Application nr. 3
(Ultimate Limit State)

Resistance of member cross-section

1)Resistance of member cross-section in tension

- Examples of members in tension:

  - Diagonal of a truss-girder
  - Bottom chord of a truss girder
  - Element of an X diagonal

Application: design of member in tension with bolted end connections

Initial data:

- Design value of the tension axial load:

  N_{Ed} = + 56 392 daN = + 563 920 N
- HEA profile required
- Bolted end connection = two bolt holes of 14 mm diameter in each flange (bolts M12)
- S235 Steel with f_y = 2350 daN/cm^2

Equation used for design:

- The design value of the tension force N_{Ed} at each cross section shall satisfy:

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1{,}0$$

Where: $N_{t,Rd}$ = design tension resistance

First step: sizing of the profile

- Previous formula used as an equation of equilibrium:

$$N_{Ed} = N_{t,Rd}$$

- In which:

$$N_{t,Rd} = \frac{A_{req} \cdot f_y}{\gamma_{M0}} \Rightarrow N_{Ed} = \frac{A_{req} \cdot f_y}{\gamma_{M0}}$$

$\gamma_{M0} = 1{,}0$

Calculation of the required area (A_{req}) for the HEA tension member:

$$56392 = \frac{A_{req} \cdot 2350}{1,0} \Rightarrow A_{req} = 24,0\ cm^2$$

The actual area of the HEA profile in tension is chosen from HEA profile tables and shall be:

$$A_{act} \geq A_{req}$$

$\Rightarrow$ From profile table: HE120A profile with

$$A = A_{act} = 25,3\ cm^2 > A_{req}$$

Step two: Checking of the member

- Performed using the relation:

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1{,}0$$

- Where: N_{Ed} = + 56392 daN

$$N_{t,Rd} = \min\left\{N_{pl,Rd}\,; N_{u,Rd}\right\}$$

- $N_{pl,Rd}$ = design plastic resistance of the gross cross-section (without bolt holes)

$$N_{pl,Rd} = \frac{A_{act} \cdot f_y}{\gamma_{M0}} = \frac{25{,}3 \cdot 2350}{1{,}0} = 59455\ daN$$

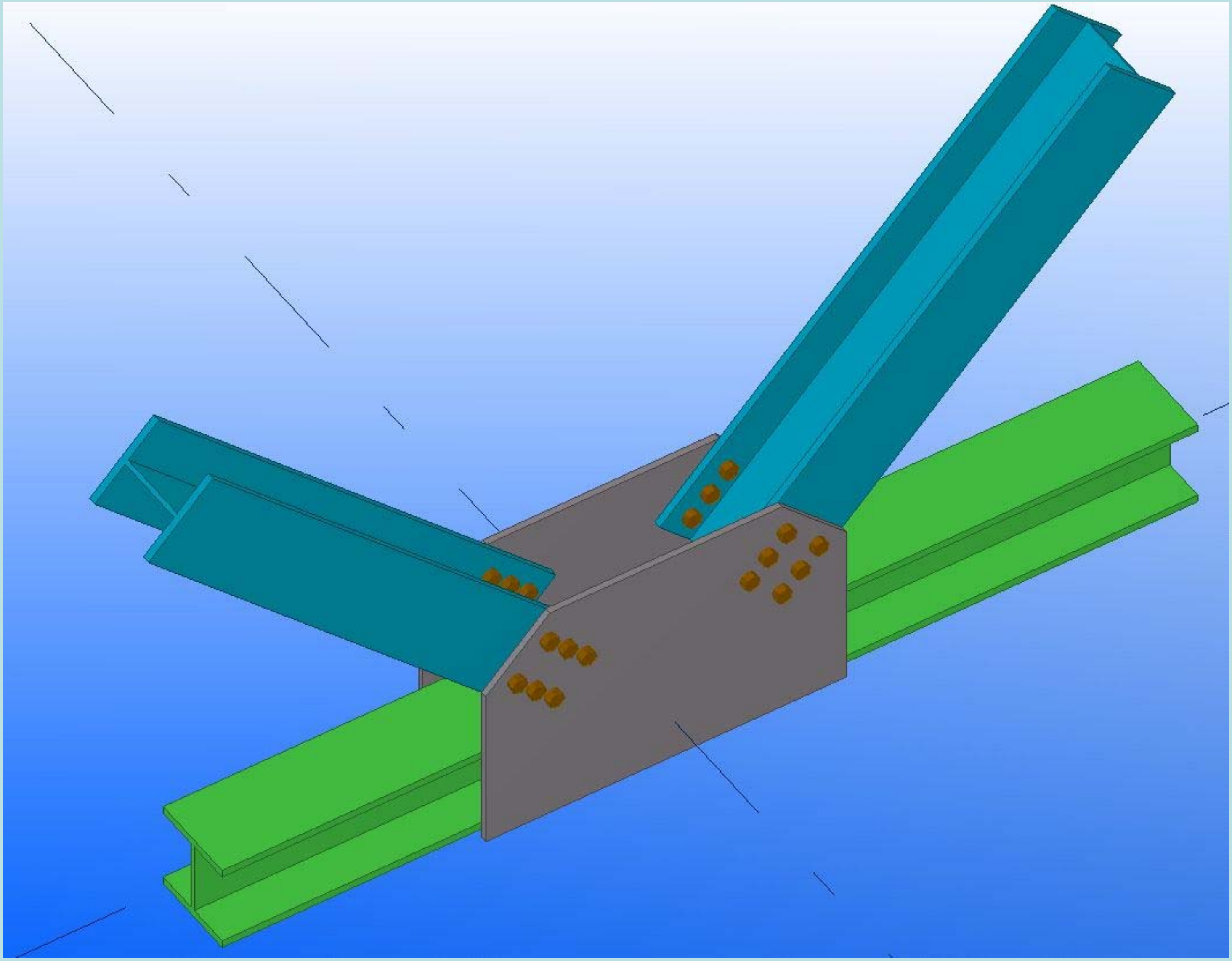
- $N_{u,Rd}$ = the design ultimate resistance at the net cross-section (end section with bolt holes)

$$N_{u,Rd} = \frac{0,9 \cdot A_{net} \cdot f_u}{\gamma_{M2}}$$

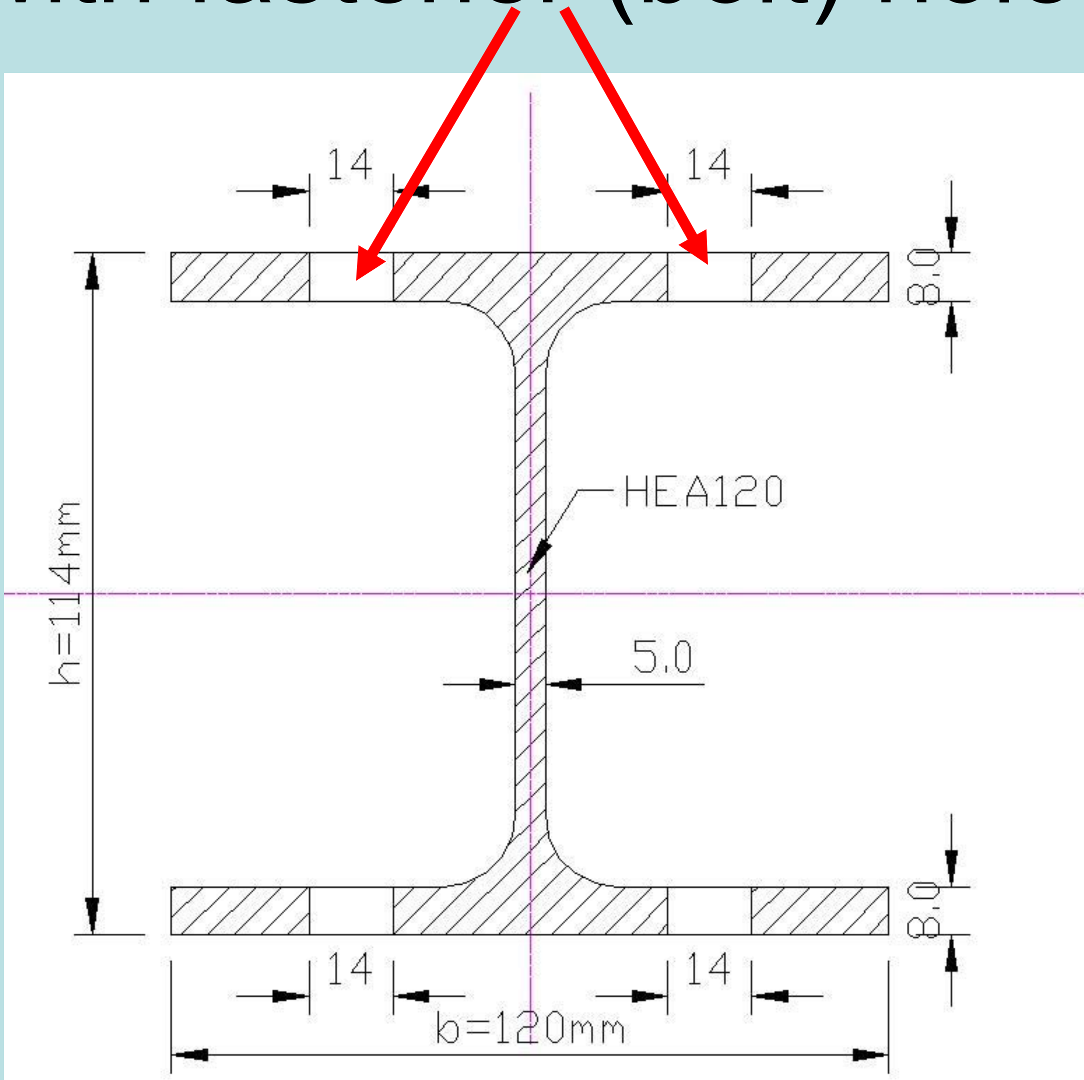
Where: f_u = 3400 daN/cm^2 for S235 steel,

and γ_{M2} = 1,25

Bolted end connection of the member



Cross-section of the chosen profile with fastener (bolt) holes



Calculation of the net area (A_{net}) = gross area minus holes for bolts

- The net cross-section area at holes for fasteners:
- $A_{net} = A_{act} – n_f \times d \times t_f = 25,3 – 4 \times 1,4 \times 0,8 = 20,82\ cm^2$
- n_f = number of fasteners per cross-section = 4 fasteners (bolts) in this case
- Evidently, $A_{net} < A_{act}$

Consequently:

$$N_{u,Rd} = \frac{0,9 \cdot 20,82 \cdot 3400}{1,25} = 50967 \ \ daN$$

Finally:

$$N_{t,Rd} = \min\left\{N_{pl,Rd} = 59455 \ daN; N_{u,Rd} = 50967 \ daN\right\}$$

Result: $N_{t,Rd}$ = 50967 daN

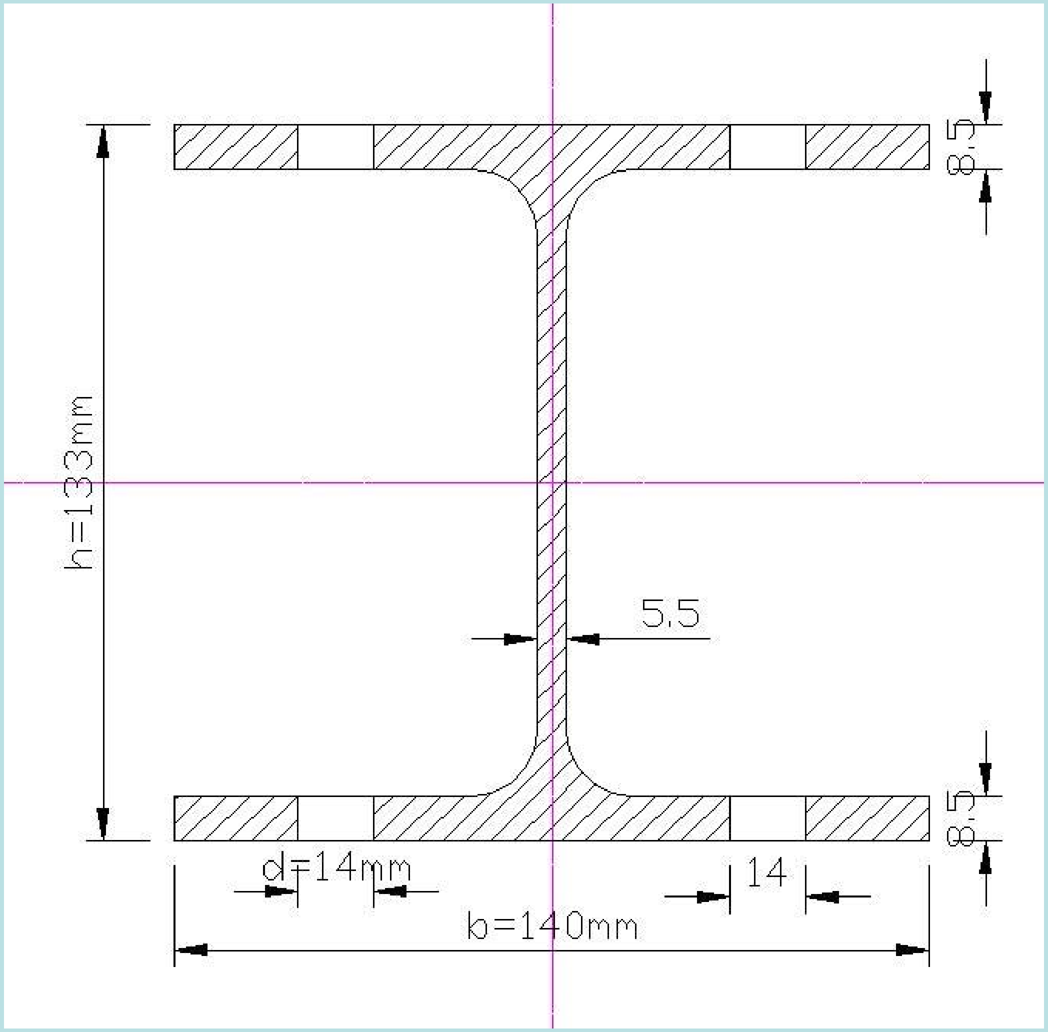
Member checking:

$$\frac{N_{Ed}}{N_{t,Rd}} = \frac{56392}{50967} = 1{,}106 > 1{,}0$$

NOT OK !!

A bad choice was made for the HEA profile from the profile table (the area of the chosen profile is too small !). A larger profile should be <u>chosen </u>!

Cross section of the NEW chosen profile with fastener holes:



Geometric characteristics and resistances for HE140A

- $A_{act} = 31{,}4\ cm^2$
- $A_{net} = 31{,}4 - 4 \times 1{,}4 \times 0{,}85 = 26{,}64\ cm^2$

$$\begin{cases} N_{pl,Rd} = \dfrac{31{,}4 \cdot 2350}{1{,}0} = 73790\ daN \\ N_{u,Rd} = \dfrac{0{,}9 \cdot 26{,}64 \cdot 3400}{1{,}25} = 65215\ daN \end{cases}$$

$$N_{t,Rd} = \min\{N_{pl,Rd}; N_{u,Rd}\} = 65215\ daN$$

Tension member final checking:

$$\frac{N_{Ed}}{N_{t,Rd}} = \frac{56392}{65215} = 0,86 < 1,0$$

$\Rightarrow$Member profile HE140A   OK !

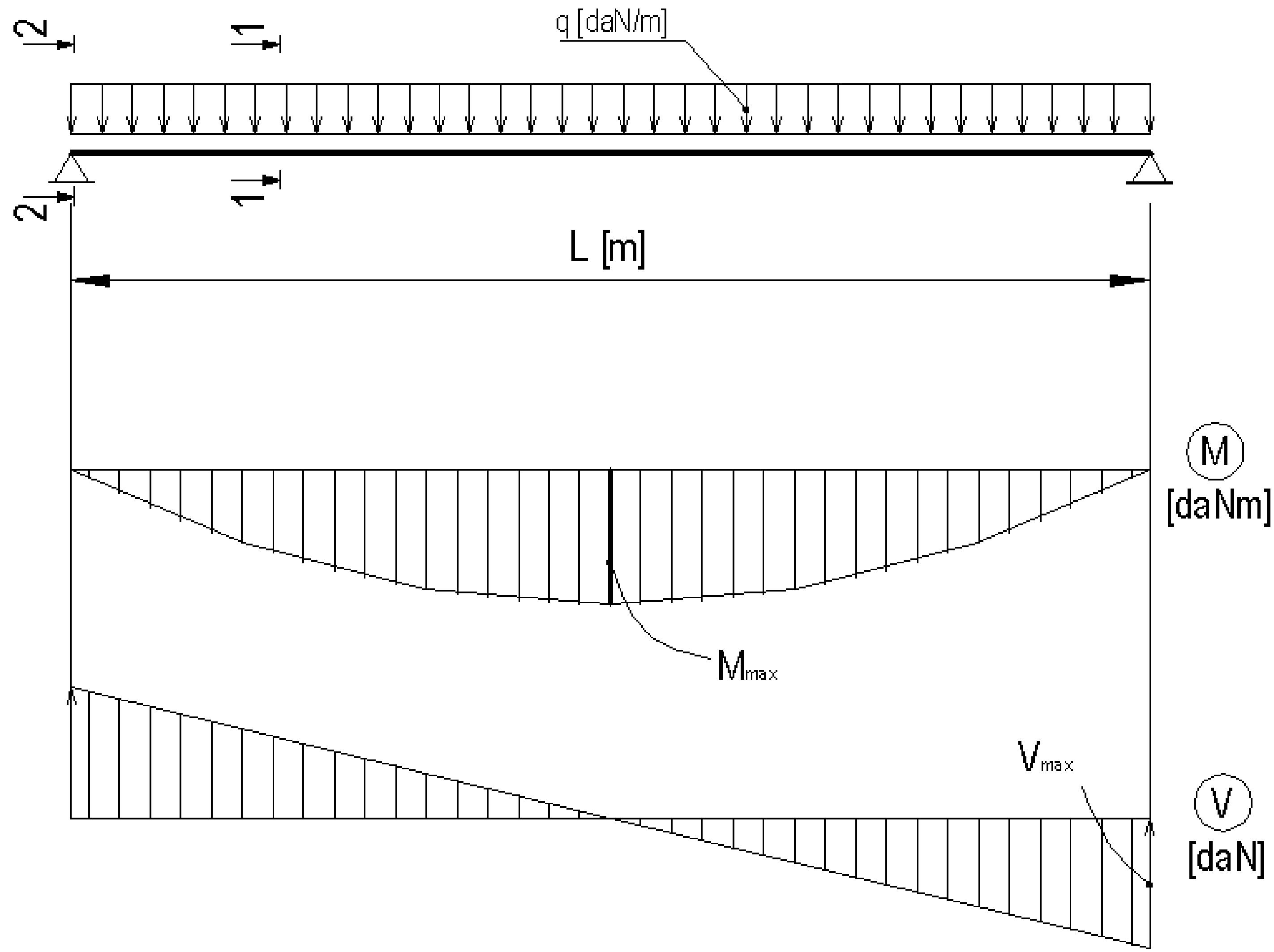
2) Resistance of member cross-section in pure bending

- Object of the application: simply supported beam under uniformly distributed load (q)

OBSERVATION:

- Mid-span cross-section (section 1-1) is submitted to maximum bending moment (M_{max}) and no shear force (V=0)
- The maximum shear force appears on the support (V_{max}) where the bending moment is null (M=0).

Static scheme of the beam:



Step1: Sizing of the beam cross-section under pure bending (section 1-1)

INITIAL DATA:

- IPE profile required (other profiles also possible !)
- Value of uniformly distributed load q=800 daN/m
- Value of the span: L=12,0 m
- Steel grade: S235 $\Rightarrow$ fy=2350 daN/cm^2

Formula to check member resistance in bending:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1,0$$

Where:

$$M_{Ed} = M_{\max}^{beam} = \frac{q \cdot L^2}{8} = \frac{800 \cdot 12,0^2}{8} = 14400\ daNm = 1440000\ daNcm$$

Sizing procedure:

- Equation of equilibrium used for sizing:

  $M_{Ed} = M_{c,Rd}$
- In case of class 1 or 2 cross sections for bending (as the IPE profiles usually are) member resistance in bending becomes:

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl} \cdot f_y}{\gamma_{M0}}$$

Where γ_{M0} =1,0 and W_{pl} = plastic modulus of beam cross-section

Consequently:

$$M_{\max} = \frac{W_{pl}^{req} \cdot f_y}{\gamma_{M0}} \Rightarrow W_{pl}^{req} = \frac{M_{\max} \cdot \gamma_{M0}}{f_y}$$

And:

$$W_{pl}^{req} = \frac{1440000 \cdot 1{,}0}{2350} = 612{,}8\ cm^3$$

Required value !

The actual value of section plastic modulus is further on chosen from the IPE profile tables.

For IPE 300 in the table we have:

$$W_{pl}^{act} = W_{pl.y}^{table} = 628{,}4\ cm^3 > W_{pl}^{req} = 612{,}8\ cm^3$$

OBSERVATION: The chosen cross-section in the table is of class 1 under pure bending (see last column of the table) which confirms the employed formula for sizing.

STEP 2: Checking of member cross-section under pure bending

- Checking performed in section (1-1) of the beam
- Checking formula:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1,0$$

In previous formula:

$$\begin{cases} M_{Ed} = M_{\max} = 1440000 \ daNcm \\ M_{c,Rd} = \dfrac{W_{pl} \cdot f_y}{\gamma_{M0}} = \dfrac{628{,}4 \cdot 2350}{1{,}0} = 1476740 \ daNcm \end{cases}$$

And consequently:

$$\frac{1440000}{1476000} = 0{,}976 < 1{,}0$$

Profile OK !

3) Checking of the beam cross-section under pure shear in section (2-2):

Checking formula to EN 1993-1-1:

- The design value of the shear force V_{Ed} at each cross section shall satisfy:

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1,0$$

- Where $V_{c,Rd}$ is the design shear resistance

In previous formula:

$$\begin{cases} V_{Ed} = V_{\max} = \dfrac{q \cdot L}{2} = \dfrac{800 \cdot 12}{2} = 4800\ daN \\ V_{c,RD} = V_{pl,Rd} \end{cases}$$

In <u>absence of the torsion</u>, the design plastic shear resistance $V_{pl,Rd}$ is given by the formula:

$$V_{pl,Rd} = \frac{A_v \cdot \left(\dfrac{f_y}{\sqrt{3}} \right)}{\gamma_{M0}}$$

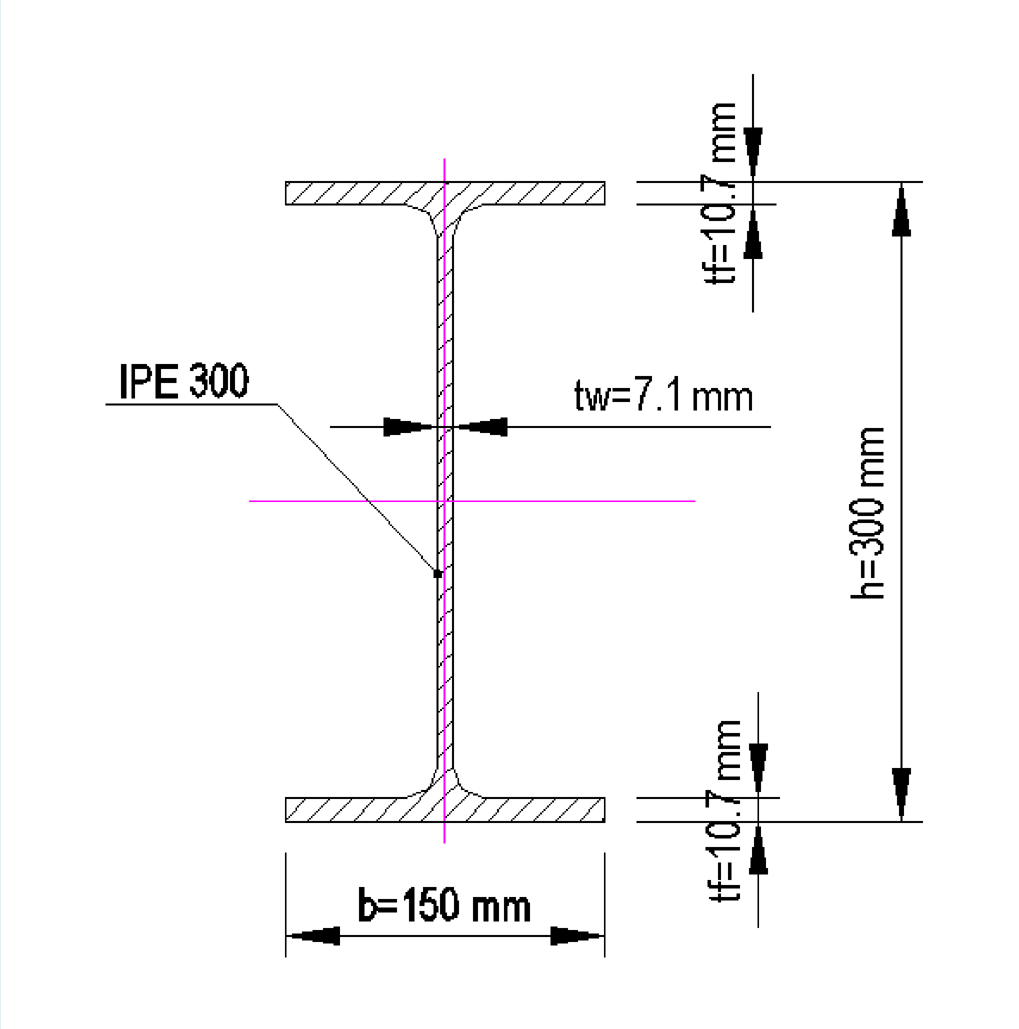
Where A_v is the shear area

Calculation of the shear area value A_v

- In case of “I” sections (i.e. IPE !) taken from the profile table, the shear area may be calculated using:

$$A_v = A - 2 \cdot b \cdot t_f + (t_w + 2r) t_f$$

Geometry of chosen profile cross-section (IPE 300):



For IPE 300 we obtain from the profile table:

- $A = 53{,}8\ cm^2$ (gross cross-section)
- r = 15 mm = 1,5 cm (inner radius of profile)

Consequently:

- $A_v = 53{,}8-2\cdot 15{,}0\cdot 1{,}07+(0{,}71+2\cdot 1{,}5)\cdot 1{,}07$
  $= 25{,}67\ cm^2$

Cross-section checking in shear:

$$V_{pl,Rd} = \frac{25{,}67 \cdot 2350}{1{,}0 \cdot \sqrt{3}} = 34828\ daN$$

Checking of the member under pure shear:

$$\frac{V_{Ed}}{V_{pl,Rd}} = \frac{4800}{34828} = 0{,}137 < 1{,}0$$

OBSERVATIONS:

- The value 0,137 obtained for the ratio shows a small influence of the shear force on profile cross-section

- Pure bending or pure shear are rather rare practical cases. The most often in practice we have combined bending and shear (see next example)

4)Member resistance under combined bending and shear

- According to code provisions, where the shear force is present simultaneously with a bending moment, allowance shall be made for its effect on the moment resistance

Actually, shear force presence may diminish the moment resistance:

a) Where the shear force is less than half the plastic shear resistance, its effect on the moment resistance may be neglected. This is equivalent with the following relation:

$$\frac{V_{Ed}}{V_{pl,Rd}} < 0,5$$

If upper relation is satisfied, we can neglect the effect of the shear force on the moment resistance

b) Where the shear force is larger than half of the plastic shear resistance, or:

$$\frac{V_{Ed}}{V_{pl,Rd}} \geq 0{,}5$$

the design value of the moment resistance will be diminished by using a reduced strength value:

$$f_y^* = (1 - \rho) \cdot f_y$$

- In the previous:

$$\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1 \right)^2$$

- Consequently, the checking formula in bending becomes:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1{,}0$$

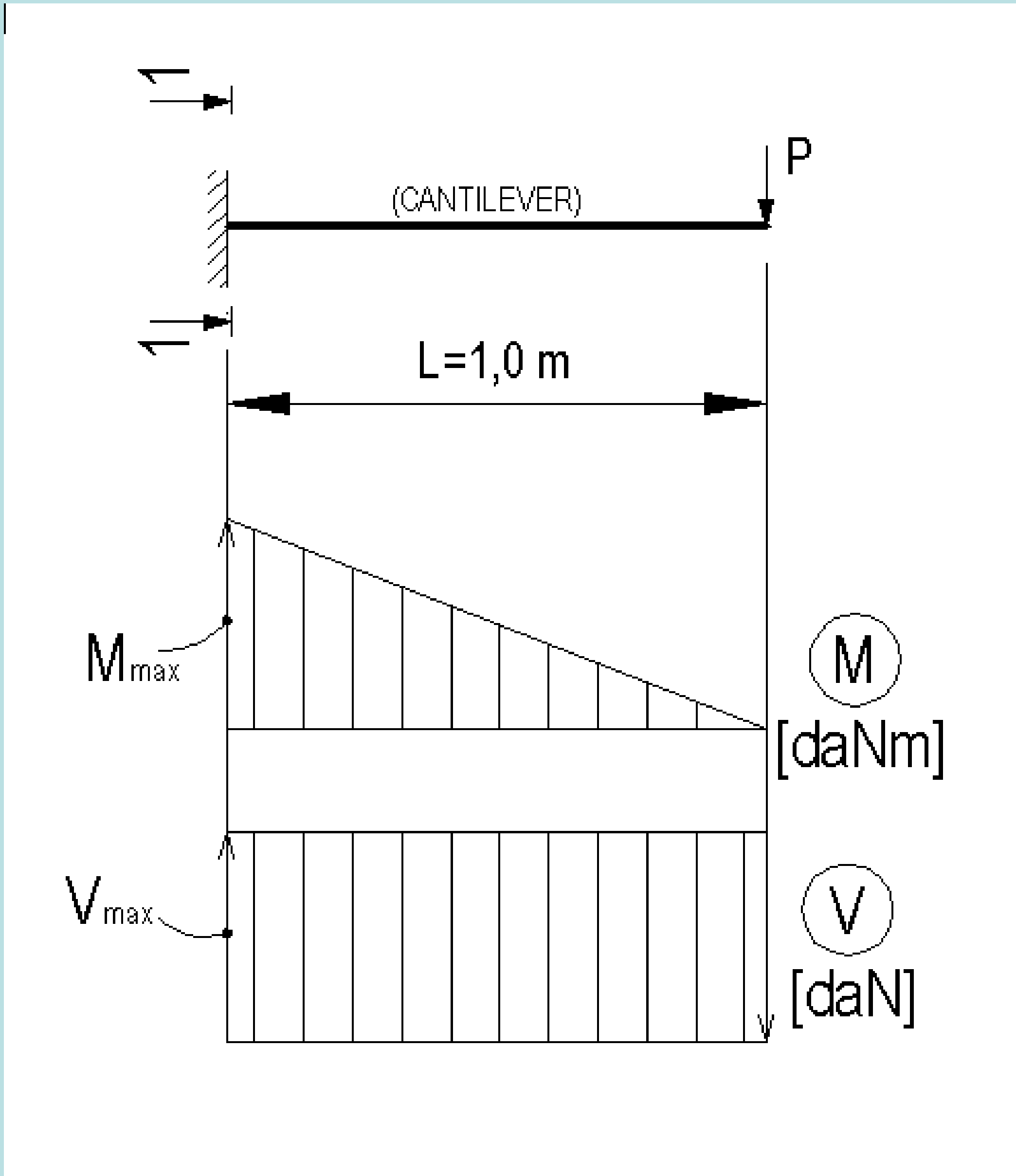
Where:

$$M_{c,Rd} = \frac{W_{pl} \cdot f_y^*}{\gamma_{M0}}$$

PRACTICAL EXAMPLE:

- Cantilever with length= 1,0 m loaded with a concentrated force "P" at its end

- Value of the concentrated force P=30000 daN = 300000 N
- IPE profile required for cantilever beam
- S235 steel grade $\Rightarrow f_y = 2350\ daN/cm^2$



STATIC SCHEME AND DIAGRAMS:

$M_{max}=P\cdot L=30000$ daNm

$V_{max}=V_{Ed}=30000$ daN

OBSERVATION: In section (1-1) on the diagram, the maximum value of the bending moment appears together with the shear force (typical to cantilever structures)

Step 1: Sizing of the cantilever cross-section in bending

- Equilibrium equation for sizing:

$$M_{Ed} = M_{\max} = M_{c,Rd} \Rightarrow M_{\max} = \frac{W_{pl}^{req} \cdot f_y}{\gamma_{M0}}$$

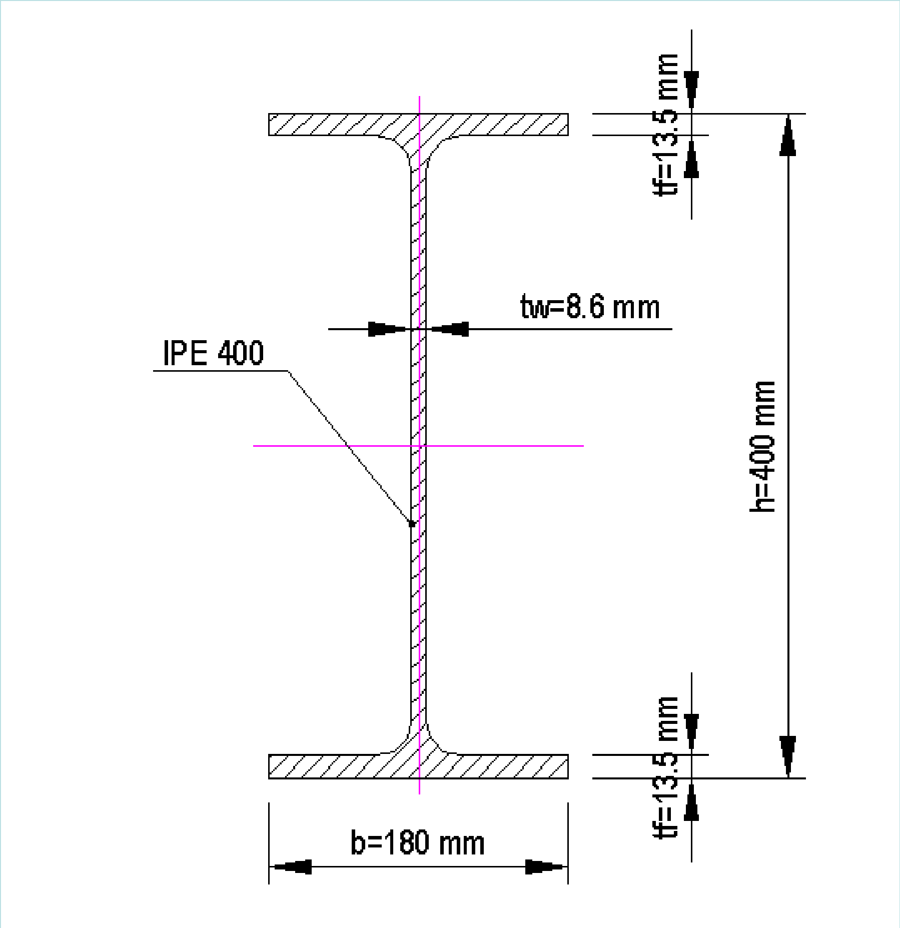
$$3000000 = \frac{W_{pl}^{req} \cdot 2350}{1,0} \Rightarrow W_{pl}^{req} = 1276,6\ cm^3$$

- From the profile table, for IPE 400 we obtain:

$$W_{pl}^{act} = W_{pl,y} = 1307\ cm^3 > W_{pl}^{req}$$

Step 2: Checking of the shear force influence

$$\begin{cases} V_{Ed} = 30000\ daN \\ V_{pl,Rd} = \dfrac{A_v \cdot f_y}{\gamma_{M0} \cdot \sqrt{3}} \end{cases}$$



Geometry of the cross section and geometric characteristics:

$A=84,5\ cm^2$

(gross area for IPE 400)

$A_v=A-2\cdot b\cdot t_f+(tw+2r)t_f=$

$= 84,5-2\cdot 18\cdot 1,35+(0,86+2\cdot 2,1)\cdot 1,35 =42,73\ cm^2$

Influence of shear force:

$$\begin{cases} V_{pl,Rd} = \dfrac{42,73 \cdot 2350}{1,0 \cdot \sqrt{3}} = 57976 \ daN \\ \dfrac{V_{Ed}}{V_{pl,Rd}} = \dfrac{30000}{57976} = 0,517 > 0,5 \end{cases}$$

CONSEQUENCE: Allowance shall be made for the effect of shear force on the <u>moment resistance</u>

- Calculation of the diminishing factor:

$$\rho = \left(2 \cdot \frac{V_{Ed}}{V_{pl,Rd}} - 1\right)^2 = \left(2 \cdot 0{,}517 - 1\right)^2 = 0{,}0012$$

- Calculation of the diminished strength:

$$f_y^* = (1 - \rho)\, f_y = (1 - 0{,}0012) \cdot 2350 = 2347\ daN / cm^2$$

- Calculation of the diminished moment resistance:

$$M_{c,Rd} = \frac{W_{pl} \cdot f_y^*}{\gamma_{M0}} = \frac{1307 \cdot 2347}{1,0} = 3067529\ daNcm$$

- Checking of the cross-section in bending using the diminished moment resistance:

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{3000000}{3067529} = 0,978 < 1,0$$

Cross-section OK !

OBSERVATION:

- As evident for the present example, generally the influence of the shear force on the moment resistance is small or to neglect.

- Most often, in practical cases, in structures working predominant in bending, shear force influence is neglected, according to code conditions.