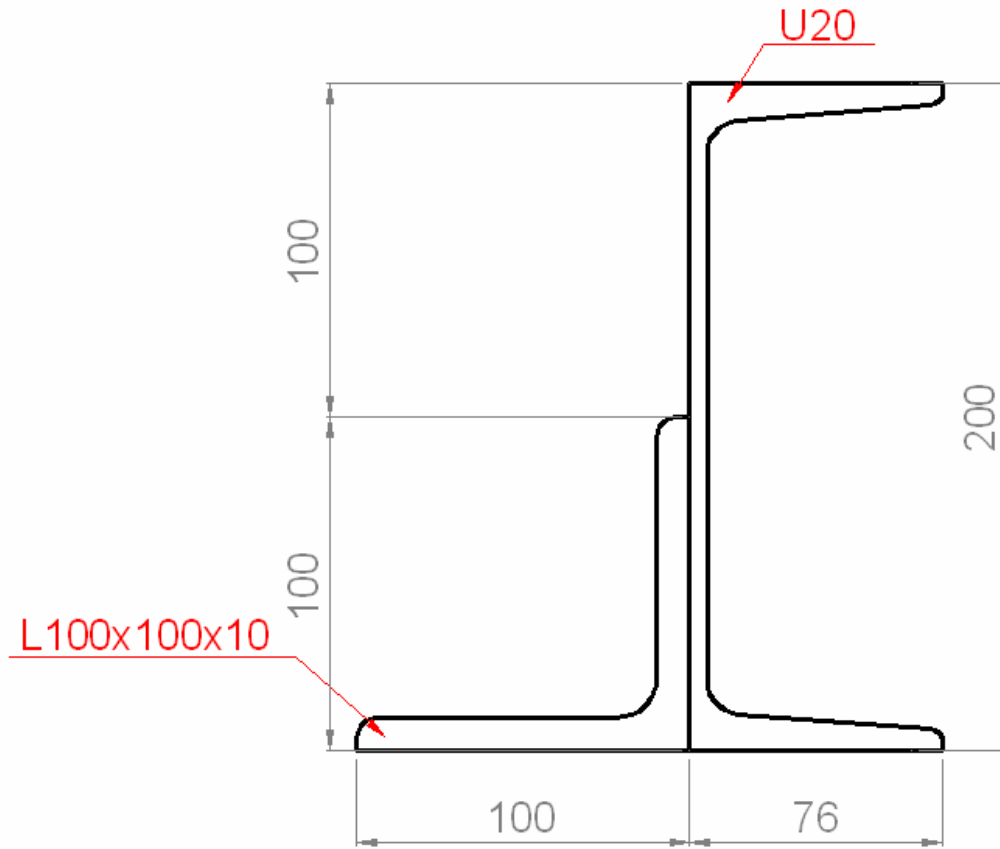


Sa se determine momentele de inertie principale si pozitia axelor de inertie principale pentru sectiunea din figura de mai jos:



- Determinarea pozitiei centrului de greutate

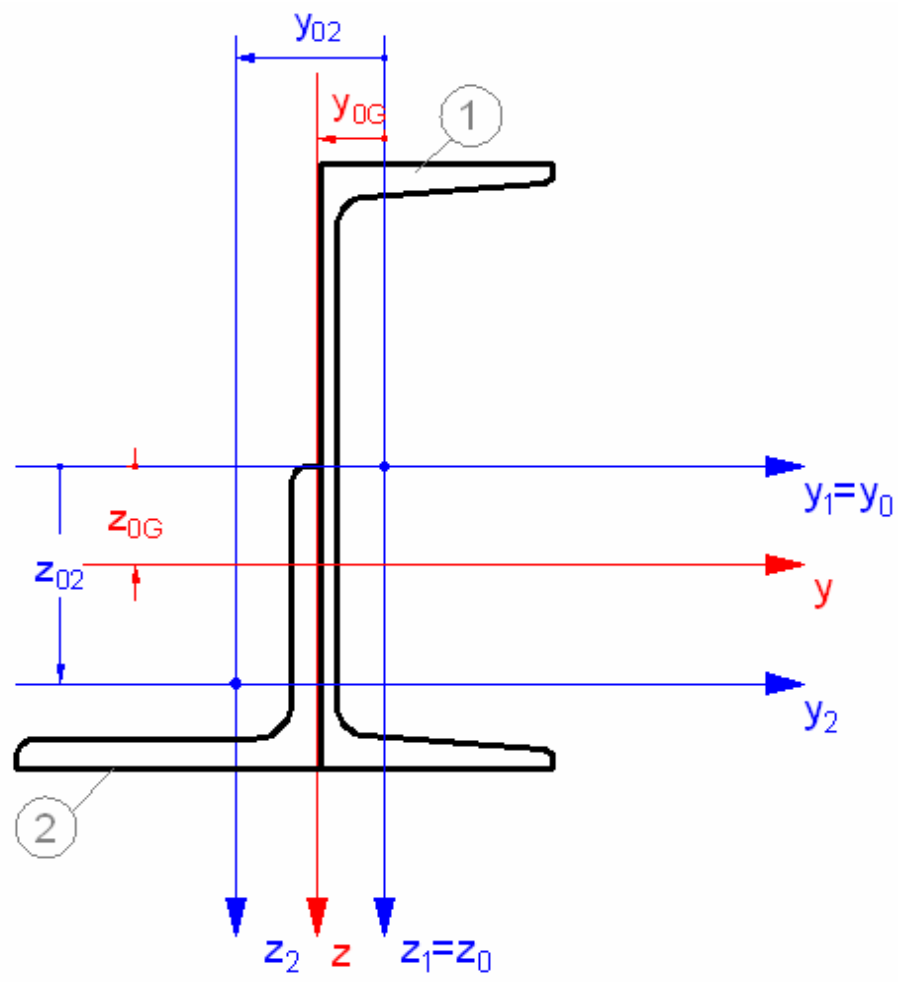
$$A_1 = 23.4 \text{ cm}^2 \quad A_2 = 19.2 \text{ cm}^2$$

$$I_{y1} = 1520 \text{ cm}^4 \quad I_{y2} = 177 \text{ cm}^4$$

$$I_{z1} = 113 \text{ cm}^4 \quad I_{z2} = 177 \text{ cm}^4$$

$$y_{0G} = \frac{\sum y_{0i} \cdot A_i}{\sum A_i} = \frac{0 \cdot 23.4 + (-4.89) \cdot 19.2}{23.4 + 19.2} = -2.203 \text{ cm}$$

$$z_{0G} = \frac{\sum z_{0i} \cdot A_i}{\sum A_i} = \frac{0 \cdot 23.4 + 7.18 \cdot 19.2}{23.4 + 19.2} = 3.236 \text{ cm}$$



- Momentele de inertie in raport cu axele centrale

$$I_y = I_{y1} + I_{y2} + z_1^2 \cdot A_1 + z_2^2 \cdot A_2$$

$$z_1 = -z_{0G} = -3.326 \text{ cm}$$

$$z_2 = (z_{0G} - z_{02}) = (7.18 - 3.326) = 3.854 \text{ cm}$$

$$\Rightarrow I_y = 1520 + 177 + (-3.326)^2 \cdot 23.4 + 3.854^2 \cdot 19.2 = 2241 \text{ cm}^4$$

$$I_z = I_{z1} + I_{z2} + y_1^2 \cdot A_1 + y_2^2 \cdot A_2$$

$$y_1 = y_{0G} = 2.203 \text{ cm}$$

$$y_2 = -(y_{02} - y_{0G}) = -(4.89 - 2.203) = -2.687 \text{ cm}$$

$$\Rightarrow I_z = 113 + 177 + (2.203)^2 \cdot 23.4 + (-2.687)^2 \cdot 19.2 = 542 \text{ cm}^4$$

$$I_{yz} = I_{y1z1} + I_{y2z2} + y_1 \cdot z_1 \cdot A_1 + y_2 \cdot z_2 \cdot A_2$$

$$I_{y1z1} = I_{y2z2} = 0 \quad \text{deoarece au cel puțin o axa de simetrie}$$

$$\Rightarrow I_z = (-3.326)(2.203) \cdot 23.4 + (3.854)(-2.687) \cdot 19.2 = -370.28 \text{ cm}^4$$

- Momentele de inertie principale

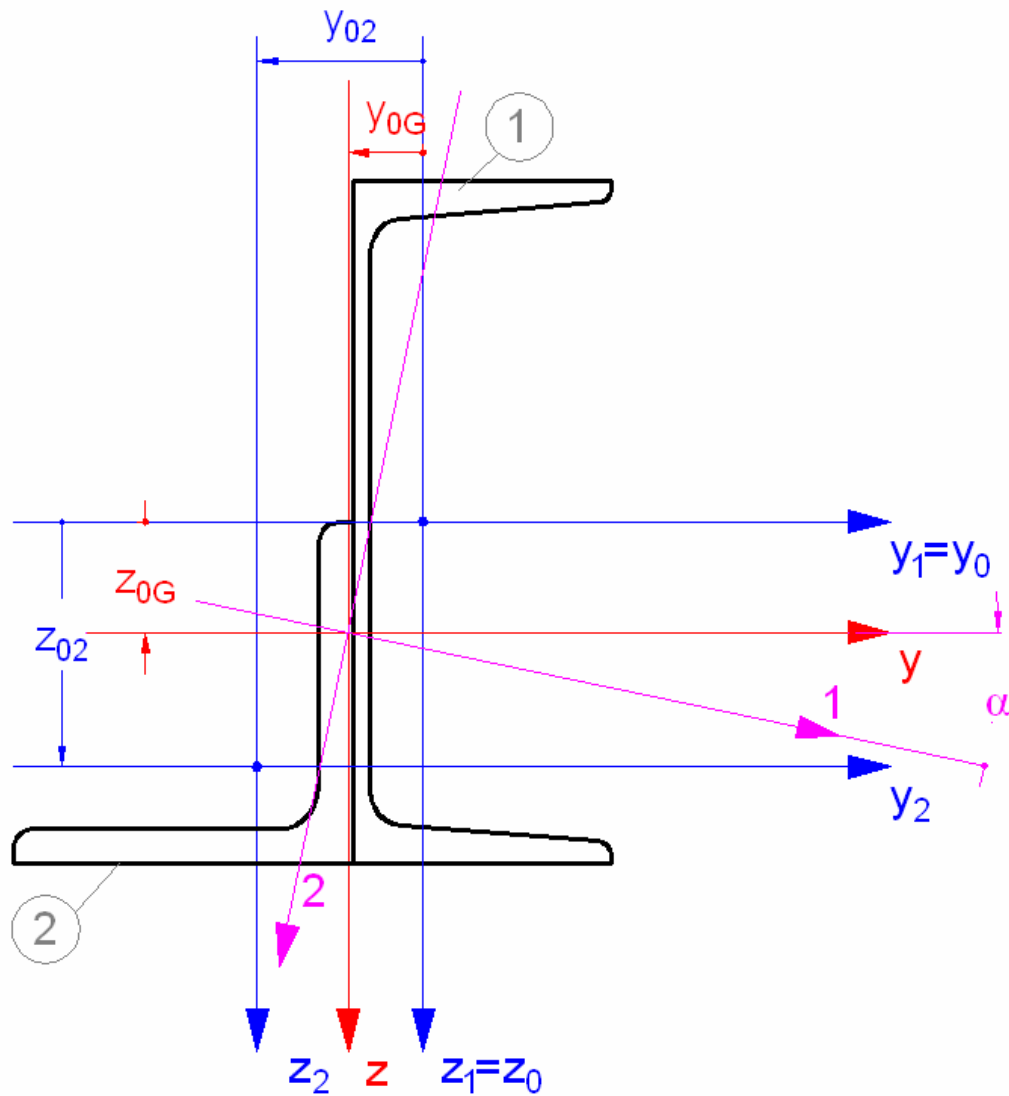
$$I_{1,2} = \frac{I_y + I_z}{2} \pm \frac{1}{2} \cdot \sqrt{(I_y - I_z)^2 + 4 \cdot I_{yz}^2}$$

$$\Rightarrow I_{1,2} = \frac{2241 + 542}{2} \pm \frac{1}{2} \cdot \sqrt{(2241 - 542)^2 + 4 \cdot (-370.28)^2}$$

$$I_{1,2} = 1391.5 \pm \frac{1}{2} \cdot 1853.38$$

$$I_1 = 2318.19 \text{ cm}^4$$

$$I_2 = 464.81 \text{ cm}^4$$



- Pozitia axelor de inertie principale

$$\operatorname{tg} 2\alpha = -\frac{2 \cdot I_{yz}}{I_y - I_z} = -\frac{2 \cdot (-370.28)}{2241 - 542} = 0.435$$

$$\nearrow 2\alpha = 23.55^\circ$$

$$\searrow 2\alpha - \pi = -156.44^\circ$$

$$\text{dar } \frac{\operatorname{tg} 2\alpha}{I_{yz}} < 0 \text{ si } I_{yz} < 0$$

$$\Rightarrow 2\alpha = 23.55^\circ \Rightarrow \alpha = 11.775^\circ$$