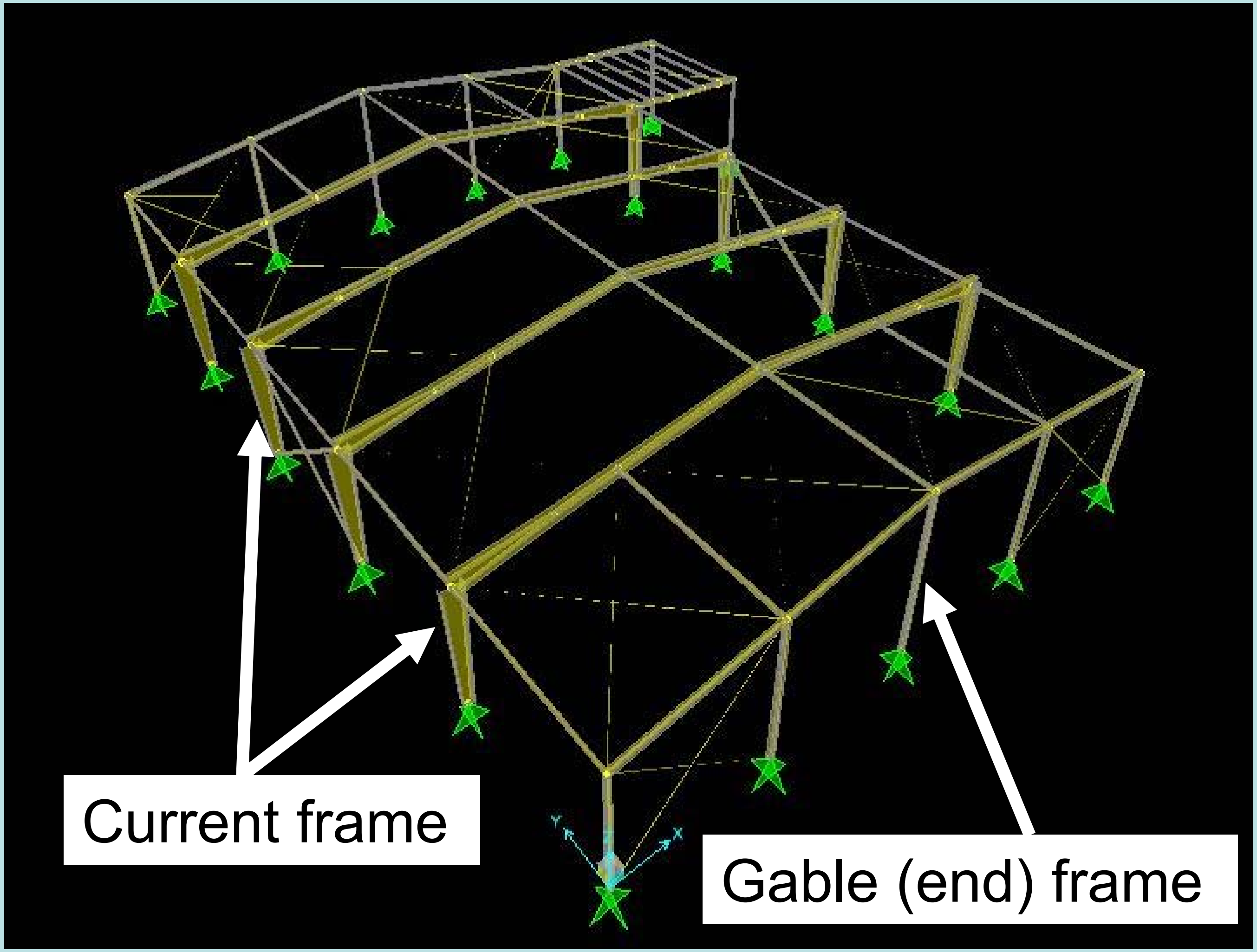
Application nr. 5
(Ultimate Limit State)

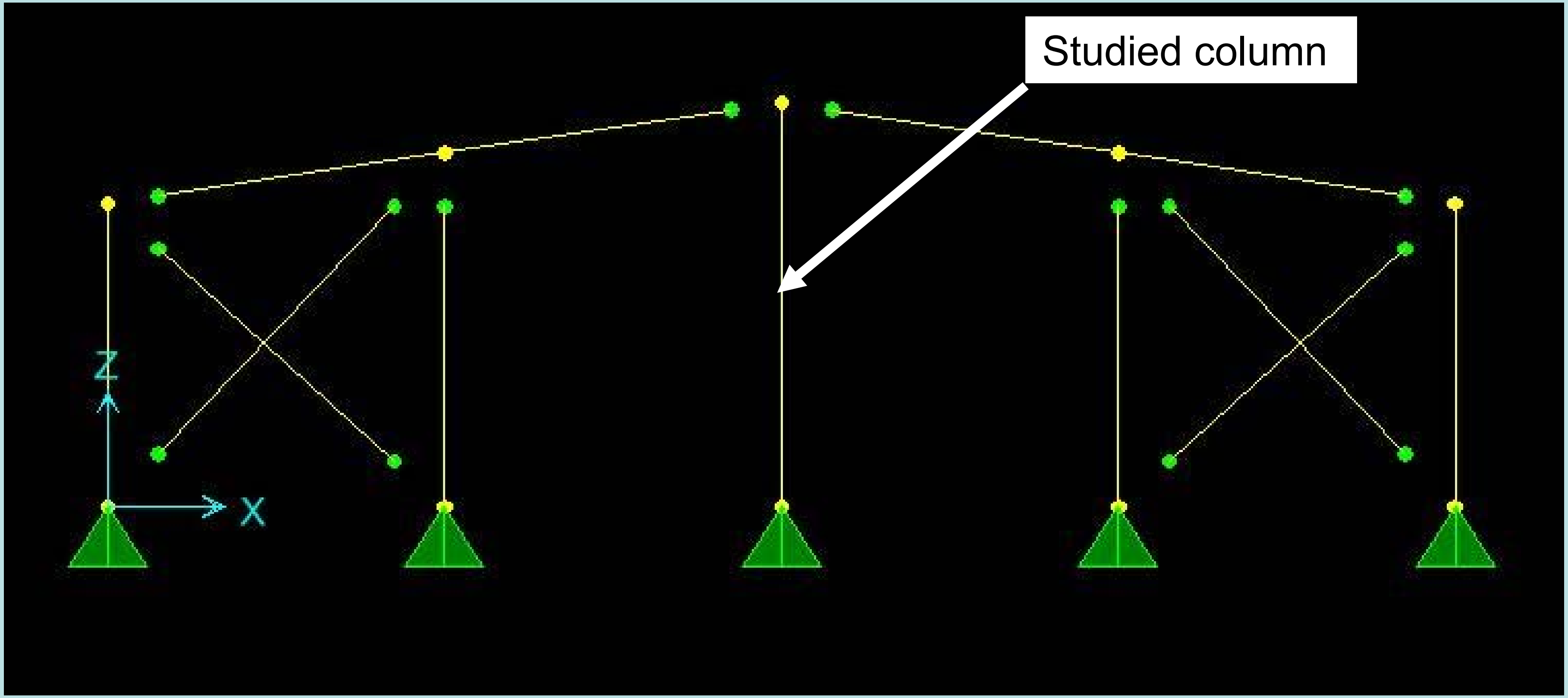
Buckling resistance of members

PART 1:
Column in axial compression

EXAMPLE: Steel structure of an industrial building



Static scheme of gable (end) frame with columns hinged at both ends



Axial compression on columns (NO bending)!

Required: design of gable columns in pure compression (buckling resistance)

Initial data:

- Steel grade S235 (f_y = 235 N/mm^2);
- European profile of “HEA” type required;
- Axial load (compression): P=150 KN
- Column height: H = 6.5 m

a) Sizing of column cross-section

- The required area of column cross-section under <u>pure compression</u> should be first determined (ch.6.2.4) :
- Employed formula:

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1,0$$

- Where N_{Ed} = 150 kN =design value of compression force

For sizing, previous formula is put in form of an equilibrium equation:

$N_{Ed} = N_{c,Rd}$

- In which:

$$N_{c,Rd} = \frac{A_{req} \cdot f_y}{\gamma_{M0}}$$

- And consequently:

$$N_{Ed} = \frac{A_{req} \cdot f_y}{\gamma_{M0}} \Rightarrow A_{req} = \frac{N_{Ed} \cdot \gamma_{M0}}{f_y}$$

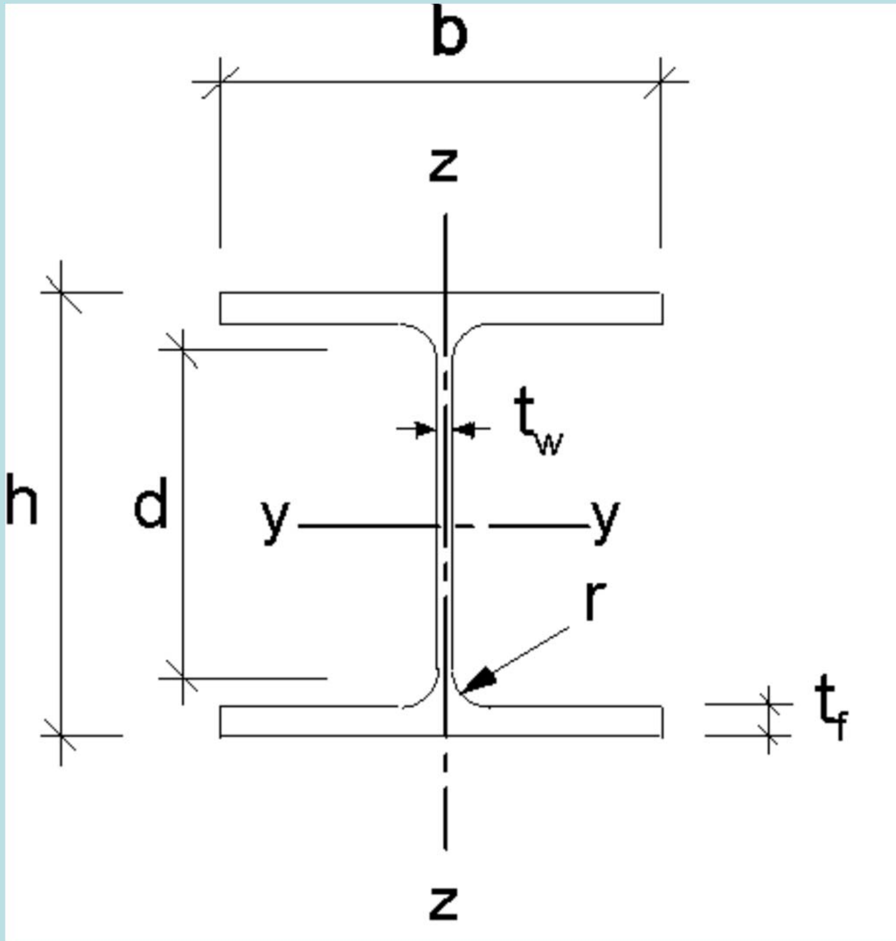
In previous equation:

- A_{req} (required area of column cross-section) is the only unknown, while:
- N_{Ed} = 150 KN;
- f_y=235 N/mm^2
- γ_{M0} = 1,0 (partial safety factor);
- Consequently:

$$A_{req} = \frac{150 \cdot 1,0}{235} = 638\ mm^2$$

- from the HEA profile table HE160A, with actual area A_{act} = 3880 mm^2 is chosen (experience based)

Geometry of HE160A profile:



From profile table:

b =160 mm

h = 152 mm

t_w = 6 mm

t_f = 9 mm

A = 3880 mm^2

i_y = 65.7 mm (gyration radius)

i_z = 39.8 mm (gyration radius)

Profile class in compression = 1

b) Resistance checking of the profile:

- Performed by using the relation (eq. 6.9):

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1{,}0$$

- Where (eq. 6.10):

$$N_{c,Rd} = \frac{A_{act} \cdot f_y}{\gamma_{M0}} = \frac{3880 \cdot 235}{1{,}0} = 911.8\ KN$$

- Column resistance checking:

$$\frac{N_{Ed}}{N_{c,Rd}} = \frac{150}{911.8} = 0{,}165 < 1{,}0$$

Section OK!

c) Buckling resistance of the column

- The column, built of an HE160A hot rolled profile, of S235 steel grade, is an uniform member with class 1 in pure compression (see profile tables, last column);
- The checking formula is (eq. 6.46):

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1{,}0$$

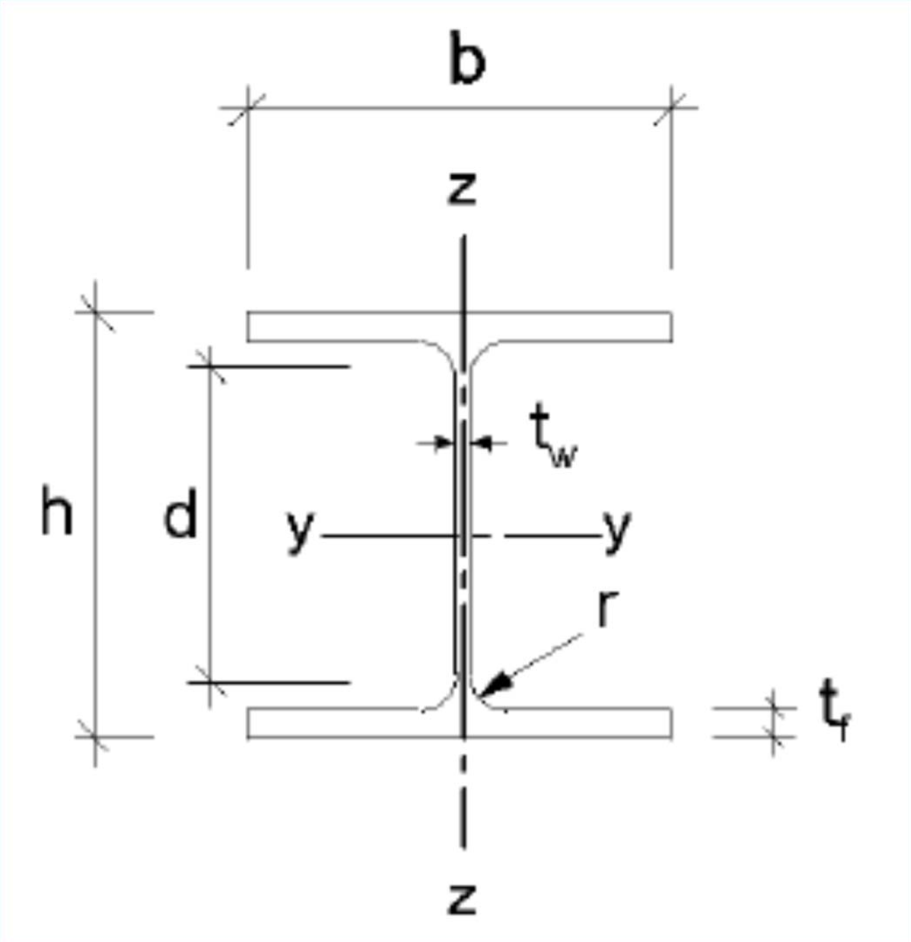
In previous checking formula:

- N_{Ed} = design value of the compressive force;
- $N_{b,Rd}$ = is the design buckling resistance of the compression member;
- For symmetric cross-section of class 1, the design buckling resistance writes (eq. 6.47):

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \quad \text{for Class 1, 2 and 3 cross-sections}$$

where "χ" is the reduction factor for the relevant buckling mode (γ_{M1} = 1,0)

Geometrical characteristics of HE160A column cross-section (after sizing):



A = 3880 mm^2 (gross-area)

i_y = 65.7 mm

(major axis gyration radius)

i_z = 39.8 mm

(minor axis gyration radius)

- The reduction factor value "χ" should be determined using the following formula (ec.6.49):

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi \leq 1{,}0$$

- Where:

$$\Phi = 0{,}5\left[1 + \alpha\left(\bar{\lambda} - 0{,}2\right) + \bar{\lambda}^2\right]$$

- and the non-dimensional slenderness:

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 cross-sections}$$

In our particular case:

- The profiles of HEA type are symmetrical about both inertia axes. Consequently, the flexural buckling is the relevant buckling mode;

- In such situation, the non-dimensional slenderness formula becomes (eq. 6.50):

$$\begin{cases} \bar{\lambda} = \sqrt{\dfrac{A \cdot f_y}{N_{cr}}} = \dfrac{L_{cr}}{i} \cdot \dfrac{1}{\lambda_1} \\ \lambda_1 = \pi \cdot \sqrt{\dfrac{E}{f_y}} = 93,9 \cdot \varepsilon \rightarrow \varepsilon = \sqrt{\dfrac{235}{f_y}} \end{cases}$$

In previous formula of non-dimensional slenderness:

- (L_{cr}) is the buckling length of the uniform member in the buckling plane considered (i.e. along y-y or along z-z axis)
- In this application the member is a gable column, hinged at both ends in both buckling planes (y-y) and (z-z) and therefore:

$$L_{cr}^{y-y} = L_{cr}^{z-z} = 1,0 \cdot H = 1,0 \cdot 6,5 = 6,5\, m$$

- i =is the gyration radius i_y or i_z about the relevant axis, according to cross-section geometry

In the formula for Φ factor we also have an (α) factor:

- This is an imperfection factor taken from table 6.1 of the code depending on the buckling curve to which the member profile corresponds

The buckling curve is indicated in table 6.2, considering that:

- HE160A is a hot rolled section;
- The depth per width ratio:

$$\frac{h}{b} = \frac{152}{160} = 0,95 < 1,2$$

- The flange thickness: t_f = 9mm < 100 mm

In case of steel grade S235 this gives (see table 6.2 of the code):

- For S235 steel grade we have:
  - Curve “b” about (y-y) axis:
  - $\Rightarrow \alpha = 0,34$
  - Curve “c” about (z-z) axis:
  - $\Rightarrow \alpha = 0,49$

Calculation of the non-dimensional slenderness:

$$\begin{cases} \bar{\lambda}_y = \dfrac{L_{cr}^{y-y}}{i_y} \cdot \dfrac{1}{\lambda_1} = \dfrac{650}{6{,}57} \cdot \dfrac{1}{93{,}9} = 1{,}054 \\ \bar{\lambda}_z = \dfrac{L_{cr}^{z-z}}{i_z} \cdot \dfrac{1}{\lambda_1} = \dfrac{650}{3{,}98} \cdot \dfrac{1}{93{,}9} = 1{,}739 \end{cases}$$

Flexural buckling about z-z axis appears to be the relevant instability mode of the member since :

$$\bar{\lambda}_z > \bar{\lambda}_y$$

Calculation of the Φ factor values:

$$\begin{cases} \Phi_y = 0,5 \cdot \left[1 + 0,34 \cdot (1,054 - 0,2) + 1,054^2\right] = 1,201 \\ \Phi_z = 0,5 \cdot \left[1 + 0,49 \cdot (1,739 - 0,2) + 1,739^2\right] = 2,389 \end{cases}$$

The reduction factor values on both directions result:

$$\begin{cases} \chi_y = \dfrac{1}{1,201 + \sqrt{1,201^2 - 1,054^2}} = 0,563 < 1,0 \\ \chi_z = \dfrac{1}{2,389 + \sqrt{2,389^2 - 1,739^2}} = 0,197 < 1,0 \end{cases}$$

Reduction factor value and design buckling resistance:

- The reduction factor value for the relevant buckling mode will be:

$$\chi = \min\{\chi_y ; \chi_z\} = 0{,}197$$

- The design buckling resistance of the compression member results as:

$$N_{b,Rd} = \frac{\chi_z \cdot A \cdot f_y}{\gamma_{M1}} = \frac{0{,}197 \cdot 3880 \cdot 235}{1{,}0} = 179.62\,KN$$

Checking of the compression member:

- The uniform member resistance in buckling is checked using the relation:

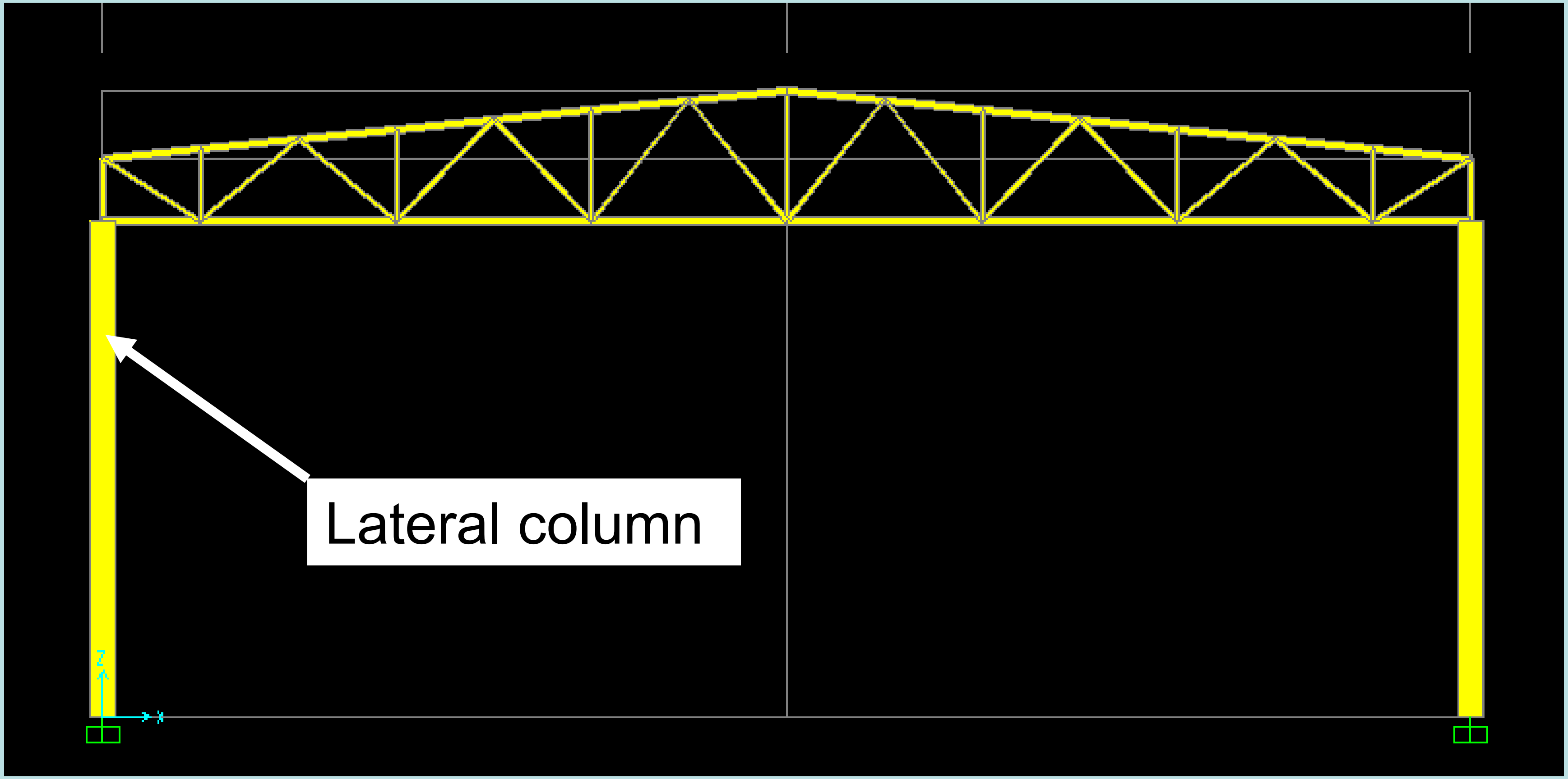
$$\frac{N_{Ed}}{N_{b,Rd}} = \frac{150}{179.62} = 0,835 < 1,0$$

Section OK!

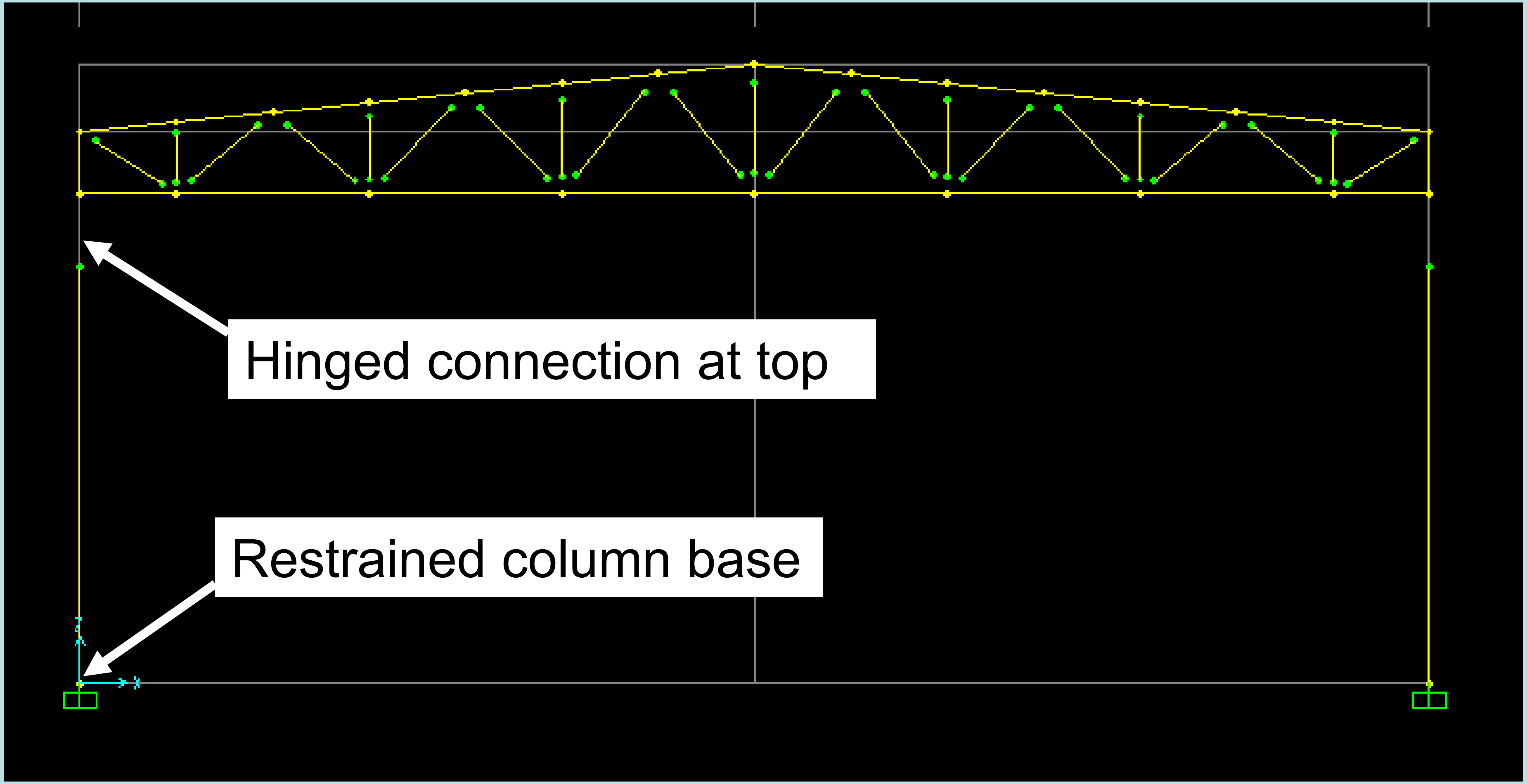
- In case the checking ratio results > 1,0 a NEW larger profile should be chosen from profile table and all the procedure repeated until it checks

PART 2:
Column in combined axial compression and bending

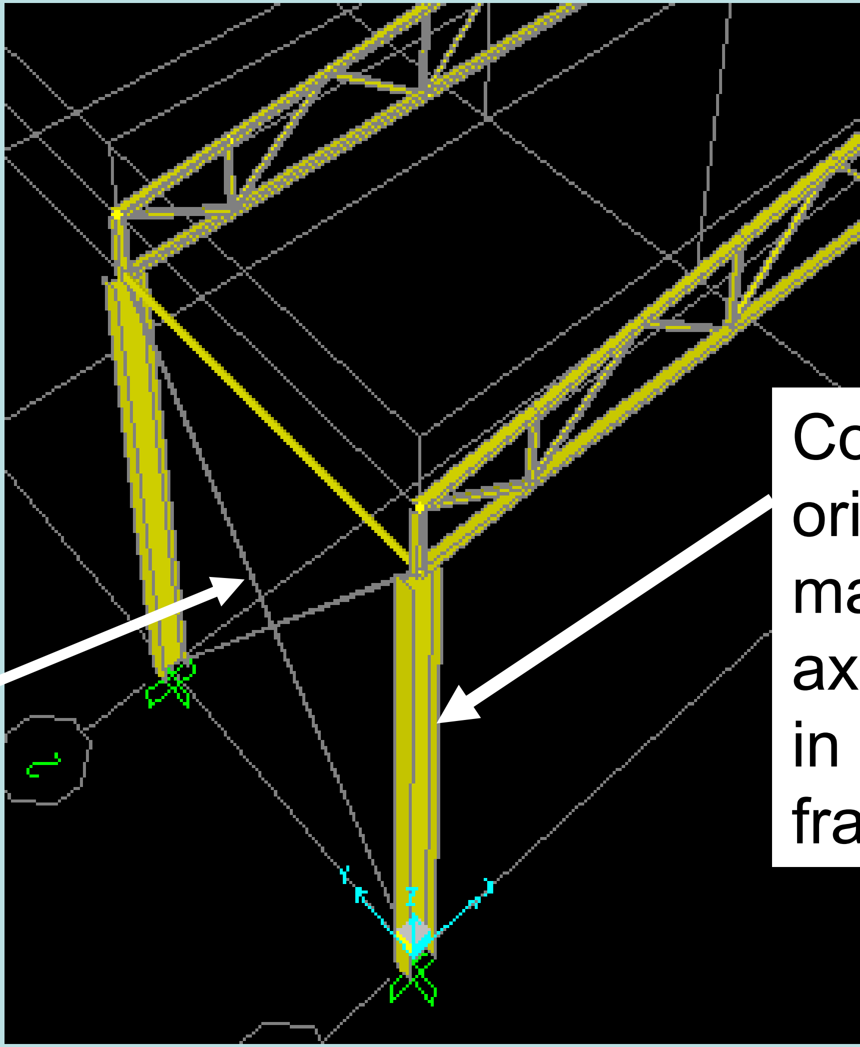
Example of a member under combined axial compression and bending: lateral column, part of current frame in an industrial building



Static scheme of the frame with restrained base and hinged connection at the top of the lateral column



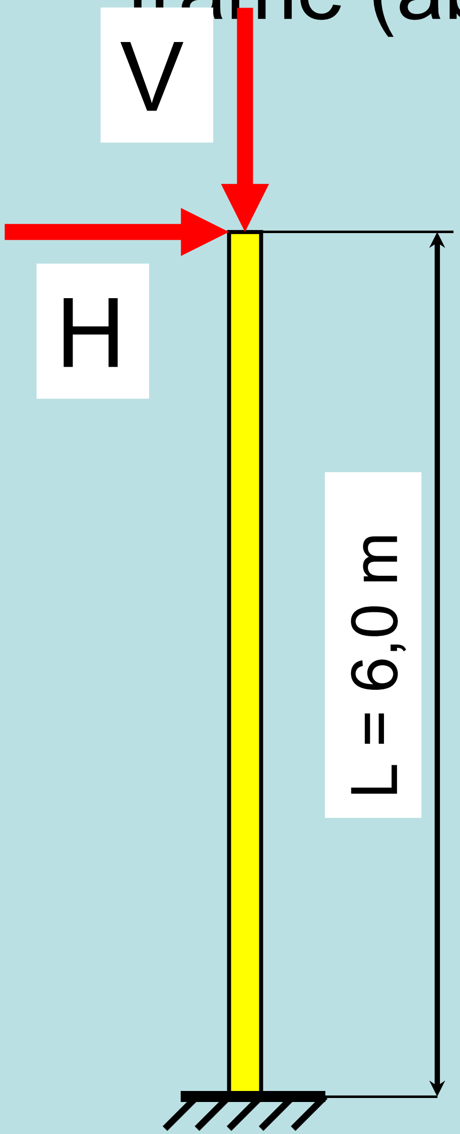
Position of the column profile in the frame (corner of the building in image):



Bracing on the longitudinal direction

Column profile oriented with maximum inertia axis (web plane) in transversal frame

Static scheme of the column in transversal frame (about y-y axis) and initial data:



Static scheme: CANTILEVER (because in the transverse frame, the top of the column is free to move laterally)

$\Rightarrow L_{cr} = 2 \cdot L = 2 \cdot 6,0 = 12,0$ m

Required profile type for column: HEB, steel grade S235

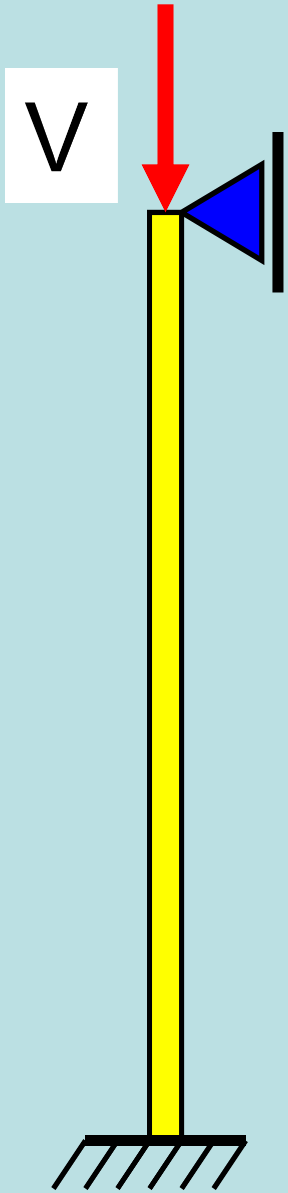
Value of the vertical load:

V = 30.000 daN = 300 kN

Value of the horizontal load:

H = 12.000 daN =120 kN

Static scheme of the column in longitudinal direction (about z-z axis)



On longitudinal direction of the building, a simple support is applied at column top because the presence of the X bracing in the longitudinal wall prevents top lateral movement.

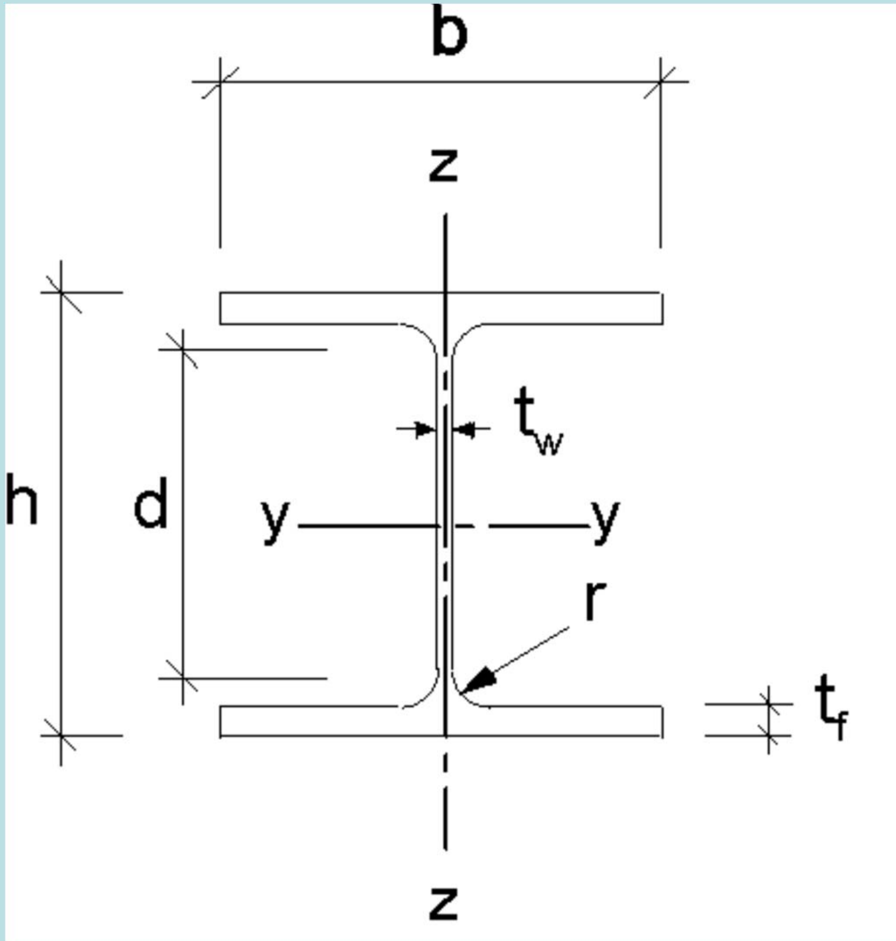
$\Rightarrow L_{cr} = 0{,}7 \cdot L = 0{,}7 \cdot 6{,}0 = 4{,}20$ m

(The horizontal load is not visible because it is acting in the other plane)

First step of the design procedure: sizing of profile cross-section

- Because of the loading complexity and simultaneous presence of several internal efforts, a simple sizing procedure is NOT possible in this case;
- To start the checking process, the column profile is proposed based on design experience;
- If the proposed profile does not fulfill the checking criteria, a NEW profile will be proposed and the procedure repeated!

Proposed European profile to start the design: HE400B



From profile table:

b =300 mm

h = 400 mm

t_w = 13,5 mm

t_f = 24 mm

h_w = 352 mm

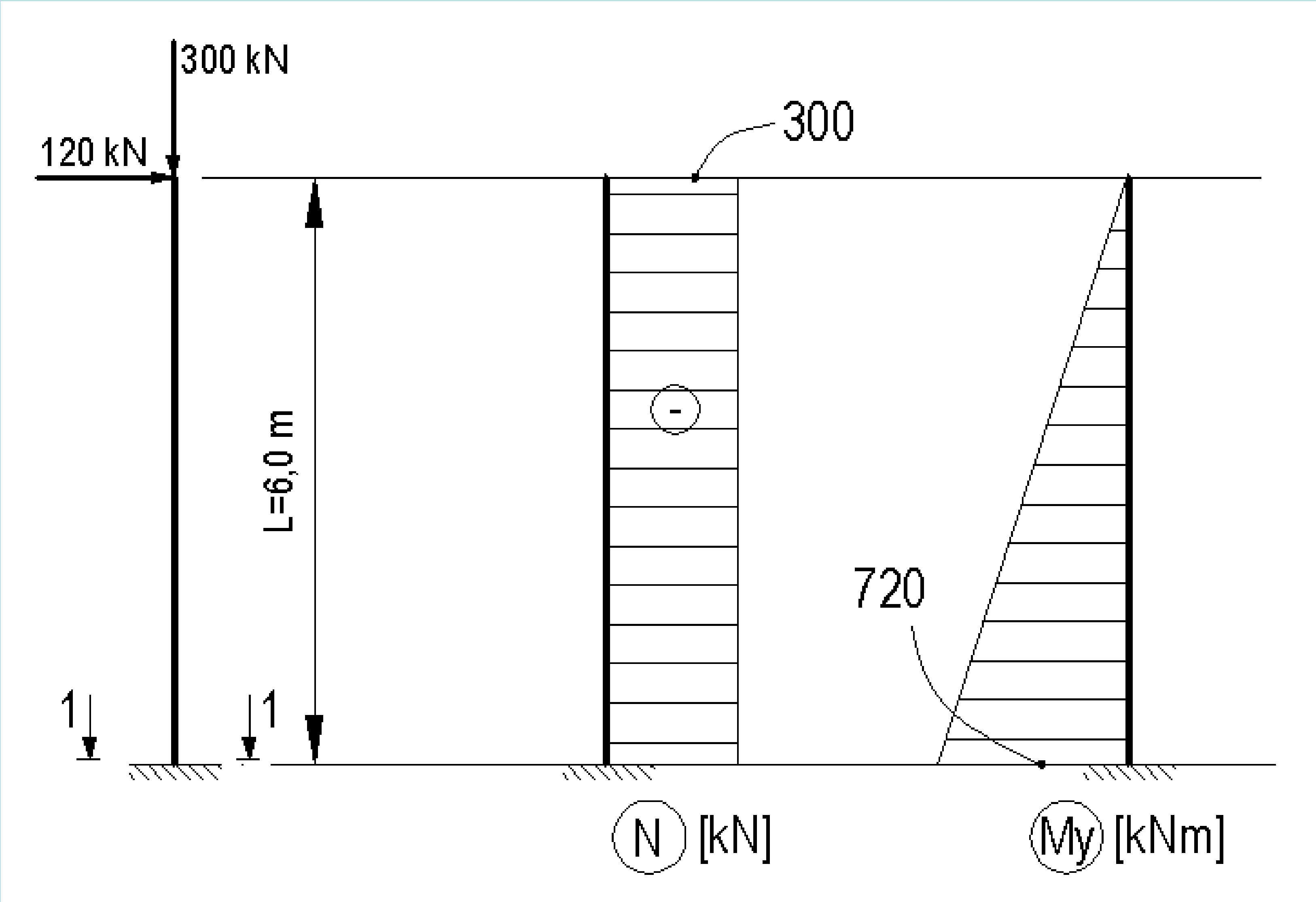
Profile class in compression = 1

Profile class in bending = 1

Other relevant geometrical characteristics of HE400B profile:

- $A = 159*10^2$ mm^2
- $i_y = 170.8$ mm (gyration radius)
- $i_z = 74.0$ mm (gyration radius)
- $W_{pl.y} = 3232*10^3$ mm^3 (plastic modulus to y-y)
- $W_{pl.z} = 1104*10^3$ mm^3 (plastic modulus to z-z)
- $A_{vz} = 6998$ mm^2 (shear area of profile)

Axial force and bending moment diagrams:



Second step: Resistance checking of the member in section (1-1), at column base, under bending and axial force (ch.6.2.9):

- For class 1 and 2 cross sections, the following criterion should be satisfied (ec. 6.31):

$$M_{Ed} \leq M_{N,Rd}$$

- where $M_{N,Rd}$ is the design plastic moment resistance reduced due to the axial force N_{Ed}.

Reduced design plastic moment:

- For a rectangular solid section without bolt holes $M_{N,Rd}$ is given by (ec. 6.32):

$$M_{N,Rd} = M_{pl,Rd}\left[1-\left(N_{Ed} / N_{pl,Rd}\right)^2\right]$$

Allowance for axial force effect?

- For I- and H-sections symmetrical about the z-z axis, allowance should be made for the effect of the axial force on the plastic resistance moment about the y-y axis when one of the following criteria are satisfied (whichever smaller) (ec 6.33 and 6.34):

$$N_{Ed} > 0,25\, N_{pl,Rd}$$

$$N_{Ed} > \frac{0,5\, h_w\, t_w\, f_y}{\gamma_{M0}}$$

Checking of the criteria:

- N_{Ed} = 300 kN

$$0.25 \cdot N_{pl,Rd} = 0.25 \cdot \frac{A \cdot f_y}{\gamma_{M0}} = 0.25 \cdot \frac{15900 \cdot 235}{1,0} = 934 kN$$

$$\frac{0.5 \cdot h_w \cdot t_w \cdot f_y}{\gamma_{M0}} = \frac{0.5 \cdot 352 \cdot 13.5 \cdot 235}{1,0} = 558.36 kN$$

$$\Rightarrow N_{Ed} < \min\left\{0.25 N_{pl,Rd}; \frac{0.5 h_w t_w f_y}{\gamma_{M0}}\right\}$$

NO allowance for the effect of axial force on moment resistance is necessary. Separate resistance checking in bending and compression will be performed!

Separate resistance checking of HE400B column in axial compression (Class 1)

- Checking relation (ec. 6.9):

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1{,}0$$

- In which: N_{Ed} = 300 kN, and (ec. 6.10):

$$N_{c,Rd} = \frac{A \cdot f_y}{\gamma_{M0}} = \frac{15900 \cdot 235}{1{,}0} = 3736.5\,kN$$

- Checking:

$$\frac{300}{3736.5} = 0.081 < 1.0$$

Separate resistance checking of HE400B column in bending (Class 1):

- Checking relation (ec. 6.12):

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1,0$$

- In which M_{Ed} = 720 kNm = 7200000 daNcm, and (ec. 6.13):

$$M_{c,Rd} = \frac{W_{pl.y} \cdot f_y}{\gamma_{M0}} = \frac{3232 \cdot 10^3 \cdot 235}{1,0} = 759.5\,KNm$$

- Checking:

$$\frac{720}{759.52} = 0.947 < 1.0$$

Section OK !

Step 3: Buckling resistance of the column under combined bending and axial compression (ch.6.3.3).

- Members which are loaded by combined bending and axial compression should satisfy (Ec. 6.61 and 6.62):

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

Previous relation has a general character being valid for all cross-section classes and bi-axial bending.

- As we are in case of mono-axial bending we have $M_{y,Ed} \neq 0$ and $M_{z,Ed} = 0$
- Also, $\Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$ (moments due to the shift of the centroidal axis for class 4 sections)
- This leads to simplified checking relations as the following:

Simplified checking relations:

$$\begin{cases} \dfrac{N_{Ed}}{\left(\dfrac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}\right)} + k_{yy} \cdot \dfrac{M_{y,Ed}}{\left(\chi_{LT} \cdot \dfrac{M_{y,Rk}}{\gamma_{M1}}\right)} \leq 1{,}0 \\ \dfrac{N_{Ed}}{\left(\dfrac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}\right)} + k_{zy} \cdot \dfrac{M_{y,Ed}}{\left(\chi_{LT} \cdot \dfrac{M_{y,Rk}}{\gamma_{M1}}\right)} \leq 1{,}0 \end{cases}$$

Where, for HE400B section (of Class 1):

$$N_{Rk} = f_y \cdot A$$

$$M_{y,Rk} = f_y \cdot W_{pl.y}$$

Other notations in previous checking relations:

- k_{yy} and k_{zy} = interaction factors from Annex A or from **Annex B** of EN 1993-1-1;
- χ_y and χ_z = reduction factors due to flexural buckling;
- χ_{LT} = reduction factor due to lateral-torsional buckling;
- As we consider the HE400B member not susceptible to torsional deformation $\chi_{LT} = 1,0$

Calculation of k_{yy} and k_{zy} coefficients

- According to EN1993-1-1, Annex B, for I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy} = 0$.
- The simplified relations are:

Simplified relations to check:

$$\begin{cases} \dfrac{N_{Ed}}{\left(\dfrac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}\right)} + k_{yy} \cdot \dfrac{M_{y,Ed}}{\left(\chi_{LT} \cdot \dfrac{M_{y,Rk}}{\gamma_{M1}}\right)} \leq 1{,}0 \\ \dfrac{N_{Ed}}{\left(\dfrac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}\right)} \leq 1{,}0 \end{cases}$$

Calculation of k_{yy} value according to Table B1 of EN 1993-1-1

$$\begin{cases} k_{yy} = C_{my}\left(1 + \left(\bar{\lambda}_y - 0{,}2\right)\cdot \dfrac{N_{Ed}}{\left(\dfrac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}\right)}\right) \\ k_{yy} \le C_{my}\left(1 + 0{,}8 \cdot \dfrac{N_{Ed}}{\left(\dfrac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}\right)}\right) \end{cases}$$

C_{my} = equivalent uniform moment factor from Table B.3

C_{my} = 0,9 (for members with sway buckling mode)

Calculation of χ_y and χ_z values of the reduction factors is first necessary for the checking

Calculation of the non-dimensional slenderness about (y-y) and (z-z) axes:

$$\begin{cases} \bar{\lambda}_y = \dfrac{L_{cr}^y}{i_y} \cdot \dfrac{1}{\lambda_1} = \dfrac{2{,}0 \cdot L}{i_y} \cdot \dfrac{1}{93{,}9} = \dfrac{2 \cdot 600}{17{,}08} \cdot \dfrac{1}{93{,}9} = 0{,}748 \\ \bar{\lambda}_z = \dfrac{L_{cr}^z}{i_z} \cdot \dfrac{1}{\lambda_1} = \dfrac{0{,}7 \cdot L}{i_z} \cdot \dfrac{1}{93{,}9} = \dfrac{0{,}7 \cdot 600}{7{,}40} \cdot \dfrac{1}{93{,}9} = 0{,}604 \end{cases}$$

Evaluation of the (α) imperfection factor values about (y-y) and (z-z) axes:

- Profile type: HE400B
- Steel grade S235
- Geometrical characteristics: h=400 mm, b=300 mm
- t_f = 24 mm < 40 mm

$$\Rightarrow \frac{h}{b} = \frac{400}{300} = 1{,}33 > 1{,}2$$

$$\begin{cases} \Rightarrow (y-y) \rightarrow curve\ a \rightarrow \alpha_y = 0{,}21 \\ \Rightarrow (z-z) \rightarrow curve\ b \rightarrow \alpha_z = 0{,}34 \end{cases}$$

Calculation of factors Φ_y and Φ_z

$$\begin{cases} \Phi_y = 0,5\left[1+\alpha_y\left(\bar{\lambda}_y - 0,2\right)+\bar{\lambda}_y^2\right] = 0,5\left[1+0,21\left(0,748-0,2\right)+0,748^2\right] = 0,837 \\ \Phi_z = 0,5\left[1+\alpha_z\left(\bar{\lambda}_z - 0,2\right)+\bar{\lambda}_z^2\right] = 0,5\left[1+0,34\left(0,604-0,2\right)+0,604^2\right] = 0,751 \end{cases}$$

Calculation of the reduction factors χ_y and χ_z

$$\begin{cases} \chi_y = \dfrac{1}{\Phi_y + \sqrt{\Phi_y^2 - \bar{\lambda}_y^2}} = \dfrac{1}{0,837 + \sqrt{0,837^2 - 0,748^2}} = 0,825 < 1,0 \\ \chi_z = \dfrac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^2}} = \dfrac{1}{0,751 + \sqrt{0,751^2 - 0,604^2}} = 0,835 < 1,0 \end{cases}$$

Calculation of the k_{yy} value using previously determined values:

$$k_{yy} = 0{,}9\left(1 + (0{,}748 - 0{,}2)\cdot\frac{30000}{\left(\frac{0{,}825\cdot 197{,}8\cdot 2350}{1{,}0}\right)}\right) = 0{,}938$$

Condition checking:

$$0{,}9\left(1 + 0{,}8\cdot\frac{30000}{\left(\frac{0{,}825\cdot 197{,}8\cdot 2350}{1{,}0}\right)}\right) = 0{,}956 > k_{yy}$$

Condition OK!

Calculation of N_{Rk} and $M_{y,Rk}$ values:

$$\begin{cases} N_{Rk} = f_y \cdot A = 2350 \cdot 197{,}8 = 464.830\, daN \\ M_{y,Rk} = f_y \cdot W_{pl,y} = 2350 \cdot 3235 = 7.602.250\, daNcm \end{cases}$$

Checking of the HE400B column stability about (y-y) axis:

$$\frac{N_{Ed}}{\left(\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}\right)} + k_{yy} \cdot \frac{M_{y,Ed}}{\left(\chi_{LT} \cdot \frac{M_{y,Rk}}{\gamma_{M1}}\right)} \leq 1,0$$

$$\frac{30000}{\left(\frac{0,825 \cdot 464830}{1,0}\right)} + 0,938 \cdot \frac{7200000}{\left(1,0 \cdot \frac{7602250}{1,0}\right)} = 0,966 < 1,0$$

Checking OK !

Checking of the HE400B column stability about the (z-z) axis

$$\frac{N_{Ed}}{\left(\dfrac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}\right)} \leq 1,0$$

$k_{zy} = 0$!

$$\frac{30000}{\left(\dfrac{0,835 \cdot 464830}{1,0}\right)} = 0,077 < 1,0$$

Checking OK !