance should be verified using:

$$\frac{1}{\chi_{LT}}\left(\frac{M_{y,Ed}}{W_{eff,y}}+\frac{N_{Ed}}{A_{eff}}\right)+\frac{M_{fz,Ed}}{W_{fz}}\leq f_{yb}/\gamma_{M1}$$

in which χ_{LT} is the reduction factor for lateral-torsional buckling (flexural buckling of the free flange), using buckling curve b ($\alpha_{LT}=0.34$; $\bar{\lambda}_{LT,0}=0.4$, $\beta=0.75$ according to §§6.3.2.3 of EN1993-1-1) for the relative slenderness $\bar{\lambda}_{fz}$.

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Buckling resistance of free flange

The relative slenderness $\bar{\lambda}_{fz}$ for flexural buckling of the free flange should be determined from:

$$\bar{\lambda}_{fz} = \frac{l_{fz} / i_{fz}}{\lambda_1} \qquad (4.80)$$

with $\lambda_1 = \pi\left[E / f_{yb}\right]^{0.5}$,

where

l_{fz} is the buckling length for the free flange;

i_{fz} is the radius of gyration of the gross cross section of the free flange plus the contributing part of the web for bending about the z–z axis.

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Buckling resistance of free flange

For gravity loading, provided that $0 \leq R \leq 200$, the buckling length of the free flange for a variation of the compressive stress over the length L as shown in Figure 4.57 may be obtained from:

$$l_{fz} = \eta_1 L_a \left(1 + \eta_2 R^{\eta_3}\right)^{\eta_4} \qquad l_{fz} = 0.7 L_0 \left(1 + 13.1 R_0^{1.6}\right)^{-0.125}$$

where

L_a is the distance between anti-sag bars or, if none are present, the span L of the purlin;

R is as given by

$$R = \frac{K L_a^4}{\pi^4 E I_{fz}} \qquad R_0 = \frac{K L_0^4}{\pi^4 E I_{fz}}$$

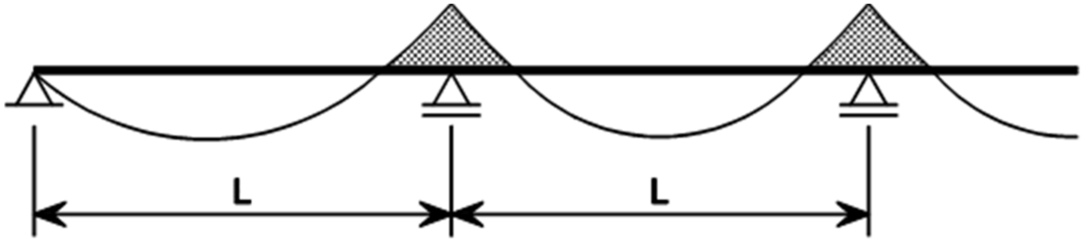
η_1 to η_4 are coefficients that depend on the number of anti-sag bars

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Buckling resistance of free flange



Coefficients η_i for down load with 0, 1, 2, 3, 4 anti-sag bars

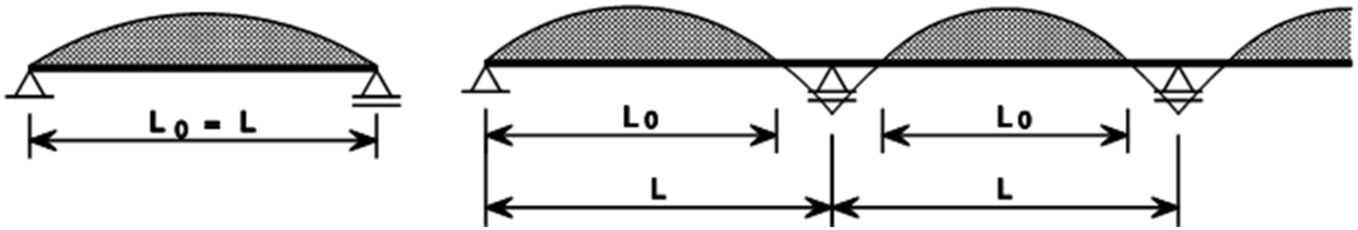
Situation	Anti-sag bar Number	η_1	η_2	η_3	η_4
End span	0	0.414	1.72	1.11	-0.178
Intermediate span		0.657	8.17	2.22	-0.107
End span	1	0.515	1.26	0.868	-0.242
Intermediate span		0.596	2.33	1.15	-0.192
End and intermediate span	2	0.596	2.33	1.15	-0.192
End and intermediate span	3 and 4	0.694	5.45	1.27	-0.168

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Buckling resistance of free flange



Coefficients η_i for uplift load with 0, 1, 2, 3, 4 anti-sag bars

Situation	Anti-sag bar Number	η_1	η_2	η_3	η_4
Simple span	0	0.694	5.45	1.27	-0.168
End span		0.515	1.26	0.868	-0.242
Intermediate span		0.306	0.232	0.742	-0.279
Simple and end spans	1	0.800	6.75	1.49	-0.155
Intermediate span		0.515	1.26	0.868	-0.242
Simple span	2	0.902	8.55	2.18	-0.111
End and intermediate spans		0.800	6.75	1.49	-0.155
Simple and end spans	3 and 4	0.902	8.55	2.18	-0.111
Intermediate span		0.800	6.75	1.49	-0.155

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Rotational restraint given by the sheeting

The total lateral spring stiffness K per unit length should be determined from:

$$\frac{1}{K} = \frac{1}{K_A} + \frac{1}{K_B} + \frac{1}{K_C}$$

where

K_A is the lateral stiffness corresponding to the rotational stiffness of the joint between the sheeting and the purlin;

K_B is the lateral stiffness due to distortion of the cross section of the purlin;

K_C is the lateral stiffness due to the flexural stiffness of the sheeting.

$$K = \frac{1}{\frac{1}{K_A} + \frac{1}{K_B}}$$

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Rotational restraint given by the sheeting

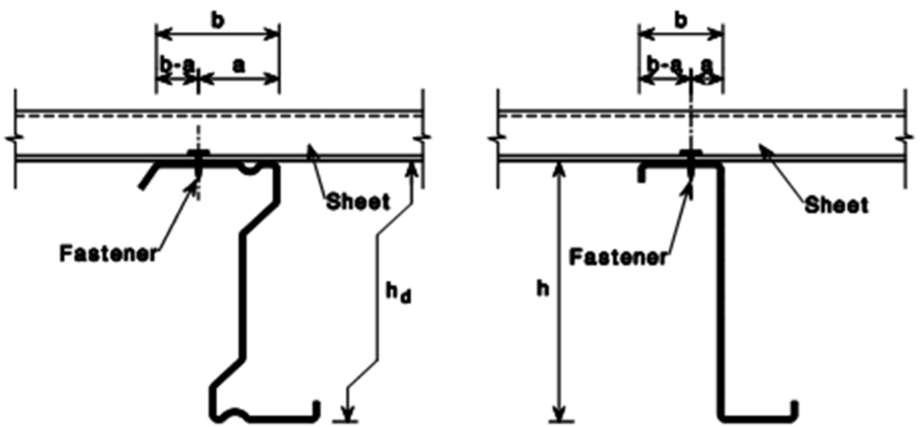
The value of $(1/K_A + 1/K_B)$ may be obtained either by testing or by calculation.

The lateral spring stiffness K per unit length may be determined by calculation using:

$$\frac{1}{K} = \frac{4(1-\nu^2)h^2(h_d + b_{mod})}{Et^3} + \frac{h^2}{C_D} \qquad (4.86)$$

in which th

- for c … ie purlin into
  cont:
- for c … ie purlin into
  cont: … $_{mod} = 2a + b$



- **Design of beams restrained by sheeting according to EN1993-1-3**

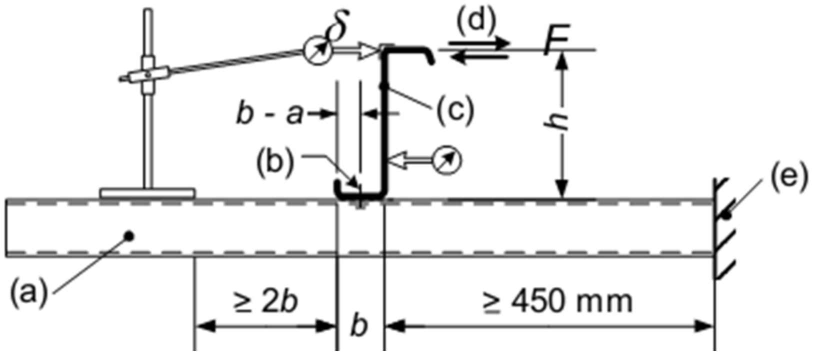
Design resistance – Rotational restraint given by the sheeting

The value of $(1/K_A + 1/K_B)$ may be obtained either by testing or by calculation.

The test set-up to determine $(1/K_A + 1/K_B)$, the amount of torsional restraint given by adequately fastened sheeting or by another member perpendicular to the span of the beam

a) The lateral stiffness K_A per unit length corresponding to the rotational stiffness of the connection between the sheeting and the beam;

b) The lateral stiffness K_B per unit length due to distortion of the cross section of the purlin.



- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Rotational restraint given by the sheeting

The combined restraint per unit length may be determined from:

$$\left(1/K_A + 1/K_B\right) = \delta / F$$

where

F is the load per unit length of the test specimen to produce a lateral deflection of $h/10$;

h is the overall depth of the specimen;

δ is the lateral displacement of the top flange in the direction of the load F.

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Rotational restraint given by the sheeting

The rotational restraint given to the purlin by the sheeting should be modelled as a rotational spring acting at the top flange of the purlin

$$C_D = \frac{1}{\left(1/C_{D,A} + 1/C_{D,C}\right)}$$

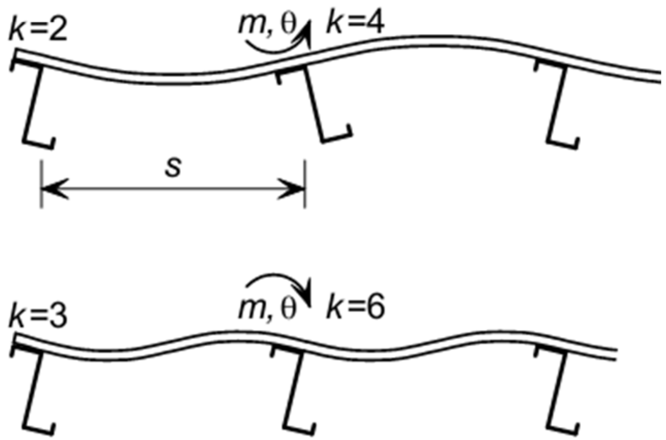
where

$C_{D,A}$ is the rotational stiffness of the connection between the sheeting and the purlin;

$C_{D,C}$ is the rotational stiffness corresponding to the flexural stiffness of the sheeting.

using:

$$C_{D,C} = m/\theta$$



- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Rotational restraint given by the sheeting

The rotational restraint given to the purlin by the sheeting should be modelled as a rotational spring acting at the top flange of the purlin

$$C_D = \frac{1}{\left(1/C_{D,A} + 1/C_{D,C}\right)}$$

where

$C_{D,A}$ is the rotational stiffness of the connection between the sheeting and the purlin;

$C_{D,C}$ is the rotational stiffness corresponding to the flexural stiffness of the sheeting.

Alternatively a conservative value of $C_{D,C}$ may be obtained from:

$$C_{D,C} = \frac{k\ E\ I_{eff}}{s}$$

I_{eff} is the effective second moment of area per unit width of sheeting;

s is the spacing of the purlins.

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Rotational restraint given by the sheeting

Generally $C_{D,A}$ may be calculated provided that the sheet-to-purlin fasteners are positioned centrally on the flange of the purlin.

Alternatively $C_{D,A}$ may be taken as equal to 130p [Nm/m/rad], where p is the number of sheet-to-purlin fasteners per metre length of purlin (but not more than one per rib of sheeting), provided that:

- the flange width b of the sheeting through which it is fastened does not exceed 120 mm;
- the nominal thickness t of the sheeting is at least 0.66 mm;
- the distance a or b - a between the centreline of the fastener and the centre of rotation of the purlin (depending on the direction of rotation), as shown in Figure 4.59, is at least 25 mm.

- **Design of beams restrained by sheeting according to EN1993-1-3**

Design resistance – Rotational restraint given by the sheeting

Generally $C_{D,A}$ may be calculated provided that the sheet-to-purlin fasteners are positioned centrally on the flange of the purlin.

Alternatively, values of $C_{D,A}$ may be obtained from a combination of testing and calculation. If the value of $\left(1/K_A + 1/K_B\right)$ is obtained by testing according to Annex A of EN1993-1-3 (in mm/N), the values of $C_{D,A}$ for gravity loading and for uplift loading should be determined from:

$$C_{D,A} = \frac{h^2 / l_A}{\left(1/K_A + 1/K_B\right) - 4(1-\nu^2)h^2\left(h_d + b_{mod}\right)/Et^3 l_B} \qquad (4.91)$$

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

Purlins with C-, Z- and Σ-cross sections with or without additional stiffeners in web or flange may be designed as described in this clause if the following conditions are fulfilled:

- the cross section dimensions are within the range of Table
- the purlins are horizontally restrained by trapezoidal sheeting where the horizontal restraint fulfils the condition
- the purlins are eting and the conditions of T
- the purlins have

$$S \geq \left(EI_w \frac{\pi^2}{L^2} + GI_t + EI_z \frac{\pi^2}{L^2} 0.25 h^2 \right) \frac{70}{h^2}$$

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

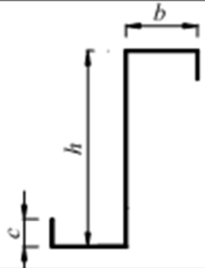
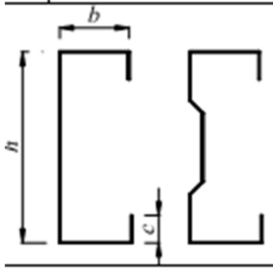
- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

Purlins with C-, Z- and Σ-cross sections with or without additional stiffeners in web or flange may be designed as described in this clause if the following conditions are fulfilled:

- the cross section dimensions are within the range of Table

Limitations to be fulfilled if the simplified design method is used

Purlins	t [mm]	b/t	h/t	h/b	c/t	b/c	L/h
	≥ 1.25	≤ 55	≤ 160	≤ 3.43	≤ 20	≤ 4.0	≥ 15
	≥ 1.25	≤ 55	≤ 160	≤ 3.43	≤ 20	≤ 4.0	≥ 15

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

Purlins with C-, Z- and Σ-cross sections with or without additional stiffeners in web or flange may be designed as described in this clause if the following conditions are fulfilled:

- the cross section dimensions are within the range of Table
- the purlins are horizontally restrained by trapezoidal sheeting where the horizontal restraint fulfils the condition
- the purlins are restrained rotationally by trapezoidal sheeting and the conditions of Table
- the purlins have equal spans and uniform loading.

This method should not be used:

- for systems using anti-sag bars;
- for sleeve or overlapping systems;
- for purlins subjected to axial forces N_{Ed}.

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

The design value of the bending moment M_{Ed} should satisfy:

$$\frac{M_{Ed}}{M_{LT,Rd}} \leq 1$$

where

$$M_{LT,Rd} = \left(\frac{f_y}{\gamma_{M1}}\right) W_{eff,y} \frac{\chi_{LT}}{k_d}$$

and

$W_{eff,y}$ is the section modulus of the effective cross section with regard to the y–y axis;

χ_{LT} is the reduction factor for lateral-torsional buckling in terms of $\bar{\lambda}_{LT}$, where α_{LT} is substituted by $\alpha_{LT,eff}$;

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y} f_y}{M_{cr}}}$$

$$\alpha_{LT,eff} = \alpha_{LT} \sqrt{\frac{W_{el,y}}{W_{eff,y}}}$$

α_{LT} is the imperfection factor;

$W_{el,y}$ is the section modulus of the gross cross section with regard to the y–y axis;

k_d is a coefficient for consideration of the non-restrained part of the purlin;

$$k_d = \left(a_1 - a_2 \frac{L}{\ldots} \right) \ldots \geq 1$$

a_1, a_2 coefficients defined in Table 4.13,

L span of the purlin;

h overall depth of the purlin

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

Coefficients a_1, a_2

System	Z-purlins		C-purlins		Σ-purlins	
	a_1	a_2	a_1	a_2	a_1	a_2
single span beam gravity load	1.0	0	1.1	0.002	1.1	0.002
single span beam uplift load	1.3	0	3.5	0.050	1.9	0.020
continuous beam gravity load	1.0	0	1.6	0.020	1.6	0.020
continuous beam uplift load	1.4	0.010	2.7	0.040	1.0	0

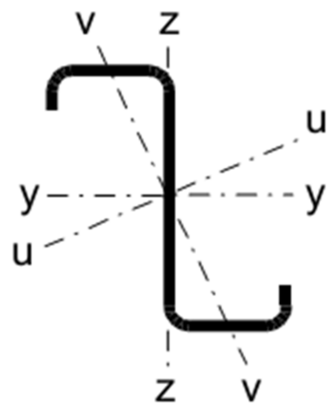
Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

$$C_D \geq \frac{M_{el,u}^2}{E I_v} k_\vartheta \quad \Rightarrow \quad \chi_{LT} = 1.0$$



where

$M_{el,u}$ is the elastic moment of the gross cross section with regard to the major *u-u* axis (see Figure 4.62);

$$M_{el,u} = W_{el,u}\, f_y$$

I_v is the moment of inertia of the gross cross section with regard to the minor *v-v* axis

k_ϑ is a factor for considering the static system of the purlin

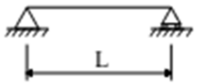
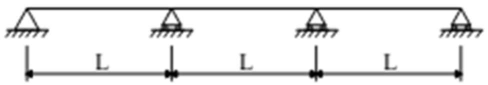
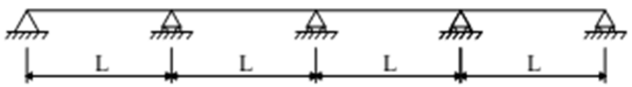
Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

k_ϑ is a factor for considering the static system of the purlin

Static system	Gravity load	Uplift load
	-	0.210
	0.07	0.029
	0.15	0.066
	0.10	0.053

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

Simplified design of purlins

$$C_D \geq \frac{M_{el}^2}{E I_v} k_\vartheta \quad \Rightarrow \quad \chi_{LT}$$

$$M_{cr} = \frac{k}{L} \sqrt{G I_t^* \, E I_v}$$

where

I_t^* is the fictitious St. Venant torsion constant considering the effective rotational restraint

$$I_t^* = I_t + C_D \frac{L^2}{\pi^2 G}$$

I_t is the St. Venant torsion constant of the purlin;

- Design of beams restrained by sheeting according to EN1993-1-3

Simplified design of purlins

Beams restrained by sheeting

- Design of beams restrained by sheeting according to EN1993-1-3

Simplified design of purlins

Behaviour and Design Resistance of Bar Members

Beams restrained by sheeting

- **Design of beams restrained by sheeting according to EN1993-1-3**

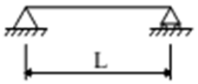
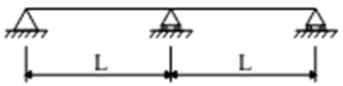
Simplified design of purlins

$$\frac{1}{C_D} = \frac{1}{C_{D,A}} + \frac{1}{C_{D,B}} + \frac{1}{C_{D,C}}$$

where

$C_{D,A}$, $C_{D,C}$	rotational stiffnesses
$C_{D,B}$	rotational stiffness due to distortion of the cross section of the purlin, $C_{D,B} = K_B\, h^2$, where h = depth of the purlin and K_B
k	lateral torsional buckling coefficient

Lateral-torsional buckling coefficients k for purlins restrained horizontally at the upper flange

Static system	Gravity load	Uplift load
	∞	10.3
	17.7	27.7

Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- **General**

The rules for serviceability limit states given in Section 7 of EN1993-1-1 should also be applied to cold-formed members and sheeting.

The properties of the effective cross section for serviceability limit states should be used in all serviceability limit state calculations for cold-formed members and sheeting.

The second moment of area may be calculated alternatively by interpolation of gross cross section and effective cross section using the expression

$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{\sigma}\left(I_{gr} - I(\sigma)_{eff}\right) \qquad (4.103)$$

where

I_{gr} is second moment of area of the gross cross section;

σ_{gr} is maximum compressive bending stress in the serviceability limit state, based on the gross cross section (positive in formula);

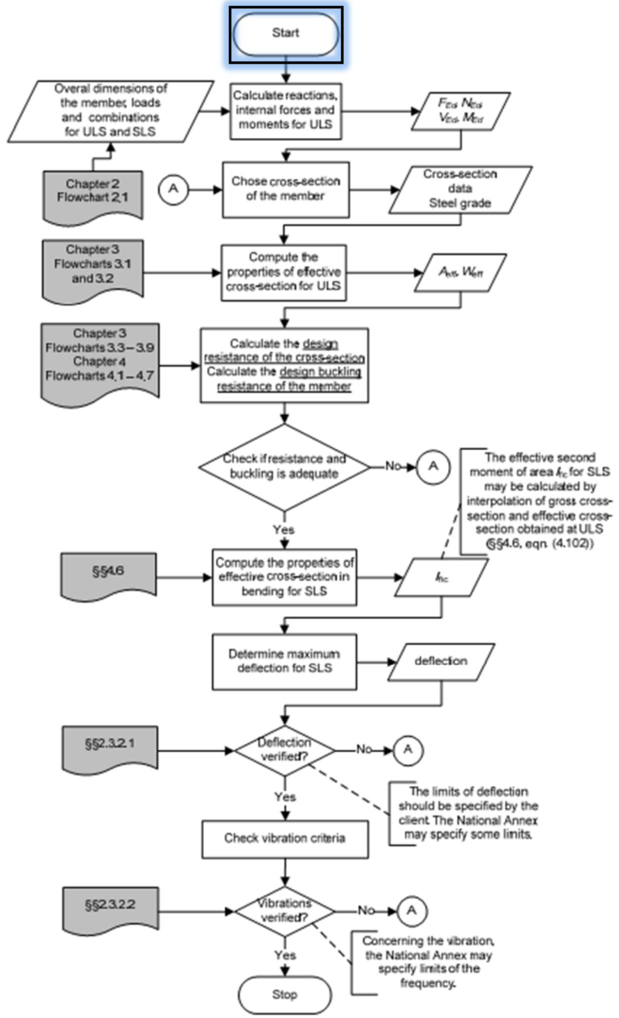
$I(\sigma)_{eff}$ is the second moment of area of the effective cross section with allowance for local buckling calculated for a maximum stress $\sigma \geq \sigma_{gr}$ in which the maximum stress is the largest

Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- *Example – the SLS check of a cold-formed steel member in bending*

This example continues the Example done for **ULS** check for a cold-formed steel beam in bending, with the serviceability limit state (**SLS**) check. The beam has pinned end conditions and is composed of two thin-walled cold-formed steel back-to-back lipped channel sections, with a span L = 4.5 m.



Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- ***Example – Design of an cold-formed steel beam in bending***

Basic Data

Span of beam $L = 4.5\ \text{m}$

Spacing between beams $S = 3.0\ \text{m}$

Distributed loads applied to the joist:

self-weight of the beam $q_{G,beam} = 0.14\ \text{kN/m}$

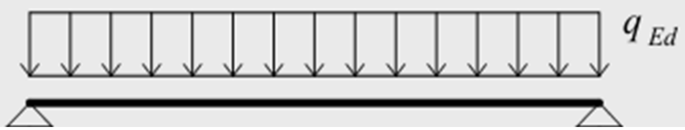
weight of the floor and $0.6\ \text{kN/m}^2$

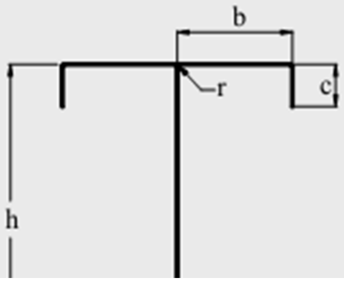
$q_{G,slab} = 0.55 \times 3.0 = 1.65\ \text{kN/m}$

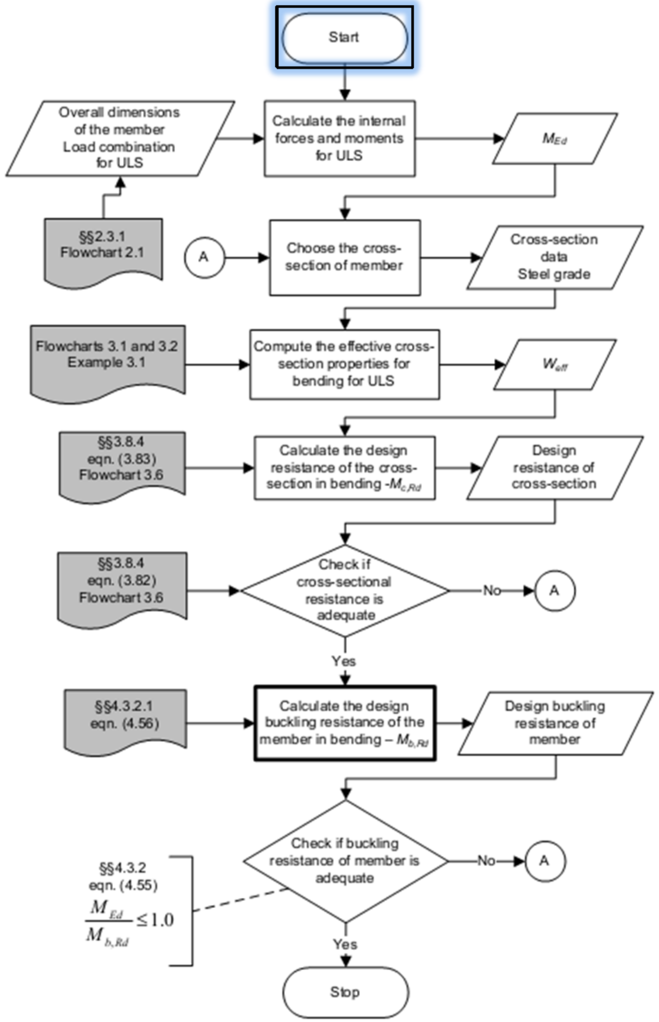
total dead load $q_G = q_{G,beam} + q_{G,slab} = 1.79\ \text{kN/m}$

imposed load $1.50\ \text{kN/m}^2$

$q_Q = 1.50 \times 3.0 = 4.50\ \text{kN/m}$







- *Example – Design of an cold-formed steel beam in bending*

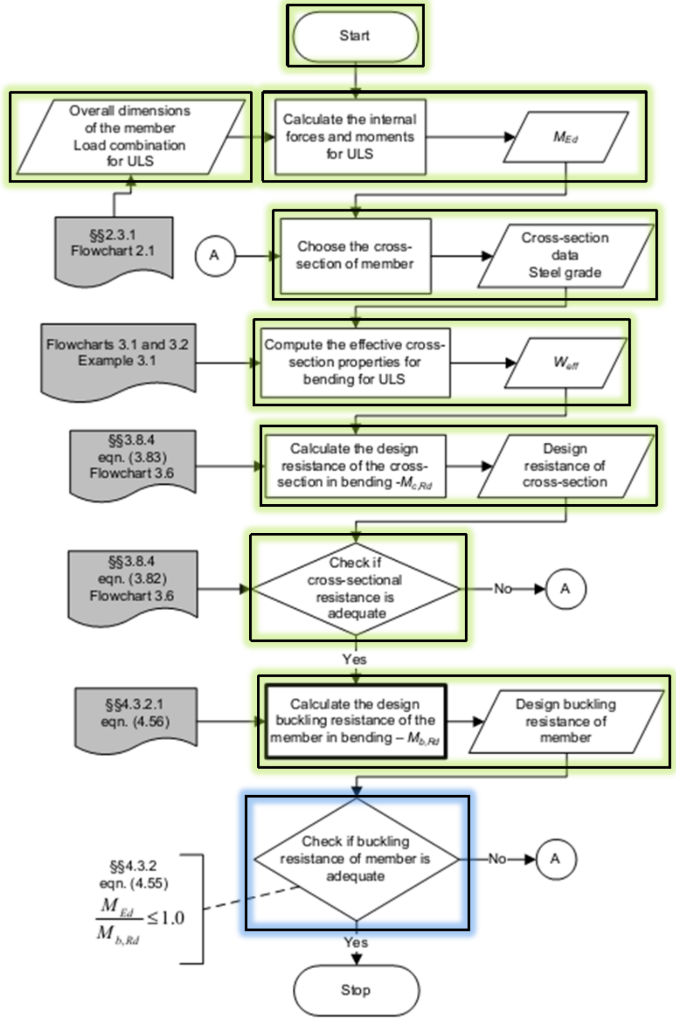
Check of buckling resistance at ULS

Design moment resistance of the cross section for bending

$$M_{b,Rd} = \chi_{LT} W_{eff,y} f_{yb} / \gamma_{M1} =$$
$$= 0.369 \times 182091 \times 10^{-9} \times 350 \times 10^{3} / 1.0 =$$
$$= 23.52 \text{ kNm}$$

Verification of buckling resistance

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{23.21}{23.52} = 0.987 < 1 \quad - \text{OK}$$



Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- *Example – the SLS check of a cold-formed steel member in bending*

Verification for Serviceability Limit State

Applied loading on the joist at SLS

$$q_{d,ser} = q_G + q_Q = 1.79 + 4.50 = 6.29\ \text{kN/m}$$

The maximum applied bending moment:

$$M_{Ed,ser} = q_{d,ser} L^2/8 = 6.29 \times 4.5^2/8 = 15.92\ \text{kNm}$$

Effective section properties at the serviceability limit state

Second moment of area for SLS (§§4

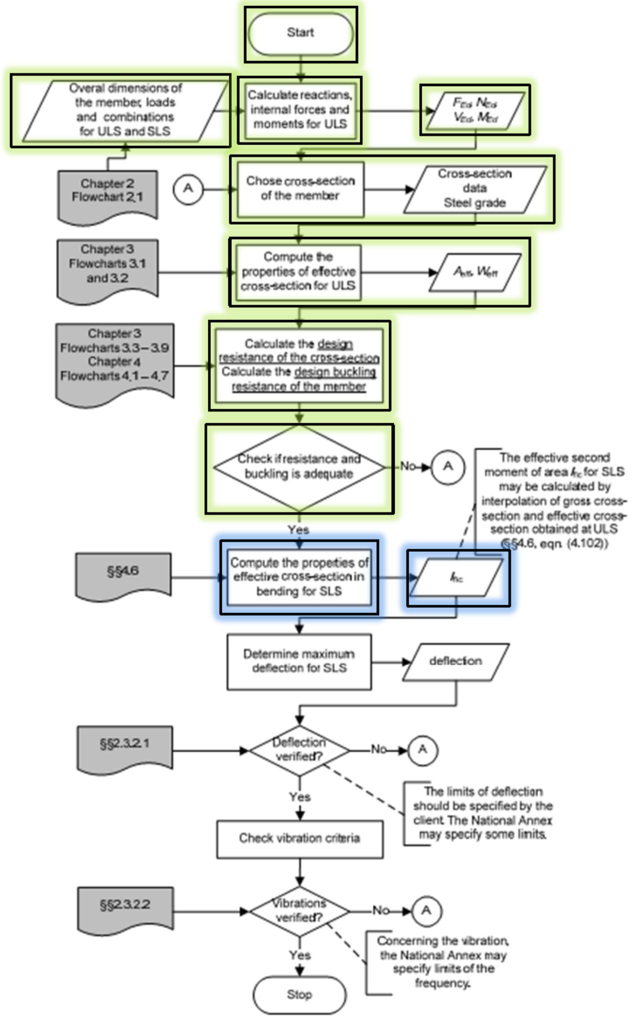
$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{\sigma}\left(I_{gr} - I(\sigma)_{eff}\right)$$

with:

$I_{gr} = 2302.15 \times 10^4\ \text{mm}^4$

σ_{gr} – maximum compressive bending stress in SLS

$z_{c,gr} = 125\ \text{mm}$ – distance from the centroidal axis to the compressed flange



Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- *Example – the SLS check of a cold-formed steel member in bending*

Verification for Serviceability Limit State

Applied loading on the joist at SLS

$$q_{d,ser} = q_G + q_Q = 1.79 + 4.50 = 6.29\ \text{kN/m}$$

The maximum applied bending moment:

$$M_{Ed,ser} = q_{d,ser} L^2 / 8 = 6.29 \times 4.5^2 / 8 = 15.92\ \text{kNm}$$

Effective section properties at the serviceability limit state

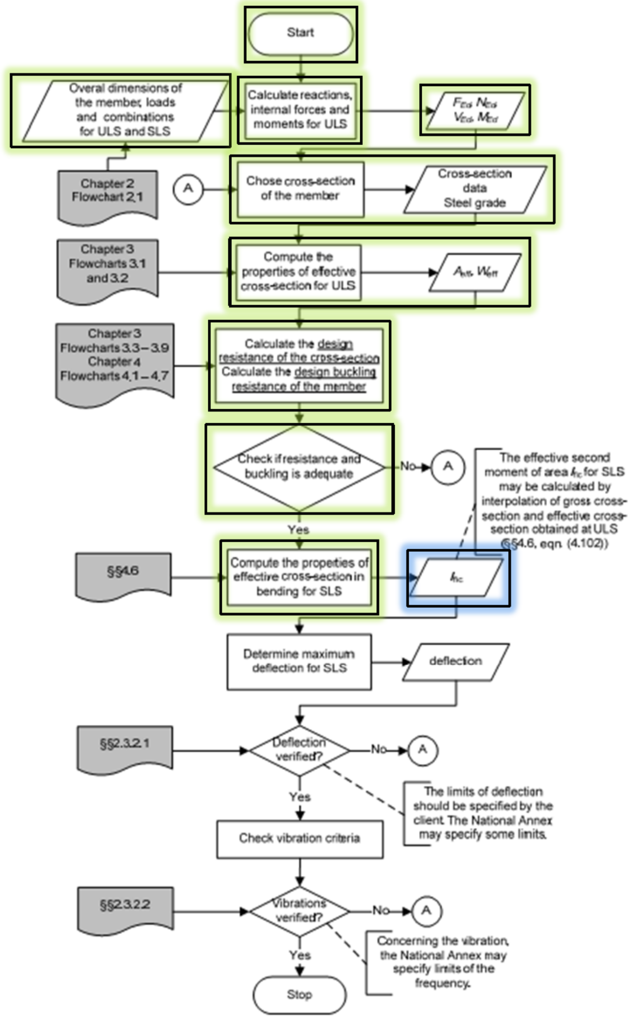
Second moment of area for SLS

$$\sigma_{gr} = \frac{M_{Ed,ser}}{W_{gr}} = \frac{M_{Ed,ser}}{I_{gr}/z_{c,gr}} = \frac{15.92 \times 10^6}{2302.15 \times 10^4 / 125} = 86.45\ \text{N/mm}^2$$

$$\sigma = f_{yb} = 350\ \text{N/mm}^2$$

...ssive bending stress in SLS

$$I(\sigma)_{eff} = I_{eff,y} = 2268.89 \times 10^4\ \text{mm}^4$$ ...entroidal axis to the compressed flange



Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- *Example – the SLS check of a cold-formed steel member in bending*

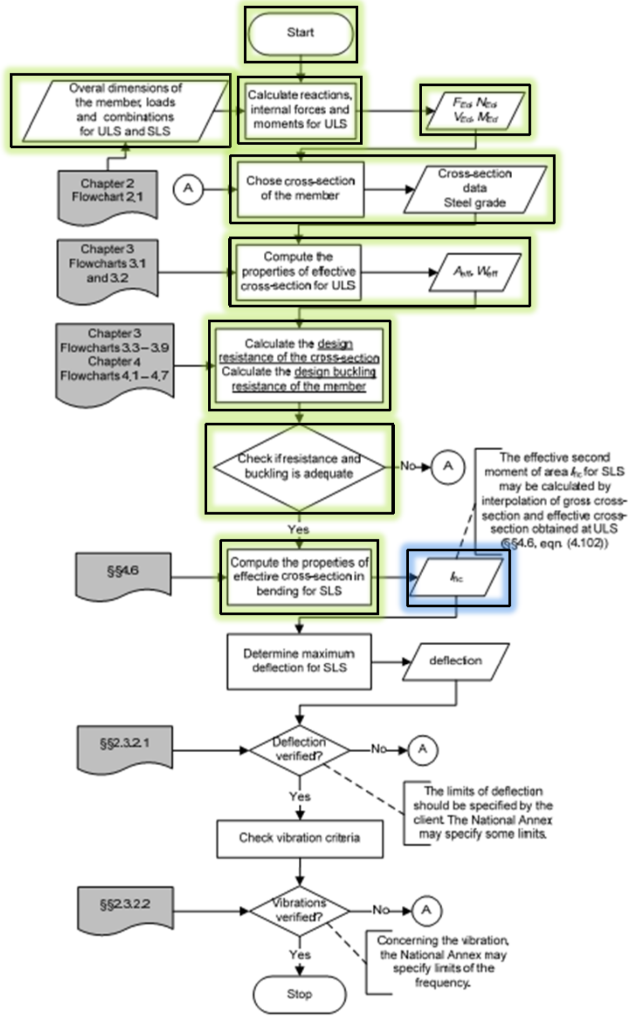
Effective section properties at the serviceability limit state

Second moment of area for SLS

$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{\sigma}\left(I_{gr} - I(\sigma)_{eff}\right)$$

$$I_{fic} = 2302.15\times10^4 - \frac{96.45}{350}\times\left(2302.15 - 2269.89\right)\times10^4 =$$

$$= 2293.26\times10^4 \text{ mm}^4$$



Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- *Example – the SLS check of a cold-formed steel member in bending*

Effective section properties at the serviceability limit state

Second moment of area for SLS

$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{\sigma}\left(I_{gr} - I(\sigma)_{eff}\right)$$

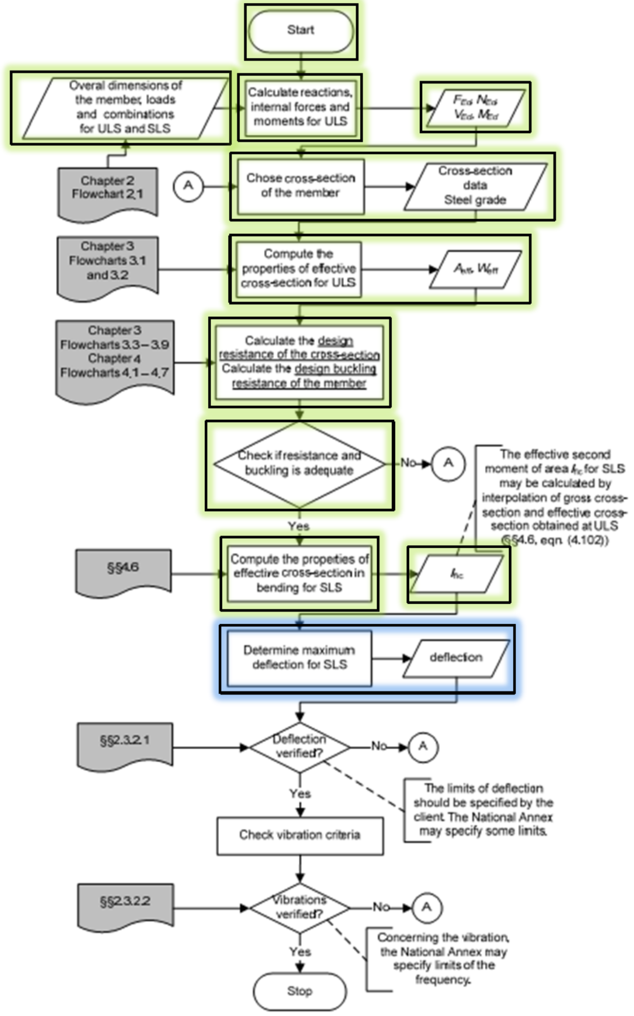
$$I_{fic} = 2302.15\times10^4 - \frac{96.45}{350}\times(2302.15 - 2269.89)\times10^4 =$$

$$= 2293.26\times10^4 \text{ mm}^4$$

Deflection check

Deflection at the centre of the joist:

$$\delta = \frac{5}{384}\frac{q_{d,ser}L^4}{EI_{fic}} = \frac{5}{384}\times\frac{6.29\times4500^4}{210000\times2293.26\times10^6} = 5.36 \text{ mm}$$



Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- *Example – the SLS check of a cold-formed steel member in bending*

Effective section properties at the serviceability limit state

Second moment of area for SLS

$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{\sigma}\left(I_{gr} - I(\sigma)_{eff}\right)$$

$$I_{fic} = 2302.15 \times 10^4 - \frac{96.45}{350} \times (2302.15 - 2269.89) \times 10^4 =$$

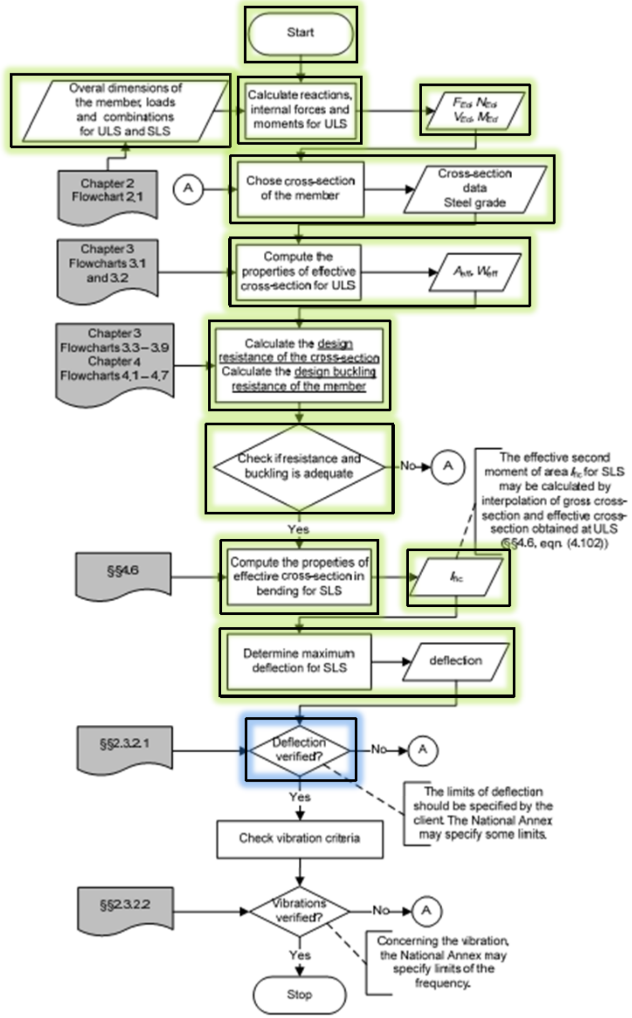
$$= 2293.26 \times 10^4 \text{ mm}^4$$

Deflection check

Deflection at the centre of the joist:

$$\delta = \frac{5}{384}\frac{q_{d,ser}L^4}{EI_{fic}} = \frac{5}{384} \times \frac{6.29 \times 4500^4}{210000 \times 2293.26 \times 10^6} = 5.36 \text{ mm}$$

The deflection is $L/840$ – OK



Behaviour and Design Resistance of Bar Members

Design of beams at serviceability limit states

- *Example – the SLS check of a cold-formed steel member in bending*

Effective section properties at the serviceability limit state

Second moment of area for SLS

$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{\sigma}\left(I_{gr} - I(\sigma)_{eff}\right)$$

$$I_{fic} = 2302.15\times10^4 - \frac{96.45}{350}\times(2302.15 - 2269.89)\times10^4 =$$

$$= 2293.26\times10^4\ \text{mm}^4$$

Deflection check

Deflection at the centre of the joist:

$$\delta = \frac{5}{384}\frac{q_{d,ser}L^4}{EI_{fic}} = \frac{5}{384}\times\frac{6.29\times4500^4}{210000\times2293.26\times10^6} = 5.36\ \text{mm}$$

The deflection is $L/840$ – OK