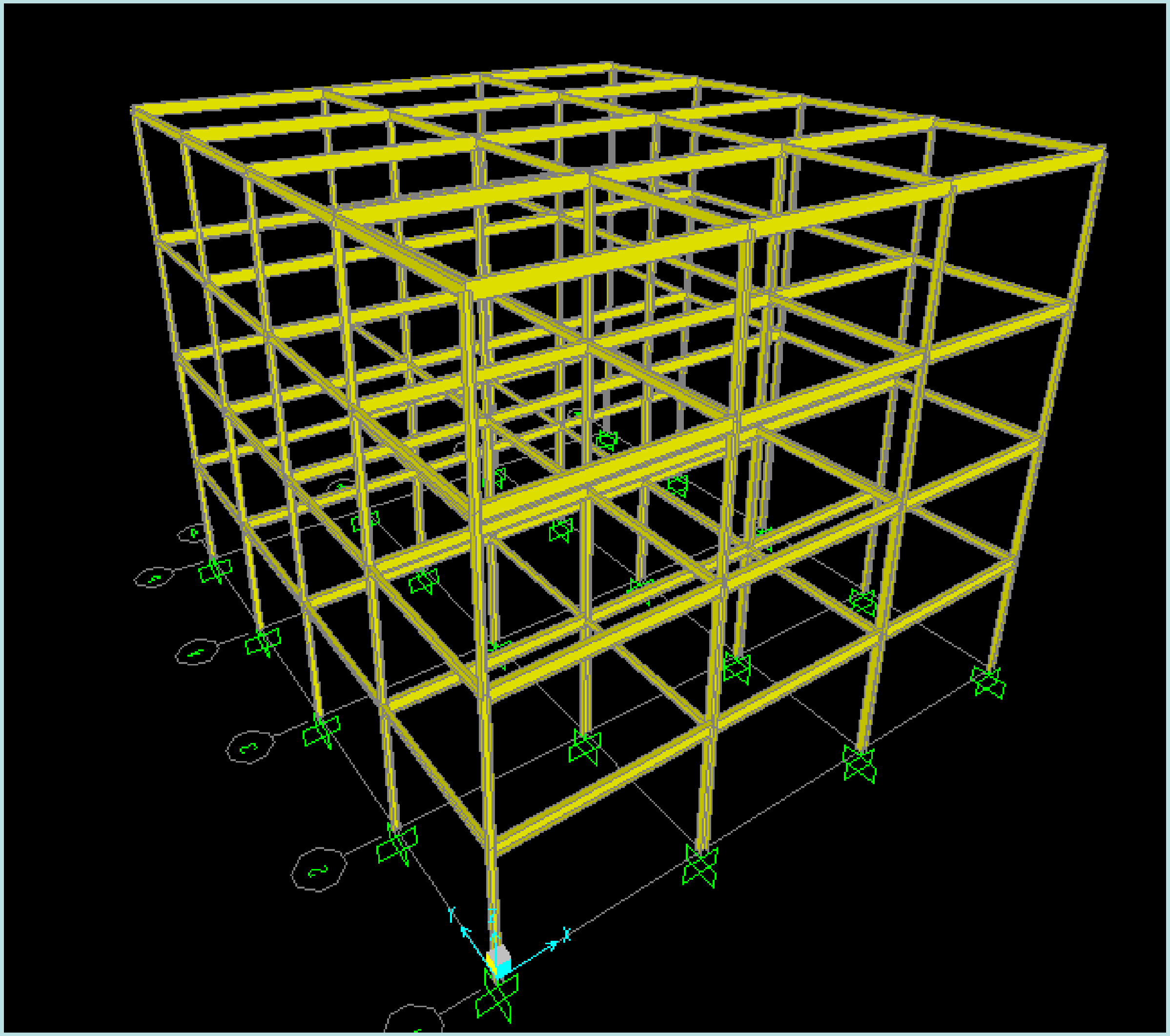
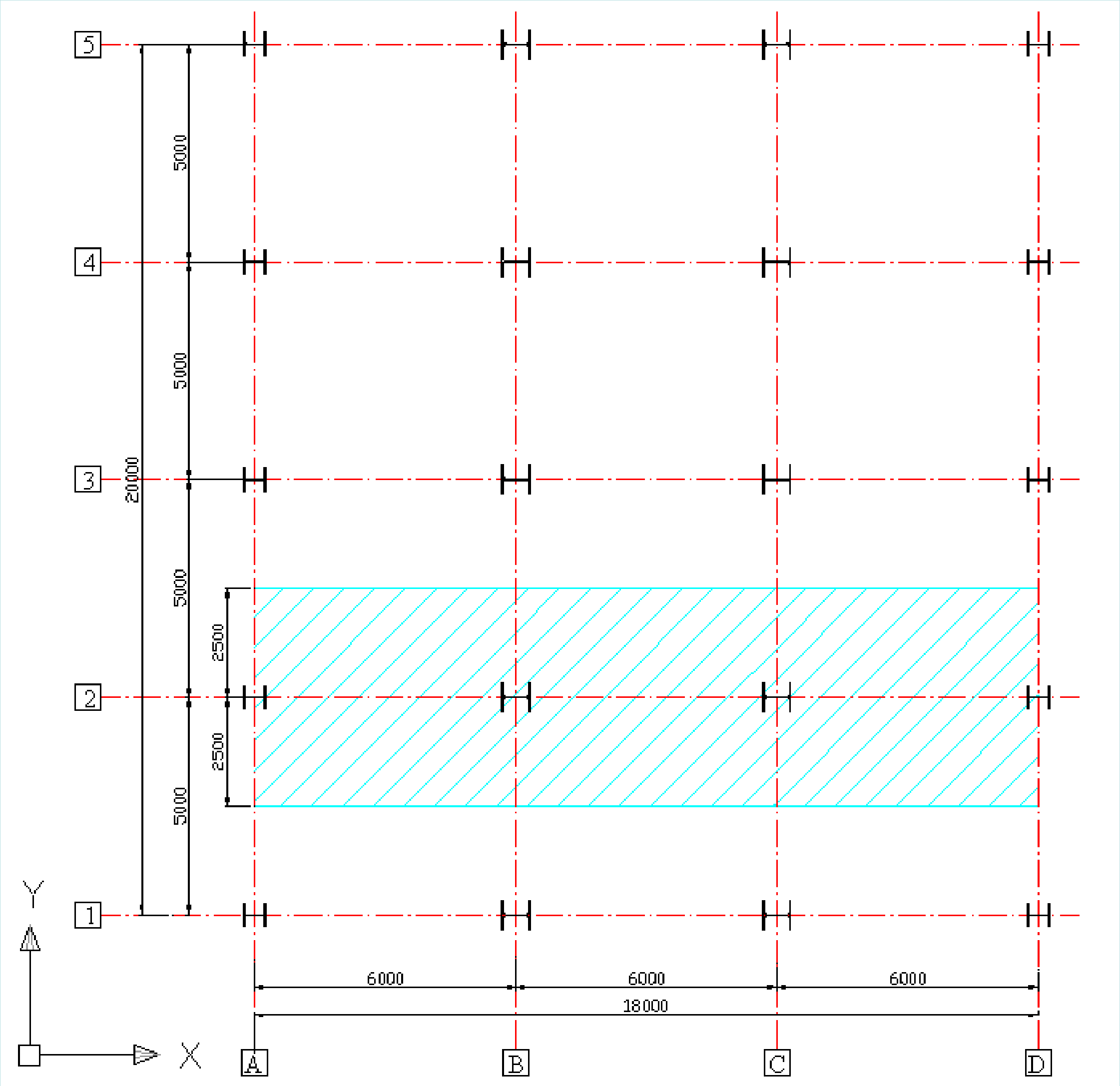
Application nr. 2
(Global Analysis)

Effects of deformed geometry of the structures. Structural stability of frames. Sway frames and non-sway frames.

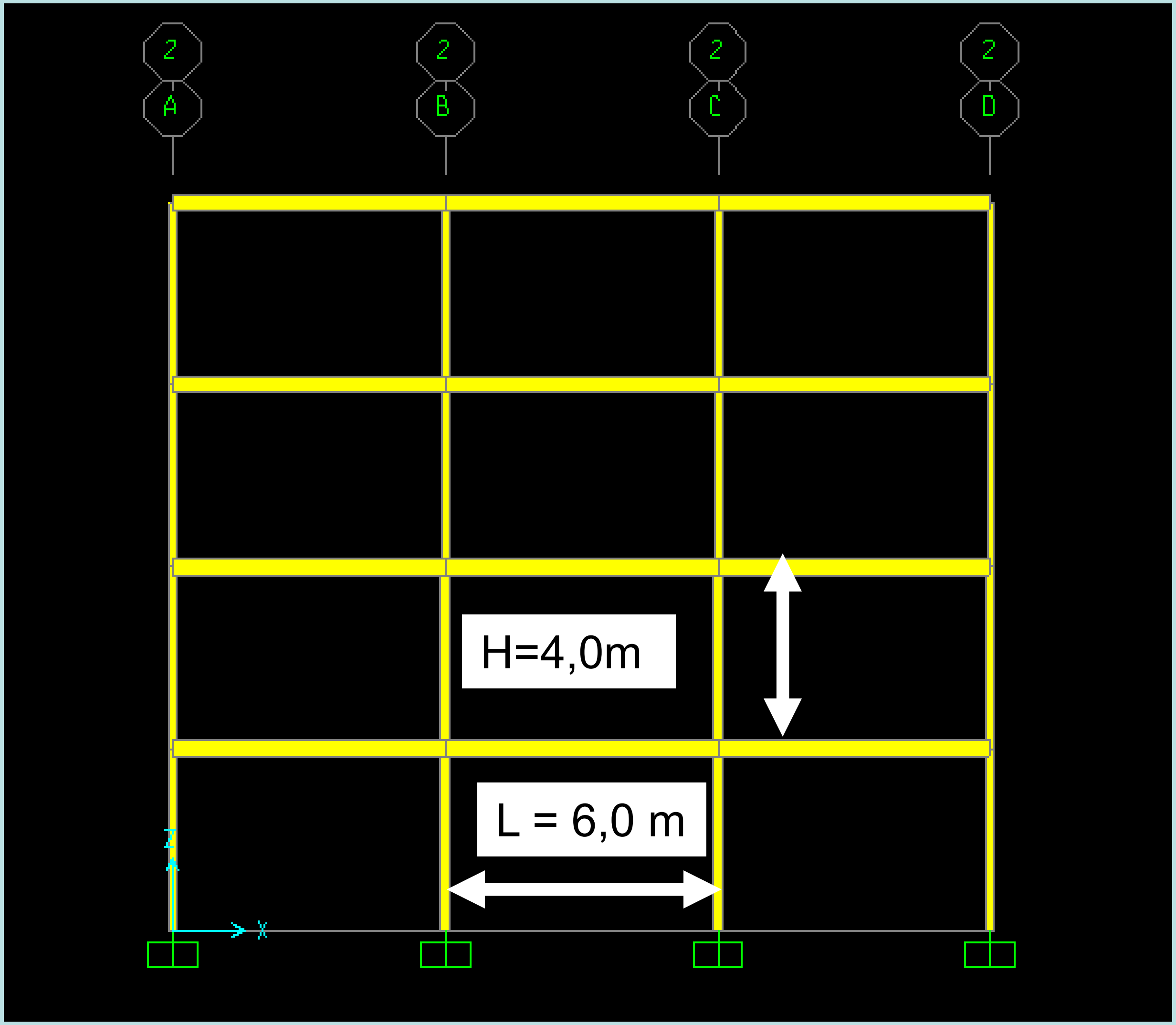
Object of study: multistorey structure (SAP 2000 Nonlinear)



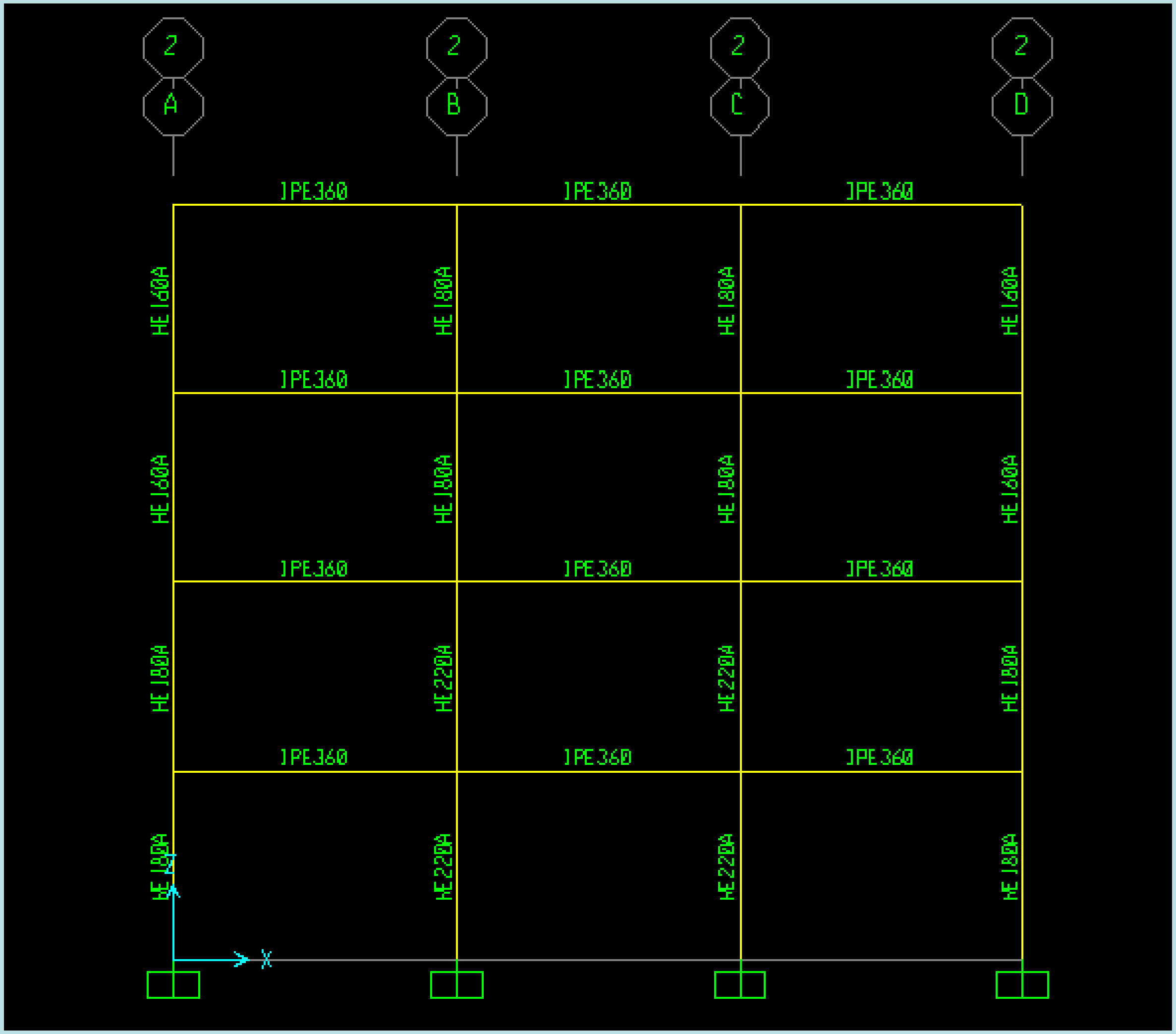
Grid of axes and positions of column cross-section in the structure



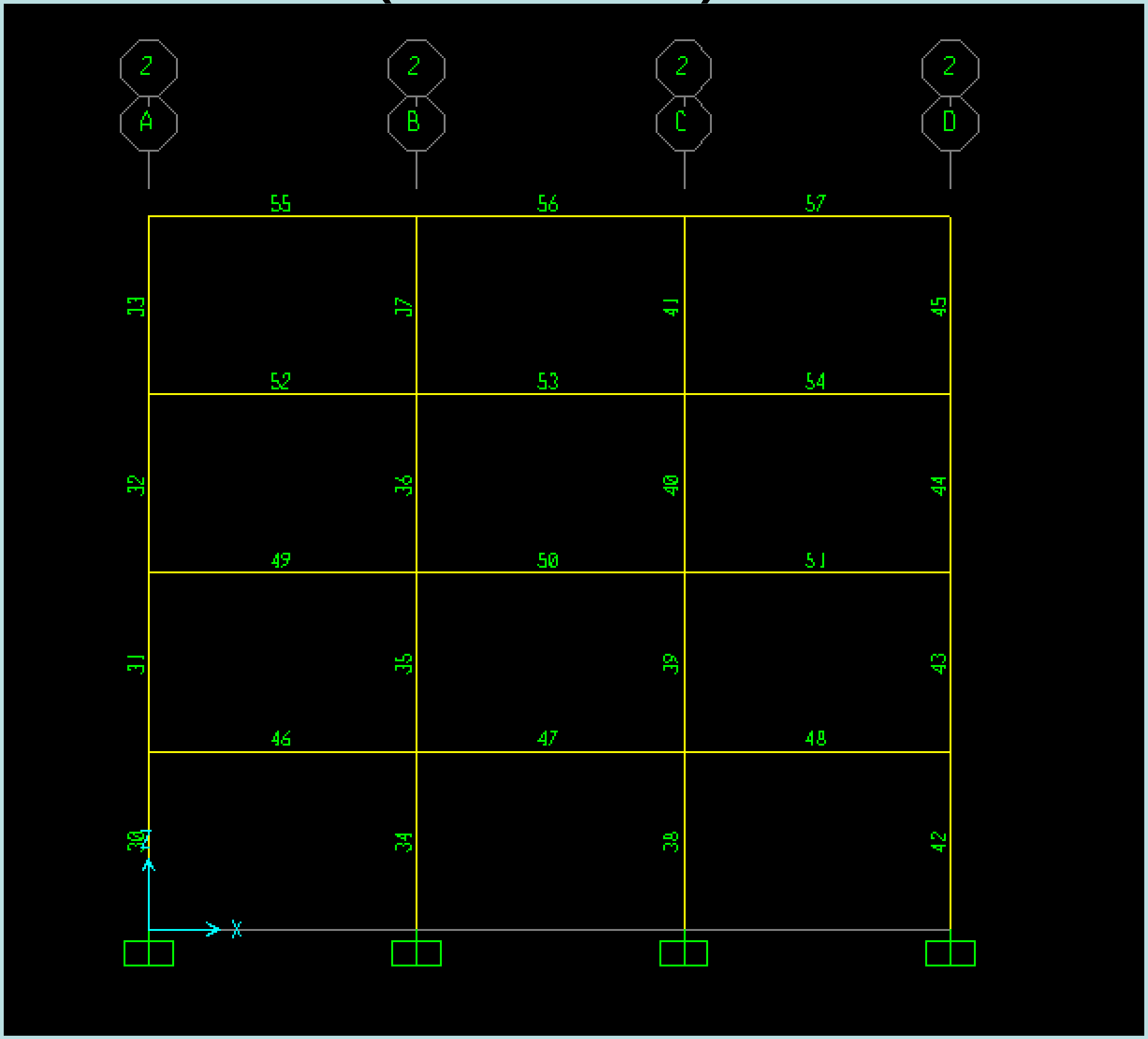
PART 1: Transversal plane frame extracted from the structure:



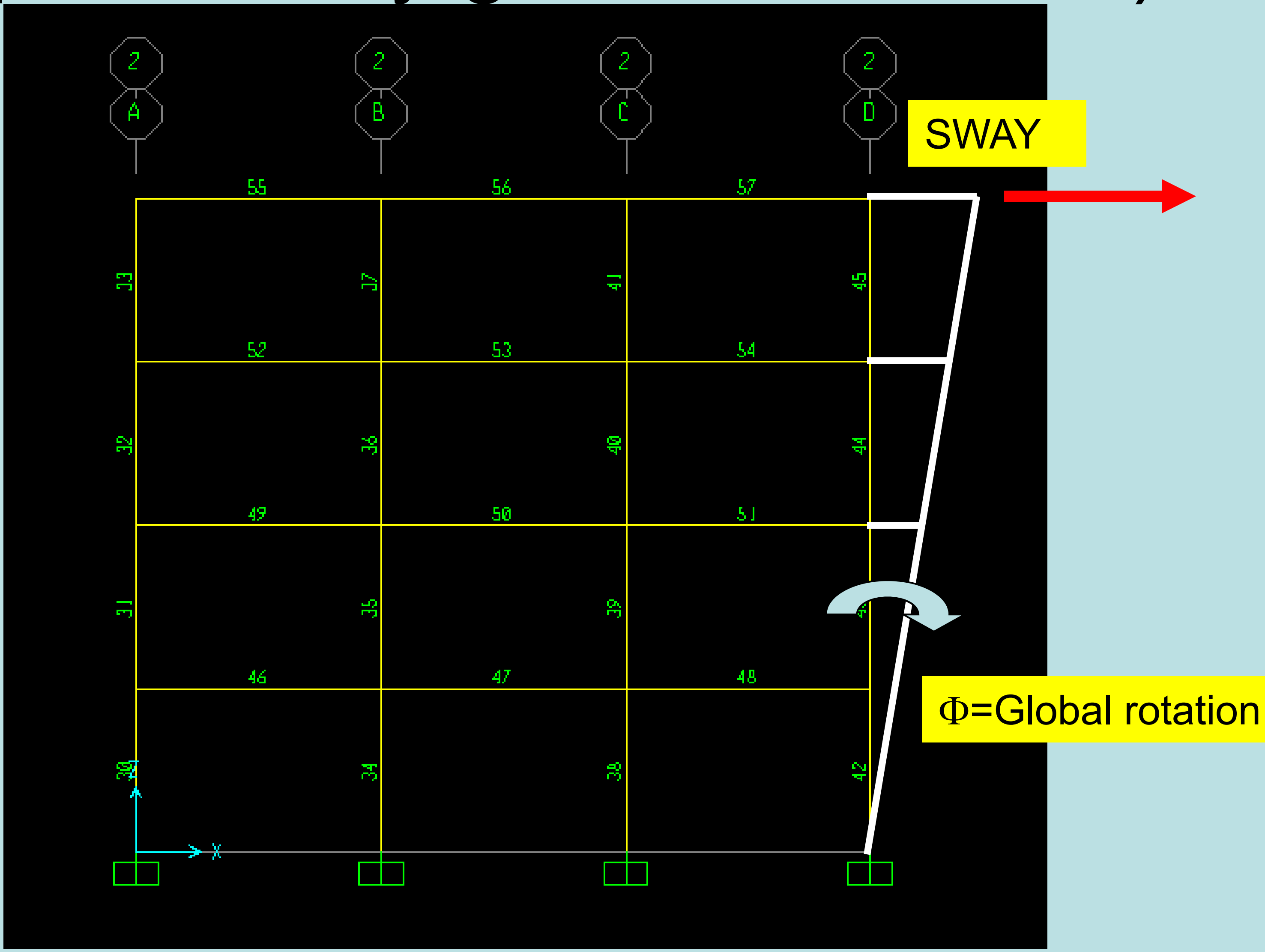
Cross-sections for plane frame columns and beams (profiles):



Labels of columns and beams (names) :



Consider global sway of the structure (expressed by global rotation Φ)



Index for the effects of deformed geometry:

Elastic instability factor in global mode:	$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}}$ F_{Ed} = design loading on the structure (including imperfection effect) F_{cr} = elastic critical buckling load for global instability mode based on initial elastic stiffness		
Design situation:	$\alpha_{cr} \geq 10{,}0$	$3{,}0 \leq \alpha_{cr} < 10{,}0$	$\alpha_{cr} < 3{,}0$
Type of analysis allowed for the structure:	Elastic first order	Elastic first order with increase of horizontal loads by the factor: $\mu = \frac{1}{1 - \frac{1}{\alpha_{cr}}}$	(ONLY!) Second order

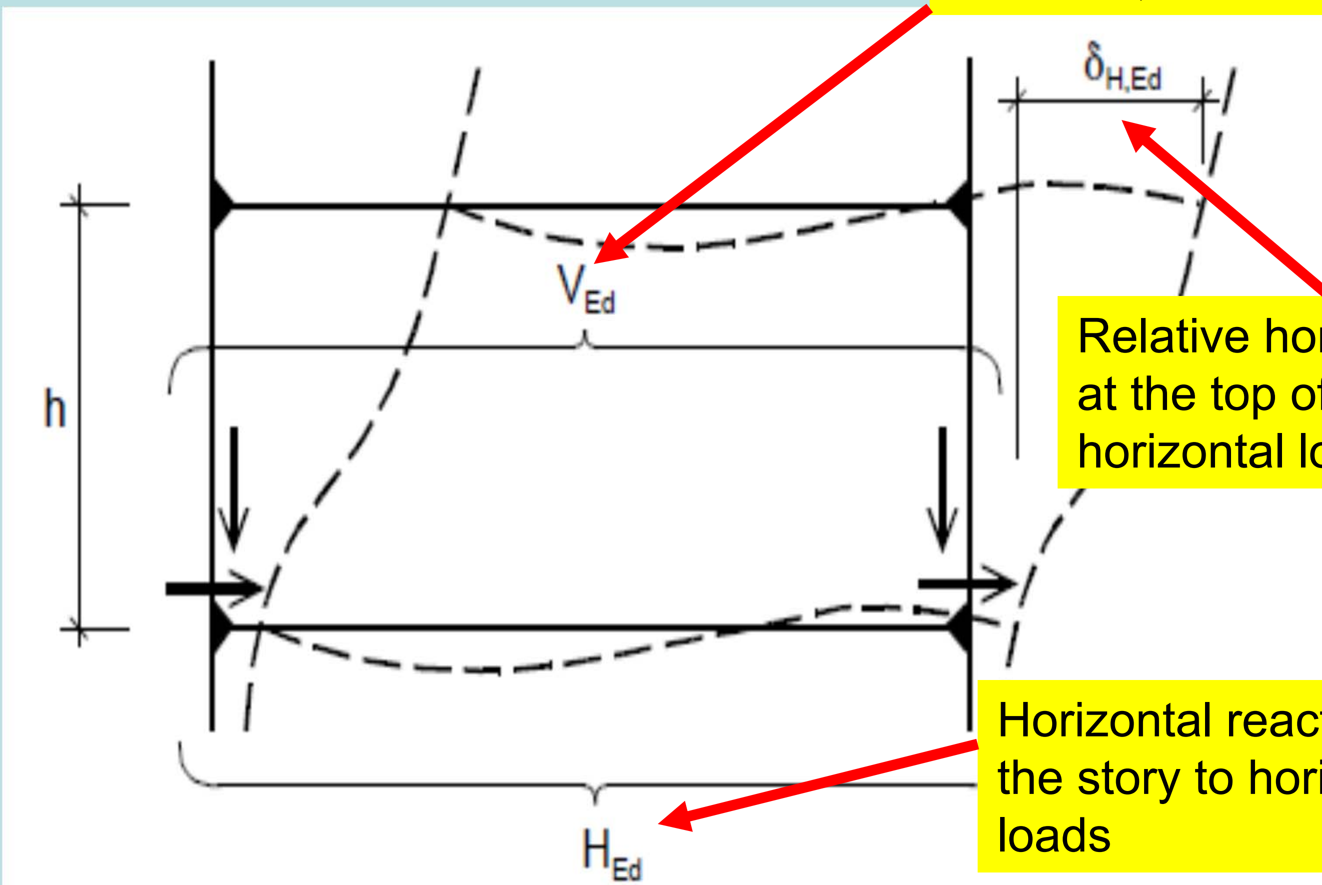
Structural behavior and terminology associated with (α_{cr}) values:

$\alpha_{cr} > 10,0$	$3,0 \leq \alpha_{cr} \leq 10,0$	$\alpha_{cr} < 3,0$
The structure is rigid or non-sway	The structure is flexible or sway	
No sensitivity to deformed geometry	Moderate or high sensitivity to deformed geometry (to implement in structural analysis)	

Methods of determining α_{cr}

Computer method of determining the critical load factor for elastic buckling of the frame	Approximate method to find (α_{cr}) by calculation, valid under limited conditions using the formula:
(α_{cr}) via (SAP 2000 N) buckling analysis with imperfections	$$\alpha_{cr} = \left(\frac{H_{Ed}}{V_{Ed}}\right) \cdot \left(\frac{h}{\delta_{H,Ed}}\right)$$

Significance of the terms in the formula: H_{Ed} and V_{Ed} (h=storey height)



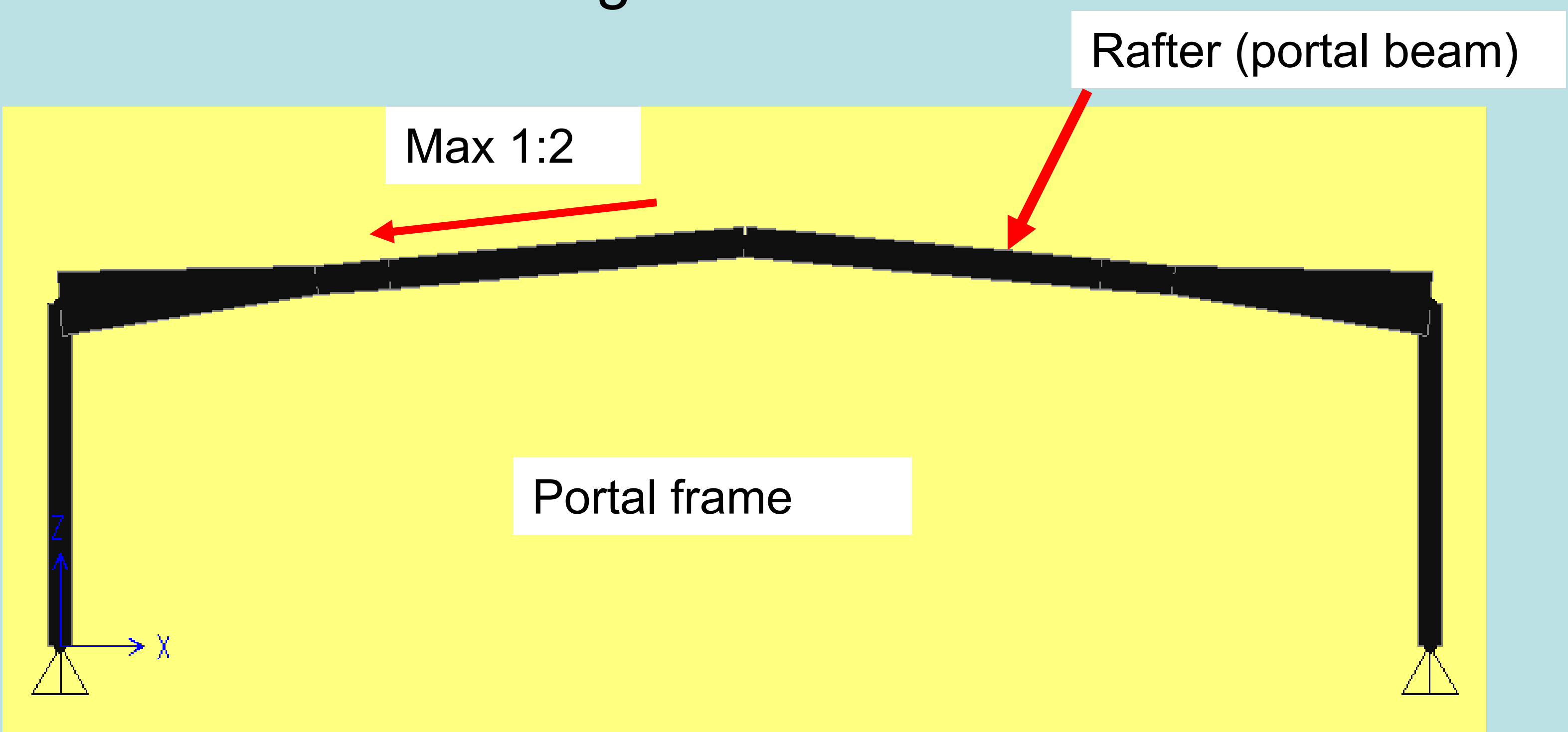
Total design vertical load on the structure, on the bottom of the storey

Relative horizontal displacement at the top of the story, under horizontal loads / fictitious

Horizontal reaction at the bottom of the story to horizontal / fictitious loads

Conditions to apply approximating formula:

•Types of structures for applicability: portal frames with shallow roof slopes (i.e. <1:2 or 26°) or beam-and-column plane frames in buildings



Conditions to formula applicability (2):

- The **compression** in beams or rafters is not significant.
- The axial compression in the beams or rafters may be assumed to be significant on the following condition:

$$\bar{\lambda} \geq 0{,}3 \cdot \sqrt{\frac{A \cdot f_y}{N_{Ed}}}$$

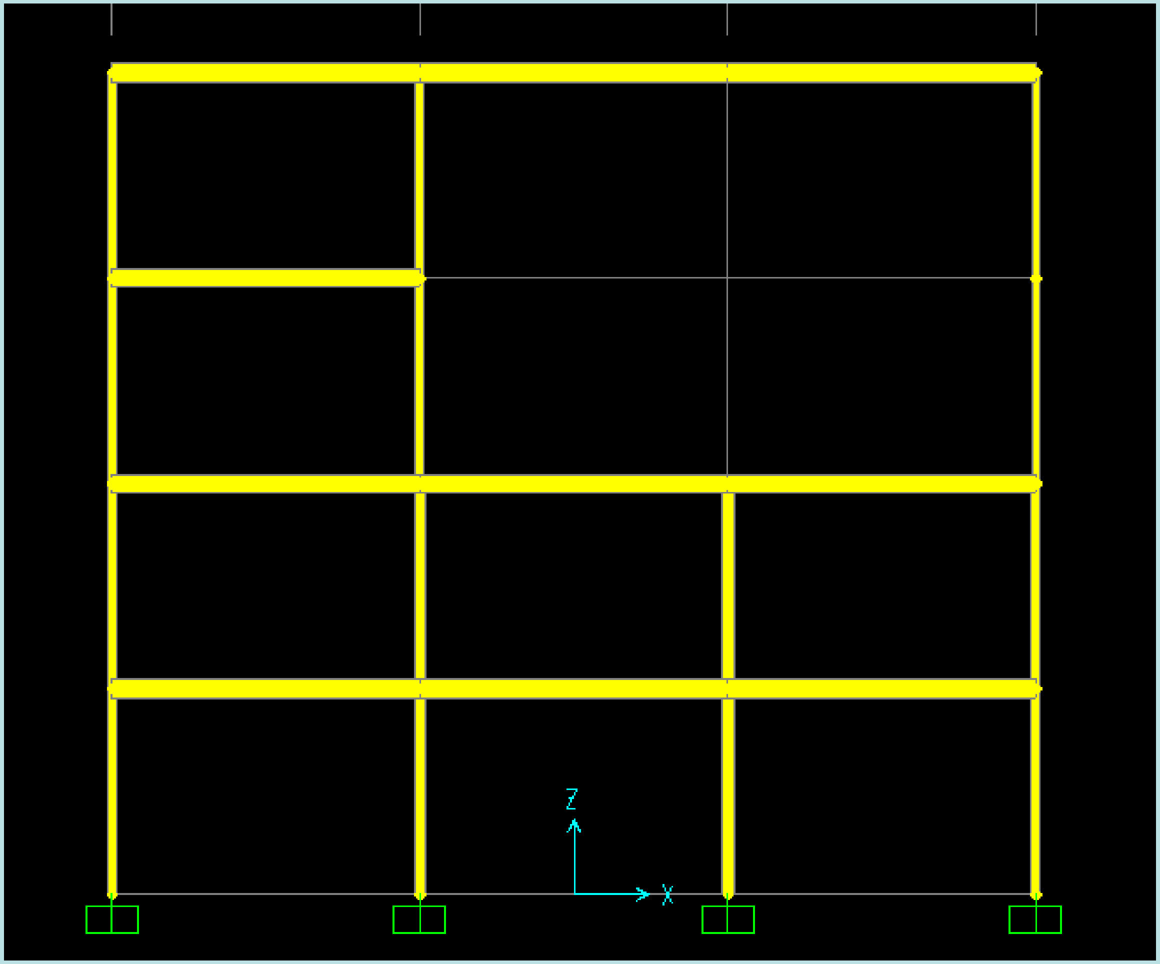
$\bar{\lambda}$ = in-plane non dimensional slenderness calculated for the beam or rafter considered as hinged at its ends

Conditions for formula aplicability(3):

For multi-storey frames, the second order effects may be calculated by means of the approximative formula provided that all storey have a similar:

- Distribution of vertical load
- Distribution of horizontal load
- Distribution of frame stiffness (frame members) with respect to applied storey shear forces

Irregular structures (with unequal distribution of frame stiffness) for which the formula is NOT applicable:

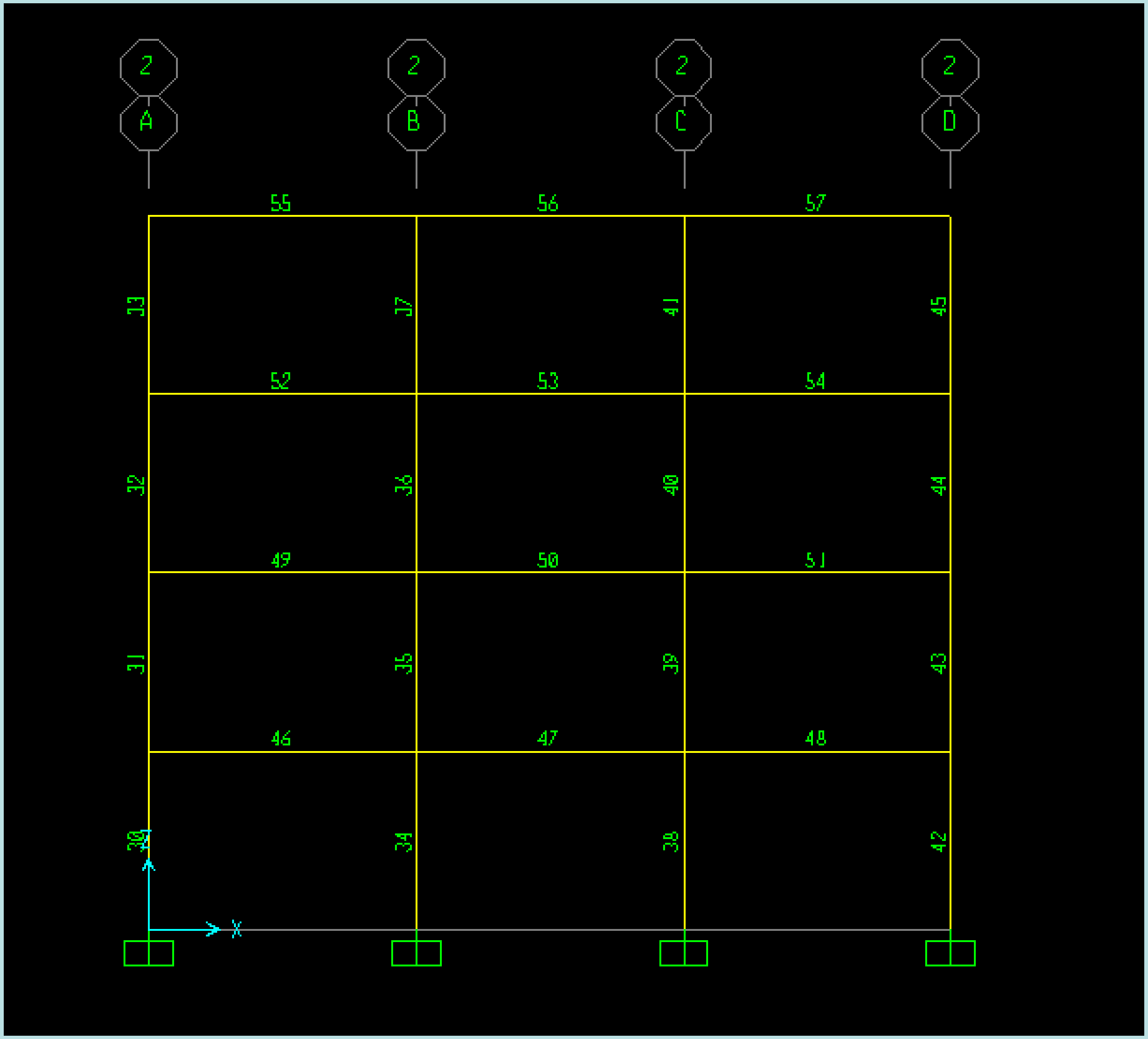


Application of the approximative formula on the multistory transverse frame:

1) Checking of the conditions of application for the approximating formula

2) Calculation of (α_{cr})

The transverse frame has a regular geometry and distribution of member stiffness:



Compression in the beams is NOT significant (very small values of axial force):

Beam Label	NEd (N)	Beam Span (L)	Beam Profile	Cross-section area [cm^2]	Gyration radius [cm]	Yield stress (fy) [daN/cm^2]	Lmd-bar	Lmd-crit
46	8004	6	IPE 360	72.7	14.96	2350	0.427	4.4
47	tension	6	IPE 361	72.7	14.96	2350	0.427	-
48	tension	6	IPE 362	72.7	14.96	2350	0.427	-
49	24092	6	IPE 363	72.7	14.96	2350	0.427	2.5
50	7788	6	IPE 364	72.7	14.96	2350	0.427	4.4
51	tension	6	IPE 365	72.7	14.96	2350	0.427	-
52	13805	6	IPE 366	72.7	14.96	2350	0.427	3.3
53	4162	6	IPE 367	72.7	14.96	2350	0.427	6.1
54	tension	6	IPE 368	72.7	14.96	2350	0.427	-
55	25258	6	IPE 369	72.7	14.96	2350	0.427	2.5
56	15904	6	IPE 370	72.7	14.96	2350	0.427	3.1
57	13934	6	IPE 371	72.7	14.96	2350	0.427	3.3

Since conditions for approximate calculation of (α_{cr}) are fulfilled, use of the formula is allowed:

$$\alpha_{cr} = \left(\frac{H_{Ed}}{V_{Ed}}\right) \cdot \left(\frac{h}{\delta_{H,Ed}}\right)$$

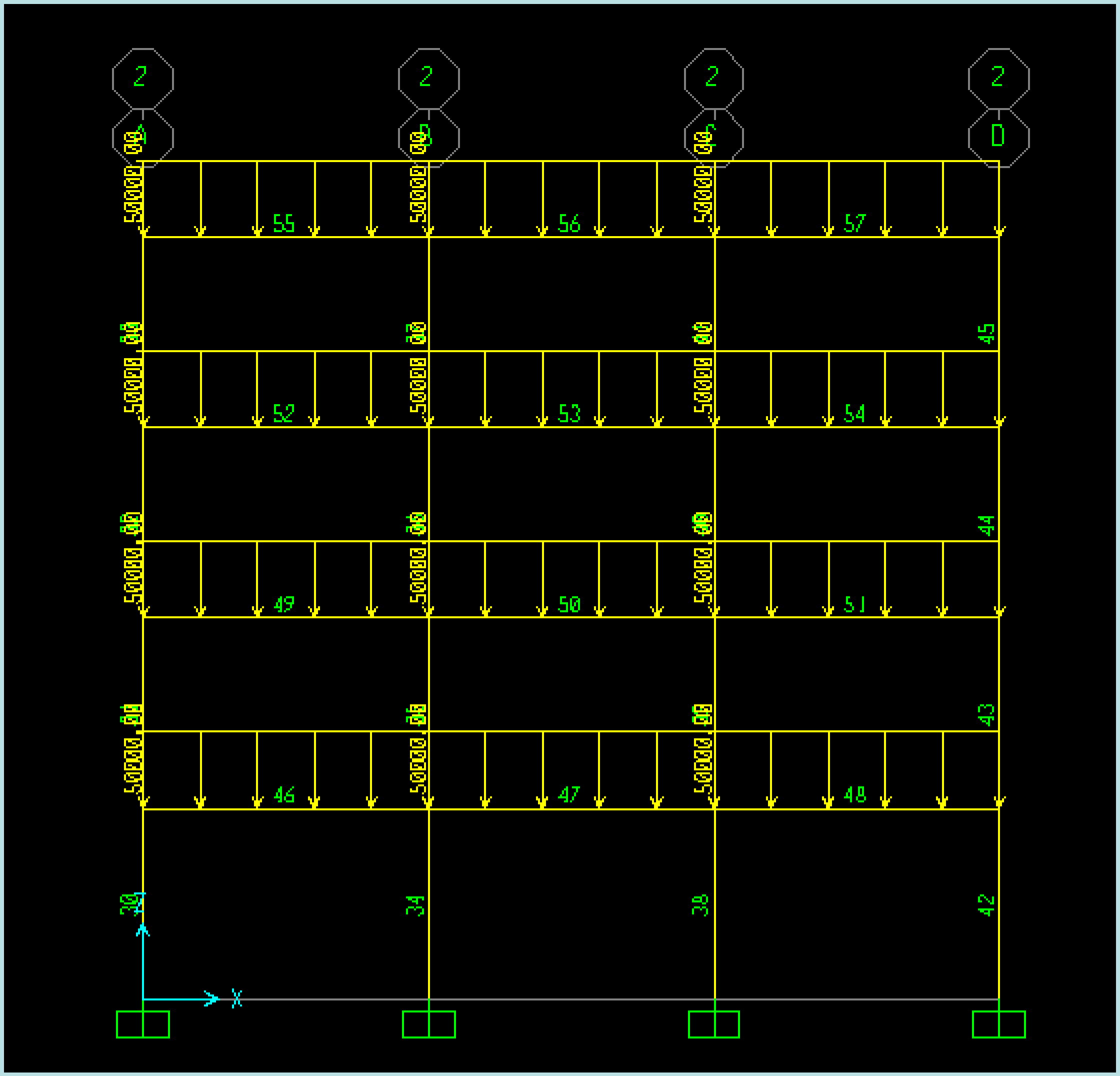
The calculation is performed in a table (EXCEL) for each storey of the steel frame.

Calculation in a table of a distinct value of (α_{cr}) for each storey. The minimum value on all storey is the analysis result

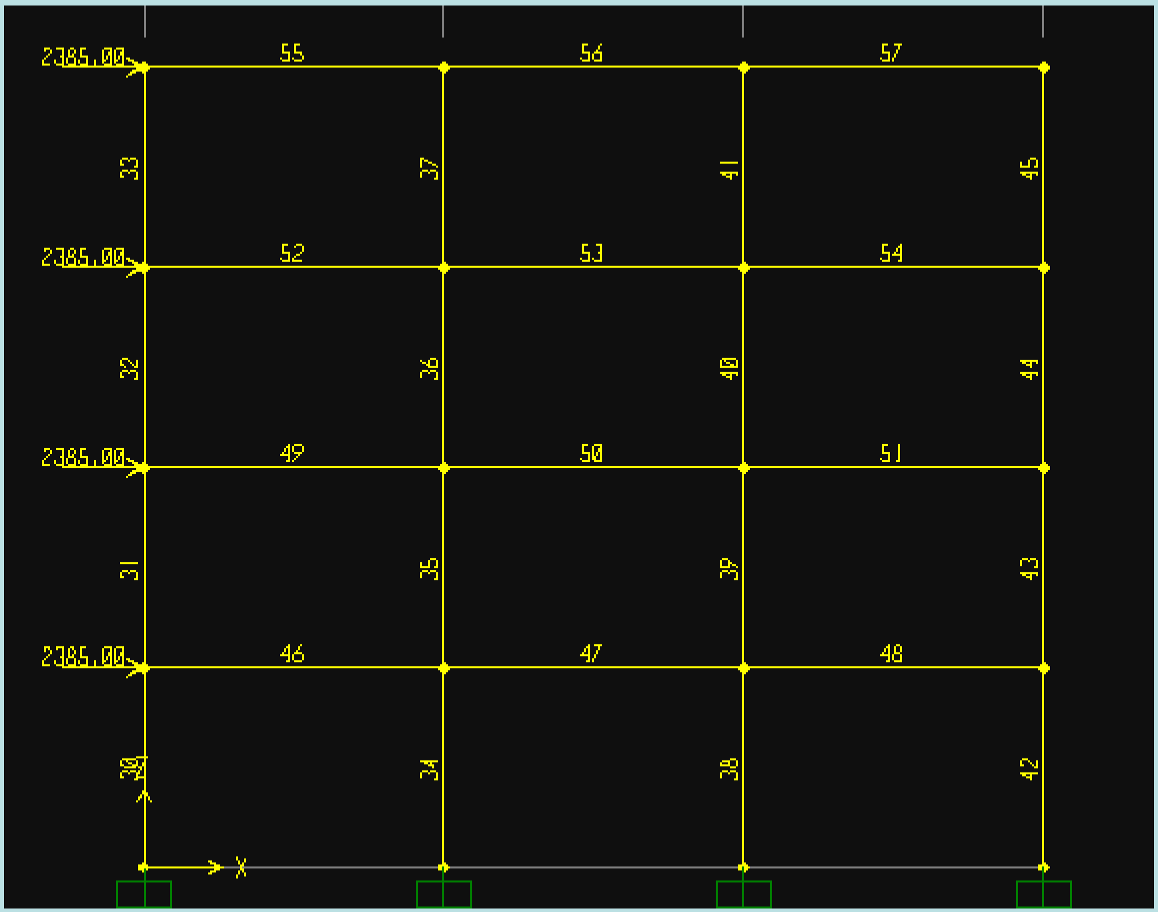
Storey number	Vertical force per level-V1 [N]	Horizontal force per level-H1 [N]	V_{Ed} [N]	H_{Ed} [N]	Level lateral displacement [m]	Delta-relative [m]	H-level [m]	Alpha-crit
1	900000	35385	3600000	141540	0.0247	0.0247	4	6.37
2	900000	35385	2700000	106155	0.046	0.0213	4	7.38
3	900000	35385	1800000	70770	0.0663	0.0203	4	7.75
4	900000	35385	900000	35385	0.0732	0.0069	4	22.79

The following data was used to find the V1 and H1 values of the table:

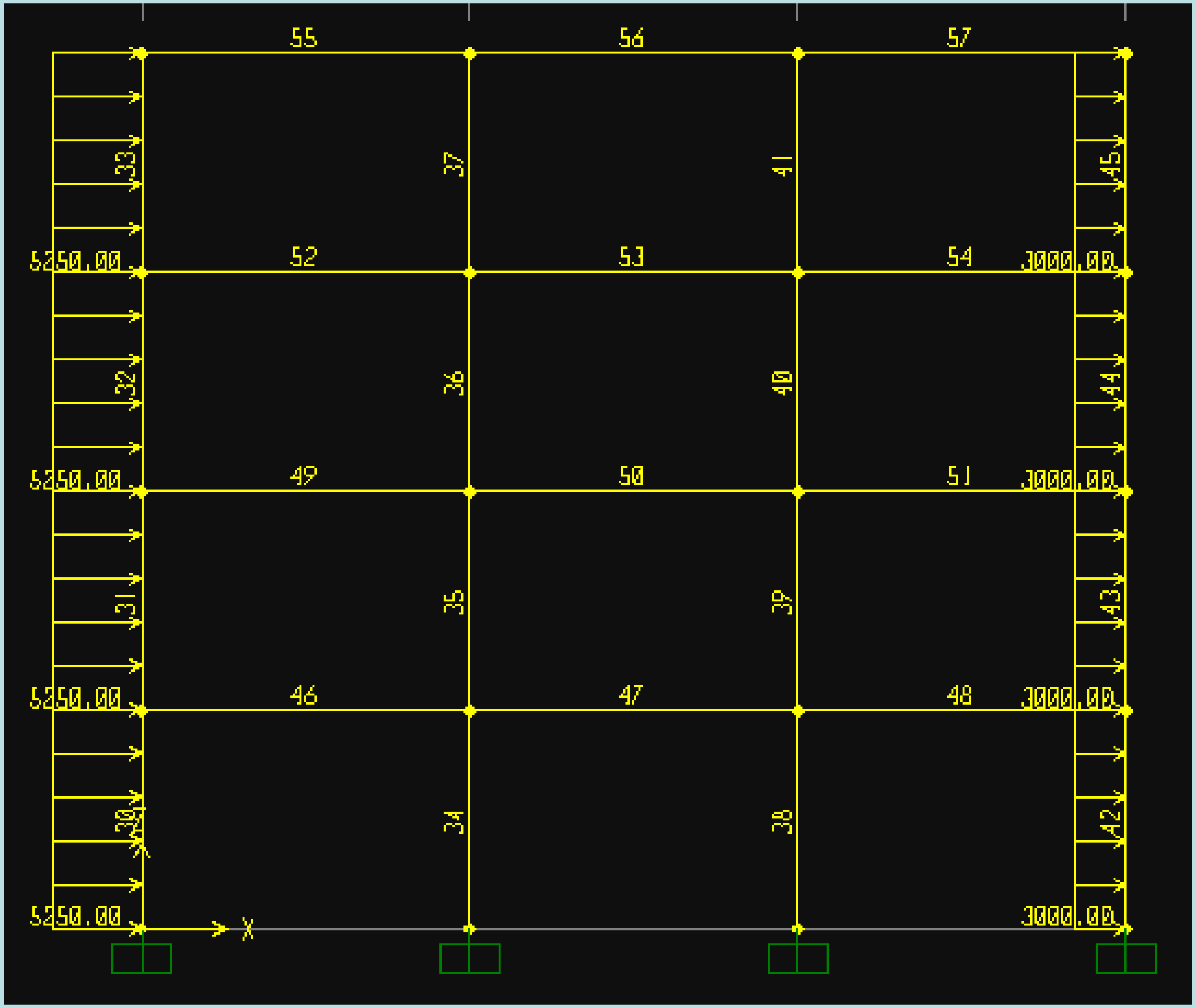
Uniformly distributed load (to calculate V1 value in table):



Equivalent horizontal forces for global sway imperfection (to calculate H1):



Lateral wind pressure -horizontal distributed load (to calculate H1) :



Calculation of the forces per each level in the table (using input data in SAP):

Vertical force per each level:

V1 = 50000N/m x 3 span x 6,0 m = 900000 N

Horizontal force per each level:

H1 = 2385 N +(5250N/m +3000N/m)x4,0m=35385 N

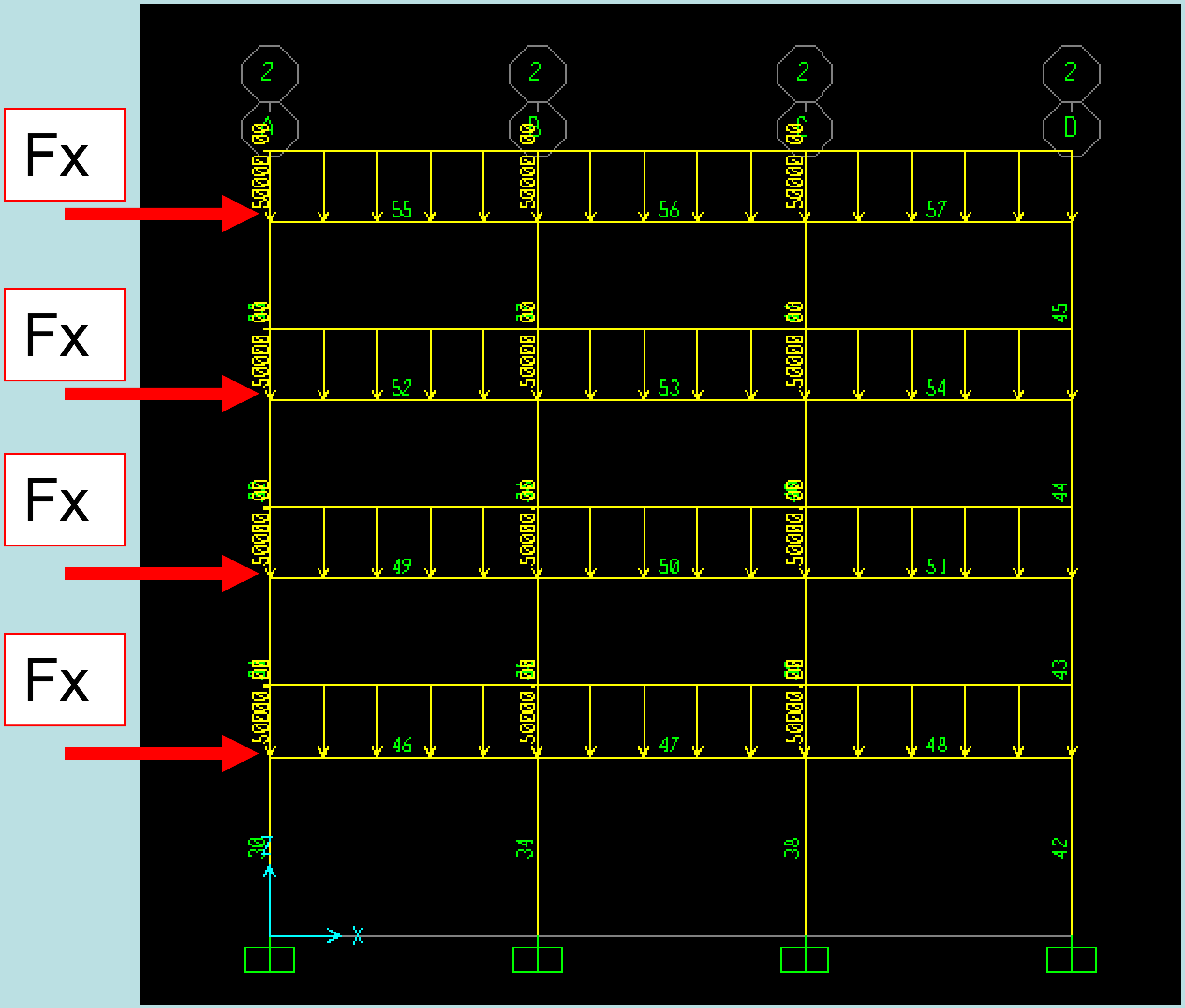
Global Sway Imperf

Lateral wind

Finding the result of the calculation procedure from the table:

$$\alpha_{cr} = \min\{6,37;\ 7,38;\ 7,75;\ 22,79\} = 6,37$$

Computer calculation of (α_{cr}) considering imperfections and second order effects:



The elastic buckling analysis is performed with SAP 2000 Nonlinear computer code, to find (α_{cr}) value

- Type of analysis: plane frame (i.e. analyzed transversal frame);
- Loads: vertical uniformly distributed+ horizontal equivalent loads to global sway imperfection;
- Result: α_{cr}=5,00 (Sway/flexible frame)

Comparison between results obtained by the two methods:

- Approximative formula: α_{cr} **= 6,37 < 10,0**

- Elastic buckling analysis using SAP computer code: α_{cr} **= 5,0 < 10.0**

- CONCLUSION: By both methods the transversal plane frame is a sway (flexible) frame

Since $\alpha_{cr} > 3,0$ the structure has a moderate sensitivity to deformed geometry

CONSEQUENTLY: A first order elastic analysis is allowed for the structure where all the horizontal lods (equivalent horizontal forces of global sway imperfections and wind forces) will be multiplied with the following factor:

$$\mu = \frac{1}{1-\frac{1}{\alpha_{cr}}} = \frac{1}{1-\frac{1}{5,0}} = 1,25$$

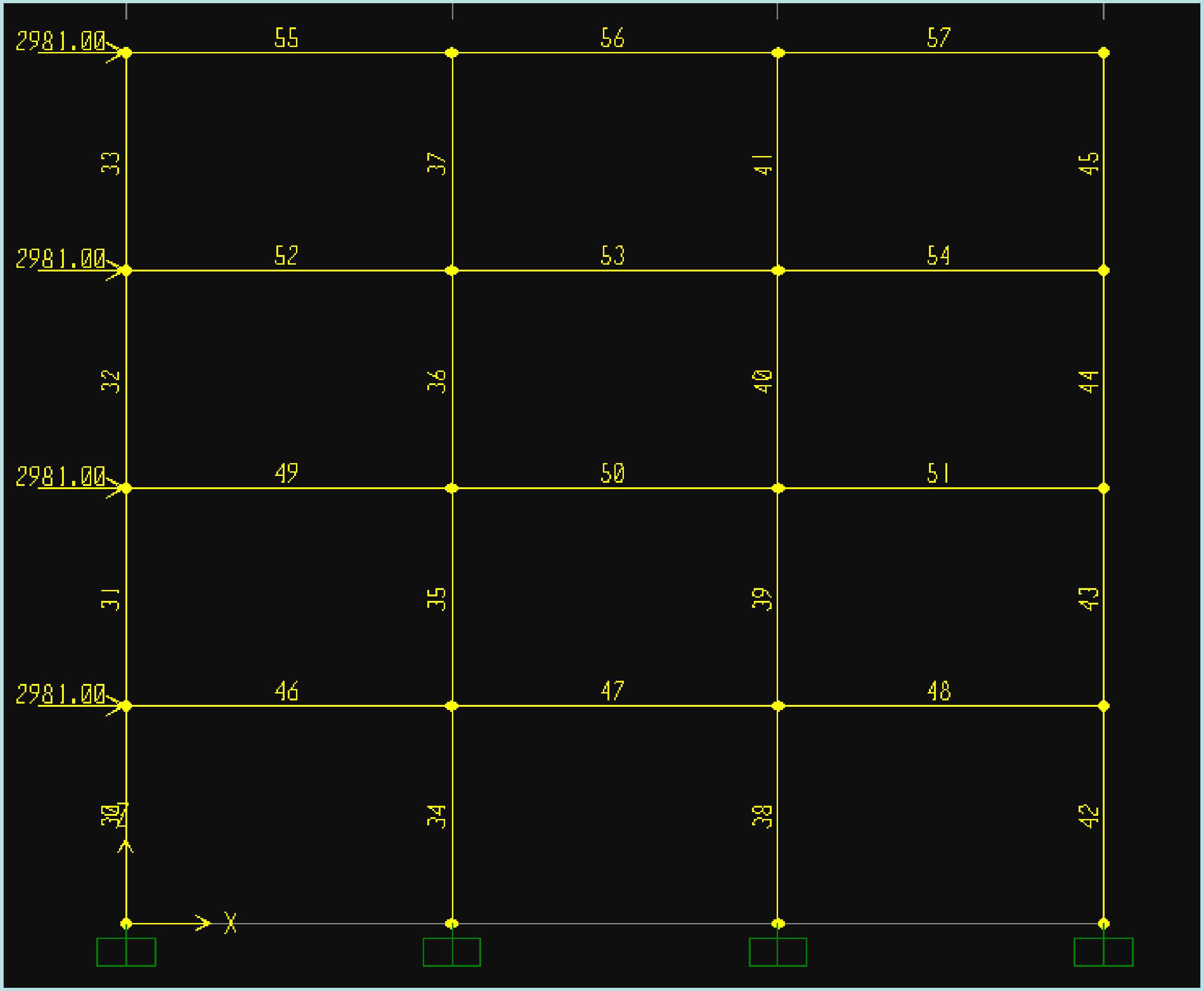
OBSERVATION: Multiplying the horizontal loads with $\mu=1,25>1,0$ will result into an amplification of internal forces obtained from the elastic first order structural analysis (M,T, N) thus taking into account structure increased sensitivity to deformed geometry.

The new values for horizontal loads are:

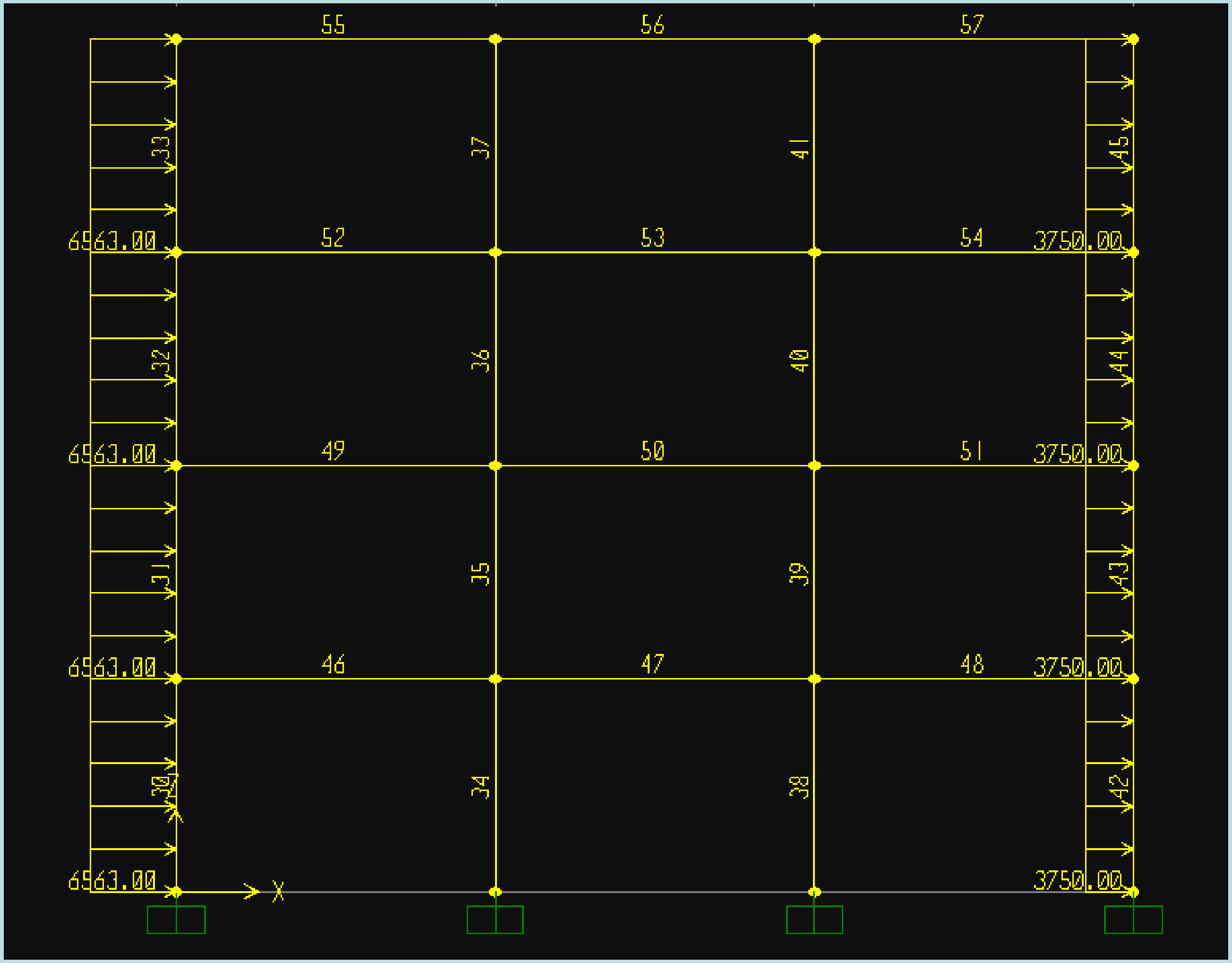
a) Equivalent horizontal forces for global sway imperfection: F_{x1}=1,25 x 2385 N =2981 N

b) Wind distributed loads: w1=5250 N/m x1,25 = 6563 N/m and w2=3000 N/m x 1,25 = 3750 N/m

New equivalent horizontal forces to global sway imperfection of the frame:



New wind load in SAP
(elastic first order analysis!)



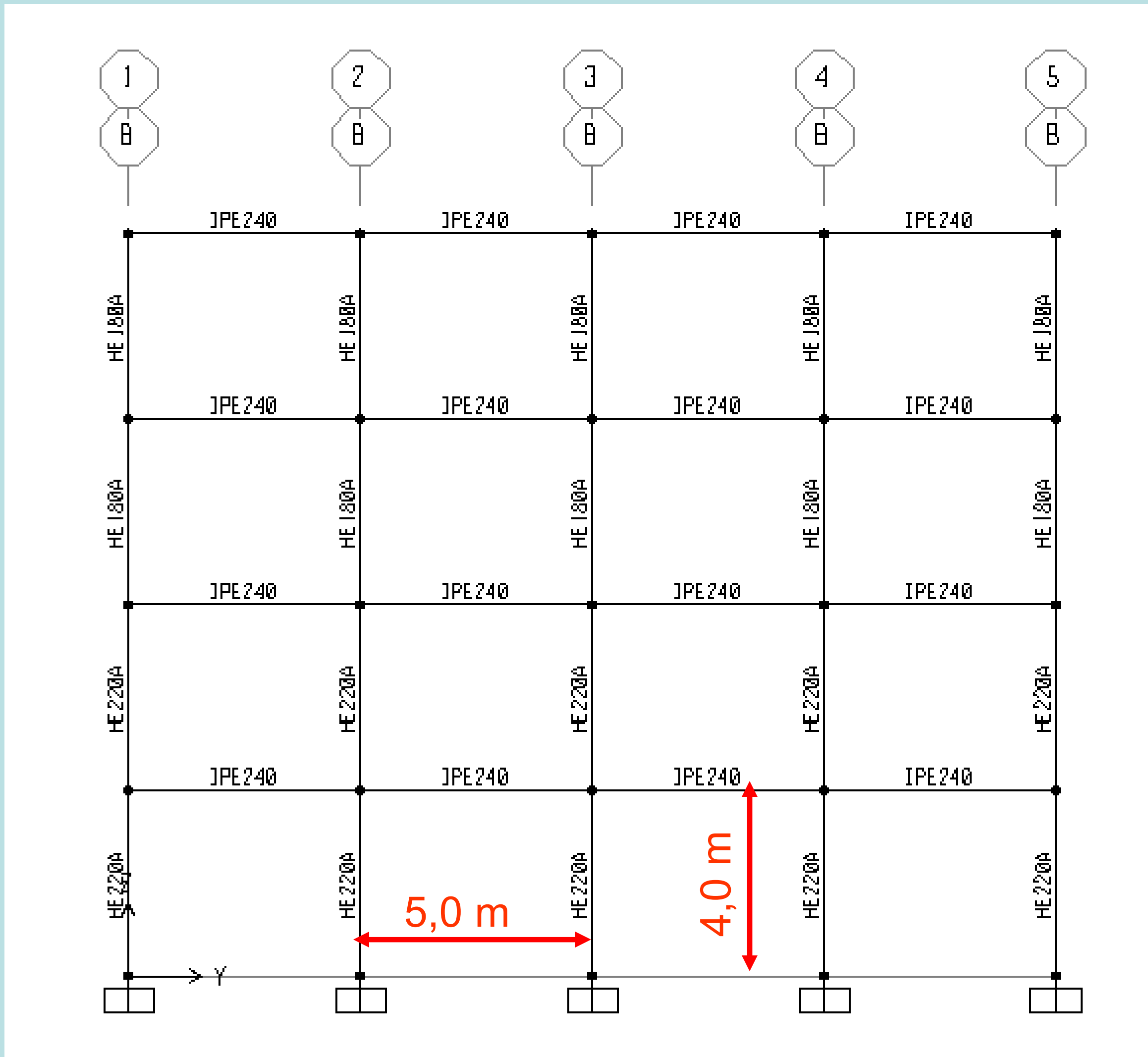
PART 2: Longitudinal frame

⇒To apply approximative formula for (α_{cr}) calculation, the same conditions as before should be checked for the longitudinal frame, also operating with the gyration radius about minimum inertia axis

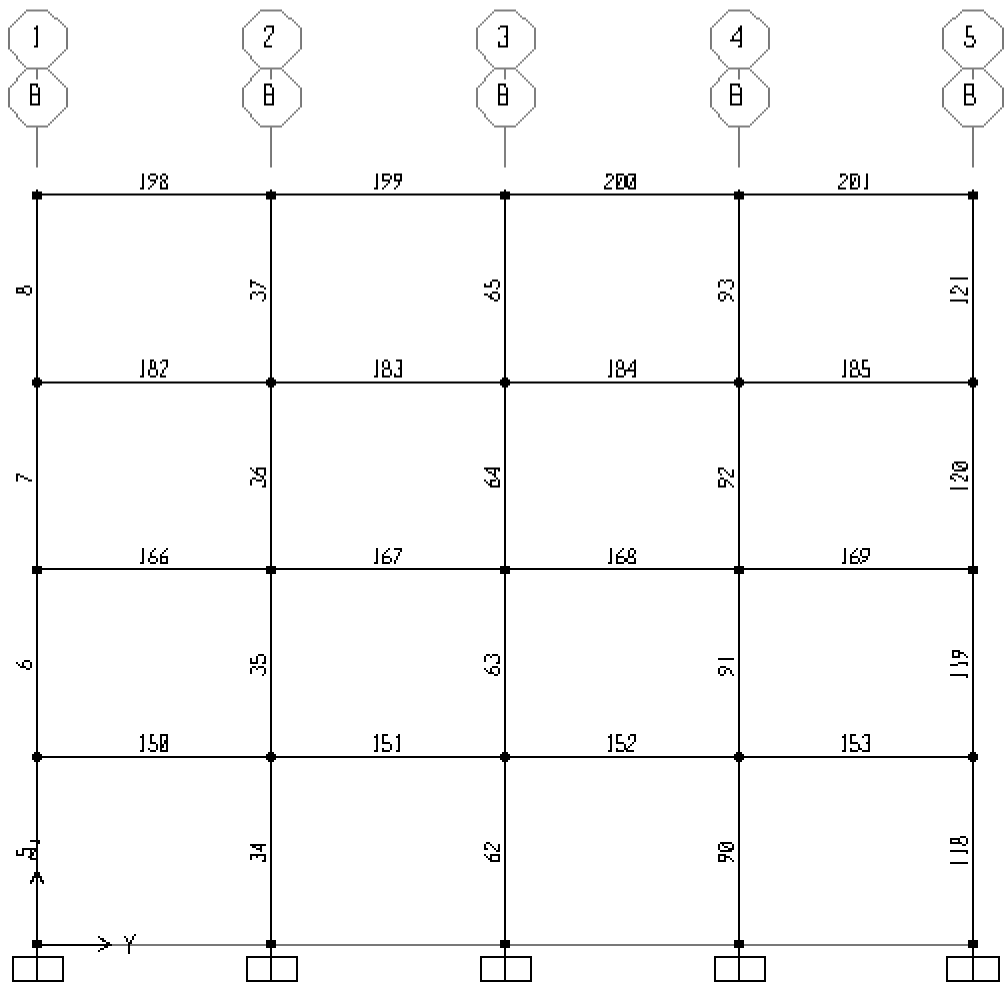
(plus profiles labels and sections accordingly!)

Application of approximative formula will be skipped and only computer analysis will be furtheron used:

Longitudinal frame: profiles and geometry:



Labels of the columns for longitudinal frame (used to find the new values of axial forces)



Calculation of (Φ): global initial sway imperfection for the longitudinal frame

$$\Phi = \Phi_0 \cdot \alpha_h \cdot \alpha_m$$

Where:

$$\Phi_0 = \frac{1}{200}$$

α_h = reduction factor for height (h) applicable to columns;

α_m = reduction factor for the number of columns in a row;

Calculation of factor (α_h)

Height of the structure = 4 storey x 4,0 m = 16,0 m

$$\Rightarrow \alpha_h = \frac{2}{\sqrt{h}} = \frac{2}{\sqrt{16}} = 0,5$$

Code supplementary condition:

$$\frac{2}{3} \leq \alpha_h \leq 1,0$$

Result: α_h = 2/3=0,667

Calculation of factor (α_m):

$$\alpha_m = \sqrt{0{,}5\left(1+\frac{1}{m}\right)}$$

Where m=4 =number of columns in a row (in our case)

$$\alpha_m = \sqrt{0{,}5\left(1+\frac{1}{5}\right)} = 0{,}775$$

Calculation of global initial sway imperfection (Φ)

$$\Phi = \frac{1}{200} \cdot 0{,}667 \cdot 0{,}775 = 0{,}00258\, radians$$

OBSERVATION: The result is a rotation angle measured in radians. This value is not simple to implement in static calculation of structures. Therefore equivalent horizontal forces (F_x) are used

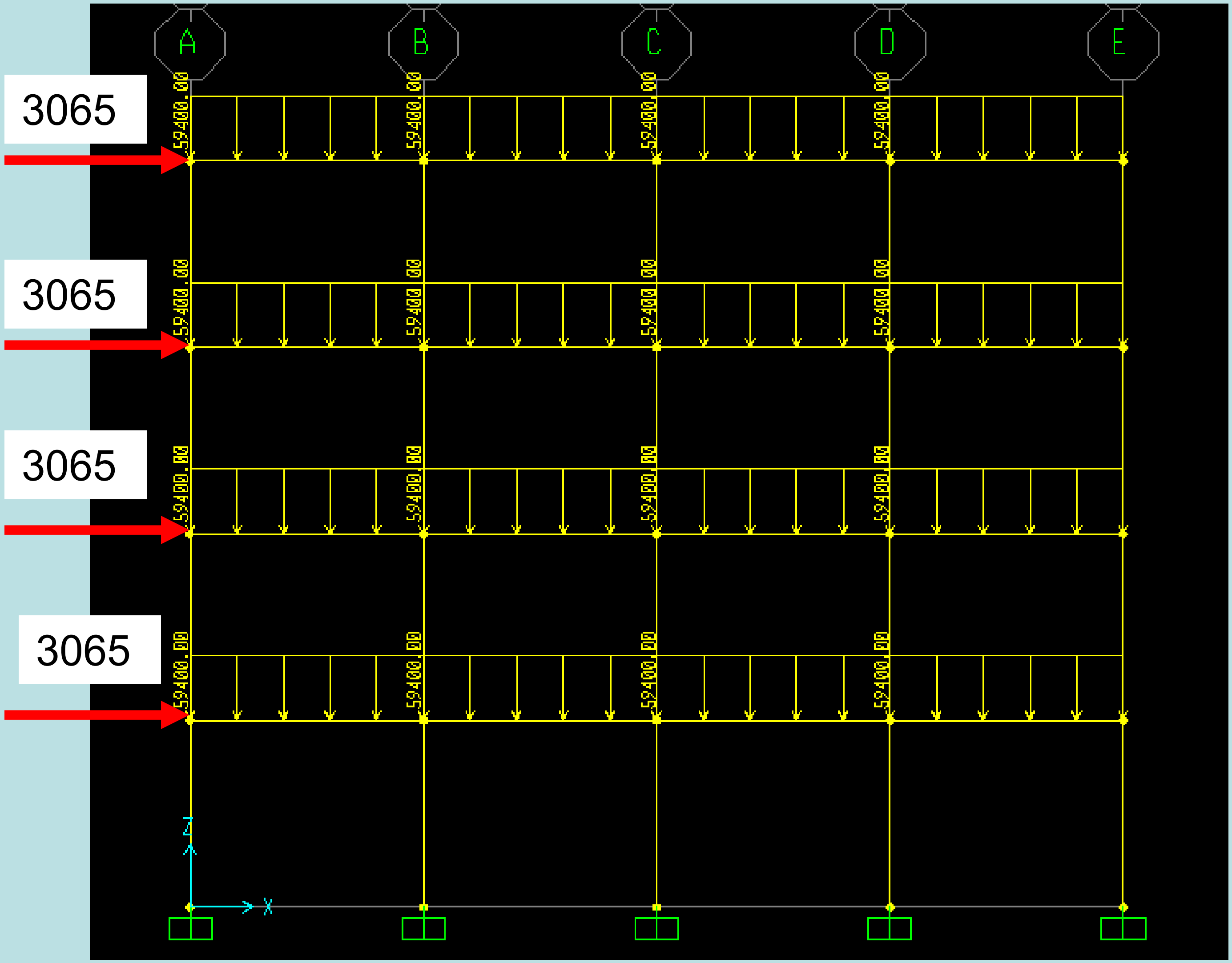
Equivalent horizontal forces shall be applied at each level to produce the same sway (they replace rotation Φ)

$$F_x = V \cdot \Phi$$

V = sum of vertical force at each storey = 4 span x 5,0m x 59400 N/m = 1188000 N

$$\Rightarrow F_x = 0{,}00258 \cdot 1188000 = 3065\ N$$

Longitudinal frame loading for elastic critical buckling analysis:



The elastic buckling analysis is performed with SAP 2000 Nonlinear computer code, to find (α_{cr}) value

- Type of analysis: plane frame (i.e. analyzed longitudinal frame);
- Loads: vertical uniformly distributed + horizontal equivalent loads to global sway imperfection;
- Result: $\alpha_{cr}=2,28 < 3,0$ (Sway/flexible frame)

CONCLUSION:

1) The longitudinal frame is a flexible (sway) structure;

2)The (usual) elastic first order analysis is NOT CORRECT on this frame since the frame is sensitive to deformed geometry;

3) ONLY second order analysis performed by a suitable computer code is correct for this frame

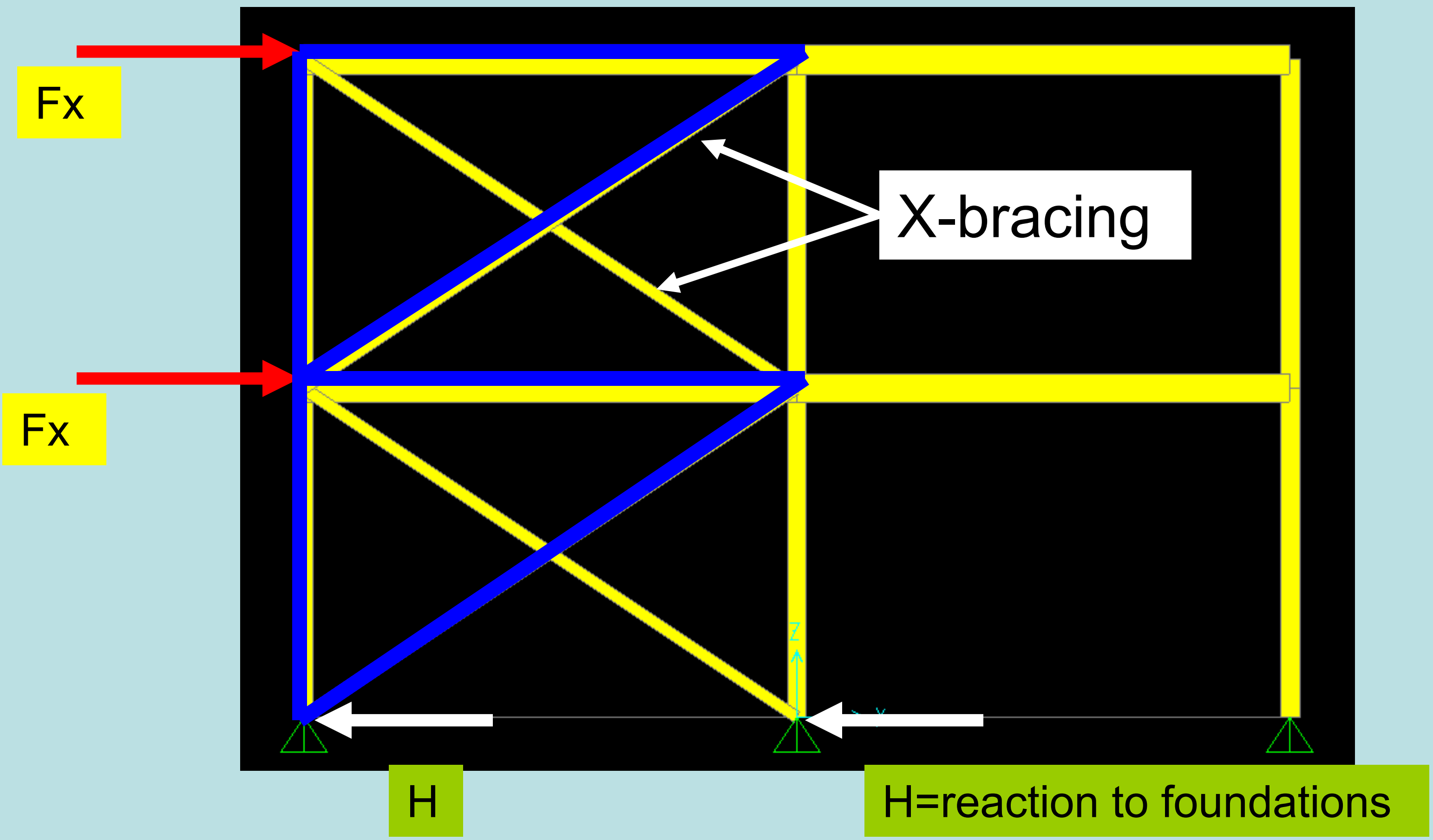
PART 3: Bracing systems. Rigid (non-sway) frames

- Bracing systems provide deformability control on structures;

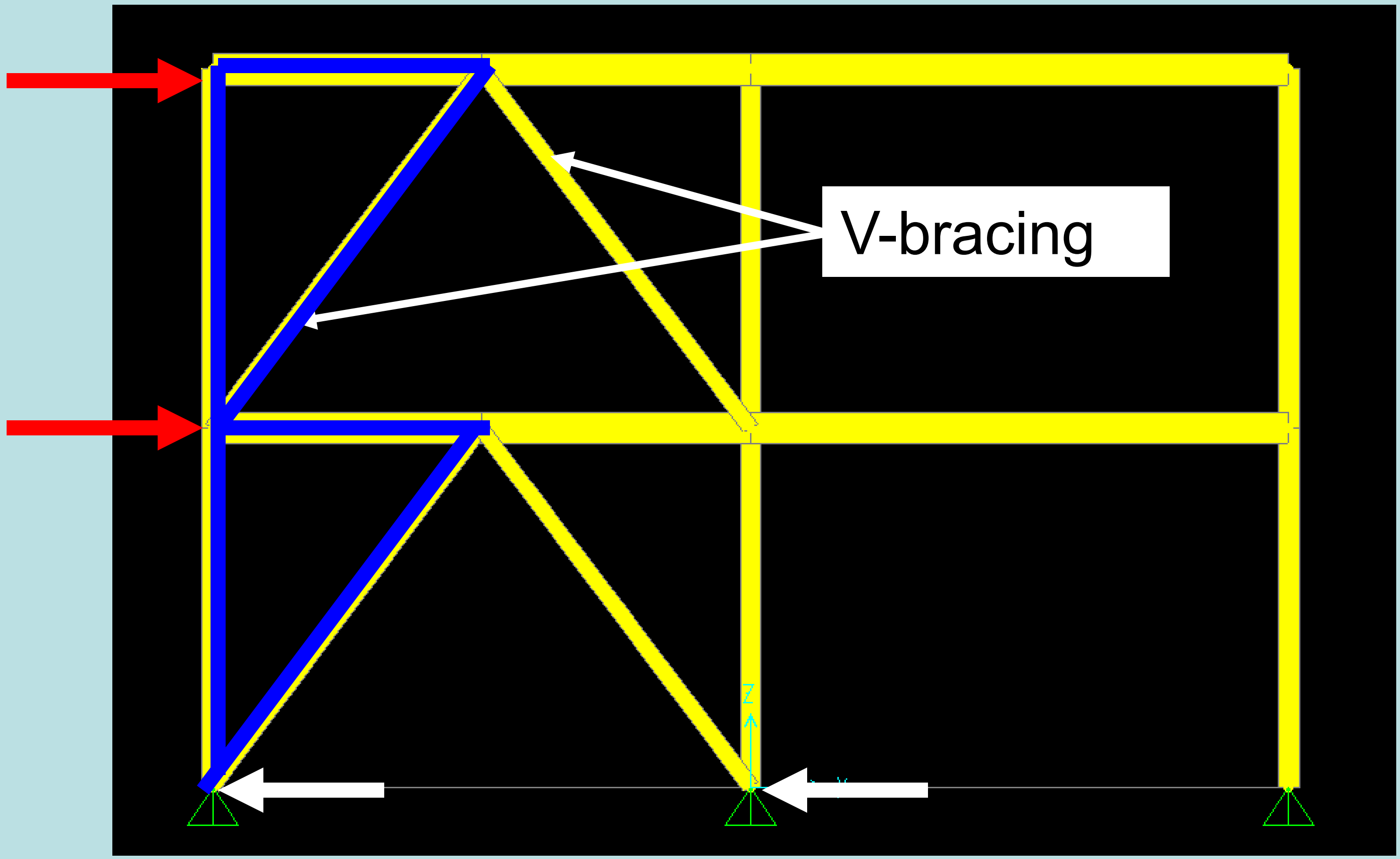
- Most of the bracing systems are based on the principle of the triangle;

- The triangle is an un-deformable geometric figure used to control structural deflection

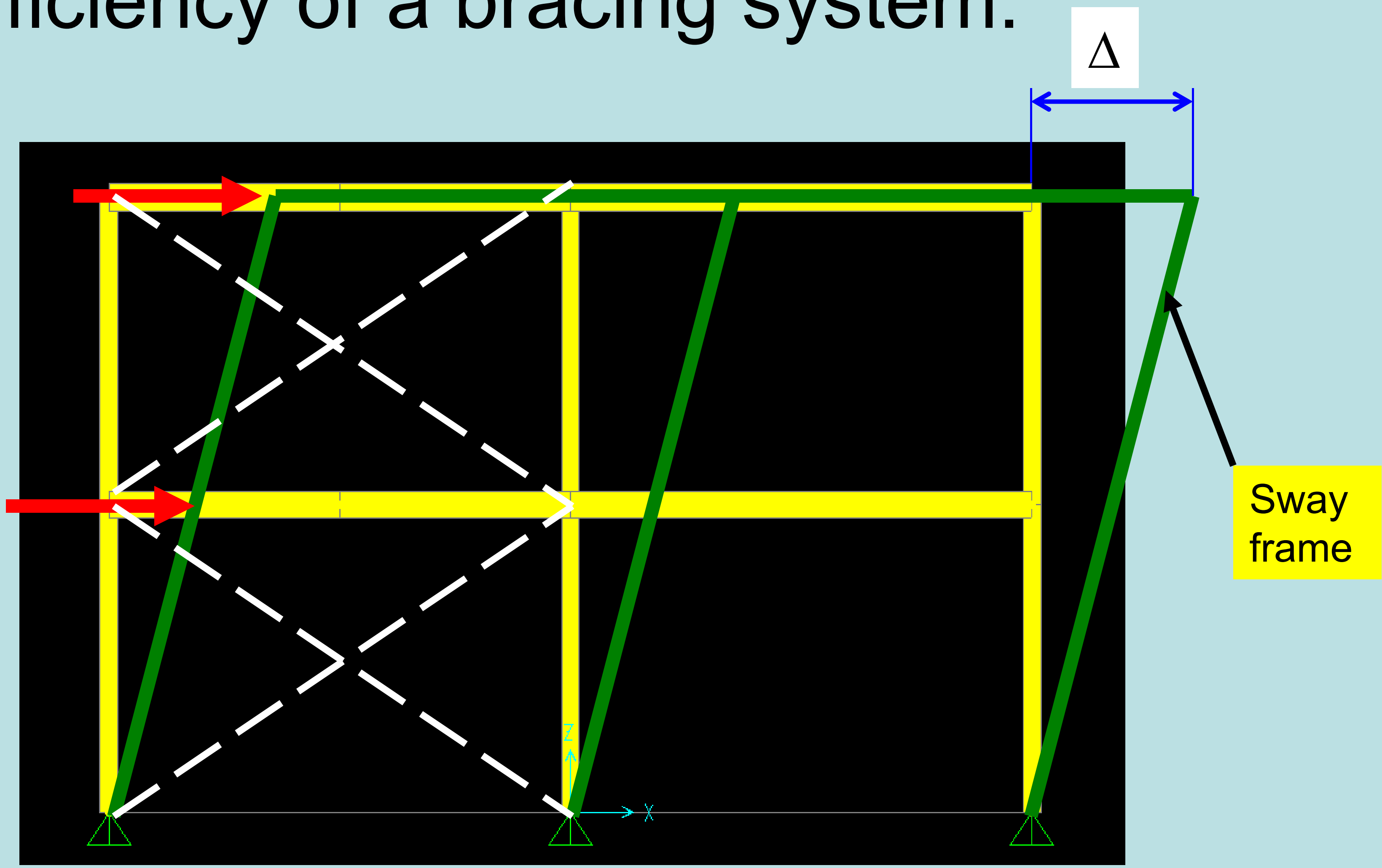
Under horizontal loading Fx, which otherwise induce deformations, bracing systems resist sway and transmit the load to foundations:



Other usual systems of bracing:

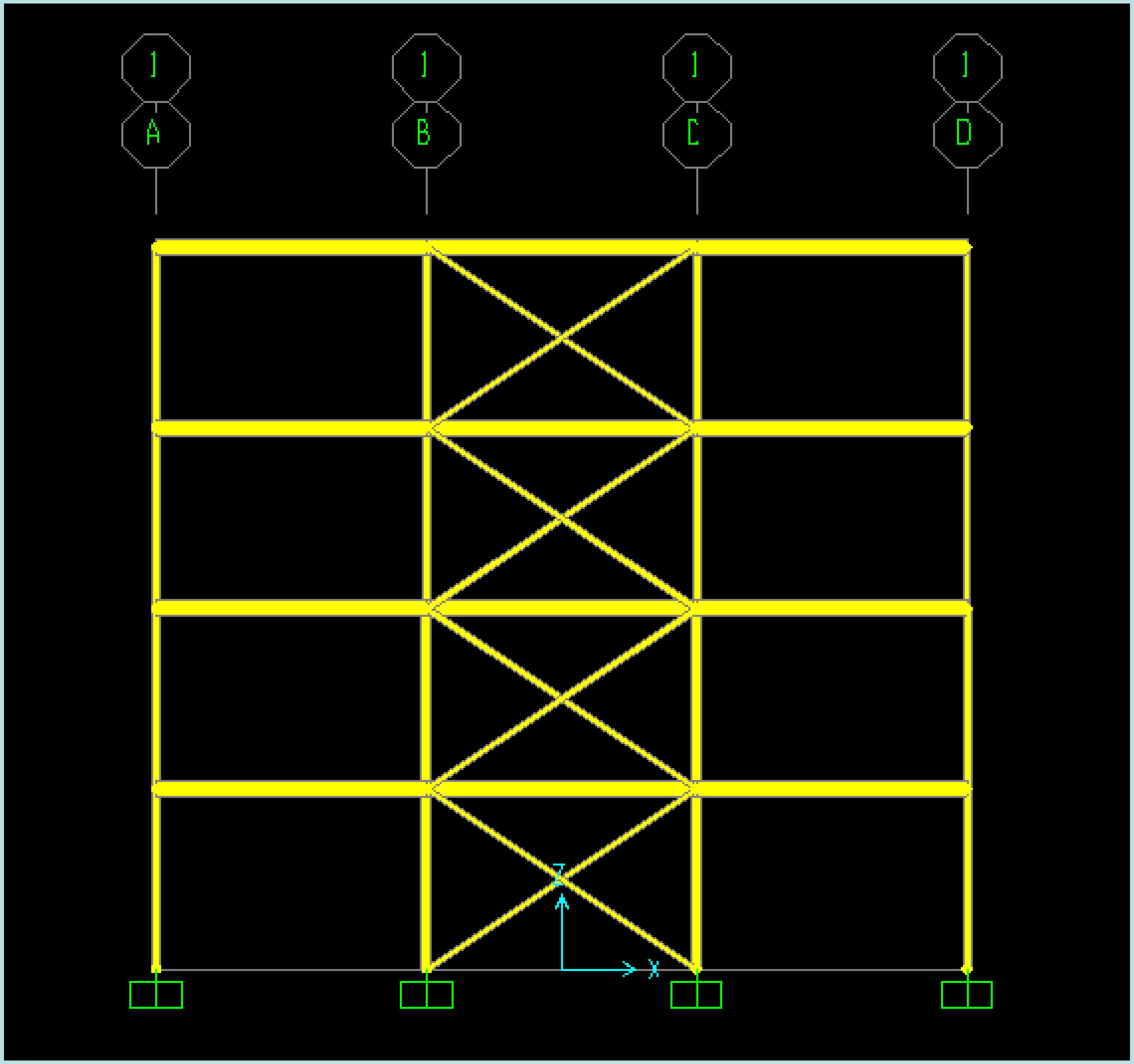


Efficiency of a bracing system:

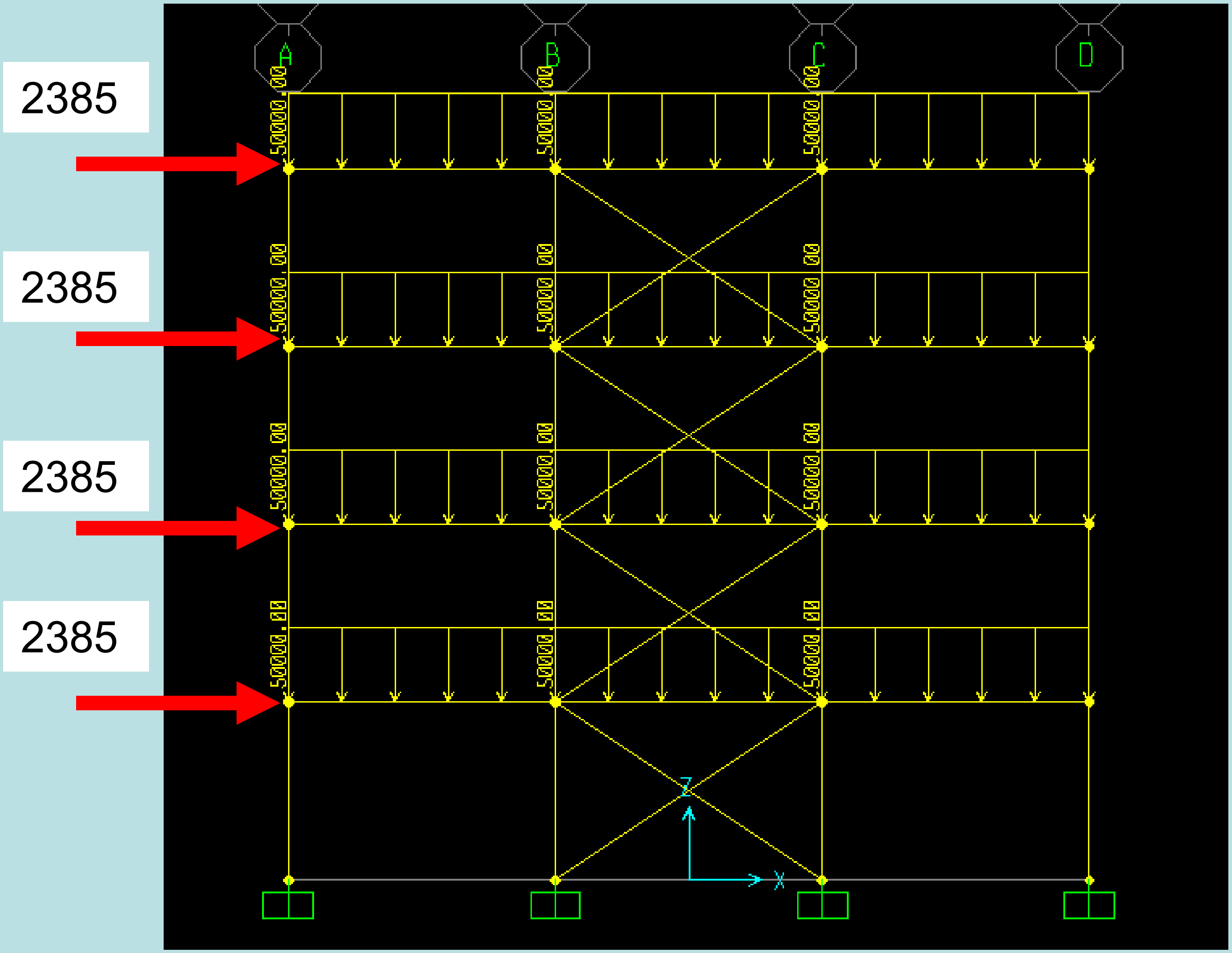


Any type of bracing system is considered efficient if, when applied to a sway frame, it reduces the maximum drift value (Δ) with 80% (i.e. 5 times)

Effect of an X-bracing system on the transverse frame of the application:



Horizontal and vertical loading of transverse frame for elastic buckling analysis:



Efficiency of the bracing system:

- Horizontal deflection (drift) at the top of the frame without bracing (from SAP analysis):

  Δ_1=0,0732 m = 7,32 cm

- Horizontal deflection (drift) at the top of the frame using the bracing system (from SAP analysis):

  Δ_2=0,0088 m = 0,88 cm

- Ratio between the two drift values:

$$\frac{\Delta_2}{\Delta_1} = \frac{0,88}{7,32} = 0,12 < \frac{1}{5} = 0,20$$

Bracing system OK !

The elastic buckling analysis is performed with SAP 2000 Nonlinear computer code, to find (α_{cr}) value

- Type of analysis: plane frame (i.e. analyzed braced transverse frame);
- Loads: vertical uniformly distributed + horizontal equivalent loads to global sway imperfection;
- Result: $\alpha_{cr}=7{,}81 < 10$ (Sway/flexible frame)

Since α_{cr} > 3,0 the structure has a moderate sensitivity to deformed geometry

CONSEQUENTLY: A first order elastic analysis is allowed for the structure where all the horizontal lods (equivalent horizontal forces to global sway imperfections and wind forces) will be multiplied with the following factor:

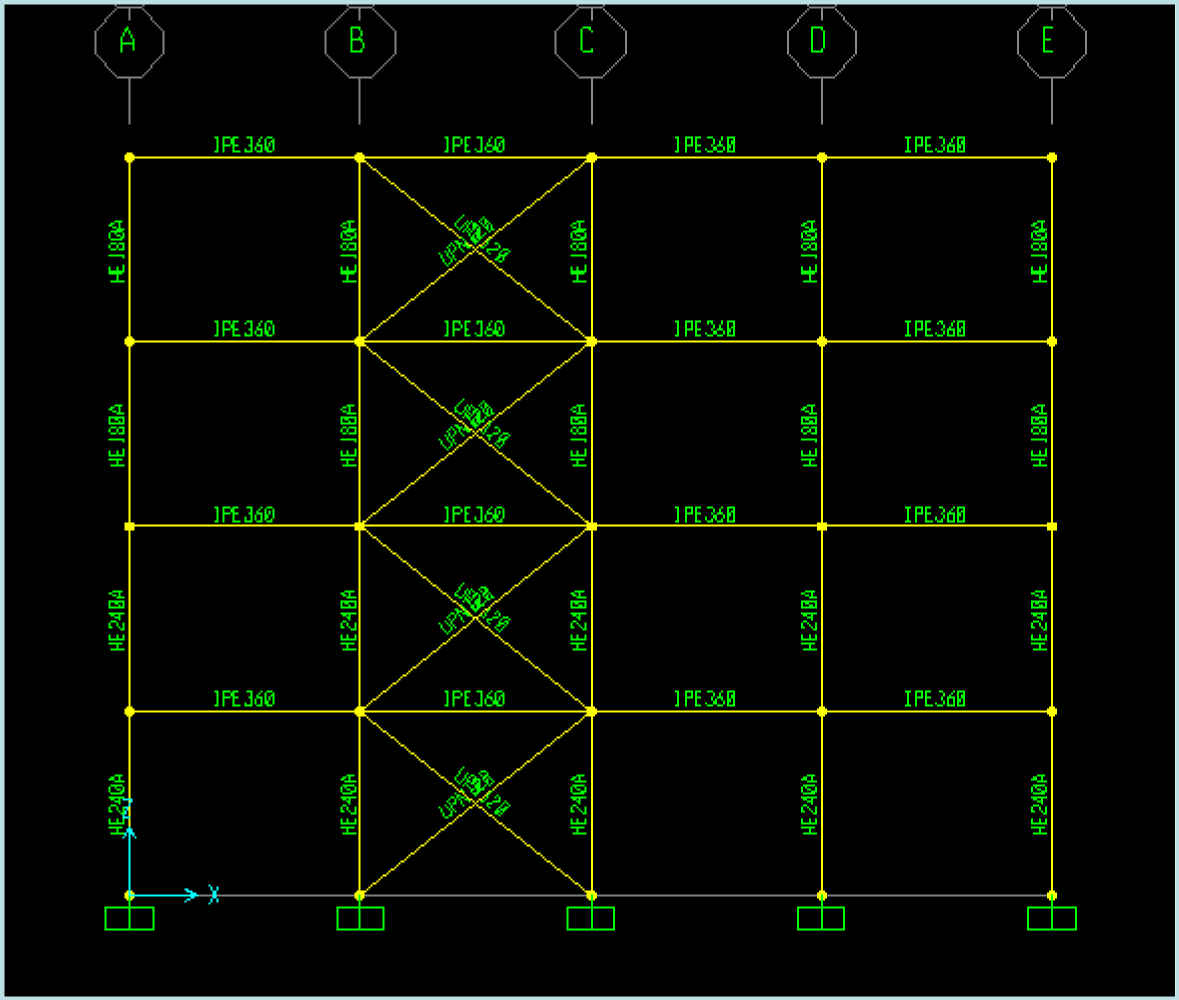
$$\mu = \frac{1}{1 - \frac{1}{\alpha_{cr}}} = \frac{1}{1 - \frac{1}{7{,}81}} = 1{,}147$$

OBSERVATION:

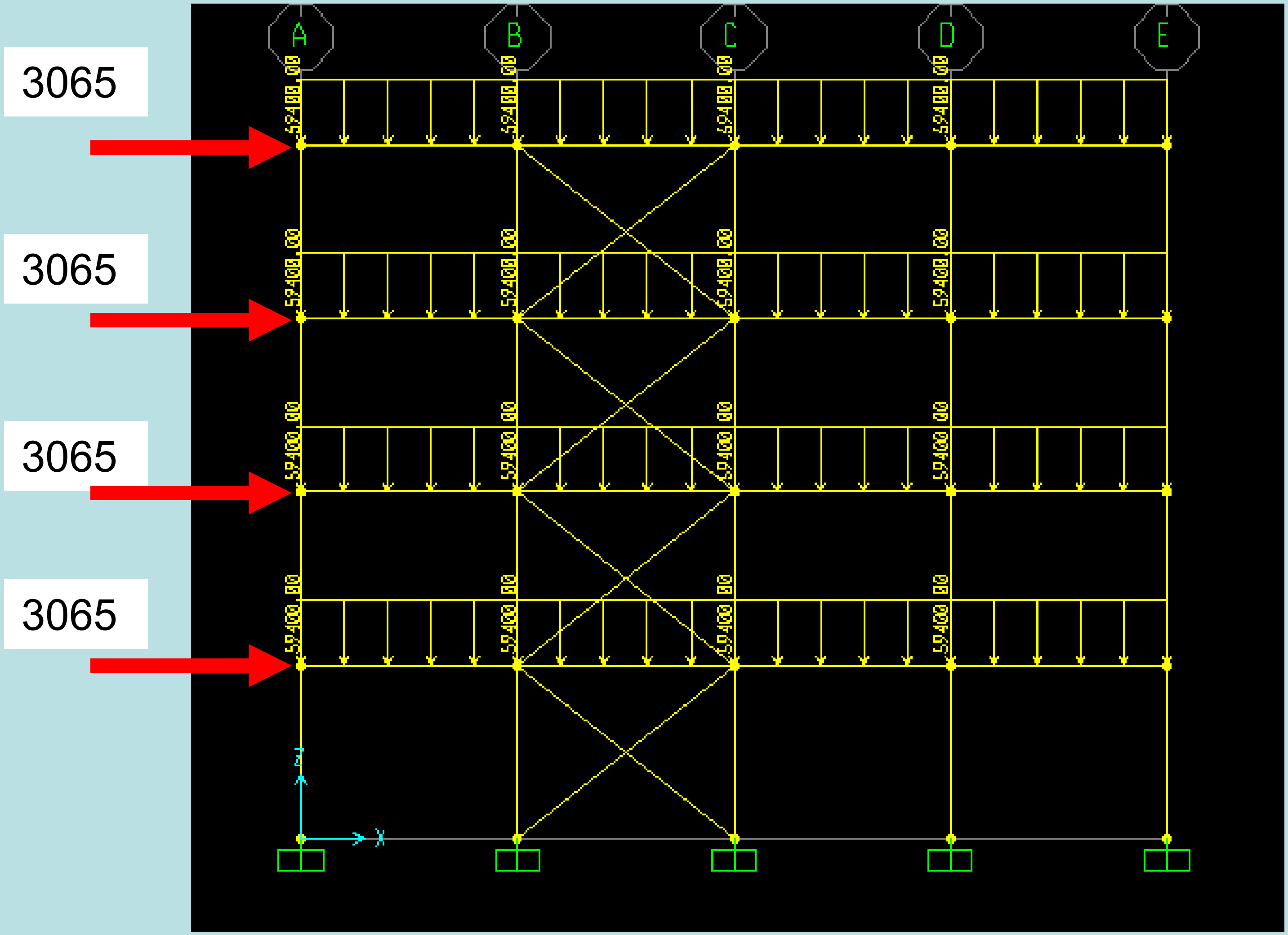
By introducing a bracing system into the frame, the value of (α_{cr}) has increased, showing less sensitivity to deformed geometry

$$\alpha_{cr}^{brace} = 7{,}81 > \alpha_{cr}^{sway} = 5{,}0$$

Effect of an X-bracing system on the longitudinal frame of the application:



Horizontal and vertical loading of transverse frame for SAP elastic buckling analysis:



The elastic buckling analysis is performed with SAP 2000 Nonlinear computer code, to find (α_{cr}) value

- Type of analysis: plane frame (i.e. analyzed braced longitudinal frame);
- Loads: vertical uniformly distributed + horizontal equivalent loads to global sway imperfection;
- Result: $\alpha_{cr}=6,71 < 10$ (Sway/flexible frame)

Since α_{cr} > 3,0 the structure has a moderate sensitivity to deformed geometry

CONSEQUENTLY: A first order elastic analysis is allowed for the structure where all the horizontal lods (equivalent horizontal forces to global sway imperfections and wind forces) will be multiplied with the following factor:

$$\mu = \frac{1}{1 - \frac{1}{\alpha_{cr}}} = \frac{1}{1 - \frac{1}{6,71}} = 1,175$$

OBSERVATION:

By introducing a bracing system into the frame, the value of (α_{cr}) has increased, showing less sensitivity to deformed geometry

$$\alpha_{cr}^{brace} = 6,71 > \alpha_{cr}^{sway} = 2,82$$

In this particular case the frame has changed category from highly sensitive to deformed geometry (requiring a second order analysis) to moderate sensitive, allowing for an elastic first order analysis via horizontal load multiplication with factor μ=1,175