Basis of Structural Design

Course 12
EN 1990:
Basic variables
The partial factor method

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EN 1990: Basic variables

- The design procedure using the limit state method consists in setting up structural and load models for the relevant ultimate and serviceability limit states which are considered in various design situations and load cases.

- Basic variables needed to set up structural and load models
  - Actions and environmental influences
  - Material and product properties
  - Geometrical data
Variability of loads

- Loads change in time.
- What loads to design a structure for?
- Most loads are determined today based on measurements taken over time.
- Example: maximum wind speed measured over 5-min periods in each month between 1884 and 1950 in New York at a height of 140 m.

Histogram

Theoretical distribution
Fractiles

- A p-fractile is defined as the x-value of the distribution which includes p*N observations, with 0<p<1 and N being the number of observations.

- Example: the 0.1-fractile of the distribution shown below is 14.6, as it includes 10% of all observations (starting from the left)
Variability of loads

- Loads are probabilistic variables (not deterministic)
- Basic value of a load can be defined in probabilistic terms only, e.g. "for annual probabilities of exceedence of 0.02, which is equivalent to a mean return period of 50 years".
- A larger or smaller probability of the loads being exceeded can be considered in design (corresponding to smaller or larger mean return periods)
- Smaller or larger design loads
- More or less economical structure
Variability of loads

- The engineer can never be absolutely sure that the loads he designs his structure for will not be exceeded in its lifetime.
The aim of the classifications is to identify the similar or dissimilar characteristics of various actions and to enable the use of appropriate theoretical action models and reliability elements in structural design.

Actions are classified by their variation in time as follows:
- permanent actions \((G)\), e.g. self-weight of structures, fixed equipment and road surfacing, and indirect actions caused by shrinkage and uneven settlements;
- variable actions \((Q)\), e.g. imposed loads on building floors, beams and roofs, wind actions or snow loads;
- accidental actions \((A)\), e.g. explosions, or impact from vehicles.

Actions can also be classified
- by their origin, as direct or indirect,
- by their spatial variation, as fixed or free, or
- by their nature and/or the structural response, as static or dynamic.
Characteristic values of actions

- **Characteristic value** of an action \((F_k)\) is a principal representative value of an action.

- When \(F_k\) can be fixed on statistical bases, it is chosen so as to correspond to a prescribed probability of not being exceeded on the unfavourable side during a "reference period" taking into account the design working life of the structure and the duration of the design situation. Depending on the available data and experience, the characteristic value of an action is specified as a mean, upper or lower value.

- In some cases there is a lack in statistical data concerning various actions. Under these circumstances, actions can be evaluated based on a fairly subjective assessment, judgment, or decision, and are assigned nominal values.
Permanent actions: characteristic values

- Usually there is sufficient statistical data for permanent actions, in particular for self-weight of traditional structural materials.

- If the variability of a permanent action is small, a single characteristic value can be considered: the mean value \( \mu_G \).

- If the variability of a permanent action cannot be considered as small, two values are used: an upper value \( G_{k,\text{sup}} \) (representing 0.95 fractile) and a lower value \( G_{k,\text{inf}} \) (representing 0.05 fractile).

- The self weight of a bridge deck has a small variability because the execution of the bridge is strictly controlled by competent personnel.

- The self-weight of items such as vehicle parapets, waterproofing, coatings, railway ballast, etc., has a large variability.
Variable actions: characteristic values

- For variable actions, the characteristic value ($Q_k$) corresponds to either:
  - an upper value with an intended probability of not being exceeded (the most common case) or a lower value with an intended probability of being achieved, during some specific reference period;
  - a nominal value, which may be specified in cases where a statistical distribution is not known

- The volume of statistical data for the most common variable actions allows assessment of their characteristic values by a probabilistic approach

- Two elements are used to define the characteristic value of the load:
  - the reference period during which the extreme is observed (maximum or minimum), and
  - the intended probability with which these extreme values should not exceed the characteristic value
Variable actions: reference period

- Variable action versus time
- The reference period $\tau$ (e.g. 1 year)
Variable actions: reference period

- During each reference period $\tau$ the variable action $Q$ reaches a maximum value $Q_{\text{max}}$ (e.g. the annual extreme).
- A sequence of values $Q_{1,\text{max}}, Q_{2,\text{max}}, Q_{3,\text{max}}, \ldots$ can be obtained.
- The distribution of these values of $Q_{\text{max}}$ is indicated in the right figure by a probability density function $\varphi_{Q_{\text{max}}}(Q)$.
- The characteristic value $Q_k$ can then be defined by the requirement that it will be exceeded by $Q_{\text{max}}$ (e.g. annual extremes) only with a limited probability (e.g. $p=0.02$).
- Thus, the characteristic value $Q_k$ is the $p$-fractile of the extremes values $Q_{\text{max}}$. 


Variable actions: characteristic values

- In general, the characteristic value of environmental loads and imposed loads on building floors for persistent design situations is based on an intended probability of the value **not being exceeded** of 0.98, and a reference period of 1 year.

- The probability $p$ of the characteristic value **being exceeded** and the reference period $\tau$ are linked by the equation:

$$T \approx -\frac{\tau}{\ln(1-p)} \approx \frac{\tau}{p}$$

where $T$ is the return period (expected period between two subsequent occurrences of the characteristic value).

- Example: for a probability of the characteristic value being exceeded $p=0.02$ and a reference period of 1 year, the return period of the characteristic value is

$$T \approx 1/0.02=50 \text{ years}$$
Other representative values of variable actions

- In addition to the characteristic values of actions, other representative values for actions are used:
  - The combination value $\psi_0 Q_k$ - takes into account the reduced probability of the simultaneous occurrence of two (or more) independent actions. It is associated with the combination of actions for ultimate and irreversible serviceability limit states.
  - The frequent value $\psi_1 Q_k$ - is primarily associated with the frequent combination in the reversible serviceability limit states and ultimate limit states involving accidental actions.
  - The quasi-permanent value $\psi_2 Q_k$ - is primarily used for assessment of long-term effects, for example creep effects in prestressed concrete elements. It is also used for the representation of variable actions in accidental and seismic combinations of actions (ultimate limit states) and for verification of frequent and quasi-permanent combinations (long-term effects) of serviceability limit states.
Other representative values of variable actions

Instantaneous value of $Q$

Characteristic value of $Q_k$

Combination value $\psi_0 Q_k$

Frequent value $\psi_1 Q_k$

Quasi-permanent value $\psi_2 Q_k$

Time
A material property is usually determined from standardized tests performed under specified conditions. It is sometimes necessary to apply a conversion factor to convert the test results into values that can be assumed to represent the behaviour of the structure or the ground. For traditional materials (e.g. steel and concrete), previous experience and extensive tests are available, and conversion factors are well-established and available in codes. The properties of new materials should be established from an extensive testing program, including tests on complete structures, revealing the relevant properties and conversion factors.
Variability of material properties

- The strength of a structure cannot be predicted with absolute confidence
- Structural materials, whether natural or man-made, vary in quality and strength
- Example: result of compression test on 303 cubes of concrete of supposedly the same strength, made during a certain period of construction.
- A symmetrical, bell-shaped distribution of results, which can be expressed mathematically using the "normal" or Gauss rule.
Variability of material properties

- The strength of concrete specimens taken from a built structure will be even worse, due to insufficient compaction of concrete for example.
- Tests specimens drilled from hardened concrete will show much greater variation in strength then the specimens taken during the mixing process, as well as a lower average strength.
Material and product properties

- **Basic properties of a material or product:**
  - **strength:** mechanical property of a material indicating its ability to resist actions, usually given in units of stress
  - **stiffness:** the force necessary to produce a unit displacement, characterizing the elastic response of the material/product
  - **ductility:** capacity to deform into the plastic range, without significant loss of strength

- **Examples of characteristic values of material/product properties:**
  - **strength** used to check the resistance of a structure that should respond in the elastic range: lower value is relevant
  - **stiffness** used to estimate deflections and deformations of the structure: the mean value is relevant
  - **ductility** used to prevent collapse of a structure due to excessive plastic deformations: lower value relevant
Material and product properties

- In design calculations properties of structural materials and products are represented by characteristic values.

- Generally characteristic values are obtained from statistical data, corresponding to a prescribed probability of not being infringed.
  - Usually the lower value of material property is unfavourable, and the 5% fractile \(X_{k,\text{inf}}\) is considered as the characteristic value.
  - For some properties, the mean value \((\mu_x)\) is most appropriate.

- When enough statistical data is not available, a nominal value can be used in design.
Geometrical data

- Geometrical variables describe the shape, size and overall arrangement of structures, structural members and cross-sections.
- No structure can be erected without some deviations of form, shape and dimension from the ones assumed in design.
- Example: steel sections are rolled under very careful control, but some variation in thickness and depth cannot be avoided.
- Concrete formwork may be slightly out in dimensions.
- Load-bearing walls in a multi-storey building may be out of alignment one above the other.
Variability of geometrical properties

- In design, account should be taken of the possible variation of their magnitudes, which depend on the level of workmanship in the manufacture and execution process.

- In most cases the geometrical data is represented by characteristic values,
  - Corresponding to values specified in design specifications, and which are nominal values.
  - Where their statistical distribution is sufficiently known, values of geometrical quantities that correspond to a prescribed fractile of the statistical distribution may be used.

- In some cases (e.g. imperfections) geometrical data is represented directly by their design values
Structural analysis

- Generally, any structural model should be regarded as an idealization of the structural system.
- A simplified model should take account of significant factors and neglect the less important ones.
- The following is a list of factors that may be important for the structural model:
  - geometric properties (e.g. structural configuration, spans, cross-sectional dimensions, deviations, imperfections)
  - material properties (e.g. strength, constitutive relations, time and stress state dependence, plasticity, temperature and moisture dependence)
  - actions (e.g. direct or indirect, variation in time, spatial variation, and static or dynamic)
- The appropriate structural model should be chosen based on previous experience and knowledge of structural behaviour.
Verification by the partial factor method: general

- Assessment of the reliability of structures in Eurocodes is based on the concept of limit state design and verification by the partial factor method.

- Using this method, a structure is considered to be reliable if no relevant limit state is exceeded for all selected design situations, when using the design values of basic variables (actions, material properties and geometrical data) in the design models.

- Critical combinations of actions should be identified and used in order to obtain the design values of action effects. Example: a combination of permanent loading due to self weight, snow load and wind load need to be considered for design of a roof panel of an industrial hall.
Verification by the partial factor method: general

- Actions that cannot occur simultaneously due to physical reasons, should not be considered together in combination
  Example: snow and live loads need not be considered simultaneously on the terrace of a multistorey building, because they cannot act simultaneously with their extreme values (just the maintenance personnel will be on the roof in the event of a heavy snow).

- Action effects ($E$) represent the response of the structure to actions applied on it. Examples:
  - action effects on structural members: internal force, moment, stress, strain
  - action effects on the whole structure: deflection, rotation
The partial factor method: design values

- The design value \( F_d \) of an action \( F \) can be expressed in general terms as:
  \[
  F_d = \gamma_f \cdot F_{\text{rep}}
  \]
  with
  \[
  F_{\text{rep}} = \psi \cdot F_k
  \]
  where:
  - \( F_k \) is the characteristic value of the action.
  - \( F_{\text{rep}} \) is the relevant representative value of the action.
  - \( \gamma_f \) is a partial factor for the action which takes account of the possibility of unfavourable deviations of the action values from the representative values.

- \( \psi \) is either 1,00 or \( \psi_0, \psi_1 \) or \( \psi_2 \).
  - the combination value \( \psi_0 Q_k \)
  - the frequent value \( \psi_1 Q_k \)
  - the quasi-permanent value \( \psi_2 Q_k \)
The partial factor method: design values

- For a specific load case the design values of the effects of actions \((E_d)\) can be expressed in general terms as:

\[
E_d = \gamma_{Sd} \cdot E \{\gamma_{f,i} \cdot F_{rep,i} ; a_d\} \quad i \geq 1
\]

where:

- \(a_d\) is the design values of the geometrical data;
- \(\gamma_{Sd}\) is a partial factor taking account of uncertainties:
  - in modelling the effects of actions;
  - in some cases, in modelling the actions.

\(E \{\gamma_{f,i} \cdot F_{rep,i} ; a_d\}\) is the effect of action for the design value of the force \(F_d\) and the design geometrical characteristics \(a_d\).
The partial factor method: design values

- In most cases, the following simplification can be made:
  \[ E_d = E \{ \gamma_{F,i} \cdot F_{rep,i} ; a_d \} \geq 1 \]
  with: \[ \gamma_{F,i} = \gamma_{Sd} \cdot \gamma_{f,i} \]
  \( \gamma_F \) factor accounts for both uncertainties in action values and uncertainties in modelling of actions

- \[ E_d = E \{ \gamma_{F,i} \cdot F_{rep,i} ; a_d \} \geq 1 \iff \text{The design value of the effect of actions } E_d \text{ is determined by applying the representative values of actions } F_{rep,i} \text{ amplified by the partial factors } \gamma_{F,i} \text{ on the model of the structure characterised by the geometry } a_d. \]
The partial factor method: design values

- \( E_d = E \{ \gamma_{F,i} \cdot F_{rep,i} \ ; \ a_d \} \quad i \geq 1 \leftrightarrow \) The design value of the effect of actions \( E_d \) is determined by applying the representative values of actions \( F_{rep,i} \) amplified by the partial factors \( \gamma_{F,i} \) on the model of the structure characterised by the geometry \( a_d \).

### Example:

Design values of permanent (\( \gamma_{F,G} \cdot G_{rep} \)) and imposed (\( \gamma_{F,Q} \cdot Q_{rep} \)) actions

\[ + \]

Geometrical data \( a_d \) (beam span \( L \))

\[ \parallel \]

Action effects \( E_d \) (bending moment \( M_{Ed} \))

\[ \gamma_{F,1} \cdot F_{rep,1} \ (G_d = \gamma_{F,G} \cdot G_{rep}) \]

\[ \gamma_{F,2} \cdot F_{rep,2} \ (Q_d = \gamma_{F,Q} \cdot Q_{rep}) \]

\( a_d \) (\( L \))

\( E_d \) (\( M_{Ed} \))
The partial factor method: design values

- Where a distinction has to be made between favourable and unfavourable effects of permanent actions, two different partial factors shall be used ($\gamma_{G,inf}$ and $\gamma_{G,super}$).

Examples:
- when estimating the effect of permanent and snow loads on a roof, both loads have the same direction of action (gravitational), therefore the design value of the permanent load should be considered with the upper value $\gamma_{G,super} \cdot G_k$
- when estimating the effect of permanent and wind loads on a roof, if wind produces suction, the two loads have opposite direction of action, therefore the design value of the permanent load should be considered with the upper value $\gamma_{G,inf} \cdot G_k$, as it reduces the effect of the wind load
The partial factor method: design values

- The design value $X_d$ of a **material or product property** can be expressed in general as:
  
  $$X_d = \eta \left( \frac{X_k}{\gamma_m} \right)$$

  where:
  
  - $X_k$ is the characteristic value of the material property;
  - $\eta$ is the mean value of the conversion factor taking into account
    - volume and scale effects,
    - effects of moisture and temperature, and
    - any other relevant parameters;
  
  - $\gamma_m$ is the partial factor for the material or product property to take account of:
    - the possibility of an unfavourable deviation of a material or product property from its characteristic value;
    - the random part of the conversion factor $\eta$.  

The partial factor method: design values

- Design values of geometrical data such as dimensions of members that are used to assess action effects and/or resistances may be represented by nominal values:
  $$a_d = a_{nom}$$

- Where the effects of deviations in geometrical data (e.g. inaccuracy in the load application or location of supports) are significant for the reliability of the structure (e.g. by second order effects) the design values of geometrical data shall be defined by:
  $$a_d = a_{nom} \pm \Delta a$$

where:

- $\Delta a$ takes account of:
  - the possibility of unfavourable deviations from the characteristic or nominal values;
  - the cumulative effect of a simultaneous occurrence of several geometrical deviations.
The partial factor method: design values

- The design resistance $R_d$ can be expressed in the following form

$$R_d = \frac{1}{\gamma_{Rd}} R\{X_{d,i}; a_d\} = \frac{1}{\gamma_{Rd}} R\left\{\eta_i \frac{X_{k,i}}{\gamma_{m,i}}; a_d\right\} \quad i \geq 1$$

where

- $\gamma_{Rd}$ is a partial factor covering uncertainty in the resistance model, plus geometric deviations if these are not modelled explicitly;
- $X_{d,i}$ is the design value of material property $i$. 
The partial factor method: design values

- The following simplification may be made:

\[ R_d = R \left\{ \eta_i \frac{X_{k,i}}{\gamma_{M,i}} ; a_d \right\} \quad i \geq 1 \]

where: \[ \gamma_{M,i} = \gamma_{Rd} \cdot \gamma_{m,i} \]
The partial factor method: design values

- For members made of a single material (e.g. steel), the design resistance may be obtained directly from the characteristic value of a material or product resistance $R_k$, without explicit determination of design values for individual basic variables, using

$$R_d = \frac{R_k}{\gamma M}$$
- Principle of checking safety of the structure:
  \[ E_d \leq R_d \]
  where:
  \( E_d \) is the design value of the effect of actions such as internal force, moment or a vector representing several internal forces or moments;
  \( R_d \) is the design value of the corresponding resistance.