# Nonlinear static analysis
# PUSHOVER

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European Erasmus Mundus Master Course
**Sustainable Constructions under Natural Hazards and Catastrophic Events**
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# Structural analysis for seismic assesement

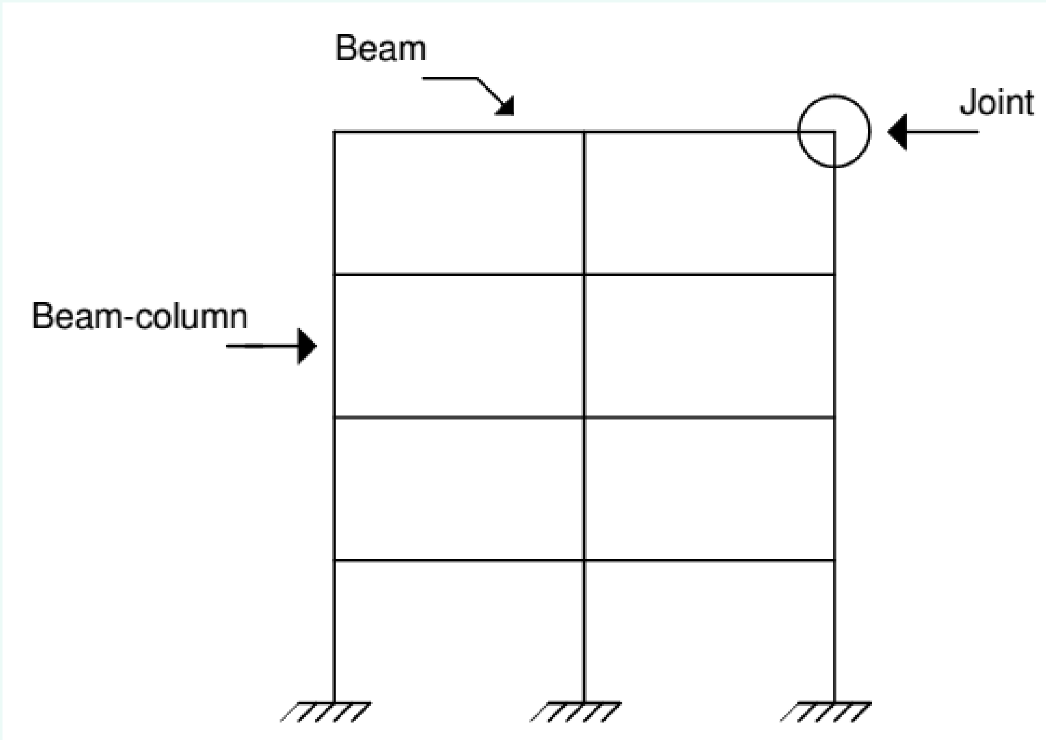
- Elastic analysis
  - Lateral force method
  - Modal response spectrum analysis (spectral analysis)
  - Modal time history analysis
  - Linear dynamic analysis
- Inelastic analysis
  - Nonlinear static analysis (pushover)
  - Nonlinear dynamic analysis

Conventional design

Advanced design

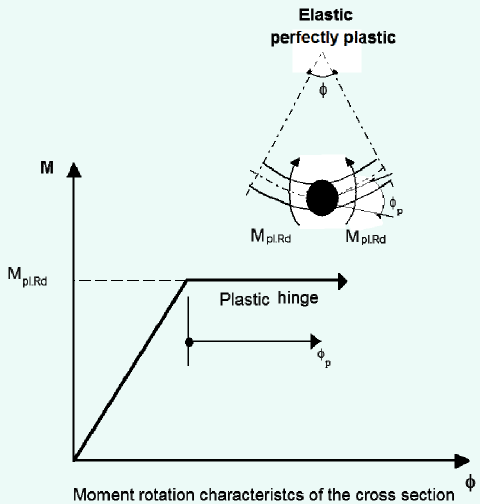
# Structural model

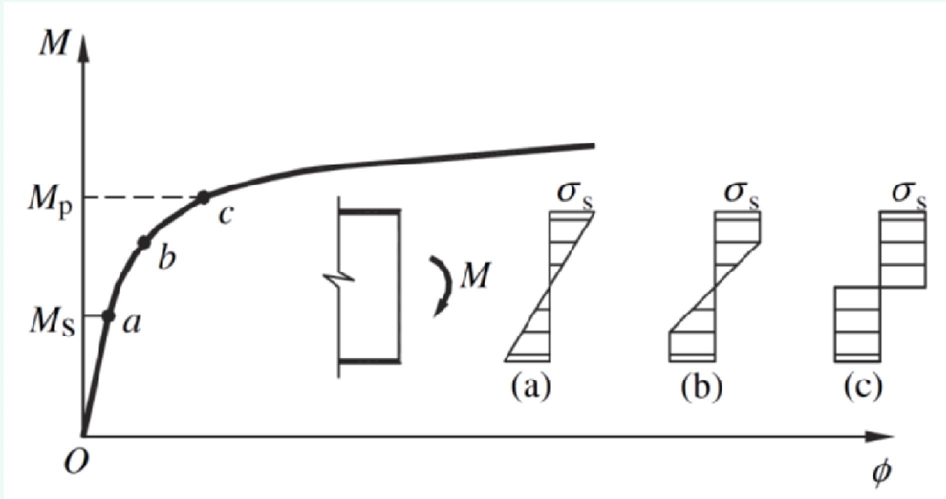
- Frames structures can be model using linear elements (beams, columns, braces) connected in nodes
- Modelling of inelastic behavior of structural components must be accounted to perform a inelastic structural analysis
- Software:
  - SAP 2000,



# Dissipative zones

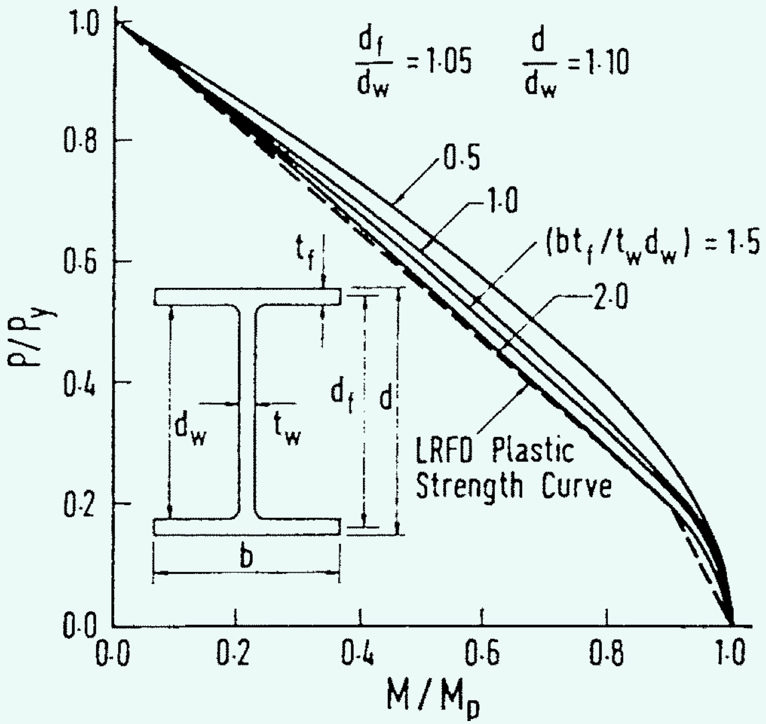
- Different plastic mechanisms are possible, depending on the type of structural action developed
- Plastic hinge:
  - Bending
  - Shear
  - Tension
- Concentrated plasticity model can be generalized for all types of action (bending, shear, axial)

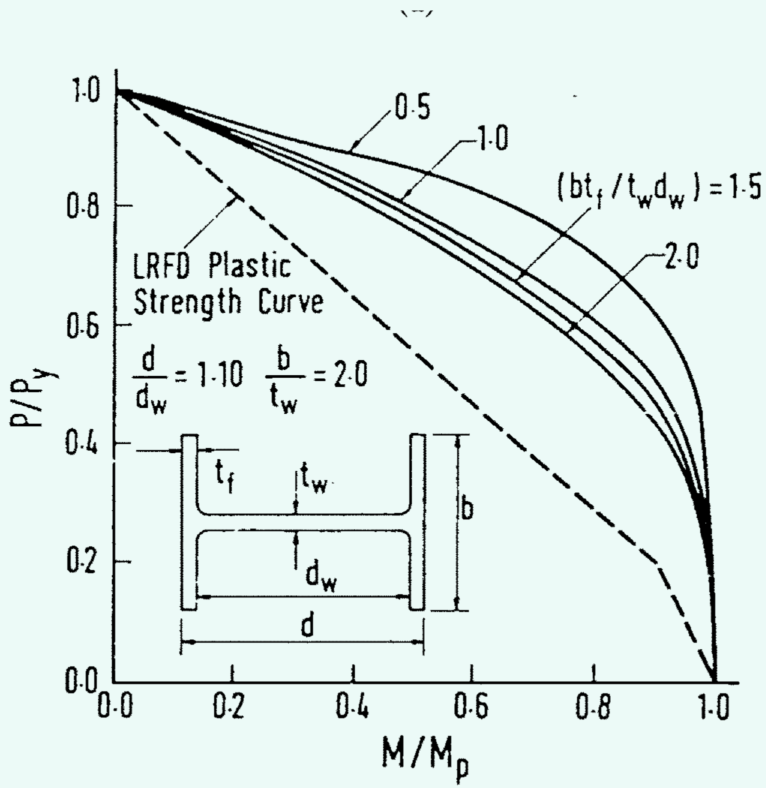


# Beam-column with plastic hinges

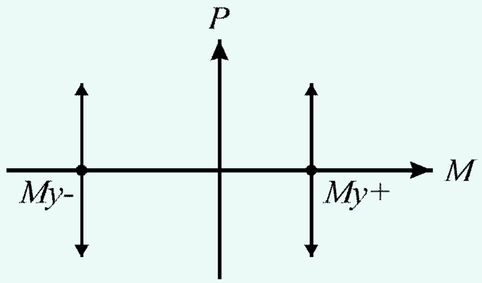
- **Axial force affects the moment capacity of the cross-section ⇒ it is necessary to account for the axial force – bending moment interaction for members subject to bending moments and axial forces**

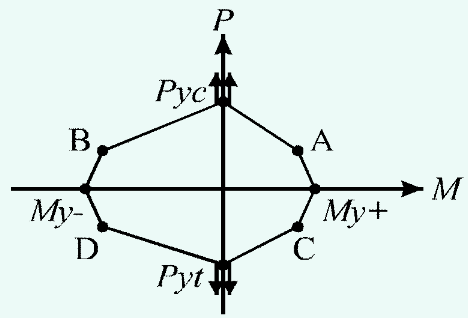


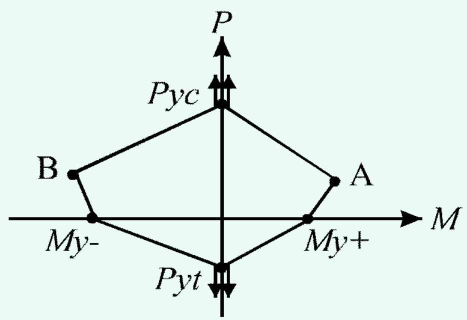



# Beam-column with plastic hinges

- Bending moment – axial force interaction in plastic hinges (concentrated plasticity): bending moment capacity affected by the axial force, but only plastic rotations are assumed to occur
  - Interaction neglected: elements with small axial force
  - Interaction curve (surface): steel members subjected to bending moment and axial force (A=0.1Py for bending about the strong axis of double T cross-sections)
  - Interaction curve (surface): reinforced concrete members subjected to bending moment and axial force



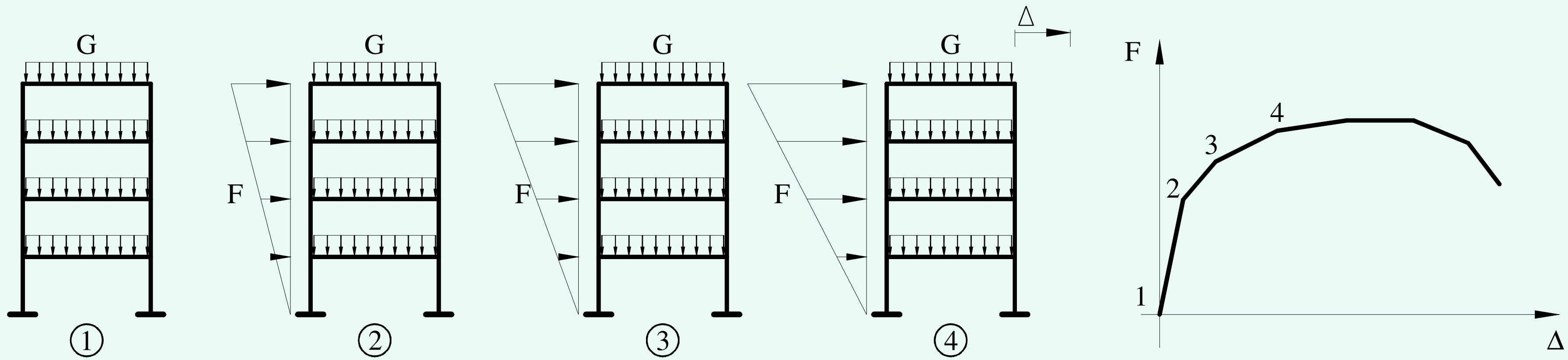



# Nonlinear static analysis (pushover)

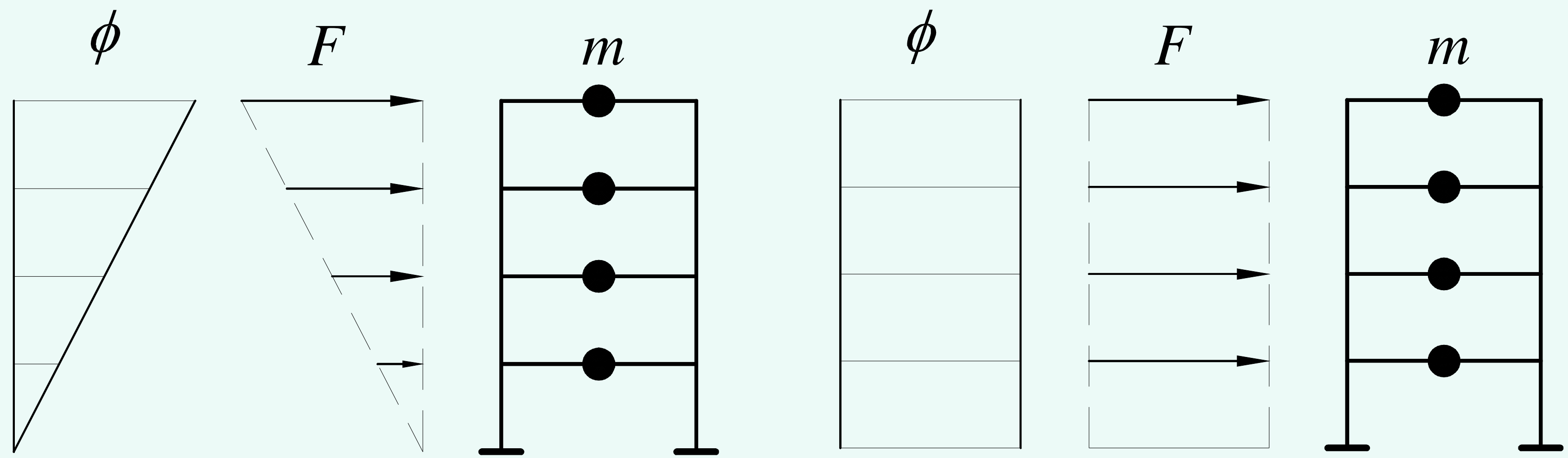
- **Nonlinear static analysis under constant gravity loading and monotonically increasing lateral forces**
- **Control elements:**
  - **Base shear force**
  - **Control displacement (top displacement)**

  } **pushover curve**
- **Provides the capacity of the structure, and does not give directly the demands associated  with a particular level of seismic action**



# Nonlinear static analysis (pushover)

- Assumes that response is governed by a single mode of vibration, and that it is constant during the analysis
- Distribution of lateral forces (applied at storey masses):
  - modal (usually first mode – inverted triangle)
  - uniform: lateral forces proportional to storey masses



# Shape of Lateral force

- **Total base force**

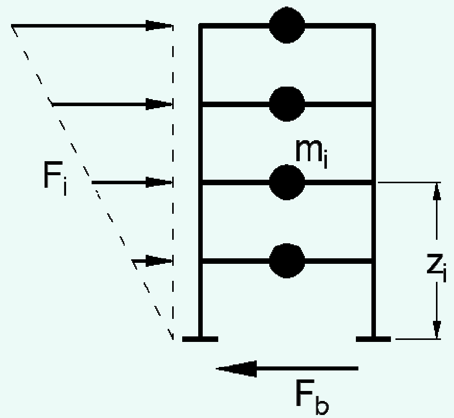
$$F_b = \gamma_{I,e} S_d\left(T_1\right) m\lambda$$

  - **$S_d(T_1)$ - ordinate of the design response spectrum corresponding to fundamental period $T_1$;**
  - **$m$ - total mass of the structure;**
  - **$\gamma_{I,e}$ – importance factor;**
  - **$\lambda$ - correction factor (contribution of the fundamental mode of vibration using the concept of effective modal mass):**

**Lateral force at storey *i***

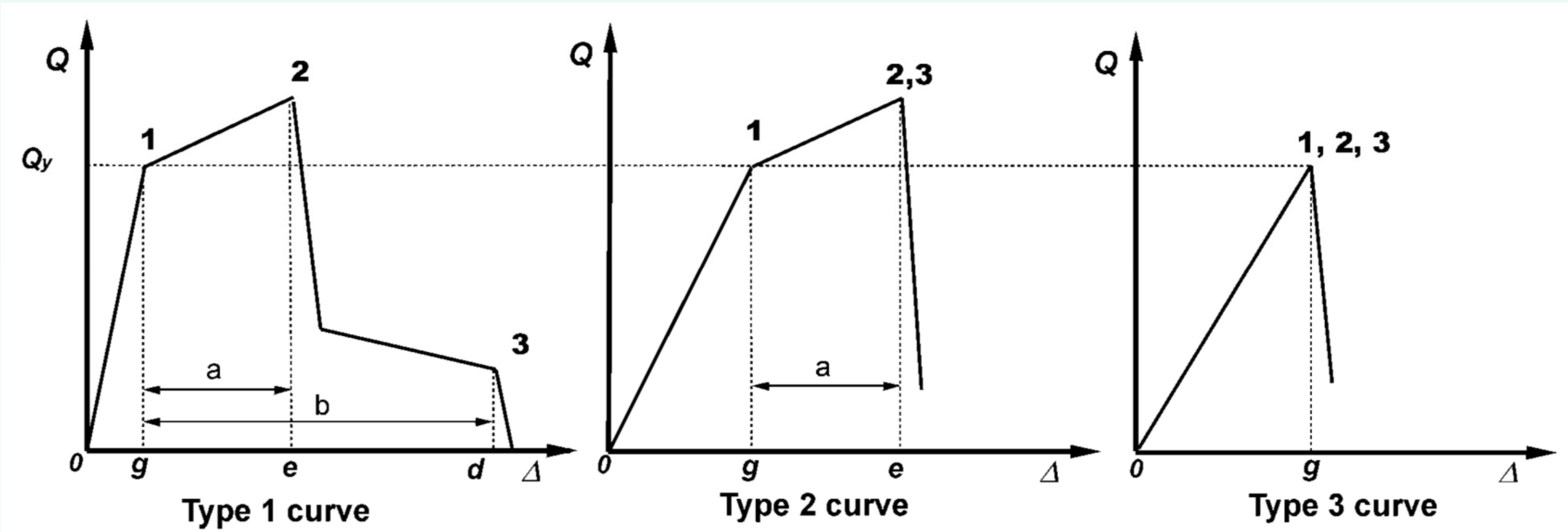
- **Triangular shape**
- **Uniform shape**

$$F_i = F_b \frac{m_i z_i}{\sum_{i=1}^{N} m_i z_i}$$



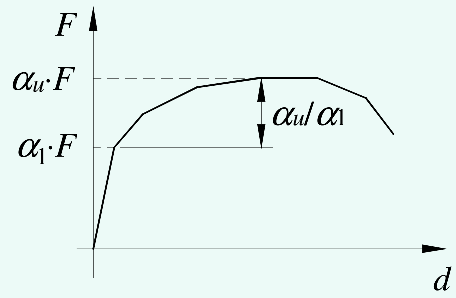
# Nonlinear static analysis (pushover)

- Applicable to low-rise regular buildings, where the response is dominated by the fundamental mode of vibration.
- Application of loading:
  - Gravity loading: force control
  - Lateral forces: displacement control
- Modelling of structural components: inelastic monotonic force-deformation obtained from envelopes of cyclic response

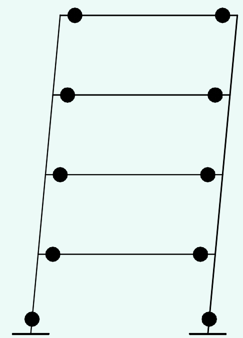


# Nonlinear static analysis (pushover)

- Represents a direct evaluation of overall structural response, not only on an element by element basis
- Allows evaluation of inealstic deformations – the most relevant response quantity in the case of inelastic response
- Allows evaluation of the plastic mechanism and redundancy of the structure ($\alpha_u/\alpha_1$ ratio)



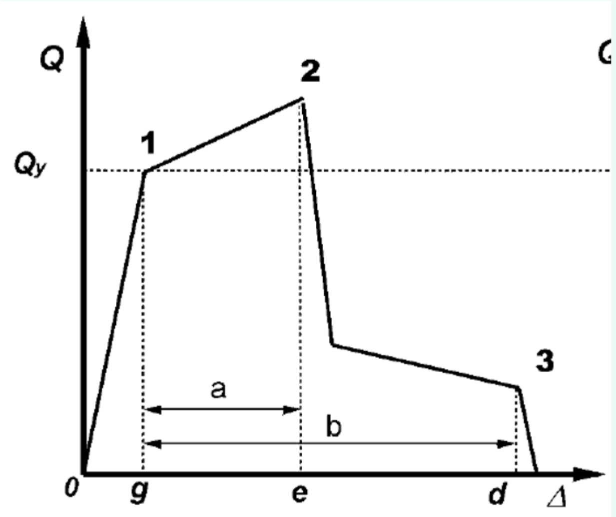

- "Local" checks:
  - Interstorey drifts
  - Strength demands in non-dissipative components
  - Ductility of dissipative components



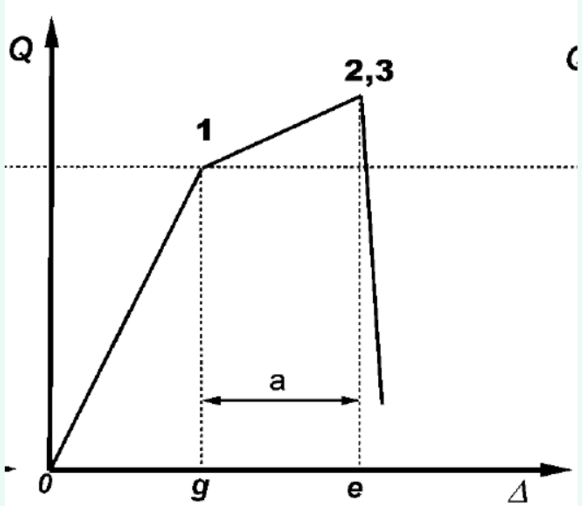
- "Global" checks – failure at the structure level
  - Failure to resist further gravity loading
  - Failure of the first vertical element essential for stability of the structure

# Deformation controlled (ductile) / force-controlled actions (brittle) actions

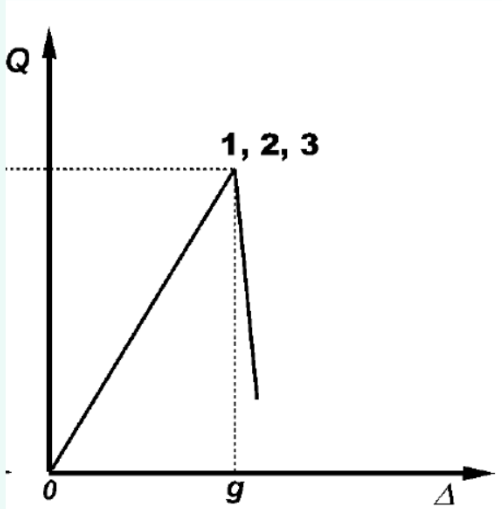
- **Type 1 curve (deformation-controlled or ductile)**
- **Type 2 curve (deformation-controlled or ductile)**
- **Type 3 curve (brittle or nonductile behavior)**
- **Type of response (ductile / brittle) affects:**
  - **Modelling of structural component**
  - **Performance criteria**



Type 1 curve



Type 2 curve



Type 3 curve

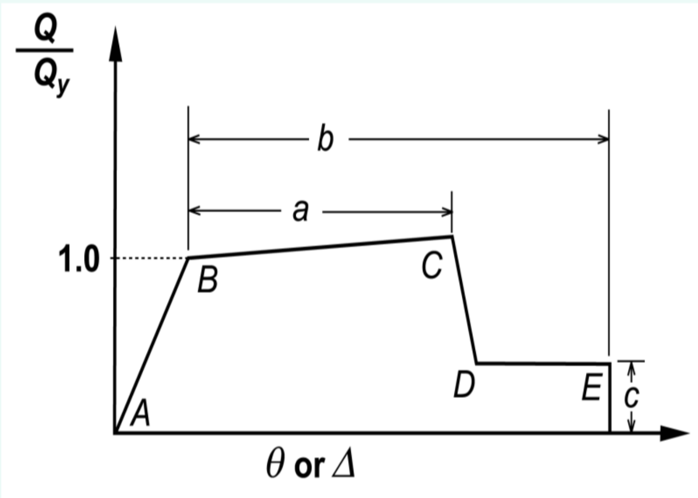
# Deformation controlled (ductile) / force-controlled actions (brittle) actions

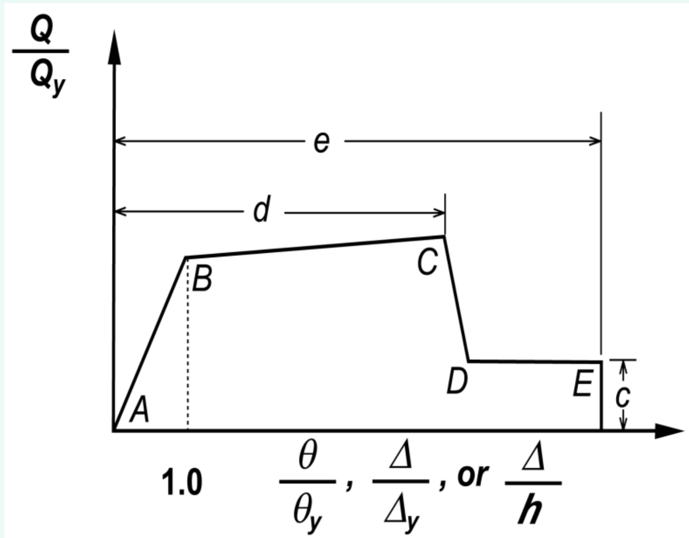
- **Examples of ductile / brittle actions:**

| Type of structure | Component | Ductile actions | Brittle actions |
|---|---|---|---|
| Steel moment resisting frames | beams | Bending (M) | Shear (V) |
| | columns | M | Axial (N), V |
| | joints | V (in general) | - |
| Steel concentrically braced frames | braces | N | - |
| | beams | - | N |
| | columns | - | N |
| Steel eccentrically braced frames | links (short) | V | M, N |
| | braces | - | M, N, (V) |
| | beams | - | M, N, V |
| | columns | - | M, N, V |

# Modelling of components

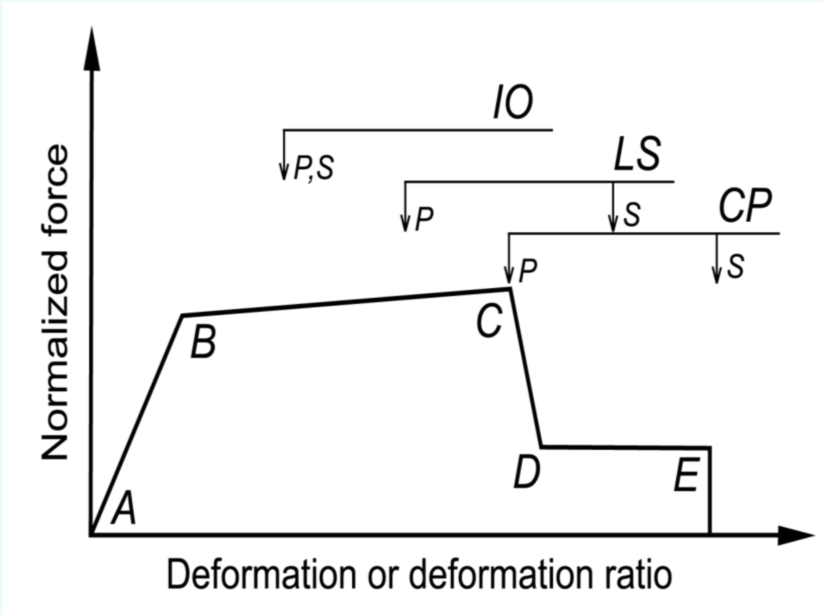
- **Modelling of ductile components:**
  - **A-B segment: elastic response**
  - **B-C segment: strain hardening**
  - **C-D segment: strength degradation**
  - **D-E segment: residual strength**
- **Modelling and performance criteria can be specified in terms of:**
  - **Absolute deformations ($\theta$ or $\Delta$) or**
  - **Normalised deformations ($\theta/\theta_y$ or $\Delta/\Delta_y$)**



# Performance criteria: components

- The degree to which a structural components fulfil a performance criteria is established based on the demand to capacity ratios.
- Generally, modelling of components and their performance criteria are obtained from experimental tests, depending on:
  - Type of structural component (primary / secondary)
  - Performance level considered
- For usual materials and structural types, data from literature or codes can be used (e.g. FEMA 356, EN 1998-3, P100-3)



# Performance criteria: components

- **Principle of checking the performance:**

  **Ed ≤ Rd**

  **Effect of action (demand) ≤ Capacity of the component**

- **In case of plastic analysis methods, performance criteria are checked in terms of deformations for ductile components and in terms of forces for brittle components**

- **Codes for existing buildings (FEMA 356, EN1998-3):**
  - **Ductile components: design deformation ≤ capacity**
  - **Brittle components: design force ≤ strength determined using characteristic material properties**

# Examples of modelling parameters and performance criteria (FEMA 356)

***Table 5-6 Modeling Parameters and Acceptance Criteria for Nonlinear Procedures—Structural Steel Components***

| | Modeling Parameters | | | Acceptance Criteria | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Plastic Rotation Angle, Radians | | | | |
| | Plastic Rotation Angle, Radians | | Residual Strength Ratio | | Primary | | Secondary | |
| **Component/Action** | **a** | **b** | **c** | **IO** | **LS** | **CP** | **LS** | **CP** |
| **Beams—flexure** | | | | | | | | |
| a. $\frac{b_f}{2t_f} \le \frac{52}{\sqrt{F_{ye}}}$ and $\frac{h}{t_w} \le \frac{418}{\sqrt{F_{ye}}}$ | $9\theta_y$ | $11\theta_y$ | 0.6 | $1\theta_y$ | $6\theta_y$ | $8\theta_y$ | $9\theta_y$ | $11\theta_y$ |
| b. $\frac{b_f}{2t_f} \ge \frac{65}{\sqrt{F_{ye}}}$ or $\frac{h}{t_w} \ge \frac{640}{\sqrt{F_{ye}}}$ | $4\theta_y$ | $6\theta_y$ | 0.2 | $0.25\theta_y$ | $2\theta_y$ | $3\theta_y$ | $3\theta_y$ | $4\theta_y$ |
| c. Other | Linear interpolation between the values on lines a and b for both flange slenderness (first term) and web slenderness (second term) shall be performed, and the lowest resulting value shall be used | | | | | | | |

# Performance criteria: structure

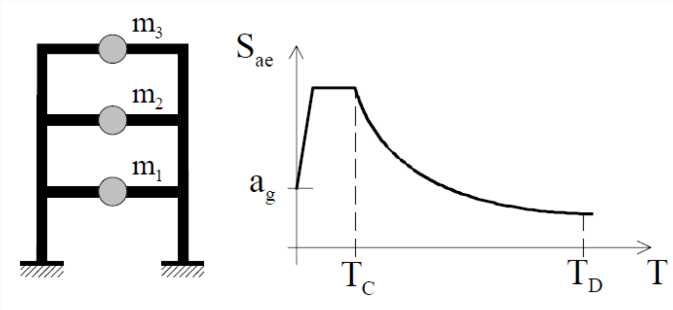
- **The structure shall be provided with at least one continuous load path to transfer seismic forces, induced by ground motion in any direction, from the point of application to the final point of resistance.**

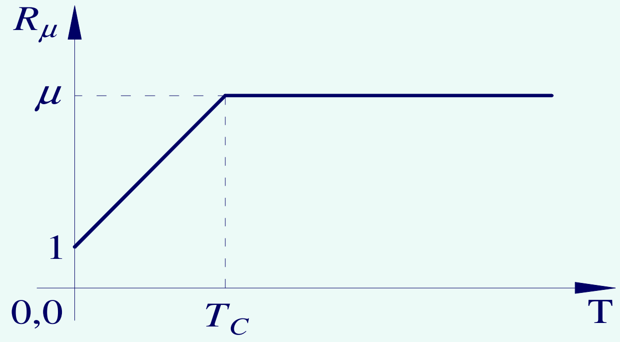
- **All primary and secondary components shall be capable of resisting force and deformation actions within the applicable acceptance criteria of the selected performance level.**

## Target displacement in a nonlinear static analysis: the N2 method

# N2: performance evaluation procedure

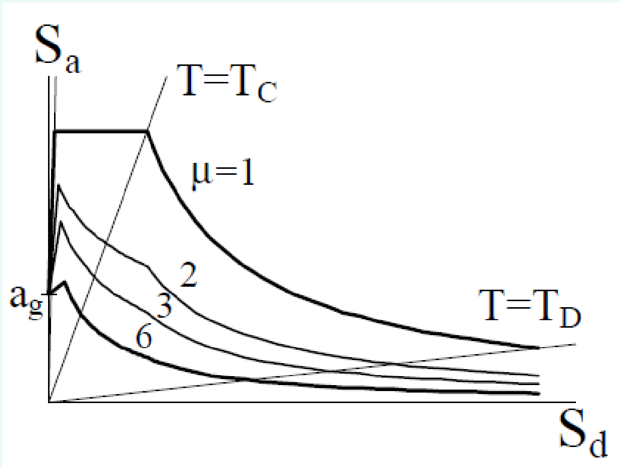
1. **Initial data**
   - **Properties of the structure**
   - **Elastic pseudo-acceleration response spectrum $S_{ae}$**
2. **Determination of spectra in AD format for constant values of ductility, e.g. μ=1, 2, 4, 6, etc. (only if graphical representation of the method is needed)**



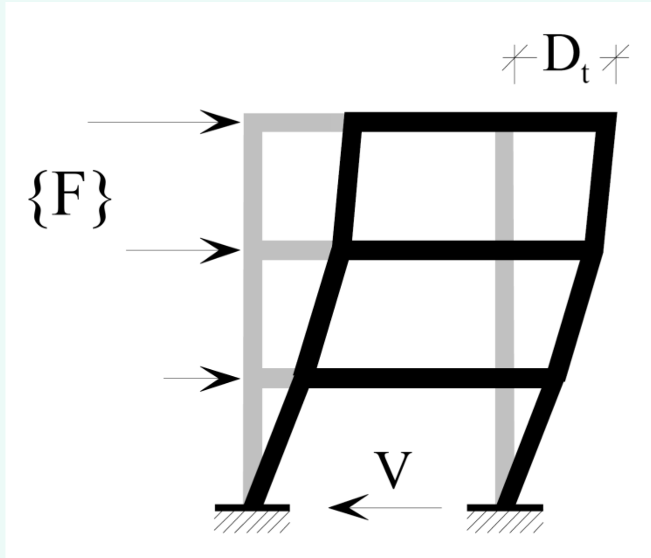

$$R_\mu = (\mu - 1)\frac{T}{T_C} + 1 \qquad T < T_C$$

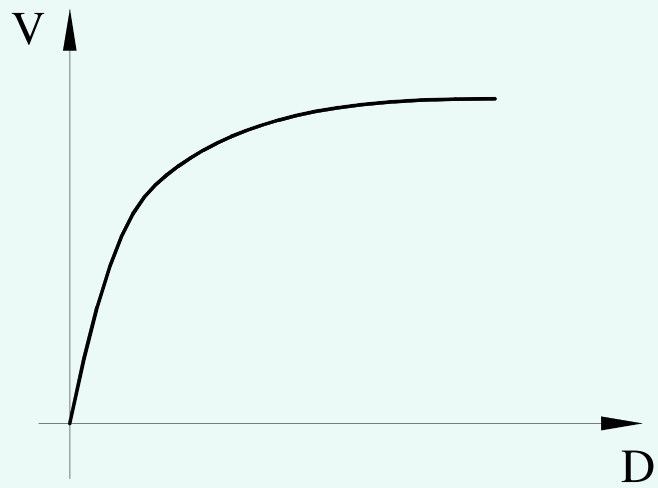
$$R_\mu = \mu \qquad T \geq T_C$$



# N2: performance evaluation procedure

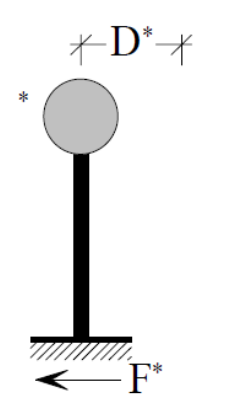
3. **Nonlinear static analysis**
   - **Assume displacement shape {$\phi$}**
     **Note: normalized in such a way that the component at the top is equal to 1**
   - **Determine vertical distribution of lateral forces**
     $F_i=[m]\{\phi\}=m_i \cdot \phi_i$
   - **Determine base shear (V)–top displacement (Dt) relationship by performing the nonlinear static analysis**



# N2: performance evaluation procedure

4. **Equivalent SDOF system**

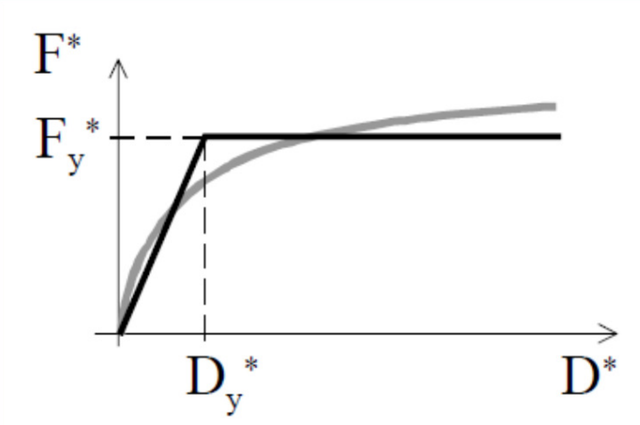



- **Determine mass *m** =** $\sum m_i \cdot \phi_i$
- **Transform MDOF quantities (Q) to SDOF quantities (Q*)**
  $Q^* = Q/\Gamma$

$$\Gamma = \frac{m^*}{\sum m_i \phi_i^2}$$

- **Determine an approximate elasto-plastic force – displacement relationship** $F^*$-$D^*$
- **Determine strength** $F_y^*$**, yield displacement** $D_y^*$**, and period** $T^*$

$$T^* = 2\pi \sqrt{\frac{m^* \cdot D_y^*}{F_y^*}}$$



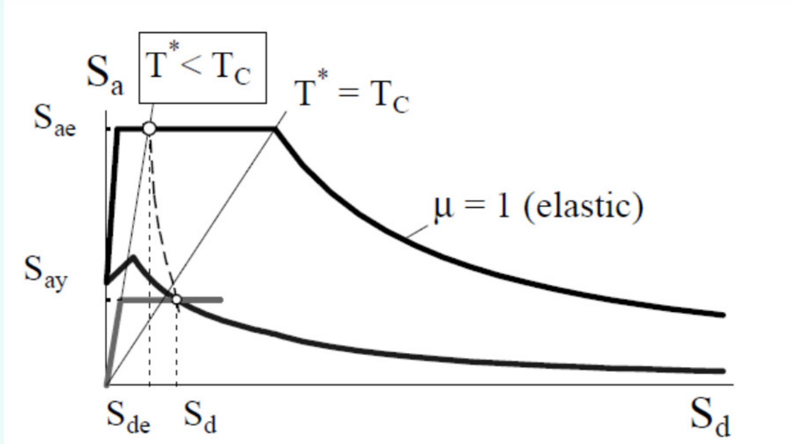

- **Determine capacity diagram** $S_a$-$S_d$ **(only if graphical representation of the method is needed)**

$$S_a = \frac{F^*}{m^*} \qquad S_d = D^*$$

# N2: performance evaluation procedure

5. **Seismic demand for SDOF system**
   - **Determine reduction factor $R_\mu$**

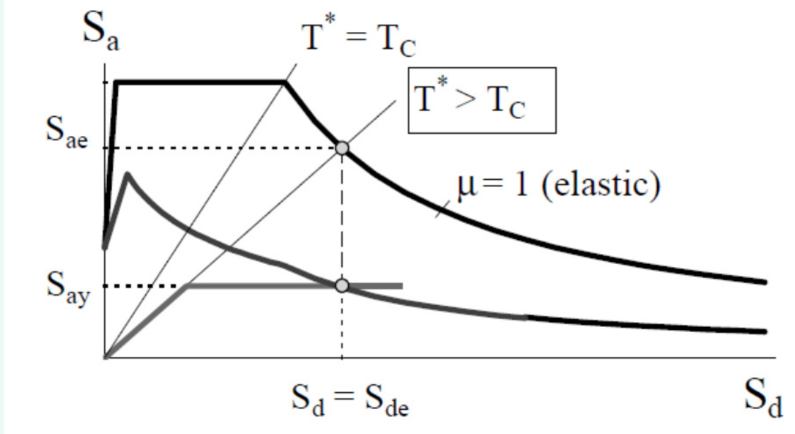
$$R_\mu = \frac{S_{ae}\left(T^*\right)}{S_{ay}}$$




   - **Determine displacement demand $S_d$=$D*$**

$$S_d = \frac{S_{de}}{R_\mu}\left(1+\left(R_\mu -1\right)\frac{T_C}{T^*}\right) \qquad T^* < T_C$$

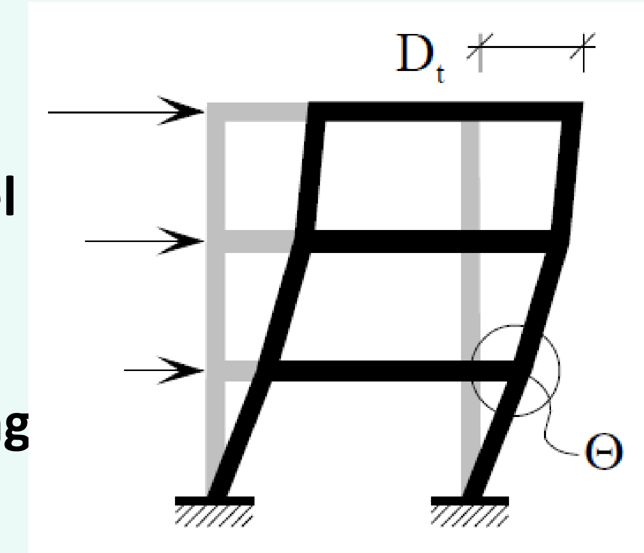
$$S_d = S_{de} \qquad T^* \geq T_C$$



# N2: performance evaluation procedure

6. **Global seismic demand for MDOF system**
   - **Transform SDOF displacement demand to the top displacement of the MDOF model**
     $D_t = \Gamma \cdot S_d$

7. **Local seismic demands for MDOF system**
   - **Perform pushover analysis of MDOF model up to the top displacement $D_t$**
   - **Determine local response quantities (e.g. story drifts, rotations $\theta$, etc.) corresponding to $D_t$**




8. **Performance evaluation**
   - **Compare local and global seismic demands with the capacities for the relevant performance level**