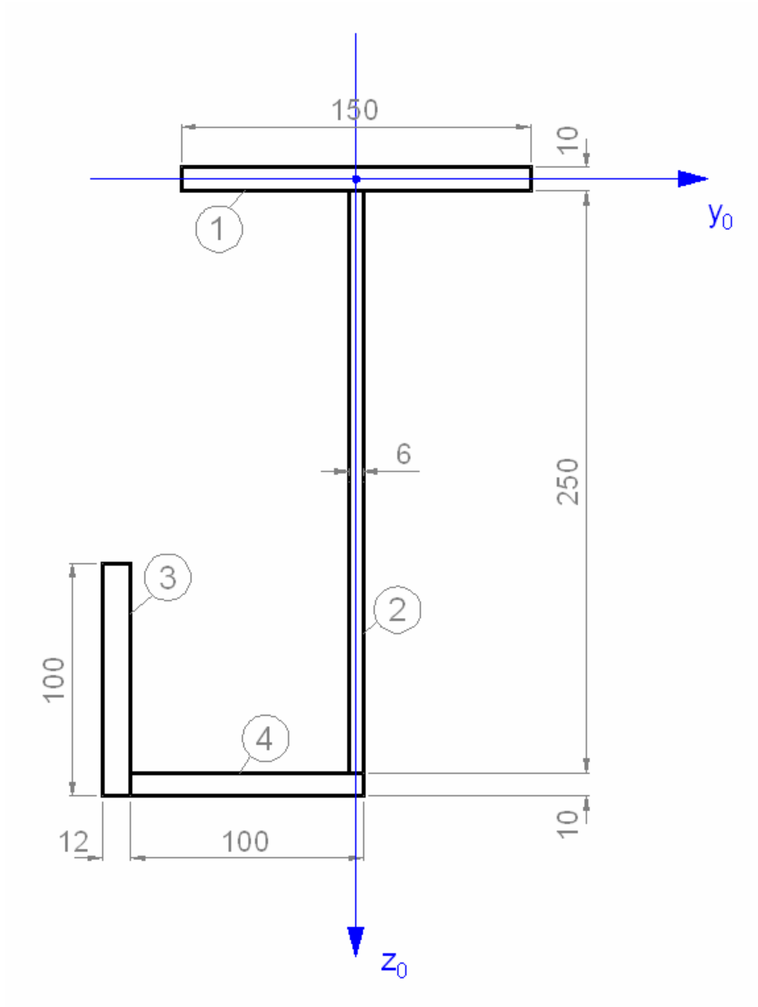
Să se determine momentele de inerție principale şi poziția axelor de inerție principale pentru secțiunea din figura de mai jos:



- Determinarea poziției centrului de greutate

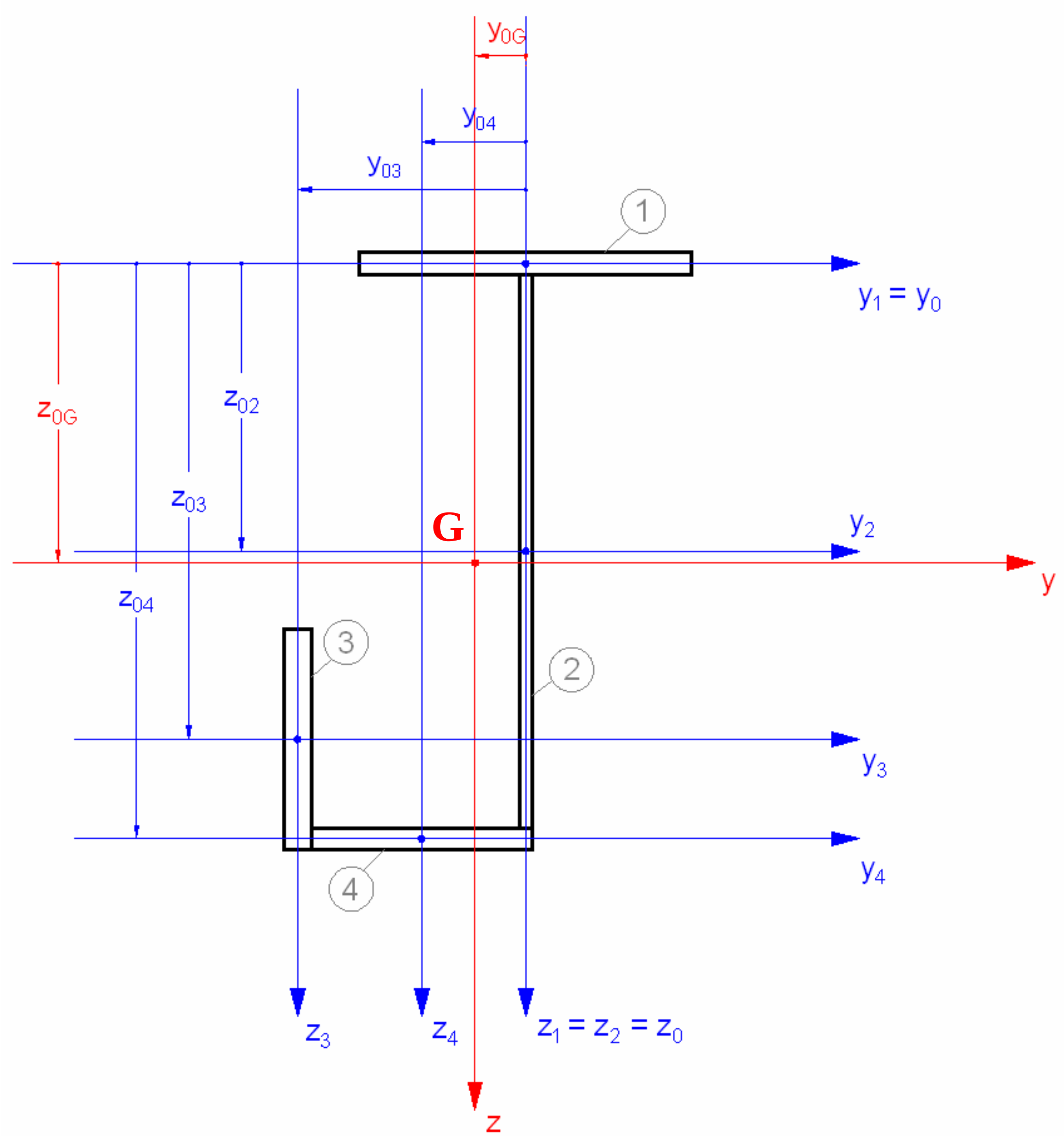
$A_1 = 15 \cdot 1 = 15\ cm^2 \qquad A_2 = 25 \cdot 0.6 = 15\ cm^2 \qquad A_3 = 10 \cdot 1.2 = 12\ cm^2 \qquad A_4 = 10 \cdot 1 = 10\ cm^2$

$I_{y1} = \dfrac{15 \cdot 1^3}{12} = 1.25\ cm^4 \qquad I_{y2} = \dfrac{0.6 \cdot 25^3}{12} = 781.25\ cm^4 \qquad I_{y3} = \dfrac{1.2 \cdot 10^3}{12} = 100\ cm^4 \qquad I_{y4} = \dfrac{10 \cdot 1^3}{12} = 0.83\ cm^4$

$I_{z1} = \dfrac{1 \cdot 15^3}{12} = 281.25\ cm^4 \qquad I_{z2} = \dfrac{25 \cdot 0.6^3}{12} = 0.45\ cm^4 \qquad I_{z3} = \dfrac{10 \cdot 1.2^3}{12} = 1.44\ cm^4 \qquad I_{z4} = \dfrac{1 \cdot 10^3}{12} = 83.33\ cm^4$

$$y_{0G} = \frac{\sum y_{0i} \cdot A_i}{\sum A_i} = \frac{(-10.3) \cdot 12 + (-4.7) \cdot 10}{15 + 15 + 12 + 10} = -1.14\ cm$$

$$z_{0G} = \frac{\sum z_{0i} \cdot A_i}{\sum A_i} = \frac{13 \cdot 15 + 21.5 \cdot 12 + 26 \cdot 10}{15 + 15 + 12 + 10} = 13.71\ cm$$



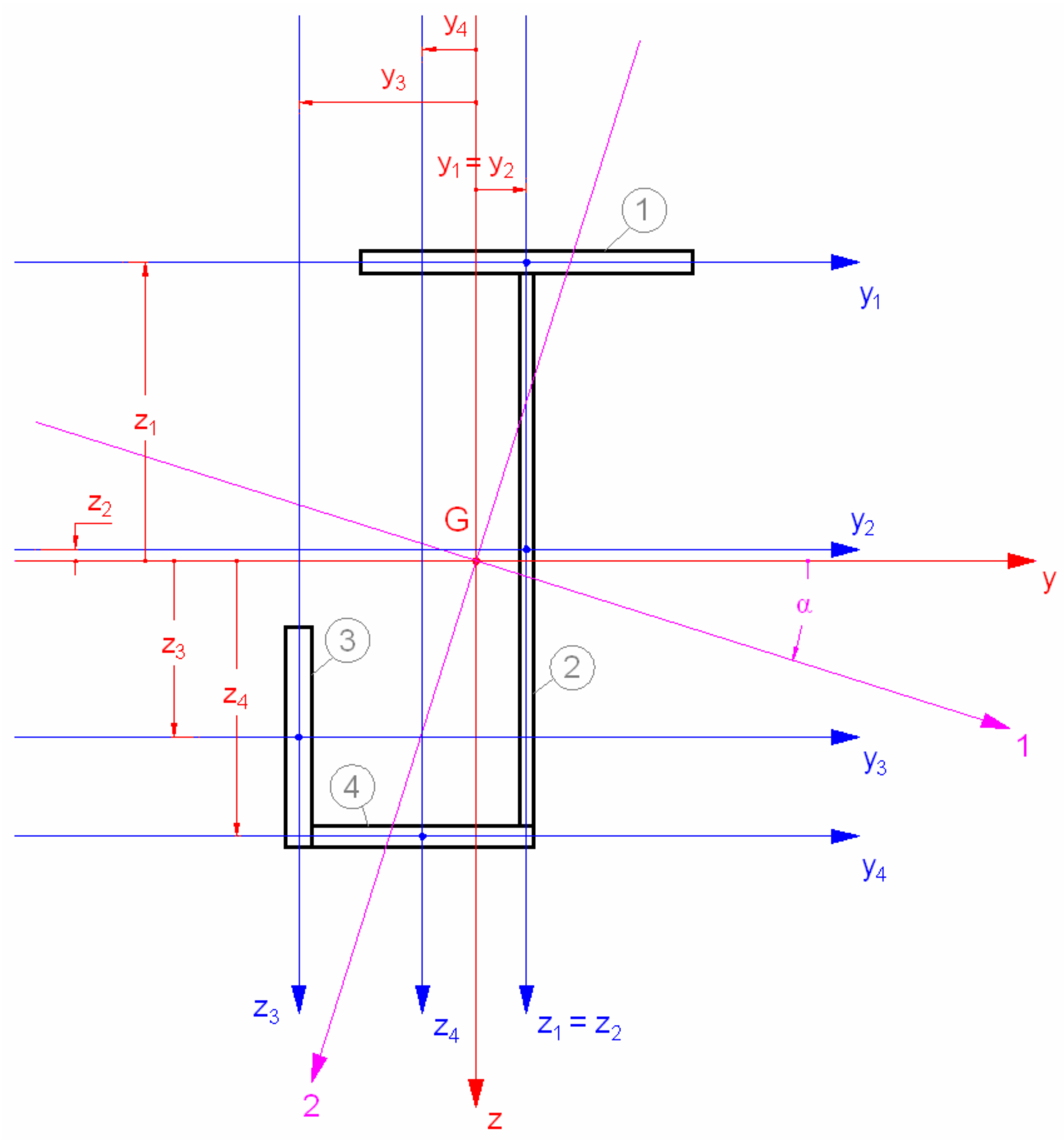
y0G
y04
y03
1
y1 = y0
z0G
z02
z03
G
y2
y
z04
3
2
y3
y4
4
z3
z4
z1 = z2 = z0
z

- Momentele de inerție in raport cu axele centrale

$$I_y = I_{y1} + I_{y2} + I_{y3} + I_{y4} + z_1^2 \cdot A_1 + z_2^2 \cdot A_2 + z_3^2 \cdot A_3 + z_4^2 \cdot A_4$$
$$z_1 = -z_{0G} = -13.71\, cm$$
$$z_2 = -\left(z_{0G} - z_{02}\right) = -\left(13.71 - 13\right) = -0.71\, cm$$
$$z_3 = z_{03} - z_{0G} = 21.5 - 13.71 = 7.79\, cm$$
$$z_4 = z_{04} - z_{0G} = 26 - 13.71 = 12.29\, cm$$
$$\Rightarrow I_y = 1.25 + 781.25 + 100 + 0.83 + \left(-13.71\right)^2 \cdot 15 + \left(-0.71\right)^2 \cdot 15 + \left(7.79\right)^2 \cdot 12 + \left(12.29\right)^2 \cdot 10 = 5949\, cm^4$$
$$I_z = I_{z1} + I_{z2} + I_{z3} + I_{z4} + y_1^2 \cdot A_1 + y_2^2 \cdot A_2 + y_3^2 \cdot A_3 + y_4^2 \cdot A_4$$
$$y_1 = y_2 = y_{0G} = 1.14\, cm$$
$$y_3 = -\left(y_{03} - y_{0G}\right) = -\left(10.3 - 1.14\right) = -9.16\, cm$$
$$y_4 = -\left(y_{04} - y_{0G}\right) = -\left(4.7 - 1.14\right) = -3.56\, cm$$
$$\Rightarrow I_z = 281.25 + 0.45 + 1.44 + 83.33 + \left(1.14\right)^2 \cdot 15 + \left(1.14\right)^2 \cdot 15 + \left(-9.16\right)^2 \cdot 12 + \left(-3.56\right)^2 \cdot 10 = 1539\, cm^4$$
$$I_{yz} = I_{y1z1} + I_{y2z2} + I_{y3z3} + I_{y4z4} + y_1 \cdot z_1 \cdot A_1 + y_2 \cdot z_2 \cdot A_2 + y_3 \cdot z_3 \cdot A_3 + y_4 \cdot z_4 \cdot A_4$$
$$I_{y1z1} = I_{y2z2} = I_{y3z3} = I_{y4z4} = 0 \quad \textit{deoarece au cel putin o axa de simetrie}$$
$$\Rightarrow I_{yz} = \left(1.14\right)\left(-13.71\right) \cdot 15 + \left(1.14\right)\left(-0.71\right) \cdot 15 + \left(-9.16\right)\left(7.79\right) \cdot 12 + \left(-3.56\right)\left(12.29\right) \cdot 10 = -1540.38\, cm^4$$

- Momentele de inerție principale

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \frac{1}{2} \cdot \sqrt{\left(I_y - I_z\right)^2 + 4 \cdot I_{yz}^2}$$
$$\Rightarrow I_{1,2} = \frac{5949 + 1539}{2} \pm \frac{1}{2} \cdot \sqrt{\left(5949 - 1539\right)^2 + 4 \cdot \left(-1540.38\right)^2}$$
$$I_{1,2} = 3744 \pm \frac{1}{2} \cdot 5379.5$$
$$I_1 = 6433.75\, cm^4$$
$$I_2 = 1054.24\, cm^4$$



- Poziția axelor de inerție principale

$$tg2\alpha = -\frac{2\cdot I_{yz}}{I_y - I_z} = -\frac{2\cdot\left(-1540.38\right)}{5949-1539} = 0.698$$

□ $2\alpha = 34.94°$

□ $2\alpha - \pi = -145.06°$

$$dar \quad \frac{tg2\alpha}{I_{yz}} \prec 0 \; si \; I_{yz} \prec 0$$

$$\Rightarrow 2\alpha = 34.94° \Rightarrow \alpha = 17.47°$$