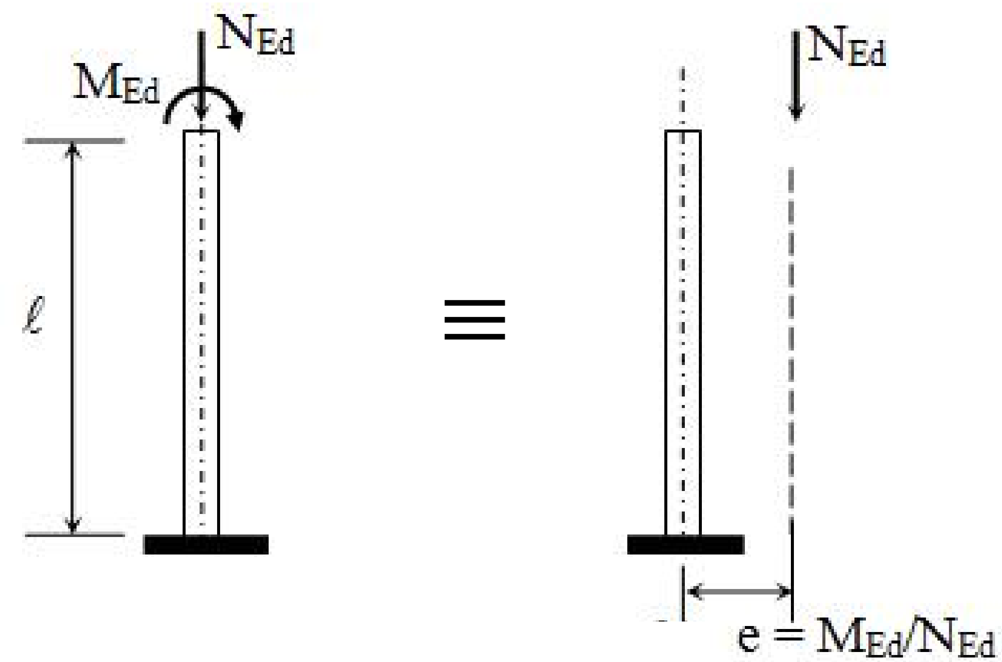
8. COLUMNS

COLUMN = ELEMENT SUBJECTED TO:

BENDING MOMENT
&
COMPRESSIVE FORCE

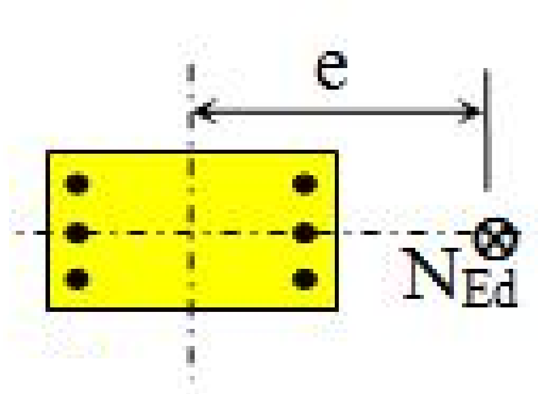
ECCENTRIC
COMPRESSIVE FORCE

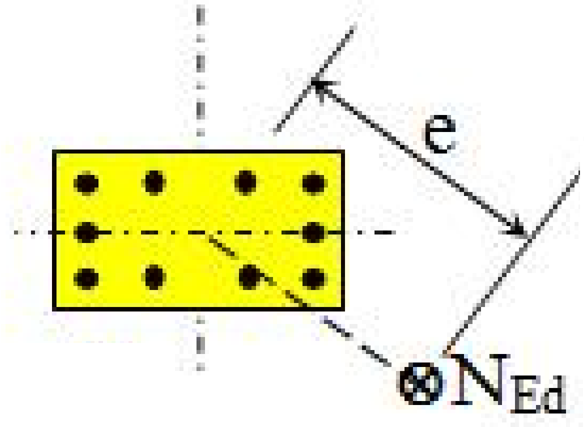


8. COLUMNS

RECTANGULAR SECTION

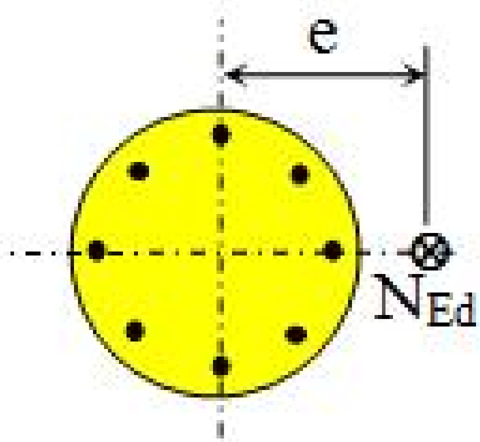
Eccentric compression



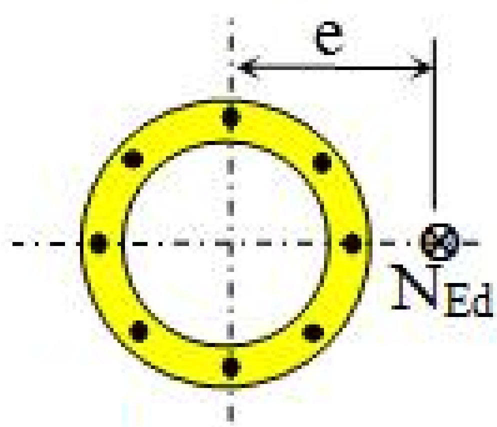


Compression with biaxial bending

CIRCULAR SECTION



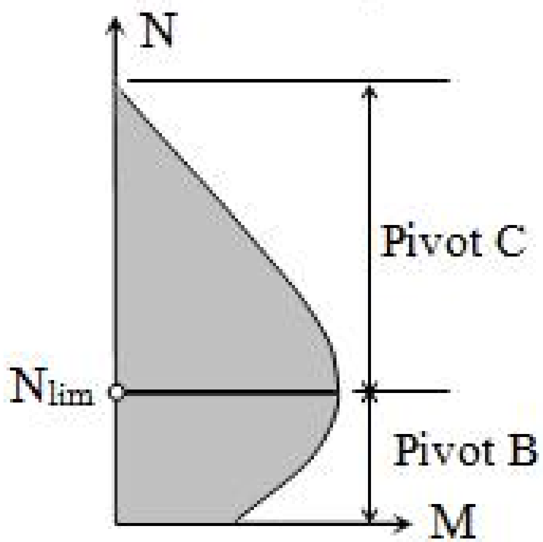
RING-SHAPED SECTION



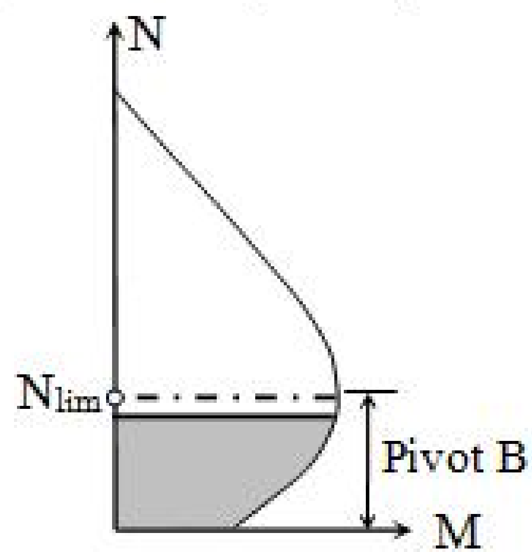
8. COLUMNS

DESIGN SITUATIONS

PERSISTENT



SEISMIC



Ductility class DCH: $N_{Ed} \leq 0{,}40 A_c f_{cd} < N_{lim}$ (P100-1/2006); $N_{Ed} \leq 0{,}45 A_c f_{cd} < N_{lim}$ (P100-1/2013)

Ductility class DCM: $N_{Ed} \leq 0{,}55 A_c f_{cd} \cong N_{lim}$ (P100-1/2006); $N_{Ed} \leq 0{,}50 A_c f_{cd} \cong N_{lim}$ (P100-1/2013)

Ductility class DCL: only in areas with $a_g \leq 0{,}10g$

P100-1/2006

P100-1/2013

8. COLUMNS

COLUMNS + GIRDERS = **FRAME**

↓

SENSITIVE TO LATERAL DISPLACEMENT

↓

HIGH VALUES OF THE BENDING MOMENTS
IN COLUMNS AND GIRDERSS

8. COLUMNS

BRACING SYSTEMS ARE USED IN ORDER TO
REDUCE THE LATERAL DISPLACEMENT

AS A RESULT OF THE ABOVE:

- BRACED COLUMNS
- UN-BRACED COLUMNS

8. COLUMNS

THE ENDS OF THE COLUMNS CAN HAVE DIFFERENT TYPES OF CONNECTIONS WITH NEIGHBORING ELEMENTS:

- RESTRAINED DISPLACEMENTS & ROTATIONS (AS FOUNDATIONS)
- PARTIALLY FREE DISPLACEMENTS & ROTATIONS DEPENDING ON:
  - stiffness of neighboring elements
  - with or without bracings
- FREE DISPLACEMENTS & ROTATIONS

8. COLUMNS

DEFINITIONS

First order effects - M_{0Ed}: action effects calculated without consideration of the effect of structural deformations, but including geometric imperfections

Second order effects - aM: additional action effects caused by structural deformations

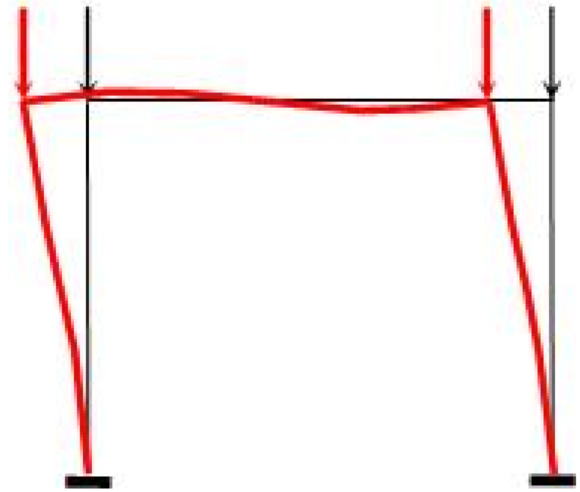
Second order moment - M_{Ed} = ¦ M_{0Ed} (¦ > 1,0) : bending moment, taking into account the influence of structural deformations

8. COLUMNS

The second order effects are produced by two types of deformations:

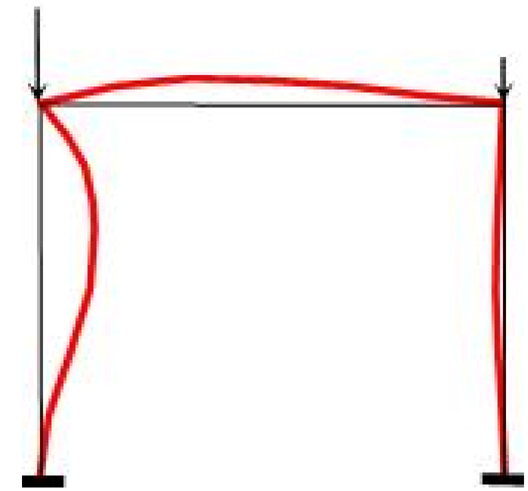
Lateral deformations of the story:

- depends on the structural stiffness,
- characteristic for unbraced structures



Individual deformations of the element:

- depends on slenderness & neighboring elements
- characteristic for braced structures
- may cause buckling



8. COLUMNS

Buckling: failure due to instability of a member or structure under perfectly axial compression and without transverse load

Buckling load: the load at which buckling occurs; for isolated elastic members it is synonymous with the Euler load

Effective length: a length used to account for the shape of the deflection curve; it can also be defined as **buckling length**.

Isolated members: members that are isolated, or members in a structure that for design purposes may be treated as being isolated

8. COLUMNS

8.1. GEOMETRIC IMPERFECTIONS

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

8.5. CIRCULAR/RING-SHAPED COLUMNS

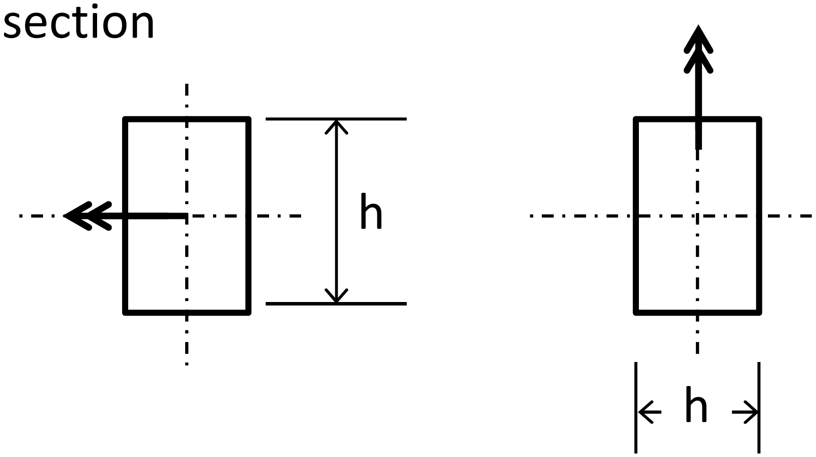
8.6. DETAILING OF COLUMNS

8.1. GEOMETRIC IMPERFECTIONS

The unfavorable effects of possible deviations shall be taken into account in the analysis of members and structures.

Deviations:
- cross section dimensions
- geometry of the structure
- position of loads

Deviations in cross section dimensions:
- are normally taken into account in the material safety factors
- these should not be included in structural analysis
- for cross section design it is necessary to assume the minimum eccentricity, e_0 = h/30 but not less than 20 mm where h is the depth of the section



8.1. GEOMETRIC IMPERFECTIONS

Deviations in the geometry of the structure:

- shall be taken into account in ultimate limit states in:
  - persistent design situations
  - accidental design situations

- need not be considered for serviceability limit states

8.1. GEOMETRIC IMPERFECTIONS

IMPERFECTIONS MAY BE REPRESENTED BY AN INCLINATION

$$\theta_i = \theta_0 \alpha_h \alpha_m$$

$\theta_0 = 1/200$ - basic value

α_h is the reduction factor for length or height: $\alpha_h = 2/\sqrt{l}$; $2/3 \le \alpha_h \le 1$

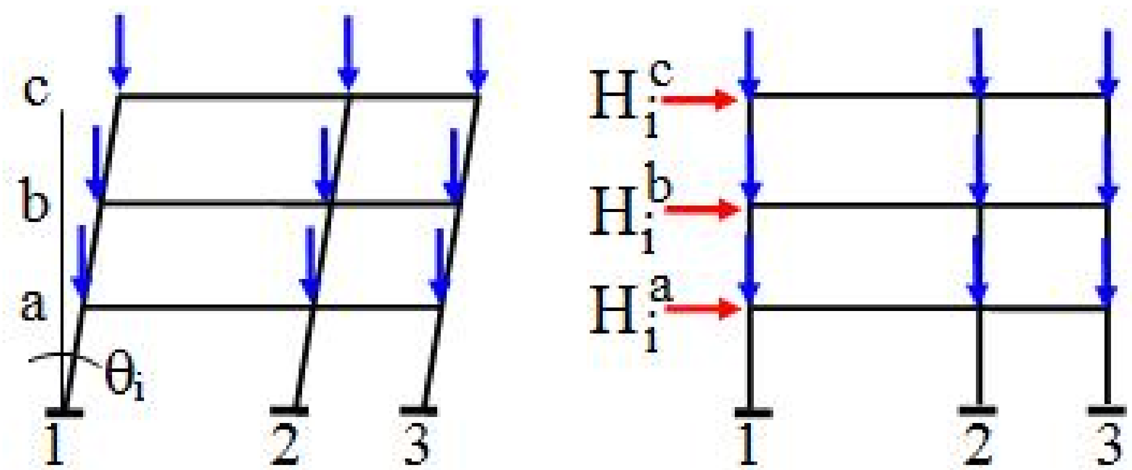
α_m is the reduction factor for number of members: $\alpha_m = \sqrt{0{,}5(1+1/m)}$

l is the length or height [m], see (4)

m is the number of vertical members contributing to the total effect

8.1. GEOMETRIC IMPERFECTIONS

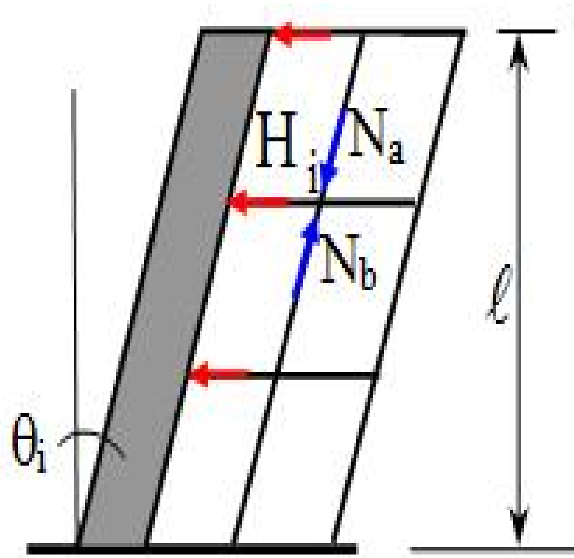
UNBRACED STRUCTURE



$$H_i = \theta_i \sum F$$

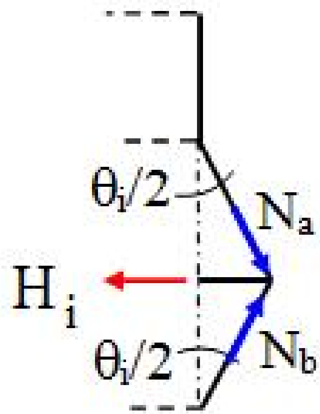
8.1. GEOMETRIC IMPERFECTIONS

BRACED STRUCTURE



$$H_i = \theta_i \left(\Sigma N_b - \Sigma N_a \right)$$

ACTION ON FLOOR



$$H_i = \theta_i \left(\Sigma N_b + \Sigma N_a \right)/2$$

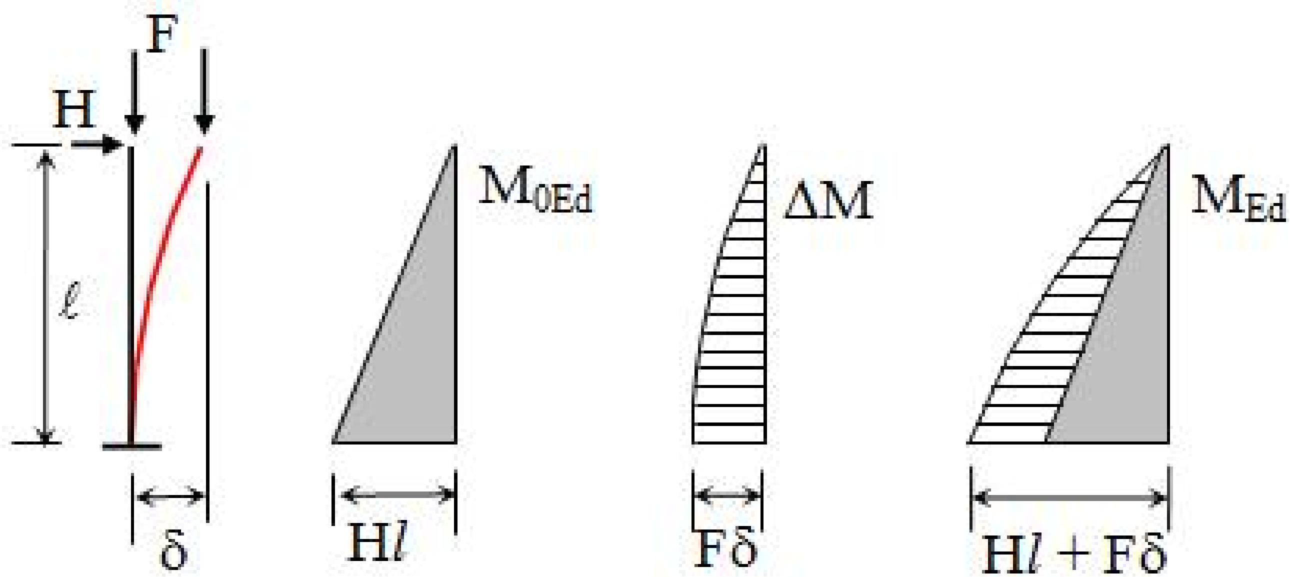
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.1. TOPIC OF SECOND ORDER EFFECTS

First order effects - M_{0Ed}: action effects calculated without consideration of the effect of structural deformations, but including geometric imperfections

Second order effects - aM: additional action effects caused by structural deformations

Second order moment - M_{Ed} = ¦ M_{0Ed} (¦ > 1,0) : bending moment, taking into account the influence of structural deformations



8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

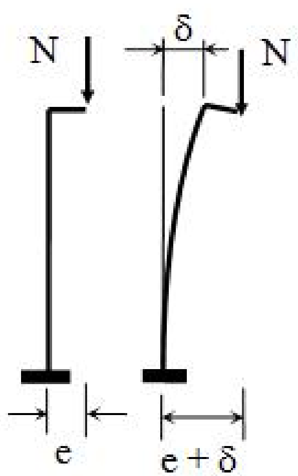
ELEMENT SENSITIVITY TO SECOND ORDER EFFECTS DEPENDS ON SLENDERNESS RATIO

$$\lambda = \frac{\ell_0}{i}$$

ℓ_0 - effective length

i – radius of gyration

THERE ARE 3 CASES OF COLUMN FAILURE DEPENDING ON SLENDERNESS RATIO



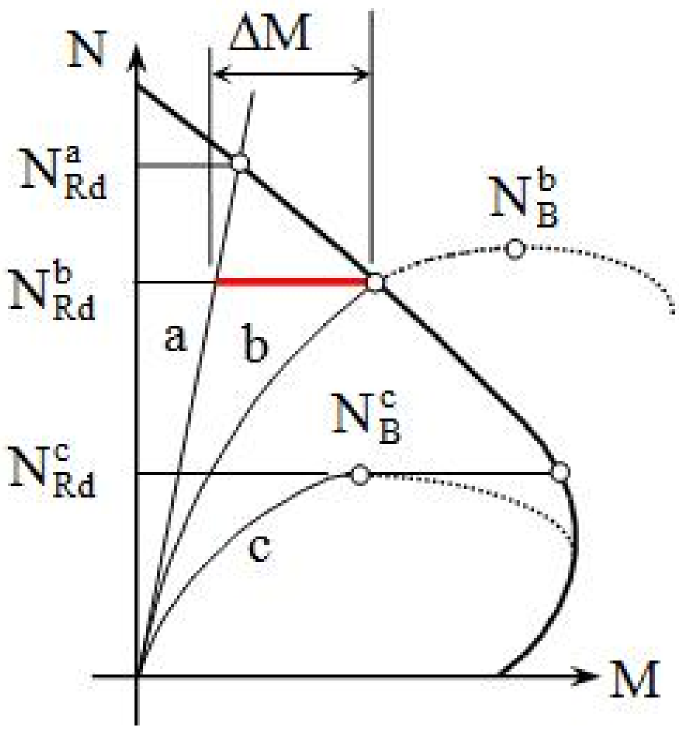
Cantilevered column

Longitudinal force increases from zero till column failure

M_{0Ed} = Ne

$\Delta M = N\delta$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE



Short columns ª á **35**

- negligible second order effects
- bending moment is proportional to the longitudinal force $\rightarrow$ line a
- element failure is produced by exhaustion of bearing capacity to a force equal to N_{Rd}^{a}

Slender columns **35 <** ª á **120**

- important second order effects
- bending moment increases faster than longitudinal force $\rightarrow$ curve b
- element failure is produced by exhaustion of bearing capacity to a force equal to $N_{Rd}^{b} < N_{B}^{b}$
- N_{B}^{b} - is the buckling force

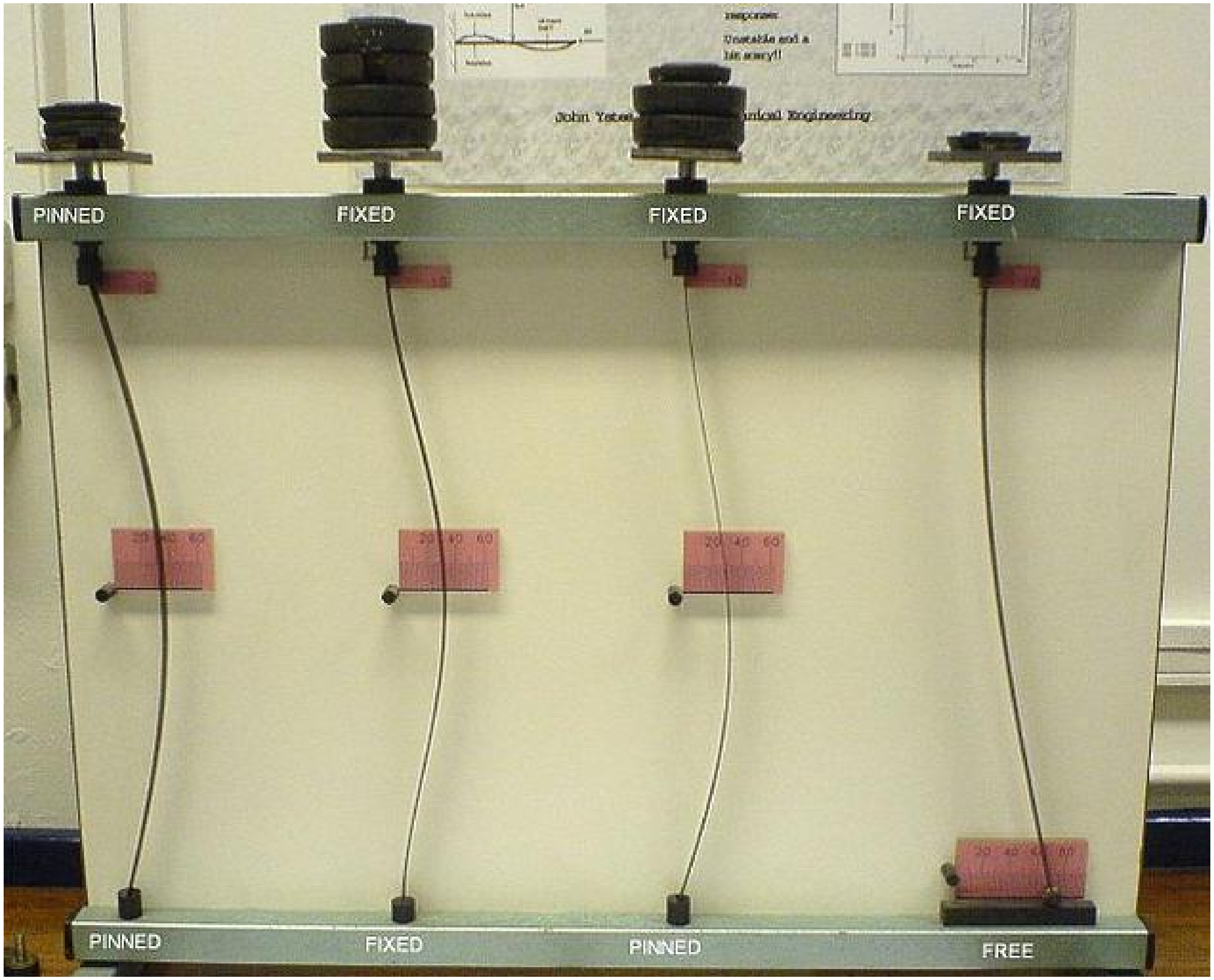
Very slender columns ª **> 120**

- buckling occurs at the force N_{B}^{c}
- deformations increase indefinitely under constant force
- in this case bearing capacity $N_{Rd}^{c} = N_{B}^{c}$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

(1707 – 1783)

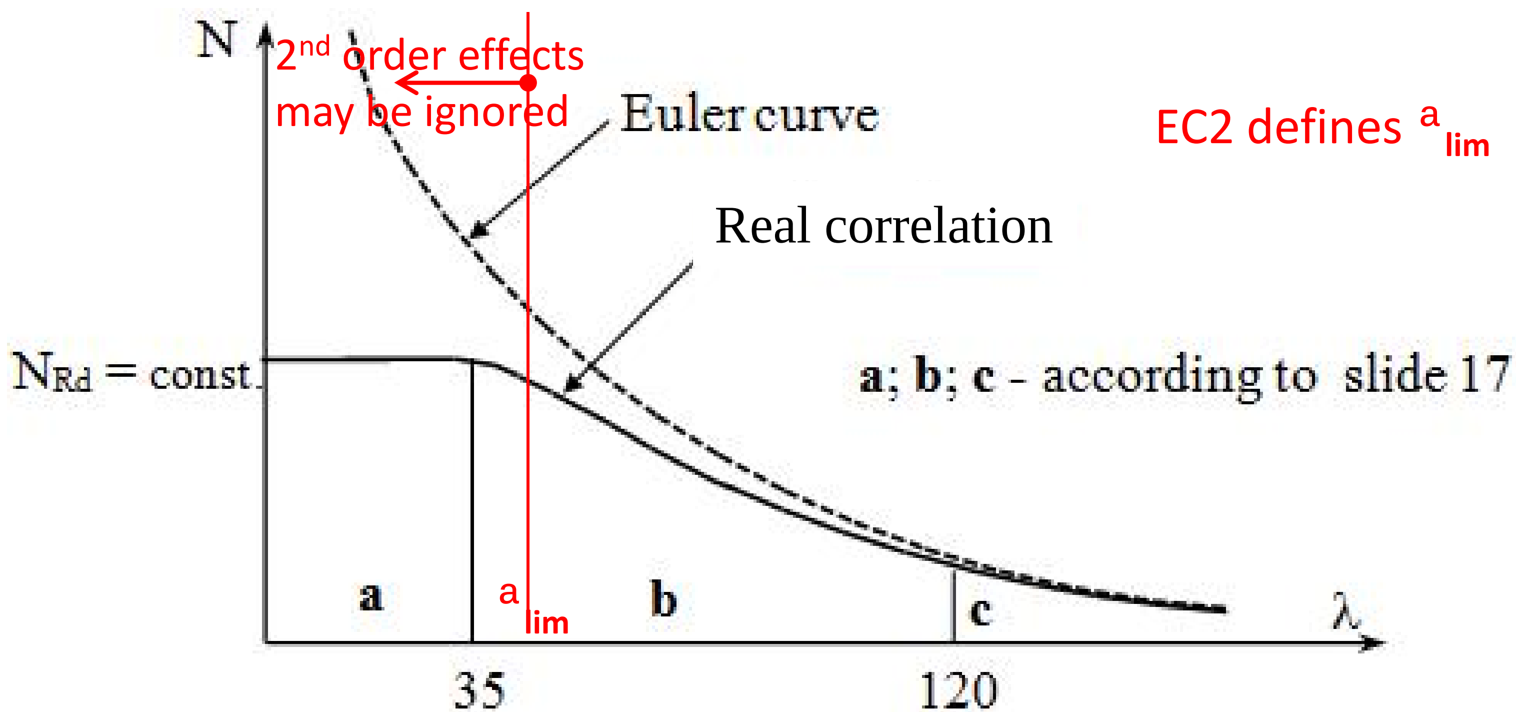
$$N_B = \frac{\pi^2 EI}{l_0^2}$$ - Euler formula for buckling load of isolated columns



8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

$$N_B = \frac{\pi^2 EI}{l_0^2}$$

Euler formula does not correctly describe the correlation between bearing capacity and element slenderness

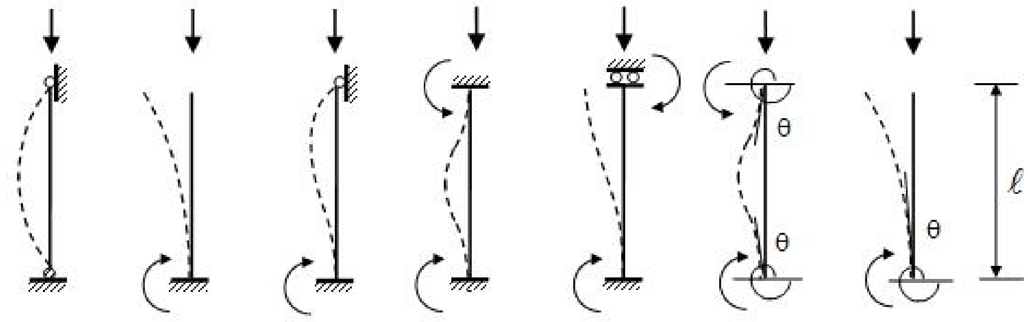


EC2 defines a_{lim}

Column:		
short	slender	very slender
Column failure:		
ultimate limit state		buckling

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.2. SLENDERNESS AND EFFECTIVE LENGTH OF ISOLATED MEMBERS



$\ell_0 = \ell$ a) $\ell_0 = 2\ell$ b) $\ell_0 = 0{,}7\ell$ c) $\ell_0 = 0{,}5\ell$ d) $\ell_0 = \ell$ e) $0{,}5\ell < \ell_0 < \ell$ f) $\ell_0 > 2\ell$ g)

a) double pined column in braced structures; not suitable in seismic areas

b) column in one level unbraced precast structure

c) column in one level braced precast structure

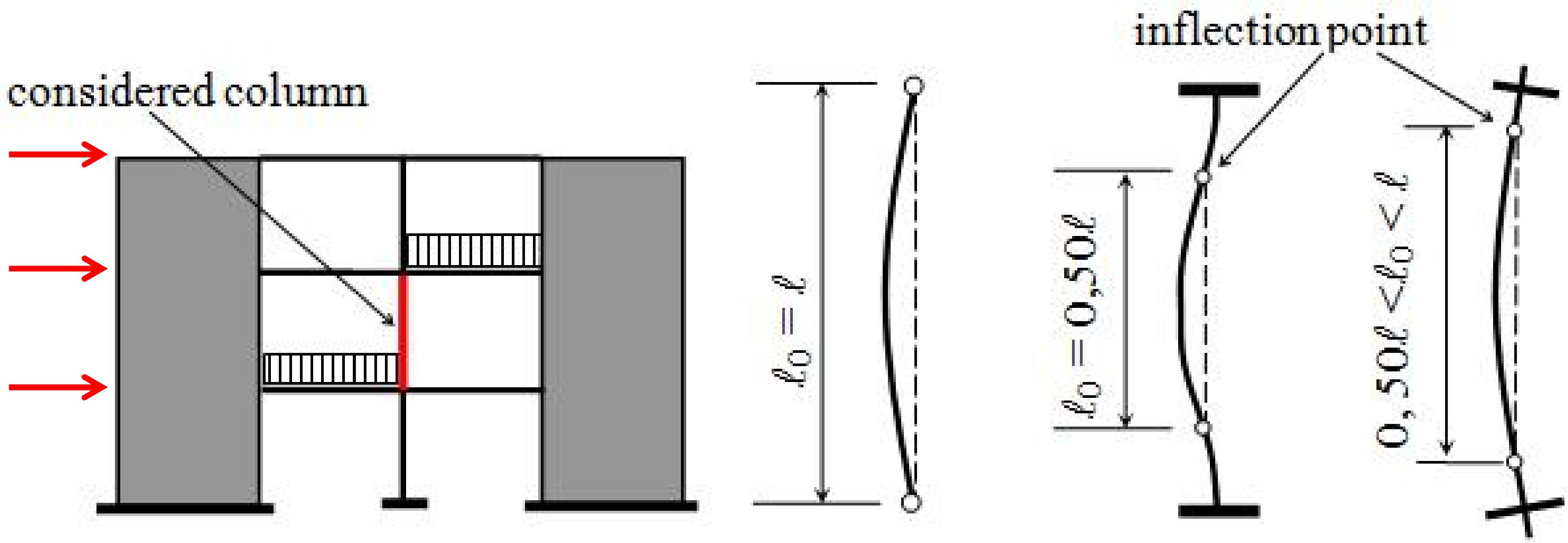
d) double fixed column in braced structure; bottom end = foundation !; top end = very stiff girder ?

e) case d in braced structure

f) column in braced structure; node rotation is possible

g) foundation rotation of case b

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE



Braced structure:
- no lateral deformations
- node rotations

Double pinned column

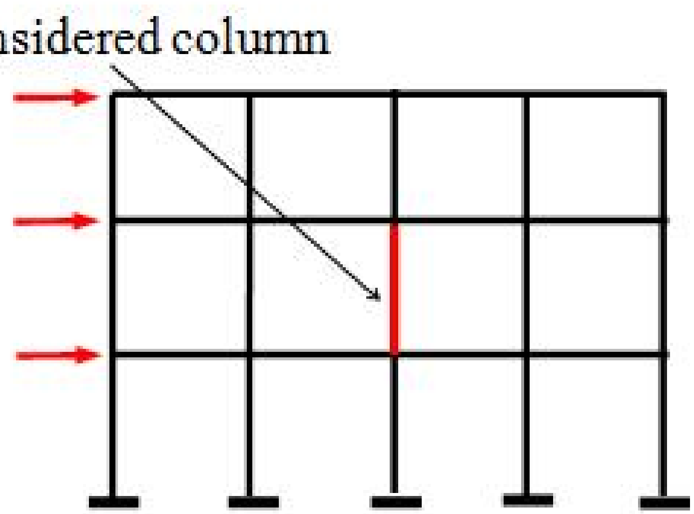
Double fix column

Real column

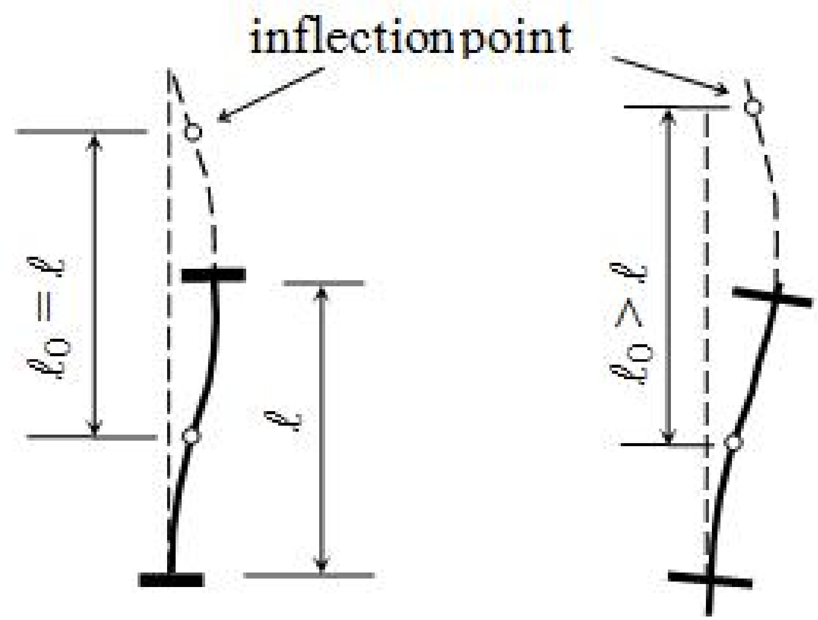
Extreme situations

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

-this is a sway structure
OR
- this structure sways



Unbraced structure:
- lateral deformations
- node rotations



Double fix column & free lateral deformations

Real column

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Regular frames

Braced columns

$$l_0 = 0{,}5l \cdot \sqrt{\left(1+\frac{k_1}{0{,}45+k_1}\right)\cdot\left(1+\frac{k_2}{0{,}45+k_2}\right)}$$

Unbraced columns

$$l_0 = l \cdot \max\left\{\sqrt{1+10\cdot\frac{k_1\cdot k_2}{k_1+k_2}}\ ;\ \left(1+\frac{k_1}{1+k_1}\right)\cdot\left(1+\frac{k_2}{1+k_2}\right)\right\}$$

k_1, k_2 are the relative flexibilities of rotational restraints at ends 1 and 2 respectively:

k $= (\theta / M)\cdot(EI / l)$

θ is the rotation of restraining members for bending moment M; ← Static analysis is required

EI is the bending stiffness of compression member,

l is the clear height of compression member between end restraints

Alternative procedure for **k** in case of braced frame

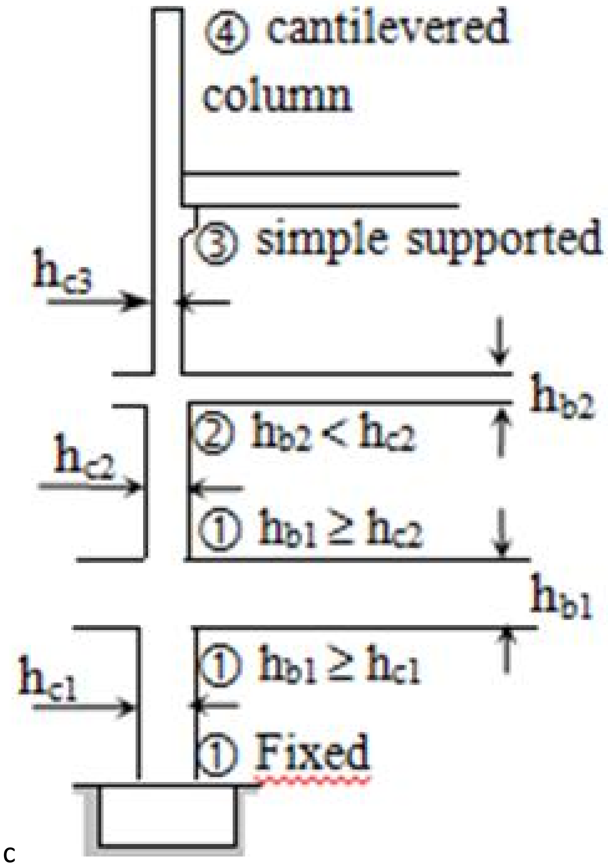
$$\mathrm{k} = \frac{(\mathrm{EI}/\ell)_\mathrm{c}}{\sum 2(\mathrm{EI}/\ell)_\mathrm{b}} \geq 0{,}1$$

c – considered column
b – adjacent girders at the top & bottom column ends

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

PRELIMINARY ASSESSMENT: $\ell_0 = \beta \cdot \ell$

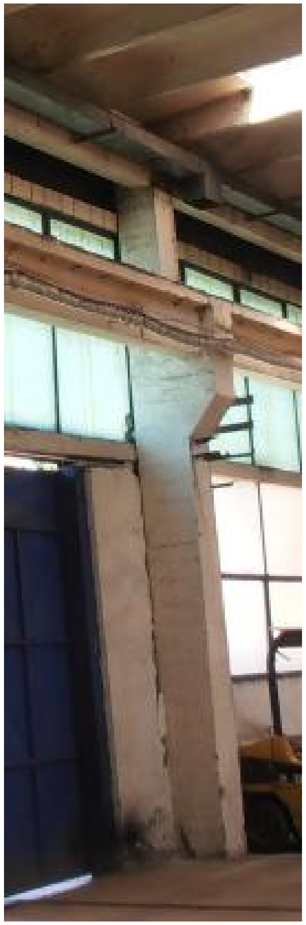
Top end condition	Bottom end condition		
	1	2	3
Braced frames			
1	0,75	0,80	0,90
2	0,80	0,85	0,95
3	0,90	0,95	1,00
Unbraced frames			
1	1,2	1,3	1,6
2	1,3	1,5	1,8
3	1,6	1,8	-
4	2,2	-	-



1 – fixed to foundation; monolithically connected to a beam $h_b \geq h_c$
2 – connected to a slab; monolithically connected to a beam $h_b < h_c$
3 - connected to simple supported beam
4 – unrestrained

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

For members with varying normal force and/or cross section

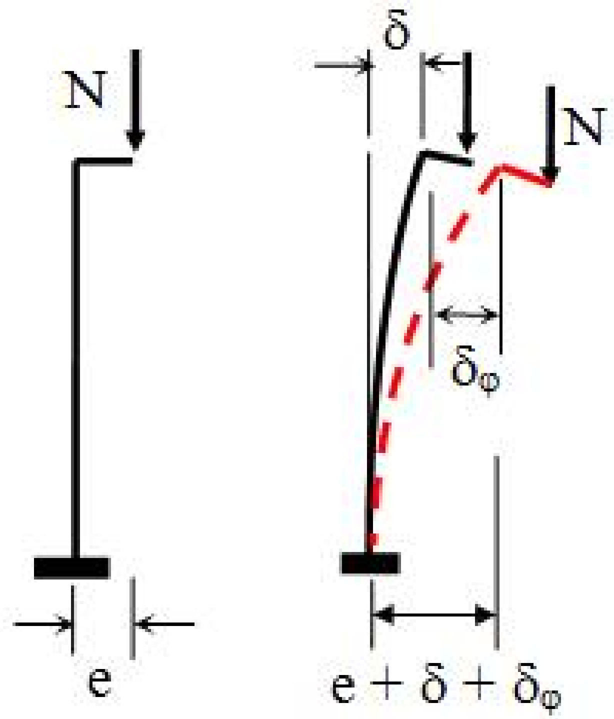


$$\ell_0 = \pi\sqrt{EI_{repr} / N_B}$$

EI_{repr} – representative stiffness
N_B – buckling load calculated by appropriate software or numerical methods

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.2. CREEP INFLUENCE



1ST order bending moment:

$$M_{0Ed} = N_{Ed} e$$

2nd order bending moment without creep influence:

$$M_{Ed} = M_{0Ed} + N_{Ed} \delta$$

2nd order bending moment with creep influence:

$$M_{Ed\varphi} = M_{0Ed} + N_{Ed} (1 + \varphi) \delta$$

The duration of loads may be taken into account by: $\varphi_{ef} = \varphi(\infty, t_0) \frac{M_{0Edqp}}{M_{0Ed}}$

$\frac{M_{0Edqp}}{M_{0Ed}} \rightarrow$

- calculated for section with maximum bending moment

or

- a representative mean value

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.3. SIMPLIFIED CRITERIA FOR SECOND ORDER EFFECTS

Second order effects may be ignored if they are less than 10 % of the corresponding first order effects

8.2.3.1. Slenderness criterion for isolated members

Second order effects may be ignored if $\lambda \leq \lambda_{lim}$

$$\lambda_{lim} = 20ABC/\sqrt{n}$$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

A $= 1 / (1+0,2\varphi_{ef})$ (if φ_{ef} is not known, $A = 0,7$ may be used)

B $= \sqrt{1+2\omega}$ (if ω is not known, $B = 1,1$ may be used)

C $= 1,7 - r_m$ (if r_m is not known, $C = 0,7$ may be used)

φ_{ef} effective creep ratio;

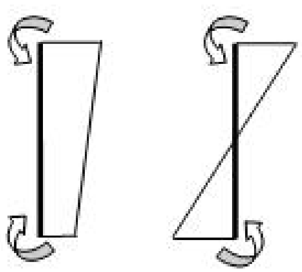
ω $= A_s f_{yd} / (A_c f_{cd})$; mechanical reinforcement ratio;

A_s is the total area of longitudinal reinforcement

n $= N_{Ed} / (A_c f_{cd})$; relative normal force

r_m $= M_{01}/M_{02}$; moment ratio

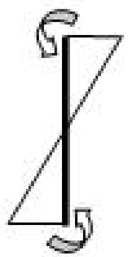
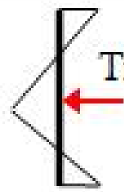
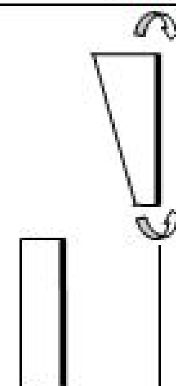
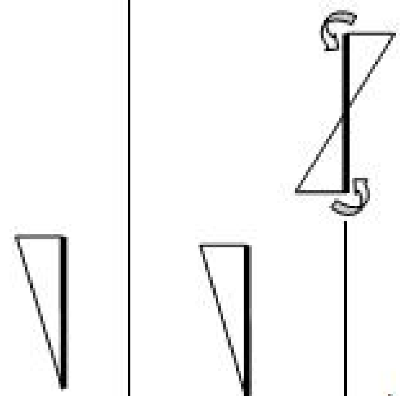
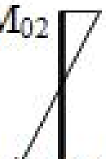
M_{01}, M_{02} are the first order end moments, $|M_{02}| \geq |M_{01}|$



$r_m > 0$ $r_m < 0$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

a_{lim} based on accepted simplifications for coefficients A, B & C

Column:	Unbraced	Braced			
Bending moment diagram		 Transverse force M_{01} & M_{02} predominant effect of geometric imperfections	 $M_{01} = M_{02}$		M_{02}  $\lvert M_{01}\rvert = M_{02}$
C	0,7			1,7	2,7
λ_{lim}	$10{,}78/\sqrt{n}$			$26{,}20/\sqrt{n}$	$41{,}60/\sqrt{n}$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.3.2. Global second order effects in buildings

Global second order effects in buildings may be ignored if

$$F_{V,Ed} \leq k_1 \cdot \frac{n_s}{n_s + 1{,}6} \cdot \frac{\sum E_{cd} I_c}{L^2}$$

$F_{V,Ed}$ is the total vertical load (on braced *and* bracing members)
n_s is the number of storeys
L is the total height of building above level of moment restraint
E_{cd} is the design value of the modulus of elasticity of concrete, see 5.8.6 (3)
I_c is the second moment of area (uncracked concrete section) of bracing member(s)

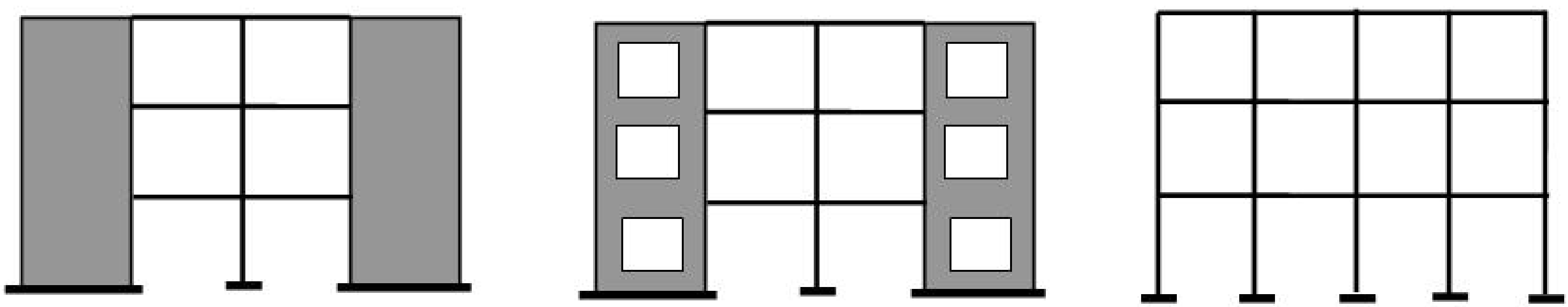
k_1 = 0,31

k_1 = 0,62 if it can be verified that bracing members are uncracked in ultimate limit state

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Previous expression is valid only if all the following conditions are met:

- torsional instability is not governing, i.e. structure is reasonably symmetrical
- global shear deformations are negligible (as in a bracing system mainly consisting of shear walls without large openings)



YES　　　NO　　　NO

- bracing members are rigidly fixed at the base, i.e. rotations are negligible
- the stiffness of bracing members is reasonably constant along the height
- the total vertical load increases by approximately the same amount per storey

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.4. Methods of analysis

General method based on nonlinear analysis

EC2 – 5.8.6

Method based on nominal curvature

Method based on nominal stiffness

Last two methods are simplified solutions.

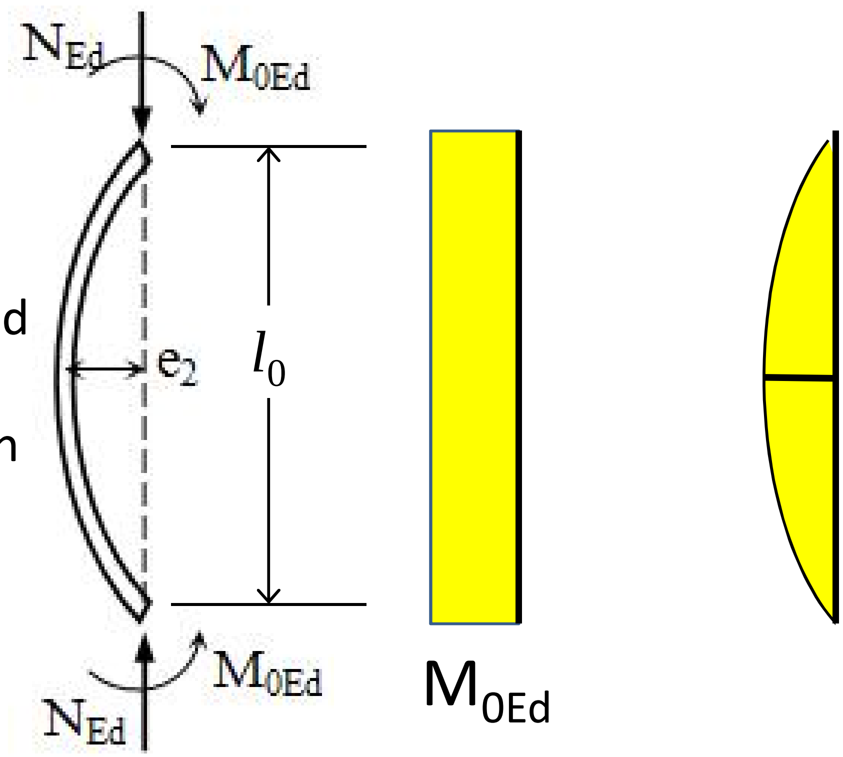
There is the possibility of the second order static analysis (nonlinear static analysis) based on nominal stiffness. Efforts resulting from this calculation include second order effects.

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.4.1. Method based on nominal curvature

Method is suitable for isolated columns with constant N_{Ed} and defined l_0

sine-shaped curvature distribution



max $M_2 = N_{Ed}e_2$

sine-shaped moment M_2 distribution

Second order effects depends on element deformed shape

Maximum deflection e_2 depends on curvature 1/r in the moment of failure

1/r depends on N_{Ed} & creep

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

CURVATURE

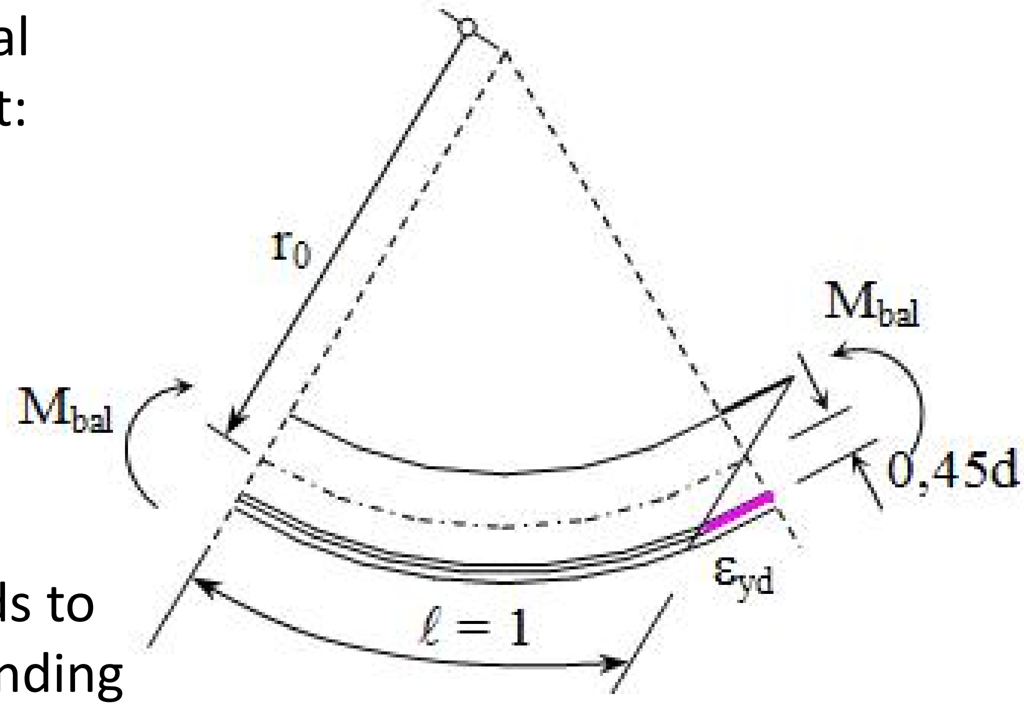
For members with constant symmetrical cross sections, including reinforcement:

$$1/r = K_r K_\varphi \cdot 1/r_0$$

K_r – correction factor for axial load

K_φ – correction factor for creep

$1/r_0$ - maximum curvature corresponds to balance situation (maximum bending moment)



$$\frac{1}{r_0} = \frac{\varepsilon_{yd}}{d - x_{lim}}$$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Correction factor K_r

Higher N_{Ed} smaller curvature 1/r

$$\frac{N_u - N_{bal}}{1/r_0} = \frac{N_u - N_{Ed}}{1/r}$$

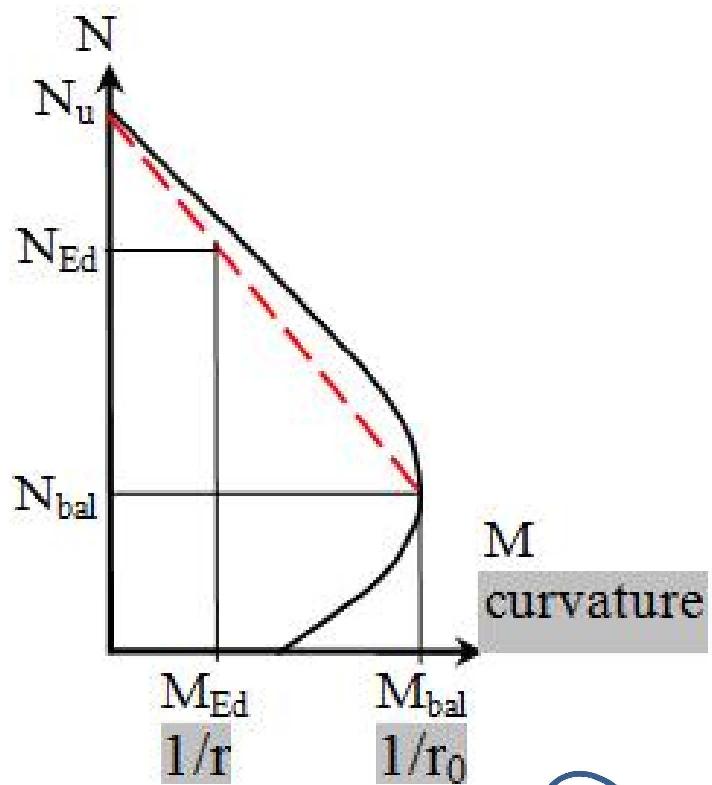
$$1/r = 1/r_0 \underbrace{\frac{N_u - N_{Ed}}{N_u - N_{bal}}}_{\text{correction}} \quad : A_c f_{cd}$$

$$n = \frac{N_{Ed}}{A_c f_{cd}}$$

$$n_u = 1 + \omega$$

$$\omega = \frac{A_{s,tot} f_{yd}}{A_c f_{cd}}$$

$$K_r = \frac{n_u - n}{n_u - n_{bal}} \le 1{,}0$$



$$N_u = A_{s,tot} f_{yd} + A_c f_{cd} \leftrightarrow N_{Rd}^c$$

$$N_{bal} = \xi_{lim} bd f_{cd} \approx 0{,}4 bd f_{cd} \leftrightarrow N_{lim}$$

Chp. 6

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Correction factor K··

$$K_{\varphi} = 1 + \beta\varphi_{ef} \geq 1{,}0$$

$$\beta = 0{,}35 + f_{ck}/200 - \lambda/150$$

$$\lambda = \frac{\ell_0}{i} \quad \leftarrow \text{slide 14}$$

$$\varphi_{ef} = \varphi(\infty, t_0)\frac{M_{0Edqp}}{M_{0Ed}} \quad \leftarrow \text{slide 24}$$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

BENDING MOMENTS

$$M_{Ed} = M_{0Ed} + M_2 \; \ldots\ldots\ldots\ldots \; (*)$$

$$M_2 = N_{Ed} e_2$$

$$e_2 = (1/r)\ell_0^2/c$$

c - factor depending on the curvature distribution; for constant cross section:

$\pi^2 \approx 10$ – for sine-shaped distribution of curvature

8 – for constant curvature distribution (constant bending moment)

1/r – curvature $\leftarrow$ slide 32

l_0 – effective length $\leftarrow$ slides 18 … 23

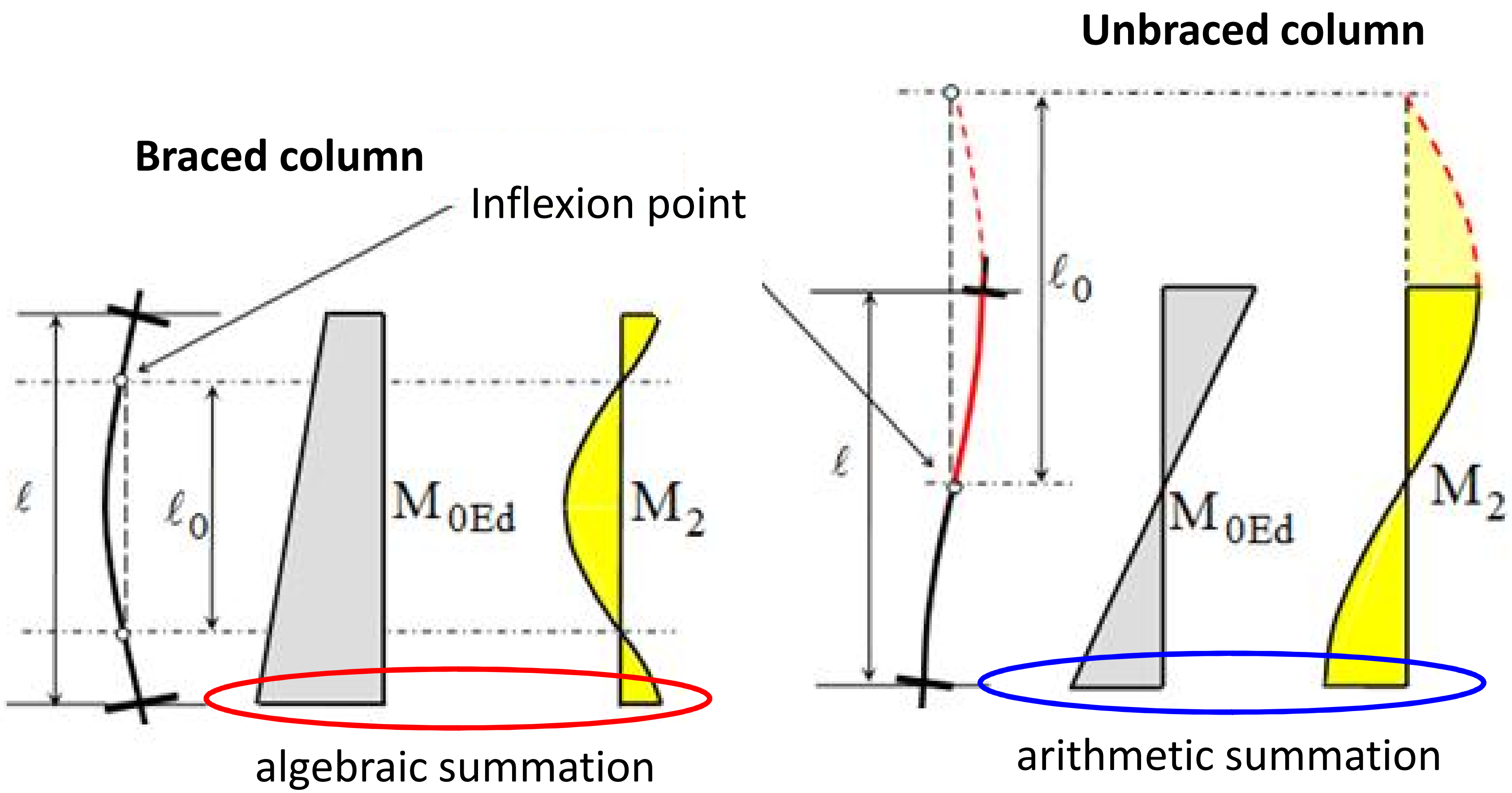
The meaning of relation $(*)$ is the summation of M_{0Ed} diagram with M_2 diagram.

The resulting diagram allows for the maximum bending moment.

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

1st order bending moment → linear diagram; maximum value at the column ends

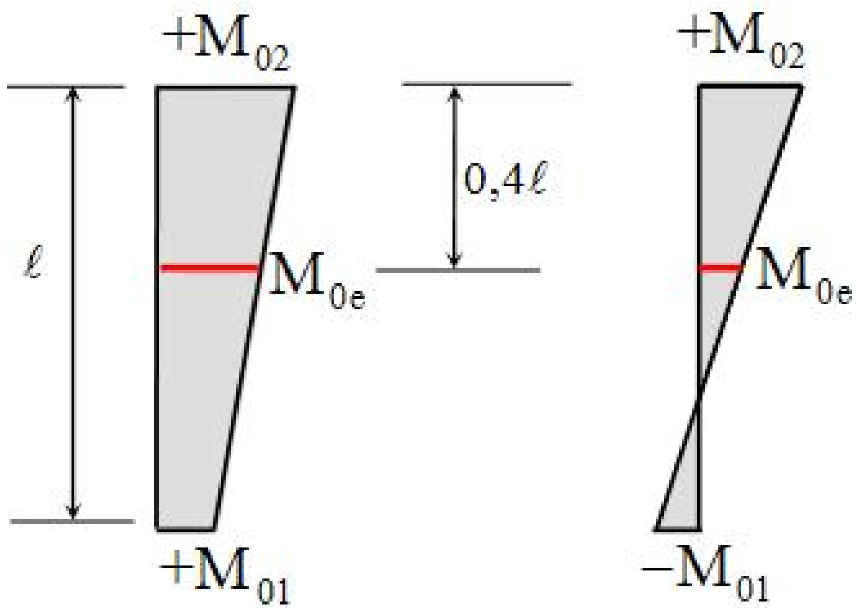
2nd order bending moment → sine-shaped diagram between inflexion points



8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Braced columns

Different first order end moments M_{01} and M_{02} may be replaced by an equivalent first order end moment M_{0e}



$$M_{0e} = 0{,}6M_{02} + 0{,}4M_{01} \geq 0{,}4M_{02}$$

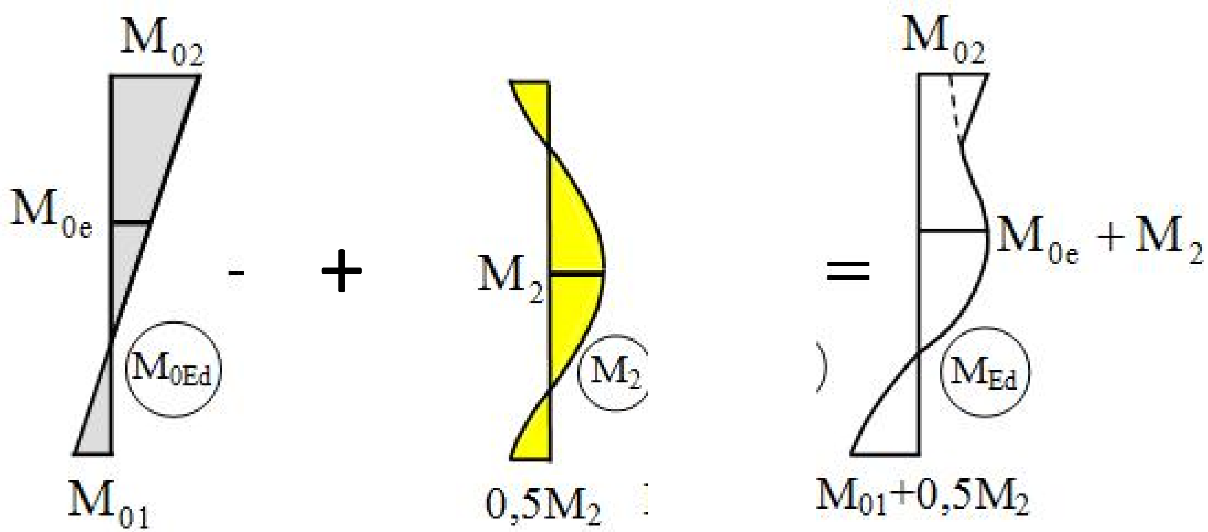
M_{01} and M_{02} should have the same sign if they give tension on the same side, otherwise opposite signs.
Furthermore, $|M_{02}| \geq |M_{01}|$.

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Maximum 1st order bending moments occur at the element ends

The maximum 2nd order bending moment occurs at about mid-length of column

Therefore it is possible that the maximum bending moment is no longer at the element ends



In such cases, the design bending moment is defined by:

$$M_{Ed} = \max\left(M_{02}; M_{0e} + M_2; M_{01} + 0{,}5M_2\right)$$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Unbraced columns

Lateral displacements may be generated by:

- asymmetry of the structure;
- horizontal seismic or wind forces.

All columns have the same mode of deformation due to high stiffness of reinforced concrete floors.

Therefore, it is reasonable to use an average curvature, even though the columns may have different rigidities.

Maximum 2nd bending moment occurs at that end of the column which has the highest stiffness.

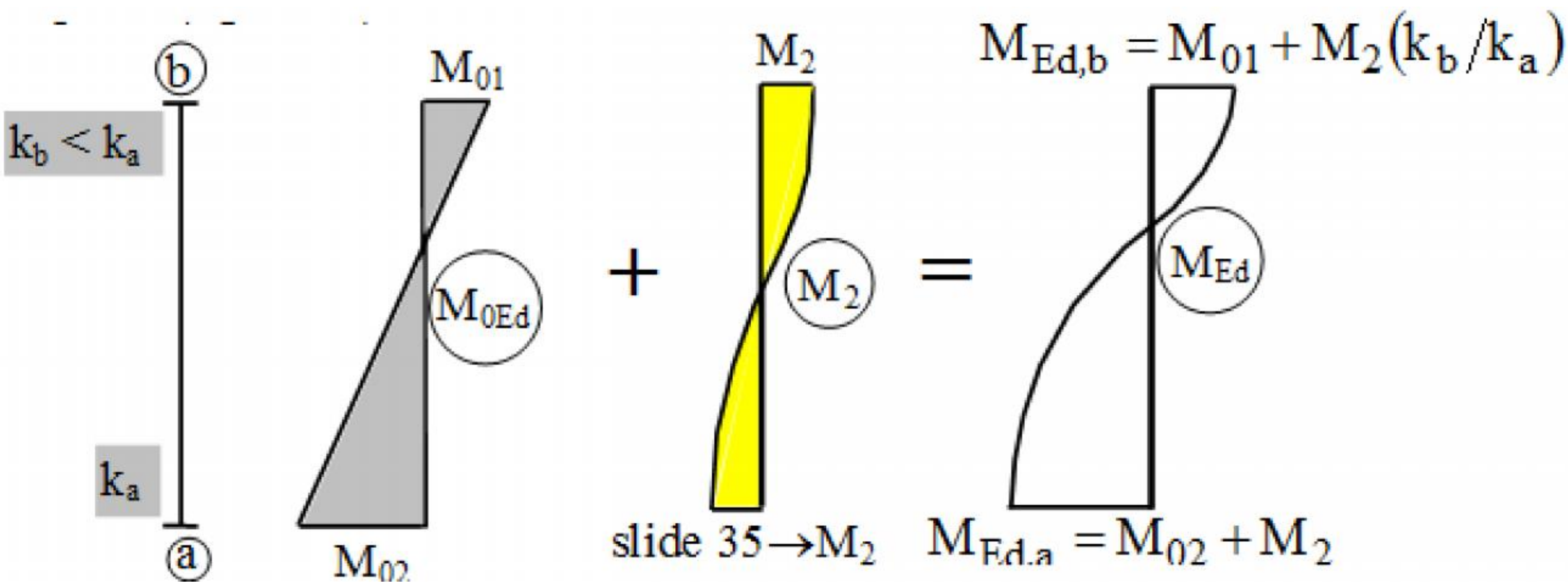
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Addition of 2nd bending moment to 1st bending moment

For the same rigidity at the both ends of column addition is done to the maximum 1st bending moment

For different rigidities of column ends the addition is done as follows:

- to the maximum 1st bending moment (which corresponds to highest rigidity)
- at the opposite end, the additional bending moment may be reduced in proportion to the ratio of the rigidities at the two ends of the column



8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.4.2. Method based on nominal stiffness

In a second order analysis based on stiffness, nominal values of the flexural stiffness should be used, taking into account the effects of

- cracking,
- material non-linearity
- creep

on the overall behavior.

This also applies to adjacent members involved in the analysis:.

- beams
- slabs.

Where relevant, soil-structure interaction should be taken into account.

The resulting design moment is used for the design of cross sections to bending moment and axial force

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

NOMINAL STIFFNESS

$$EI = K_c E_{cd} I_c + K_s E_s I_s$$

E_{cd} - Design value of the modulus of elasticity of concrete

$$E_{cd} = E_{cm} / \gamma_{cE}$$; γ_{cE} = 1,2

I_c - moment of inertia of concrete cross section

E_s - design value of the modulus of elasticity of reinforcement

I_s - second moment of area of reinforcement, about the centroid of area of the concrete

K_s = 1 - factor for contribution of reinforcement

K_c - factor for effects of cracking, creep, etc.

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

$K_c = k_1 k_2 / (1 + \varphi_{ef})$ if $\rho \geq 0{,}002$

$\rho = A_s / A_c$ - reinforcing ratio

A_s - total area of reinforcement

A_c - area of concrete section

φ_{ef} - effective creep ratio $\rightarrow$ slide 24

$k_1 = \sqrt{f_{ck}/20}$

$k_2 = n \frac{\lambda}{170} \leq 0{,}20$ with λ - slenderness ratio

$k_2 = n \cdot 0{,}3 \leq 0{,}20$ if λ is not defined

$n = N_{Ed} / A_c f_{cd}$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

In statically indeterminate structures, unfavorable effects of cracking in adjacent members should be taken into account.

Expressions from slides 45 & 46 are not generally applicable to such members. Partial cracking and tension stiffening may be taken into account according chp. 16.3. *Simplified approach of deflection control*

However, as a simplification, fully cracked sections may be assumed.

The stiffness should be based on an effective concrete modulus:

$$E_{cd,ef} = E_{cd}/(1+\varphi_{ef})$$

Note: Meaning of the text *Fully cracked section* is presented in chp. 16.3

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

MOMENT MAGNIFICATION FACTOR

The total design bending moment M_{Ed}, including second order effects, may be obtained by increasing M_{0Ed} as follows:

$$M_{Ed} = M_{0Ed}\left[1+\frac{\beta}{(N_B/N_{Ed})-1}\right] \quad \text{.......} \ (**)$$

N_{Ed} – design value of axial force

N_B – buckling load based on nominal stiffness

β – factor depending on distribution of 1st and 2nd order moments

$\beta = \pi^2/c_0$ – for sine-shaped distribution of 2nd order moments of isolated columns

c_0 – factor depending on distribution of 1st order moment:

c_0 = 8 for a constant bending moment

c_0 = 9,6 for a parabolic distribution

c_0 = 12 for symmetric triangular distribution

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Where provision for β or c_0 are not applicable, β = 1 is a reasonable simplification.

Consequently, relation (**) turns into:

$$M_{Ed} = \frac{M_{0Ed}}{1 - N_{Ed}/N_B} = \eta M_{0Ed}$$

$$\eta = \frac{1}{1 - N_{Ed}/N_B}$$

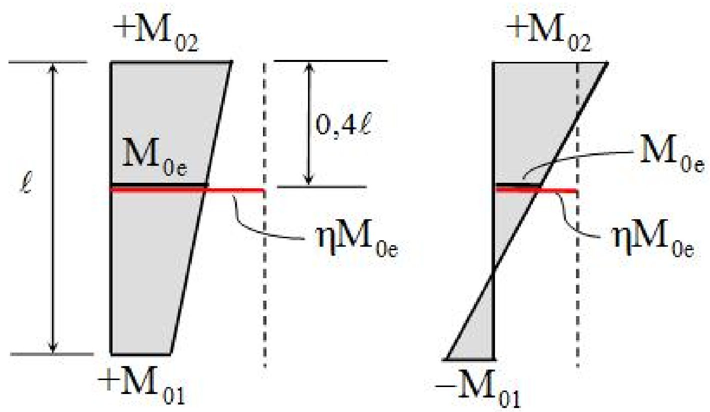
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Braced columns

For members without transverse load, different first order end moments M_{01} and M_{02} may be replaced by an equivalent *constant first order moment* M_{0e} (see slide 37).

c_0 = 8

Depending on slenderness and axial force, the end bending moments can be greater than the magnified equivalent moment ηM_{0e}



Therefore relation (**) from slide 45 is rewritten as follows:

$$M_{Ed} = M_{0e}\left[1 + \frac{\pi^2}{8(N_B/N_{Ed}) - 1}\right] \geq M_{02}$$

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Unbraced columns

The same l_0 for all columns because they "work" together due to monolithic floor

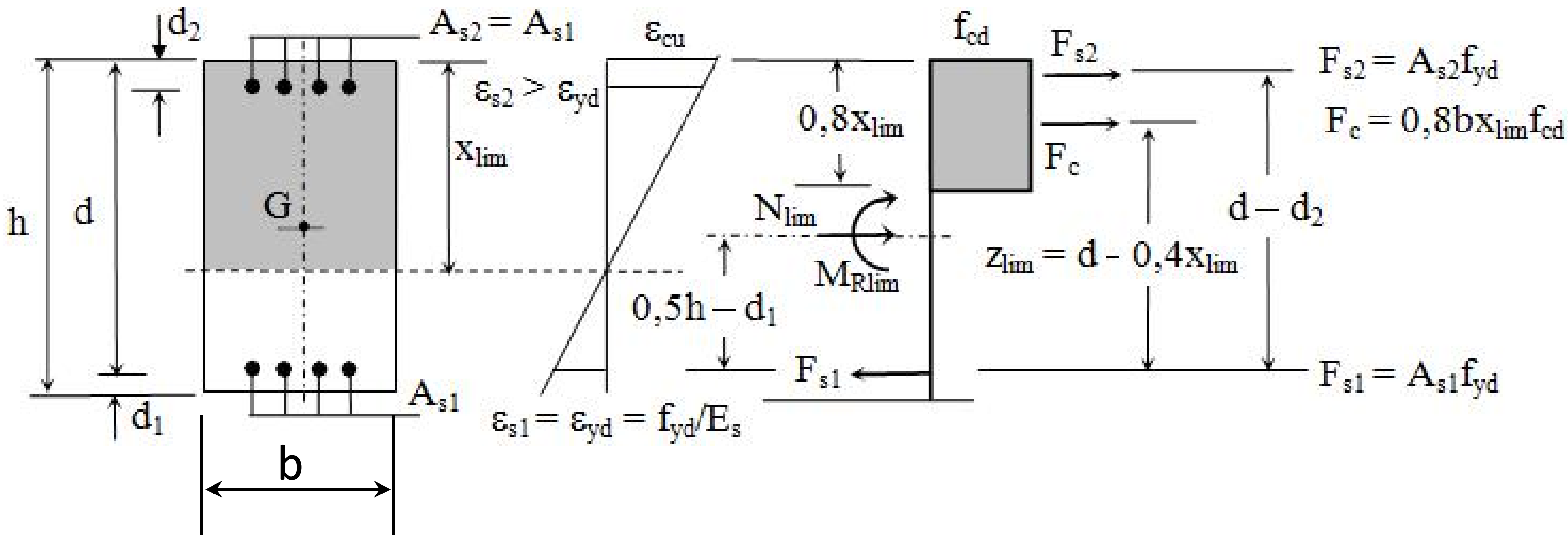
Slide 27: $\varphi_{ef} = \varphi(\infty, t_0)\frac{M_{0Edqp}}{M_{0Ed}}$

Discussion on M_{0Eqp} used for φ_{ef} : no horizontal variable loads (e.g. wind, bridge crane) are taken into account because do not induce creep

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.3.1. Balance situation

Chp. 6.5 – slide 17 $\xi_{lim} = \frac{x_{lim}}{d} = \frac{3{,}5}{3{,}5 + 1000 f_{yd}/E_s}$

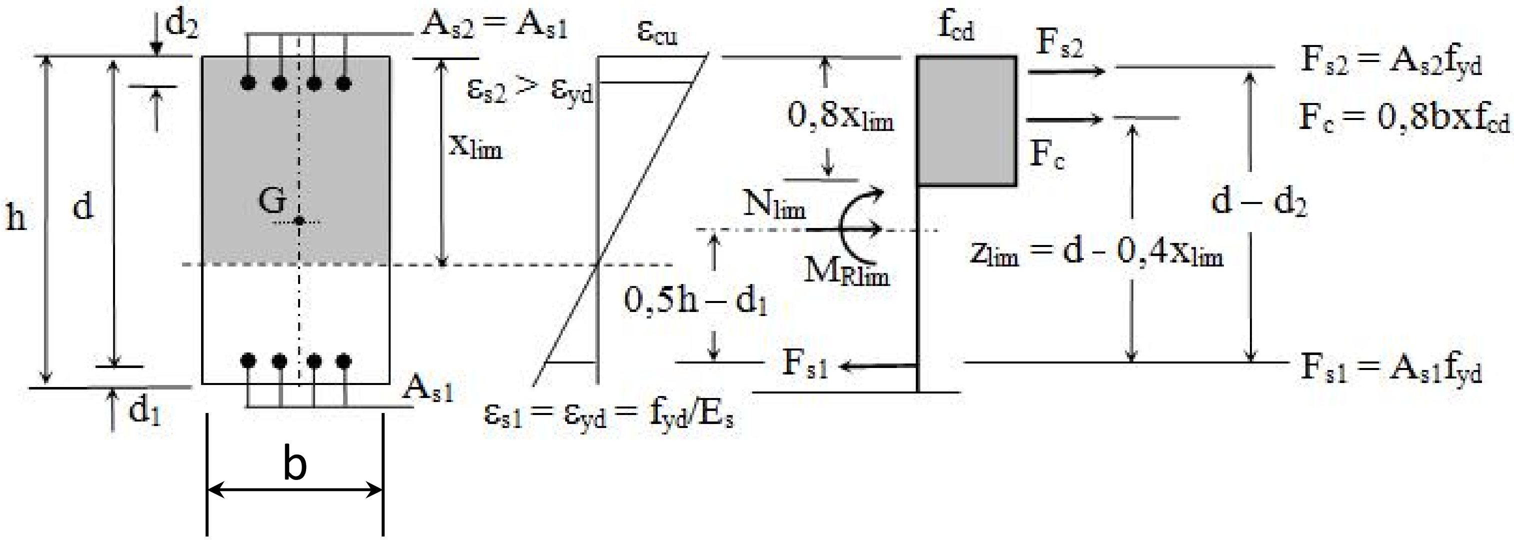


$\Sigma F = 0$

$$N_{lim} = F_c + F_{s2} - F_{s1}$$

$$N_{lim} = F_c = 0{,}8bx_{lim}f_{cd} = 0{,}8\xi_{lim}bdf_{cd}$$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION



$\Sigma M = 0 \rightarrow$ related to the $\mathbf{A_{s1}}$ **axis**

$$M_{R\,lim} + N_{lim}(0{,}5h - d_1) = F_c z_{lim} + F_{s2}(d - d_2)$$

$$M_{R\,lim} + N_{lim}(0{,}5h - d_1) = 0{,}8bx_{lim} f_{cd}(d - 0{,}4x_{lim}) + F_{s2}(d - d_2)$$

$$M_{R\,lim} + N_{lim}(0{,}5h - d_1) = 0{,}8\xi_{lim}(1 - 0{,}4\xi_{lim})bd^2 f_{cd} + F_{s2}(d - d_2)$$

$$M_{R\,lim} = \mu_{lim} bd^2 f_{cd} + A_{s2} f_{yd}(d - d_2) - N_{lim}(0{,}5h - d_1)$$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

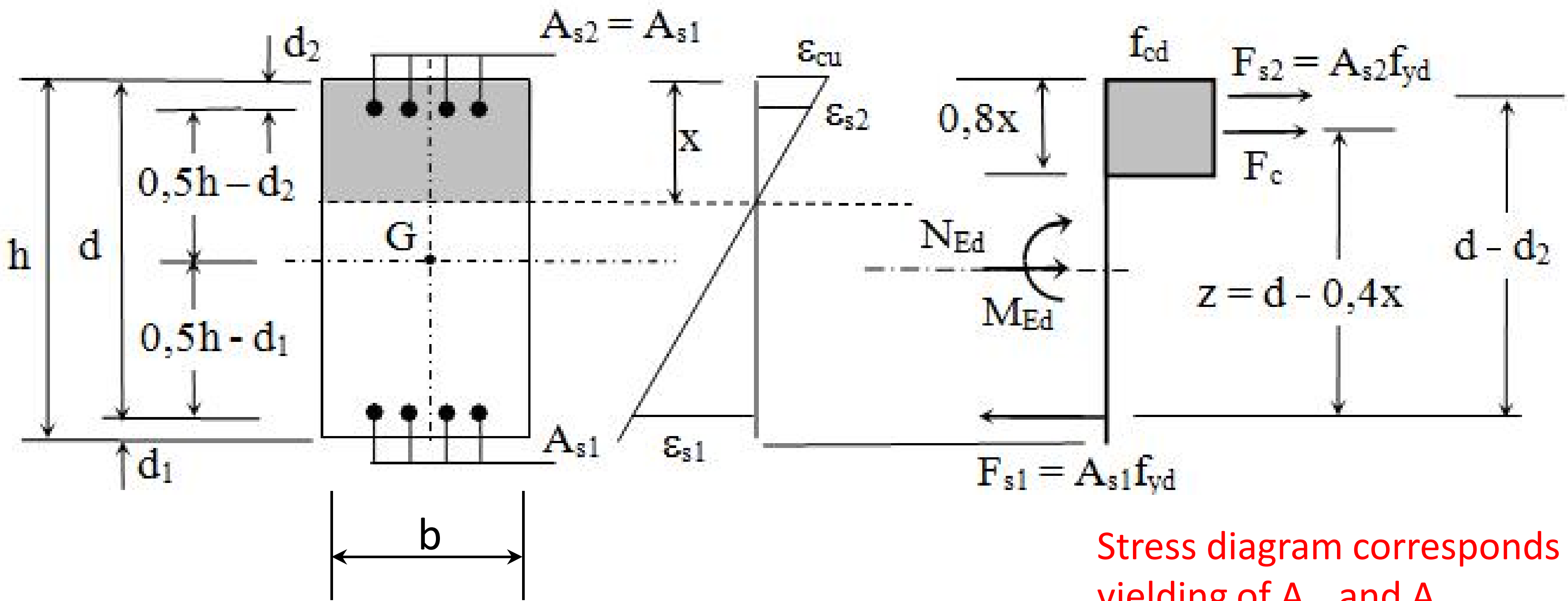
TWO WAYS OF FAILURE

$N_{Ed} \leq N_{lim} \rightarrow$ - compressive force with prevailing bending
- ductile failure due to yield of tensioned reinforcement
- compulsory in case of seismic areas

$N_{Ed} > N_{lim} \rightarrow$ - bending with prevailing compression
- brittle failure by crushing of concrete without yielding of reinforcement A_{s1} (whether it is tensioned or compressed)
- brittle character becomes stronger with the increasing of the compressive force

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.3.2. Section analysis



Stress diagram corresponds to yielding of A_{s1} and A_{s2}

TENSION REINFORCEMENT YIELDING BEFORE CONCRETE CRUSHING

¶ á ¶$_{\mathbf{lim}}$

STRESS IN COMPRESSION REINFORCEMENT

There is yielding of compression reinforcement if $\varepsilon_{s2} \geq \varepsilon_{yd}$

$$\varepsilon_{s2} = \varepsilon_{cu} \frac{x - d_{2s}}{x} \geq \varepsilon_{yd} \qquad x \geq \frac{\varepsilon_{cu}}{\varepsilon_{cu} - \varepsilon_{yd}} d_{s2} \rightarrow x_y$$

Steel	PC52	PC60	S400	S500
x_y	1,69d_2	1,91d_2	1,98d_2	2,64d_2
STAS 10107/0-97	2,0d_2			

$x \geq x_y \quad \sigma_{s2} = f_{yd}$

$x < x_y \quad \sigma_{s2} < f_{yd}$

- no yielding of compression reinforcement
- procedure in the chapter 6.4 (slide 12) applies
- simplified approach: F_c is acting at the level of F_{s2}

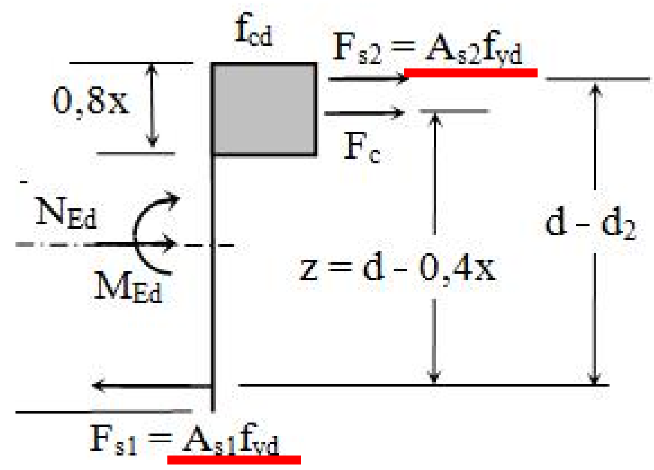
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

$$\Sigma F = 0$$

$$N_{Ed} = F_c + F_{s2} - F_{s1}$$

(1) .... $N_{Ed} = F_c$

$$x = \frac{N_{Ed}}{0{,}8bf_{cd}}$$



Let's assume yielding

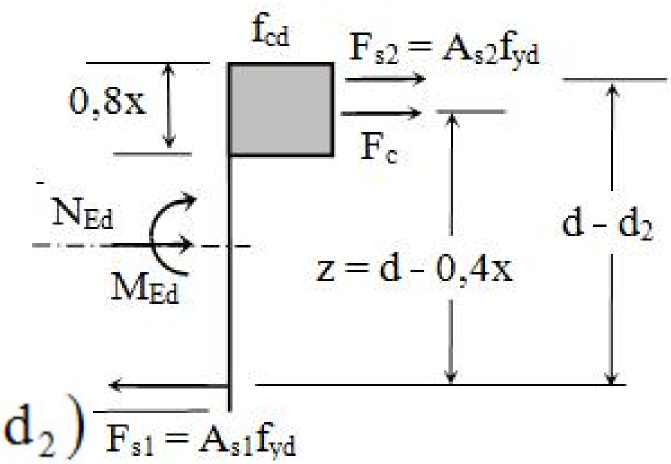
Case I: $\xi = x/d \leq \xi_{lim}$ the same as $N_{Ed} \leq N_{lim}$ $\rightarrow A_{s1}$ yields

Case II: $\xi = x/d > \xi_{lim}$ the same as $N_{Ed} > N_{lim}$ $\rightarrow A_{s1}$ does not yield

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

Case I: compression with prevailing bending - A_{s1} yields
(eccentric compression with large eccentricity)

x ñ x_y ì A_{s2} yields



$\Sigma M = 0 \rightarrow$ related to the **A_{s1} axis**

$$M_{Ed} + N_{Ed}(0{,}5h - d_1) = F_c z + F_{s2}(d - d_2)$$

slide 57: using relationship (1)

$$M_{Ed} + N_{Ed}(0{,}5h - d_1) = N_{Ed}(d - 0{,}4x) + F_{s2}(d - d_2)$$

$$M_{Ed} = N_{Ed}(d - 0{,}4x) - N_{Ed}(0{,}5h - d_1) + F_{s2}(d - d_2)$$

with $d = h - d_1$

$$M_{Ed} = N_{Ed}(h - d_1 - 0{,}4x - 0{,}5h + d_1) + F_{s2}(d - d_2)$$

(2) ….. $M_{Ed} = \underbrace{N_{Ed}(0{,}5h - 0{,}4x) + A_{s2} f_{yd}(d - d_2)}_{\text{resisting bending moment}}$

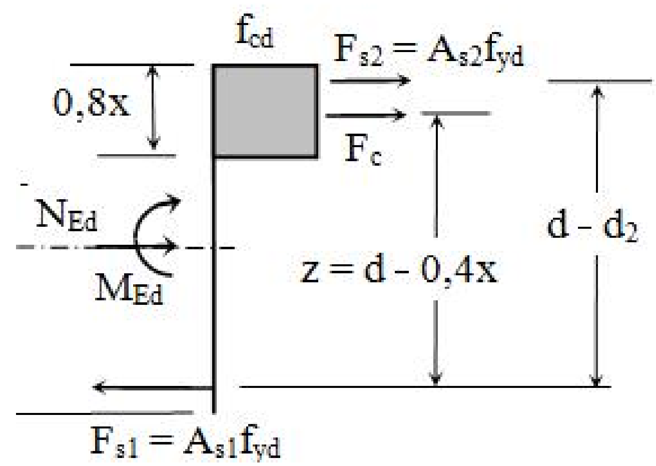
$$M_{Rd} = N_{Ed}(0{,}5h - 0{,}4x) + A_{s2} f_{yd}(d - d_2)$$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

$x < x_y$ ì A_{s2} does not yield

simplified approach: F_c is located at the level of A_{s2}

$\Sigma M = 0$ → related to the **A_{s2} axis**:



$$M_{Ed} - N_{Ed}(0,5h - d_2) = F_{s1}(d - d_2)$$

(3) …… $$M_{Ed} = \underbrace{A_{s1} f_{yd}(d - d_2) + N_{Ed}(0,5h - d_2)}_{\text{resisting bending moment}}$$

$$M_{Rd} = A_{s1} f_{yd}(d - d_2) + N_{Ed}(0,5h - d_2)$$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

Case II: bending with prevailing compression - A_{s1} does not yield
(eccentric compression with low eccentricity)

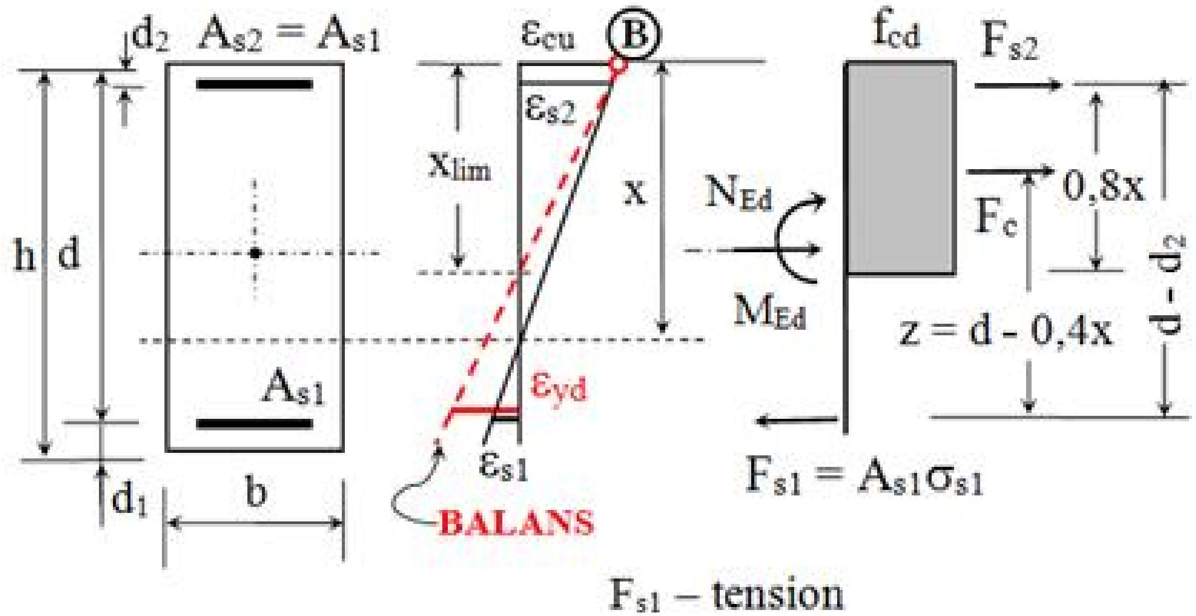
$x > x_{lim} >> x_y$ ì A_{s2} yields

Procedure described in cpt. 6.4 (slides 12, 13) should be applied using σ_c- ε_c & σ_s- ε_s diagrams

In what follows, relationships between the stress in reinforcement A_{s1} and neutral axis position are used without the need for stress-strain diagram.

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

$$x_{lim} < x \leq d$$



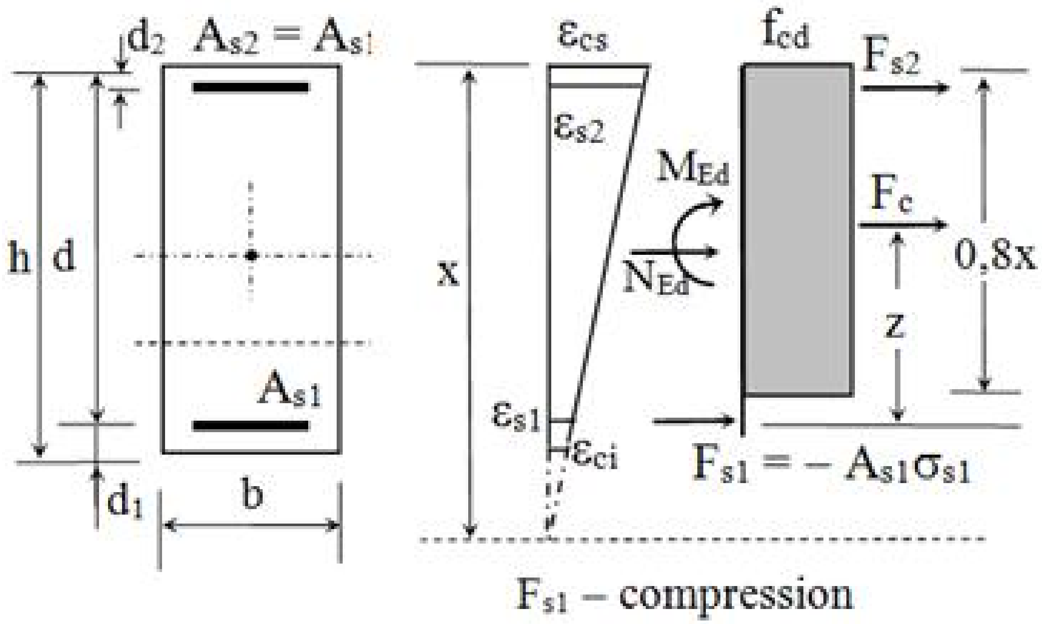
From triangles (red & black lines):

$$\varepsilon_{cu} = x_{lim}\frac{\varepsilon_{yd}}{d - x_{lim}} = x\frac{\varepsilon_{s1}}{d - x} \quad \rightarrow \quad \varepsilon_{s1} = \frac{x_{lim}}{x}\frac{d - x}{d - x_{lim}}\varepsilon_{yd}$$

$$\sigma_{s1} = \frac{x_{lim}}{x}\frac{d - x}{d - x_{lim}}f_{yd} \quad \text{(tension)}$$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

$$x > d$$



It is accepted that σ_{s1} is directly proportional to neutral axis depth

$$\sigma_{s1} = 4\frac{x-d}{d} f_{yd} \text{ (compression)}$$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

In view of the above, the stress in reinforcement A_{s1} is defined by the relationship:

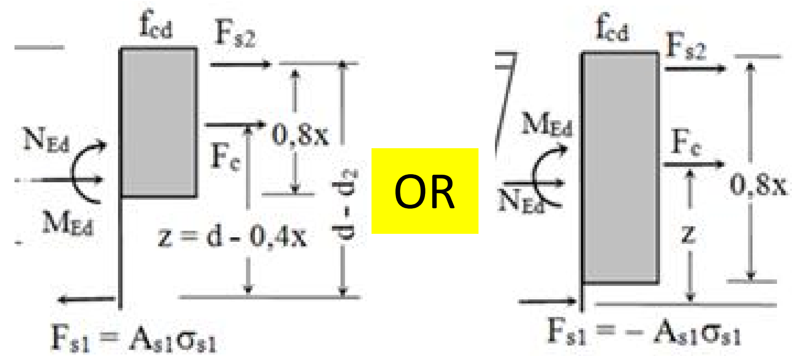
$$\sigma_{s1} = f(x) \cdot f_{yd}$$

$$f(x) = \begin{cases} x_{lim}(d - x)/x(d - x_{lim}) & \text{for } x_{lim} < x \le d \\ -4(x - d)/d & \text{for } d < x \le h \\ -1{,}0 & \text{for } x > h \end{cases}$$

NOTE:
Minus stands for compression

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

$\Sigma F = 0$



$$N_{Ed} = F_c + F_{s2} - F_{s1}$$

$$N_{Ed} = 0{,}8bxf_{cd} + A_{s2}f_{yd} - A_{s1}\sigma_{s1}$$

$$N_{Ed} = 0{,}8bxf_{cd} + A_{s2}\left(f_{yd} - \sigma_{s1}\right)$$

$$N_{Ed} = 0{,}8bxf_{cd} + A_{s2}f_{yd}\left[1 - f(x)\right]$$

$\Sigma M = 0$ $\rightarrow$ related to the **A_{s1} axis**:

$$M_{Ed} + N_{Ed}\left(0{,}5h - d_1\right) = F_c z + F_{s2}\left(d - d_2\right)$$

$$M_{Ed} = \underbrace{0{,}8bx\left(d - 0{,}4x\right)f_{cd} + A_{s2}f_{yd}\left(d - d_2\right) - N_{Ed}\left(0{,}5h - d_1\right)}_{\text{resisting bending moment}}$$

$$M_{Rd} = 0{,}8bx\left(d - 0{,}4x\right)f_{cd} + A_{s2}f_{yd}\left(d - d_2\right) - N_{Ed}\left(0{,}5h - d_1\right)$$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.3.3. Reinforcement design

Input data	Output data
M_{Ed}; N_{Ed}; b; h; f_{cd}; f_{yd}; c_{nom}	$A_{s1} = A_{s2}$; x; and eventually σ_{s1}

$$x = \frac{N_{Ed}}{0{,}8bf_{cd}} \leq \xi_{lim} d \text{ - Case I}$$

$x \geq x_y$	$x < x_y$
From relationship (2) – slide 58: $A_{s1} = A_{s2} = \frac{M_{Ed} - N_{Ed}(0{,}5h - 0{,}4x)}{f_{yd}(d - d_2)}$	From relationship (3) slide 59: $A_{s1} = A_{s2} = \frac{M_{Ed} - N_{Ed}(0{,}5h - d_2)}{f_{yd}(d - d_2)}$

$$x = \frac{N_{Ed}}{0{,}8bf_{cd}} > \xi_{lim} d \text{ - Case II}$$

Solve the system of equations to have $A_{s1} = A_{s2}$

$$\Sigma F = 0$$
$$\Sigma M = 0$$
$$\sigma_{s1} = f(x) \cdot f_{yd}$$

The system of equations is solved step by step, choosing x, because it is a non-linear system.

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

Approximate evaluation of reinforcement regardless of compression case ($d_1/d \cong 0{,}1$)

$$A_{s1} = A_{s2} = A_{s,tot}/2 \text{ with } A_{s,tot} = \omega_{tot}\, bh\, f_{cd}/f_{yd}$$

where:

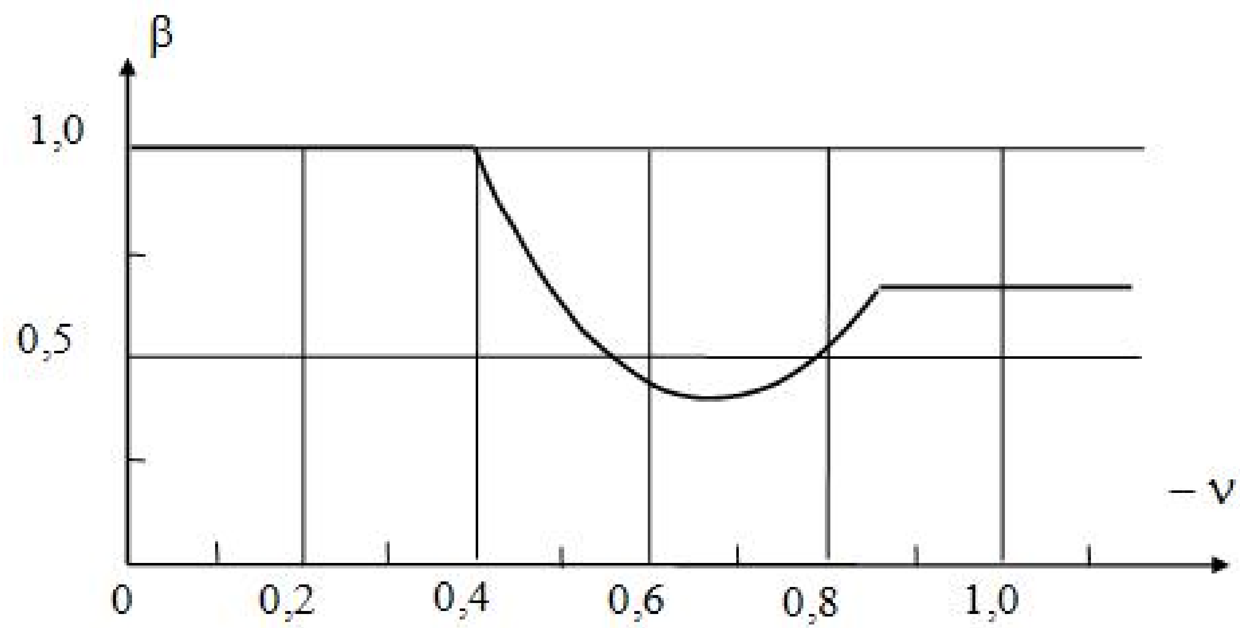
$\omega_{tot} = (\mu - 0{,}55\nu\nu_c)/\lambda\beta \quad$ if $0 > \nu \geq -0{,}85$;

$\omega_{tot} = \mu/\lambda\beta + \nu_c \quad$ if $-0{,}85 > \nu$;

$\nu = N_{Ed}/bhf_{cd}$ (negativ for compression);

$\nu_c = -0{,}85 - \nu$;

$\lambda = 0{,}50 - d_s/h$;



8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

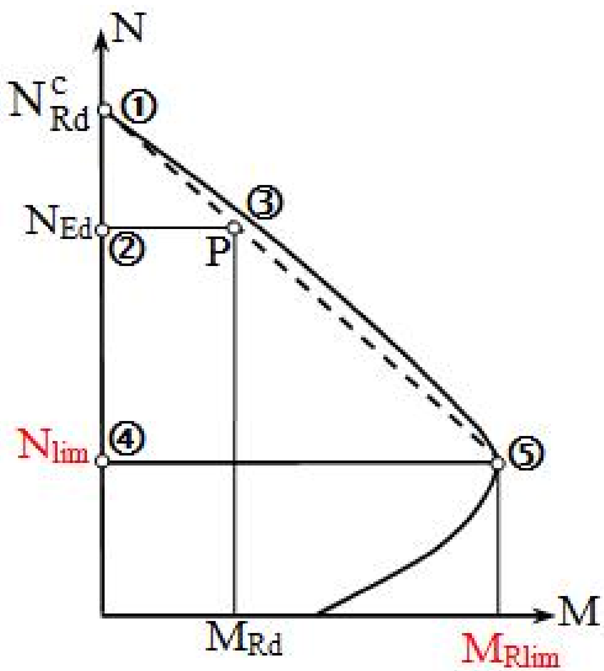
8.3.4. Cross section check

Input data	Output data
M_{Ed}; N_{Ed}; b; h; f_{cd}; f_{yd}; $A_{s1} = A_{s2}$; c_{nom}	M_{Rd}; x; and eventually σ_{s1}

<table>
<tr><td colspan="2" align="center">$$x = \frac{N_{Ed}}{0,8bf_{cd}} \leq \xi_{lim} d \text{ - Case I}$$</td></tr>
<tr><td align="center">$x \geq x_y$</td><td align="center">$x < x_y$</td></tr>
<tr><td>$M_{Ed} \leq M_{Rd} = N_{Ed}(0,5h - 0,4x) + A_{s2} f_{yd}(d - d_2)$</td><td>$M_{Ed} \leq M_{Rd} = A_{s1} f_{yd}(d - d_2) + N_{Ed}(0,5h - d_2)$</td></tr>
<tr><td colspan="2" align="center">$$x = \frac{N_{Ed}}{0,8bf_{cd}} > \xi_{lim} d \text{ - Case II}$$</td></tr>
<tr><td colspan="2">Solve the system of equations to have M_{Rd}
$\Sigma F = 0$
$\Sigma M = 0$
$\sigma_{s1} = f(x) \cdot f_{yd}$
The system of equations is solved step by step, choosing x, because it is a non-linear system.</td></tr>
</table>

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

Simplified check for case II of compression accepting M-N curve in the form of a line where $N_{Ed} > N_{lim}$



$$N^c_{Rd} = bhf_{cd} + (A_{s1} + A_{s2})f_{yd}$$

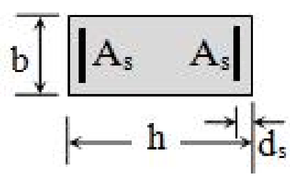
$$\frac{M_{Rd}}{N^c_{Rd} - N_{Ed}} = \frac{M_{R\,lim}}{N^c_{Rd} - N_{lim}}$$

$$M_{Ed} \le M_{Rd} = \frac{N^c_{Rd} - N_{Ed}}{N^c_{Rd} - N_{lim}} M_{R\,lim}$$

8.3.5. Alternative calculation tools

http://www.library.upt.ro/index.html?cursuri → File: 10_STALPI.pdf

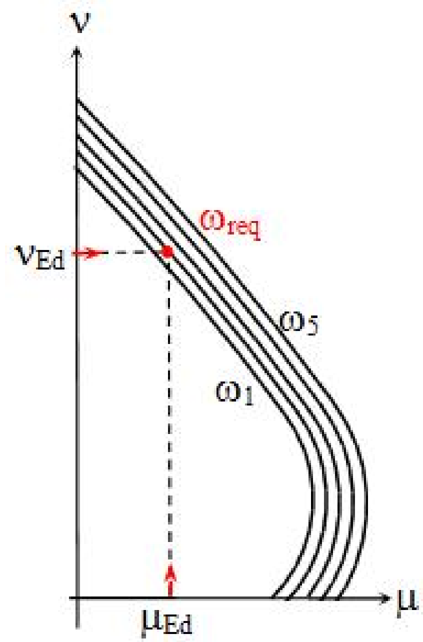
Anexa 10.1 Nomograme pentru calculul stâlpilor cu secţiune dreptunghiulară

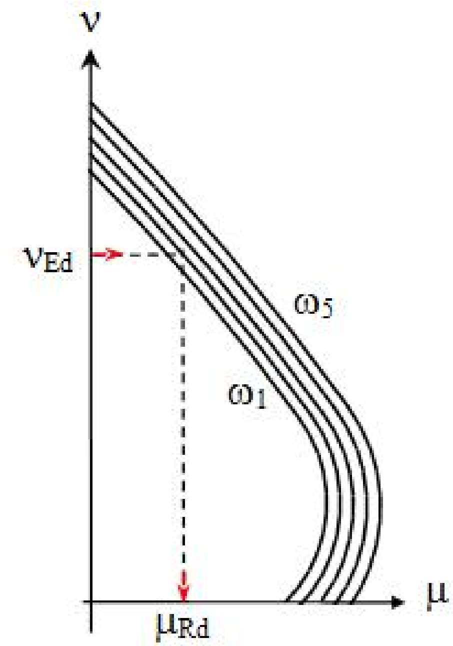


$A_{s1} = A_{s2} = A_s$

$A_{s,\,tot} = 2A_s$

$$\nu = \frac{N_{Ed}}{bhf_{cd}} \qquad \mu = \frac{M_{Ed}}{bh^2 f_{cd}} \qquad \omega = \frac{A_s f_{yd}}{bhf_{cd}} \qquad A_s = \omega\, bh \frac{f_{cd}}{f_{yd}}$$

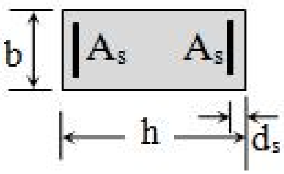




	Purposes of calculation →	$A_{s1} = A_{s2}$	M_{Rd}
1	Input data	μ_{Ed} & ν_{Ed}	ν_{Ed} & ω
2	Output data	ω_{req}	μ_{Rd}
3	Result	$A_{s1} = A_{s2} = \omega_{req} bhf_{cd}/f_{yd}$	$M_{Rd} = \mu_{Rd} bh^2 f_{cd}$

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

Anexa 10.2 Tabele pentru calculul stâlpilor cu secţiune dreptunghiulară



$A_{s1} = A_{s2} = A_s$

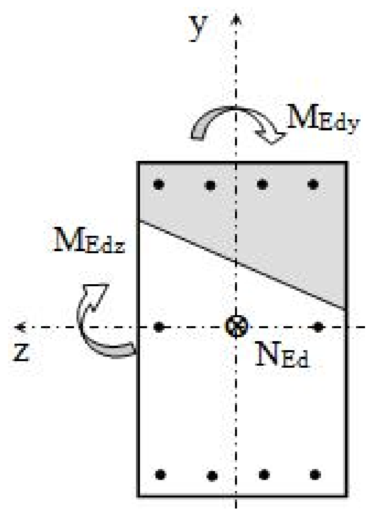
$A_{s,\,tot} = 2A_s$

$$\nu = \frac{N_{Ed}}{bhf_{cd}} \qquad \mu = \frac{M_{Ed}}{bh^2 f_{cd}} \qquad \omega = \frac{A_s f_{yd}}{bhf_{cd}} \qquad A_s = \omega\, bh \frac{f_{cd}}{f_{yd}}$$

	Purposes of calculation →	$A_{s1} = A_{s2}$	M_{Rd}
1	Input data	μ_{Ed} & ν_{Ed}	ν_{Ed} & ω
2	Output data	ω_{req}	μ_{Rd}
3	Result	$A_{s1} = A_{s2} = \omega_{req} bhf_{cd}/f_{yd}$	$M_{Rd} = \mu_{Rd} bh^2 f_{cd}$

ν	Values 1000μ for ω_{tot} =									
	0	0,20	...	ω_{req}	ω	...	ω	...	0,45	0,50
1,00										
...										
ν_{Ed}	reinforcement design	→	→	$1000\mu_{Ed}$						
...										
ν_{Ed}	M_{Rd} calculation	→	→	→	$1000\mu_{Rd}$					
...										
0										

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION



Independent design in each principal direction, disregarding biaxial bending, may be made as a first step.

Imperfections need to be taken into account only in the direction where they will have the most unfavourable effect.

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

No further check is necessary if the slenderness ratios satisfy the following condition:

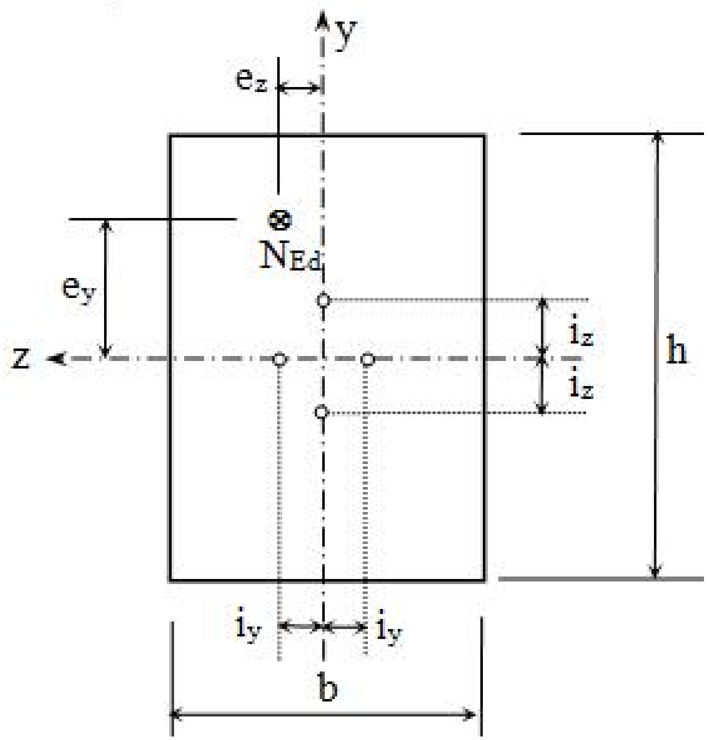
(4a) ……………… $0{,}5 \leq \lambda_y / \lambda_z \leq 2$

and if the eccentricities e_y and e_z satisfy one the following conditions:

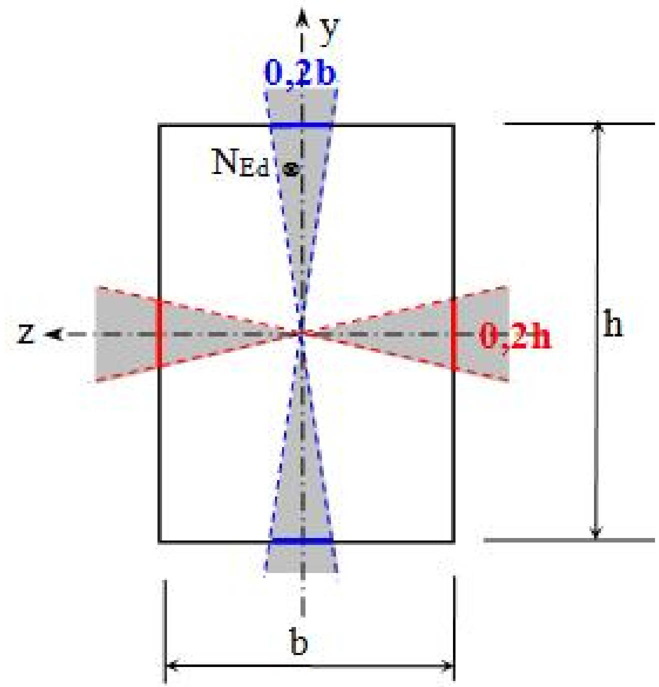
(4b) ……………… $\frac{e_y / h}{e_z / b} \leq 0{,}2$ **or** $\frac{e_z / b}{e_y / h} \leq 0{,}2$

b, h	are the width and depth of the section
λ_y, λ_z	are the slenderness ratios l_0/i with respect to *y*- and *z*-axis respectively
i_y, i_z	are the radii of gyration with respect to *y*- and *z*-axis respectively
e_z	$= M_{Edy} / N_{Ed}$; eccentricity along *z*-axis
e_y	$= M_{Edz} / N_{Ed}$; eccentricity along *y*-axis
M_{Edy}	is the design moment about y-axis, including second order moment
M_{Edz}	is the design moment about z-axis, including second order moment
N_{Ed}	is the design value of axial load in the respective load combination

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION



Definition of eccentricities e_y and e_z

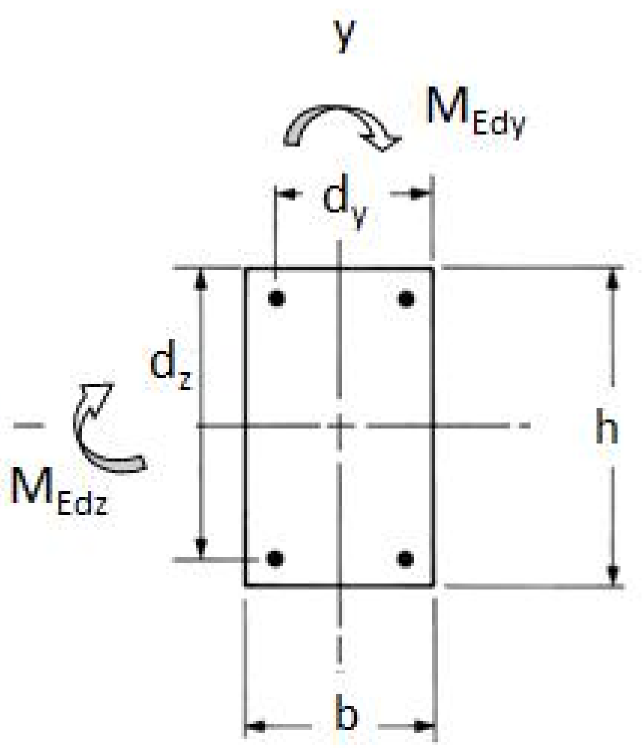


Graphical representation of the condition (4b)

If the condition of Expression (4) is not fulfilled, biaxial bending should be taken into account including the 2nd order effects in each direction

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Procedure according to BS 8100 , also accepted by **IS**truct**E**



Column may be design for a single axis bending but with an equivalent bending moment as follows:

- for: $\dfrac{M_{Edz}}{d_z} \geq \dfrac{M_{Edy}}{d_y}$

$$M_z^{ech} = M_{Edz} + \beta_N \frac{d_z}{d_y} M_{Edy}$$

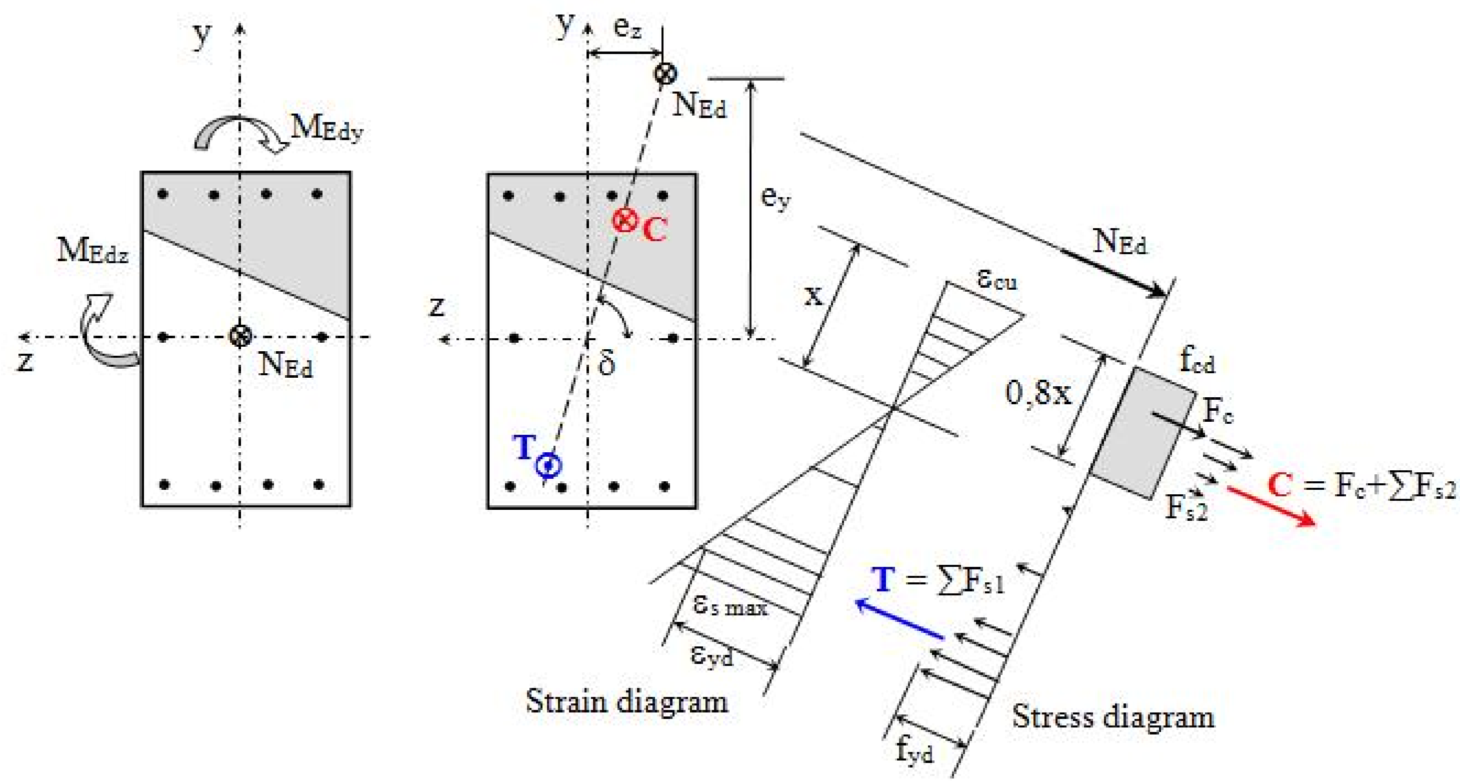
- for: $\dfrac{M_{Edz}}{d_z} < \dfrac{M_{Edy}}{d_y}$

$$M_y^{ech} = M_{Edy} + \beta_N \frac{d_y}{d_z} M_{Edz}$$

$$\beta_N = 1 - N_{Ed} / bhf_{ck} \geq 0{,}3$$

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

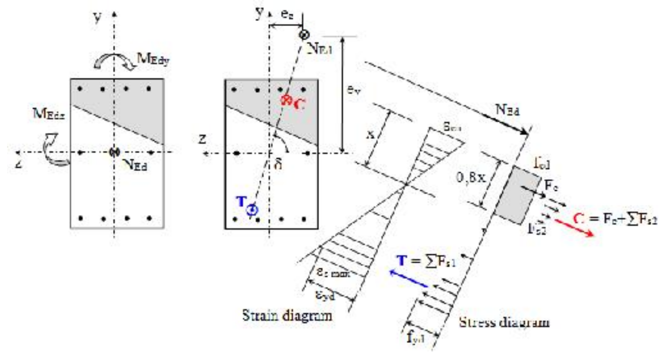
8.4.1. Basics of calculation



Reinforcement is distributed on all sides of the section

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Force line is characterized by $tg\delta = M_{Edz}/M_{Edy} = e_y/e_z$



Calculation is based on the assumptions from chp. 6.1

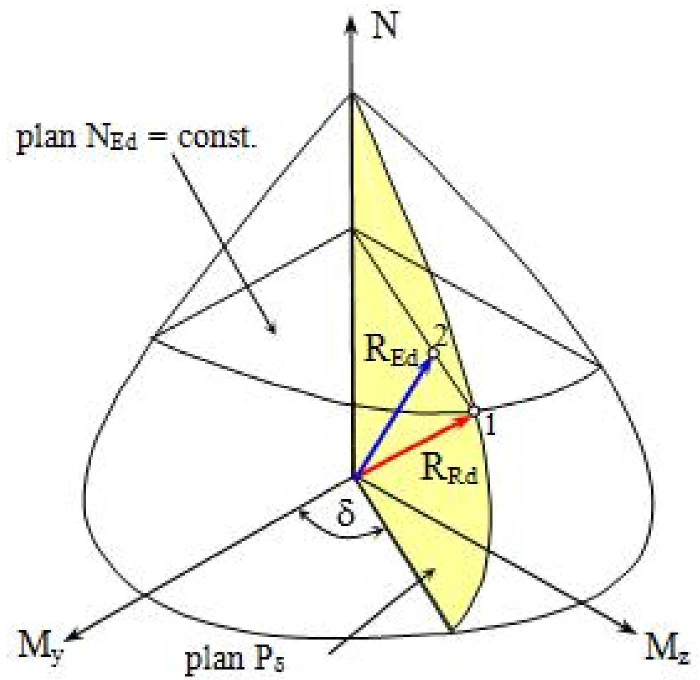
Position of the neutral axis is selected in such a way that internal forces (namely $F_c+\sum F_{s2}$ and $\sum F_{s1}$) to be located on the line of forces

Failure is produced by:

- yielding of the most tensioned bars followed by crushing of compression concrete, according to pivot B;
- crushing of compression concrete without yielding of tension bars, according to pivot C;
- whatever is the way of failure, there are bars which are not yielding

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

INTERACTON SURFACE FOR COMPRESSION WITH BIAXIAL BENDING



Static analysis: N_{Ed}; M_{Edy} ; M_{Edz}

By vectorial summation results:

$$R_{Ed} = \sqrt{N_{Ed}^2 + M_{Edy}^2 + M_{Edz}^2} = \sqrt{N_{Ed}^2 + M_{Ed}^2}$$

Bearing capacity is:

$$R_{Rd} = \sqrt{N_{Rd}^2 + M_{Rdy}^2 + M_{Rdz}^2} = \sqrt{N_{Rd}^2 + M_{Rd}^2}$$

The two vectors are in the same meridian plan P_δ

The cross section resists to loads if point 2 (corresponding to the vector R_{Ed}) is inside the interaction surface or overlapped on the point 1:

(5) ……………………………………… $$R_{Ed} \le R_{Rd}$$

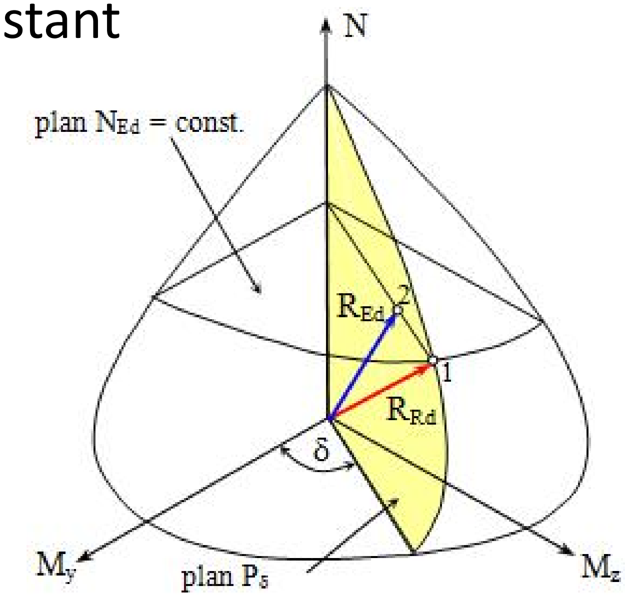
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

8.4.2. Simplified procedure of calculation

Load Contour Method

Simplified procedure, taking into account the interaction of bending moments M_{Edy} and M_{Edz} for a constant axial force N_{Ed}, may be used for calculation by hand

This method is suitable for structures located in seismic areas because the bending moments increase under constant gravitational load.



In this case, equation (5) becomes:

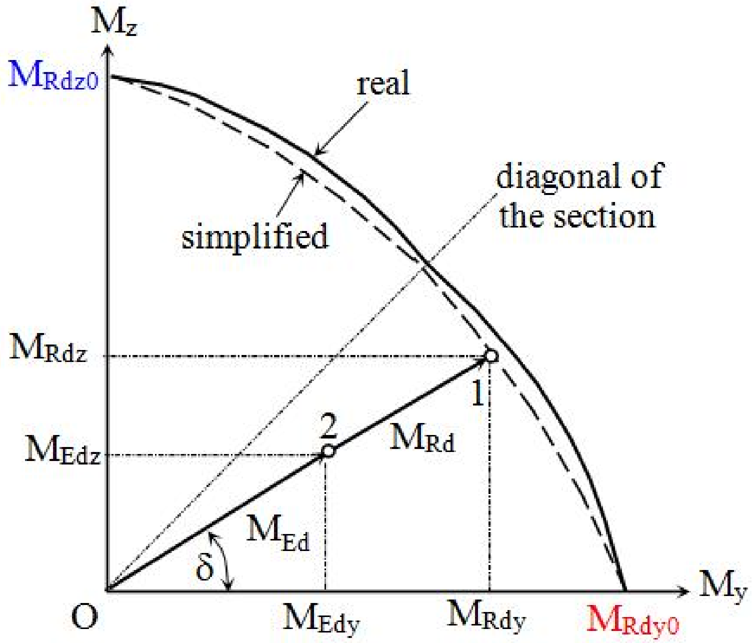
$$\sqrt{N_{Ed}^2 + M_{Ed}^2} \leq \sqrt{N_{Rd}^2 + M_{Rd}^2}$$

(6) .... $$M_{Ed} \leq M_{Rd}$$

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

The simplified procedure is based on the replacement of actual curve of interaction, dependent on angle δ, with a simplified elliptic curve

Interaction curve $M_y - M_z$



Calculation procedure is safe because simplified curve is located inside the real one

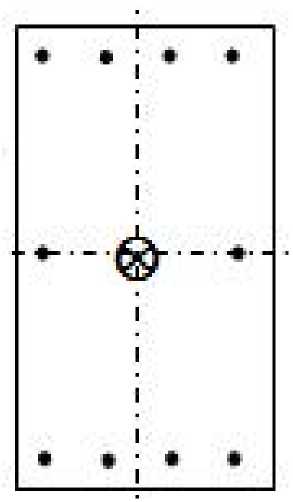
M_{Rdy0} – bearing capacity in uniaxial bending for N_{Ed} when $M_{Edz} = 0$

M_{Rdz0} – bearing capacity in uniaxial bending for N_{Ed} when $M_{Edy} = 0$

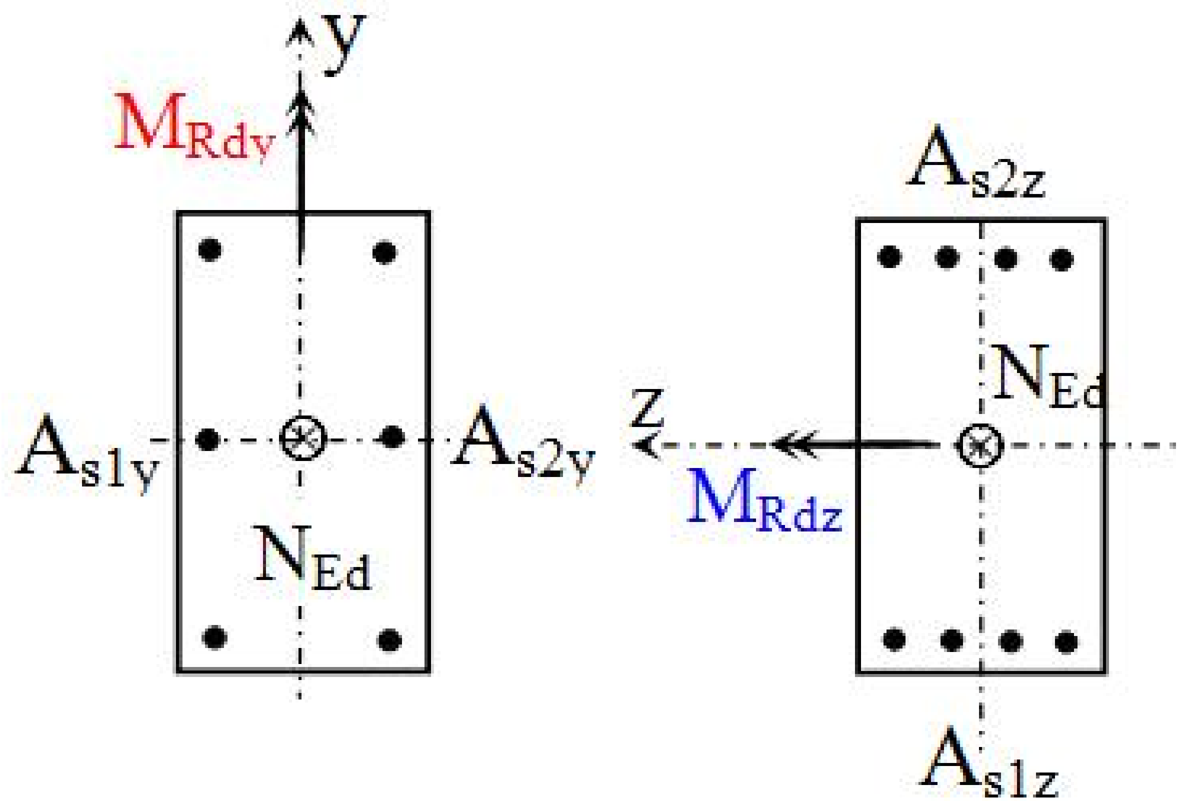
Unfavorable conclusion: due to biaxial bending there is a decreasing in uniaxial resistance

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

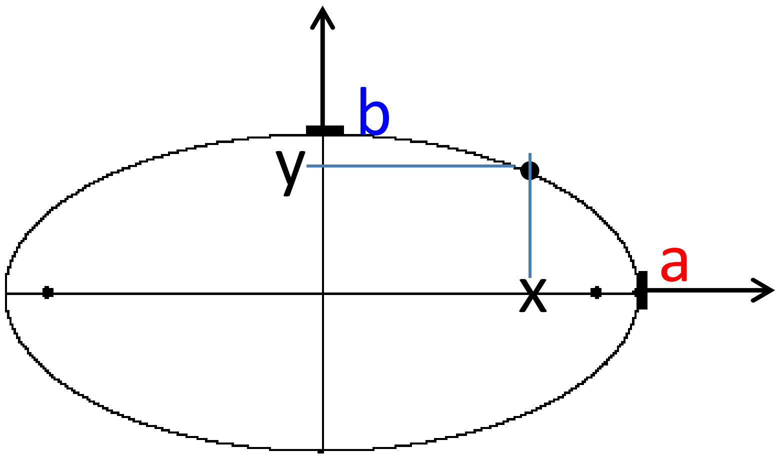
Total area $A_{s,tot}$



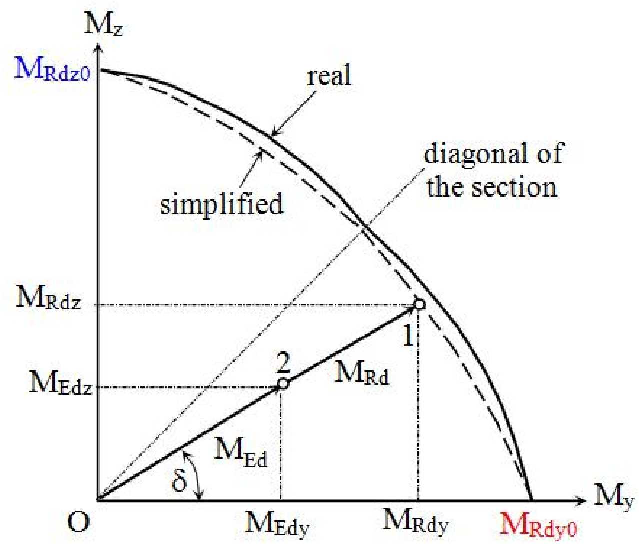
Defining areas A_{sy} and A_{sz}



8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION



$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$



$$\left(\frac{M_{Rdy}}{M_{Rdy0}}\right)^2 + \left(\frac{M_{Rdz}}{M_{Rdz0}}\right)^2 = 1$$

Checking relationship (6) becomes:

(7) ……… $$\left(\frac{M_{Edy}}{M_{Rdy0}}\right)^a + \left(\frac{M_{Edz}}{M_{Rdz0}}\right)^a \leq 1$$

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

EXPONENT *a*

SR EN 1992-1-1:2004

N_{Ed}/N_{Rd} =	0,1	0,7	1,0
a =	1,0	1,5	2,0

$$N_{Rd} = bhf_{cd} + A_{s,tot} f_{yd}$$

STAS 10107/0-90

$\frac{N_{Ed}}{bhf_{cd}}$	Bar arrangement		
	A	**B**	**C**
	4 bars, in the corner	more than 4 bars $A_{sy} \cong A_{sz}$	more than 4 bars $A_{sy} = (1,5...2,0)A_{sz}$
0,1	1,60	1,70	1,75
0,2	1,35	1,60	1,50
0,3	1,25	1,55	1,40
0,4	1,20	1,50	1,35
0,5	1,20	1,45	1,35
0,6	1,35	1,45	1,40
0,7	1,55	1,50	1,50
0,8	1,75	1,60	1,60

1. Exponent was evaluated on the basis of numerical analysis on the computer using general method (chp. 6.1).

2. The exponent was determined in such a way that, for diagonal of the section, the simplified method to give the same result as the general method (chp. 6.1).

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

8.4.3. Cross section check

Input data	Output data
b; h; $A_{s,tot}$; N_{Ed}; M_{Edy}; M_{Edz}; f_{cd}; f_{yd}; c_{nom}	Fulfillment of the condition (7)

Section verification involves the following steps:

- design axial resistance of section: $N_{Rd} = A_c f_{cd} + A_{s,tot}\ f_{yd}$
- determination of the coefficient *a* depending on the ratio N_{Rd}/N_{Ed}
- establishing reinforcements $(A_{s1} = A_{s2})_y$ and $(A_{s1} = A_{s2})_z$; bars in the corners are considered for every direction
- calculation of resisting bending moment M_{Rdy} for N_{Ed} and A_{sy}
- calculation of resisting bending moment M_{Rdz} for N_{Ed} and A_{sz}
- checking condition $\left(\frac{M_{Edy}}{M_{Rdy0}}\right)^a + \left(\frac{M_{Edz}}{M_{Rdz0}}\right)^a \le 1$

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

8.4.4. Reinforcement calculation

Input data	**Output data**
b; h; N_{Ed}; M_{Edy}; M_{Edz}; f_{cd}; f_{yd}; c_{nom}	$A_{s,tot}$

Reinforcement area is calculated for $M_{Rd} = M_{Ed}$, namely:

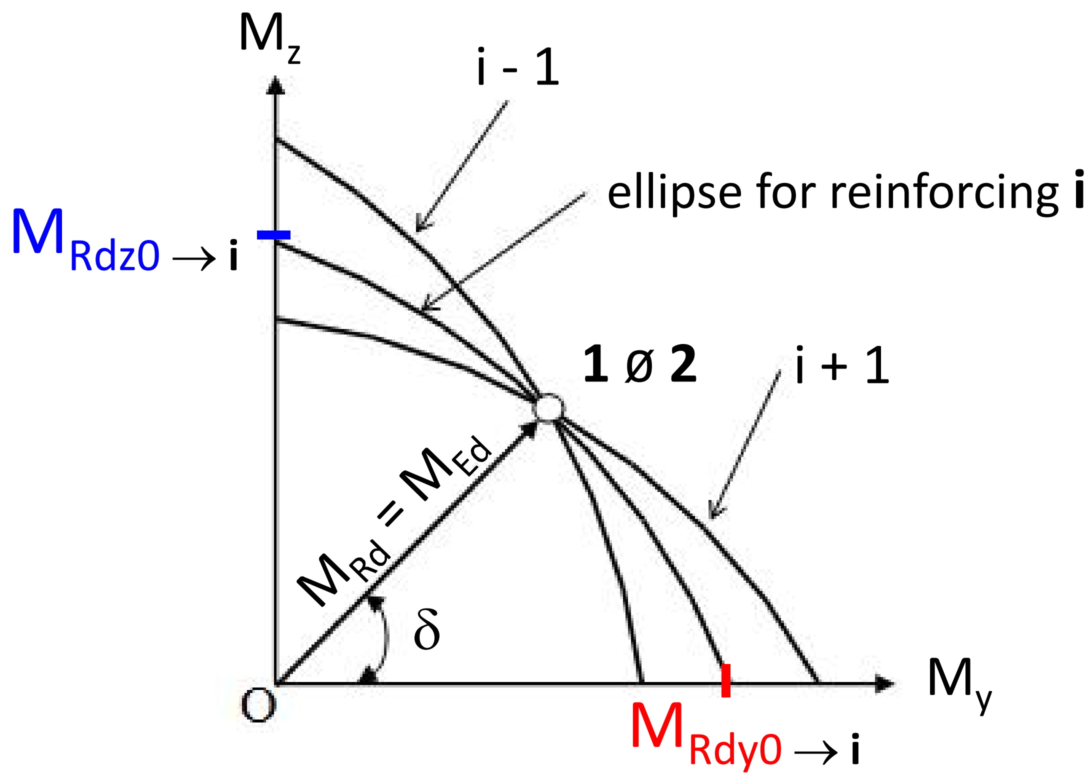
$$\left(\frac{M_{Edy}}{M_{Rdy0}}\right)^a + \left(\frac{M_{Edz}}{M_{Rdz0}}\right)^a = 1 \rightarrow$$ overlapping of points 1 and 2 (slide 79)

There is a problem: two unknowns & one equation

M_{Rdy}; actually $(A_{s1} = A_{s2})_y$
M_{Rdz}; actually $(A_{s1} = A_{s2})_z$

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Consequently, reinforcement calculation involves an infinity of solutions.



8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Additional relationship is needed between M_{Rdy} & M_{Rdz}

Between bearing capacities M_{Rdy} & M_{Rdz} to be the same ratio as between the bending moments M_{Edy} & M_{Edz} :

$$\frac{M_{Rdy}}{M_{Rdz}} = \frac{M_{Edy}}{M_{Edz}}$$

$$\frac{M_{Edy}}{M_{Rdy}} = \frac{M_{Edz}}{M_{Rdz}}$$

In this case equation (7) becomes:

(8) ……… $$\left(\frac{M_{Edy}}{M_{Rdy}}\right)^a = \left(\frac{M_{Edz}}{M_{Rdz}}\right)^a \leq 0{,}5$$

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

The calculation procedure is as follows:

- it is estimated $A_{s,tot}$
- $N_{Rd} = A_c f_{cd} + A_{s,tot}$
- choose exponent *a* depending on N_{Ed}/N_{Rd}
- according to (8), choose $\Omega = \left(\frac{M_{Edy}}{M_{Rdy}}\right)^a = \left(\frac{M_{Edz}}{M_{Rdz}}\right)^a \leq 0,5$
- $\frac{M_{Edy}}{M_{Rdy}} = \frac{M_{Edz}}{M_{Rdz}} = \sqrt[a]{\Omega}$
- required bearing capacity for y axis: $M_{Rdy} = M_{Edy} / \sqrt[a]{\Omega}$
- required bearing capacity for z axis: $M_{Rdz} = M_{Edz} / \sqrt[a]{\Omega}$

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

- calculation of reinforcement $(A_{s1} = A_{s2})_y$ shall be made for N_{Ed} and $M_{Edy} / \sqrt[a]{\Omega}$ in order to achieve required M_{Rdy}

- calculation of reinforcement $(A_{s1} = A_{s2})_z$ shall be made for N_{Ed} and $M_{Edz} / \sqrt[a]{\Omega}$ in order to achieve required M_{Rdz}

- bar detailing

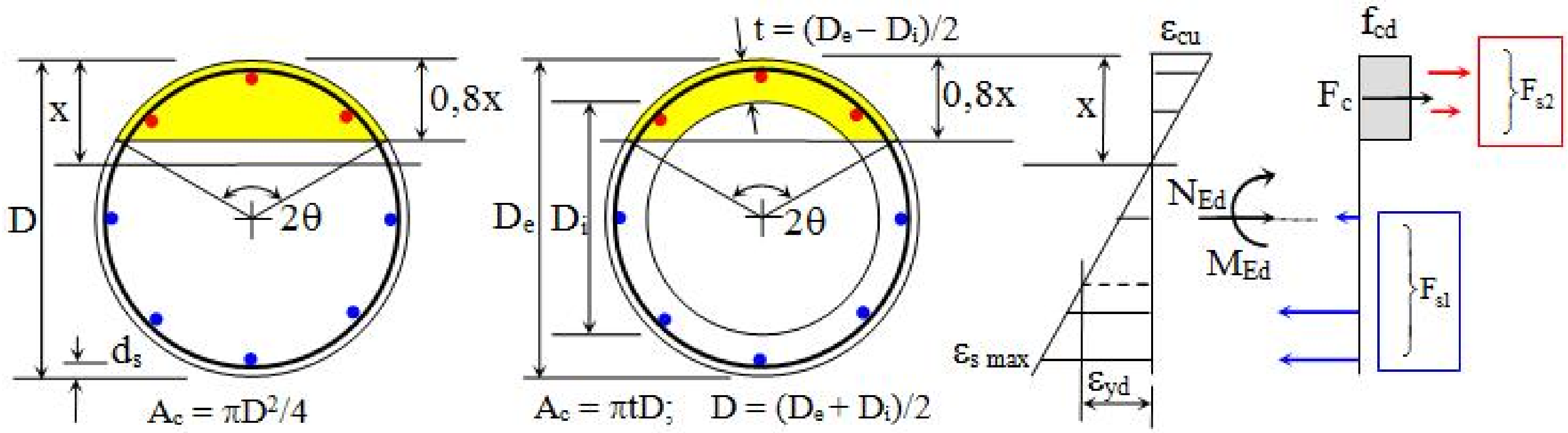
- if $(A_{s1} = A_{s2})_y$ is rounded up then $(A_{s1} = A_{s2})_z$ is rounded down

- with $A_{s,tot\ eff}$ compute the new N_{Rd}; if necessary calculation is made again

Advantage: biaxial bending is divided in two uniaxial bending with increased moments

Note: using exponent from former romanian code no recalculation is required because exponent *a* depends only on N_{Ed}/bhf_{cd}

8.5. CIRCULAR/RING-SHAPED COLUMNS



Bars are evenly distributed along the section contour

Reinforcement is considered evenly distributed on the contour if in the section there are at least six bars

Calculation is based on the assumptions from chp. 6.1

In case of ring-shaped (annular) section it is recommended that between the inner radius and the outer radius to have the following relation:

$$r_i \geq 0{,}5 r_e$$

8.5. CIRCULAR/RINGED-SHAPED COLUMNS

Failure is produced by:

- yielding of the most tensioned bars followed by crushing of compression concrete;
- crushing of compression concrete without yielding of tension bars;
- whatever is the way of failure, there are bars which are not yielding.

8.5. CIRCULAR/RINGED-SHAPED COLUMNS

Approximate evaluation of reinforcement for $0{,}15 \leq \omega_{tot} \leq 1{,}0$

$$A_{s,tot} = \omega_{tot} A_c \frac{f_{cd}}{f_{yd}}$$

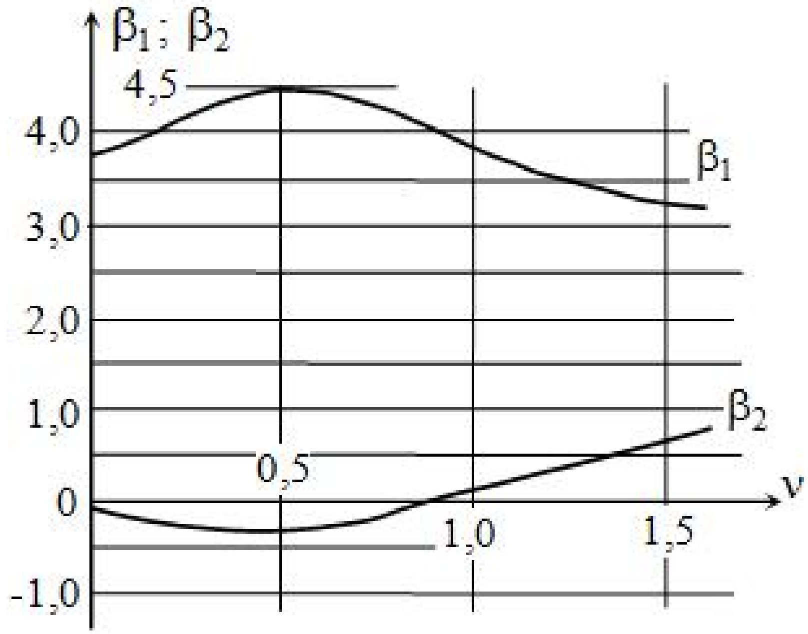
with:

$$\omega_{tot} = \beta_1 \mu + \beta_2$$

$$\mu = M_{Ed} / A_c D f_{cd}$$

$$A_c = 0{,}25 \pi D^2$$

β_1, β_2 - coefficients depending on $\nu = N_{Ed}/A_c f_{cd}$

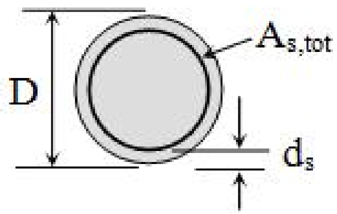


8.5. CIRCULAR/RINGED-SHAPED COLUMNS

Tools for current calculations

http://www.library.upt.ro/index.html?cursuri → File: 10_STALPI.pdf

Anexa 10.4. Tabele pentru calculul stâlpilor cu secţiune circulară



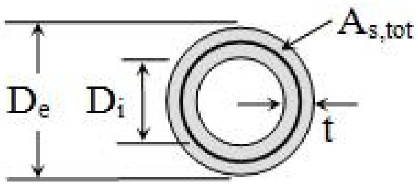
$$\nu = \frac{N_{Ed}}{A_c f_{cd}} \qquad \mu = \frac{M_{Ed}}{A_c D f_{cd}} \qquad \omega_{tot} = \frac{A_{s,tot} f_{yd}}{A_c f_{cd}} \qquad A_{s,tot} = \omega_{tot} A_c \frac{f_{cd}}{f_{yd}}$$

	Purpose of calculation →	$A_{s,tot}$	M_{Rd}
1	Input data	μ_{Ed} & ν_{Ed}	ν_{Ed} & ω
2	Output data	ω_{req}	μ_{Rd}
3	Result	$A_{s,tot} = \omega_{req} A_c f_{cd}/f_{yd}$	$M_{Rd} = \mu_{Rd} A_c D f_{cd}$

	Values 1000μ for ω_{tot} =									
	0	0,20	...	ω_{req} ↑	ω ↓	...	ω	...	0,45	0,50
1,00										
...										
ν_{Ed}	- - - reinforcement design - - - →			$1000\mu_{Ed}$						
...										
ν_{Ed}	- - - - - - M_{Rd} calculation - - - - - - →				$1000\mu_{Rd}$					
...										
0										

8.5. CIRCULAR/RINGED-SHAPED COLUMNS

Anexa 10.5 Tabele pentru calculul stâlpilor cu secţiune inelară



$D = 0{,}5(D_e + D_i)$

$t = 0{,}5(D_e - D_i)$

$A_c = \pi t D$

$$\nu = \frac{N_{Ed}}{A_c f_{cd}} \quad \mu = \frac{M_{Ed}}{A_c D f_{cd}} \quad \omega_{tot} = \frac{A_{s,tot} f_{yd}}{A_c f_{cd}} \quad A_{s,tot} = \omega_{tot} A_c \frac{f_{cd}}{f_{yd}}$$

	Purpose of calculation →	$A_{s,tot}$	M_{Rd}
1	Input data	μ_{Ed} & ν_{Ed}	ν_{Ed} & ω
2	Output data	ω_{req}	μ_{Rd}
3	Result	$A_{s,tot} = \omega_{req} A_c f_{cd}/f_{yd}$	$M_{Rd} = \mu_{Rd} A_c D f_{cd}$

	Values 1000μ for ω_{tot} =									
	0	0,20	...	ω_{req}	ω	...	ω	...	0,45	0,50
1,00										
...										
ν_{Ed}	- - - reinforcement design - - - →			$1000\mu_{Ed}$						
...										
ν_{Ed}	- - - - M_{Rd} calculation - - - - →				$1000\mu_{Rd}$					
...										
0										

8.6. DETAILING OF COLUMNS

EN 1992-1-1:2004

SR EN 1992-1-1:2006

National Annex SR EN 1992-1-1/NB:2008

P100-1/2013 → very specific provisions & highly severe

ANCHORAGE & BAR LAPS → CHP. 2.2

CROSS SECTION DIMENSIONS

Usually $h/b \leq 2{,}5$, maximum value being 4

The minimum size of the rectangular cross section is 300 mm

The minimum diameter of circular cross section is 300 mm

Usually sizes are multiples of 50 mm

LONGITUDINAL REINFORCEMENTS

ϕ_{min} = 8 mm; ….. NA: 12 mm; …. in romanian practice ¤ ñ **14 mm**

$$A_{s\,min} = \max \begin{cases} 0{,}1N_{Ed}/f_{yd} \\ 0{,}2\%A_c; \text{ …… NA: } 0{,}4\%A_c \end{cases}$$

$A_{s\,max} = 4\%A_c$

8.6. DETAILING OF COLUMNS

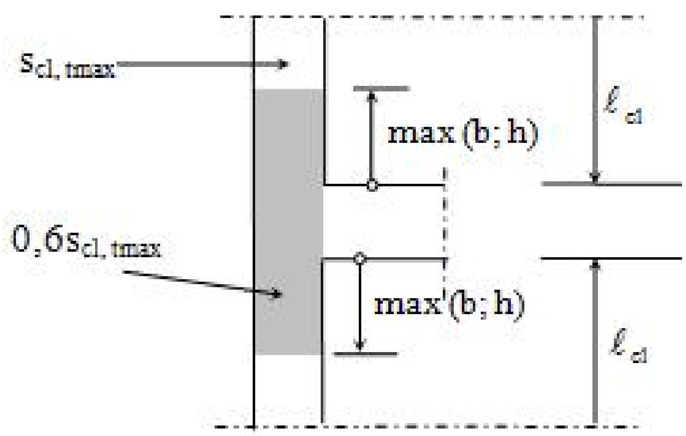
TRANSVERSAL REINFORCEMENTS
- shear force;
- compressed concrete confinement;
- no buckling of longitudinal bars between stirrups

$$\phi \geq \max \begin{cases} 6\text{ mm} \\ \phi_{long}/4 \end{cases}$$

spacing of the transverse reinforcement $s_{cl,t} \leq s_{cl,tmax} = \max \begin{cases} 20\phi_{min\_long} \\ \min(b; h) \\ 400\text{ mm} \end{cases}$

$s_{cl,tmax}$ should be reduced by a factor 0,6:

- above or below a beam or slab
- near lapped joints if $\phi_{max} > 14$ mm



8.6. DETAILING OF COLUMNS

TRANSVERSAL REINFORCEMENTS
- shear force;
- compressed concrete confinement;
- no buckling of longitudinal bars between stirrups

Weak stirrup = small ϕ & large distance between stirrups

Northridge Earthquake, 1994

Weak stirrups:
- buckling of longitudinal bars between stirrups
- no confinement of compressed concrete

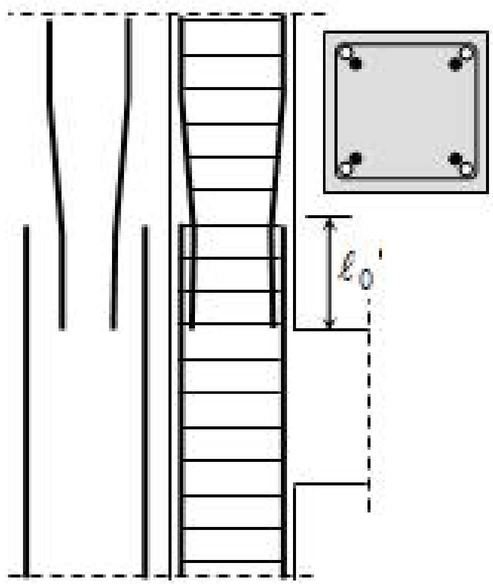
Buckling in lap zone with weak stirrups

San Fernando, 1971

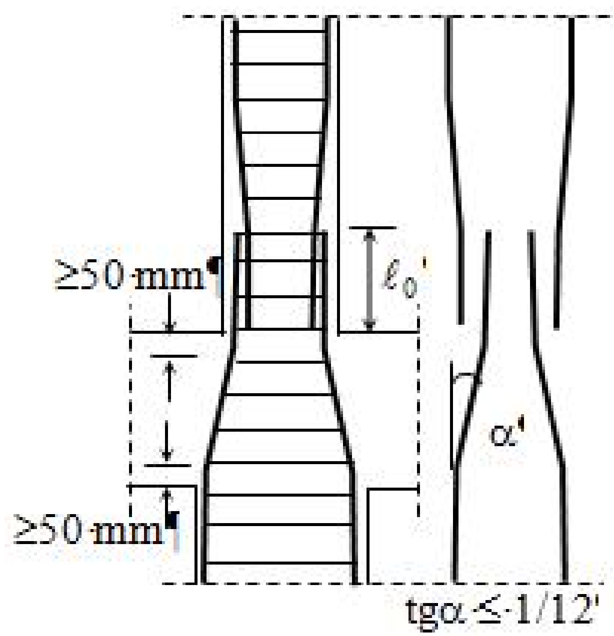
High V_{Ed} with weak stirrups (0,6 m)

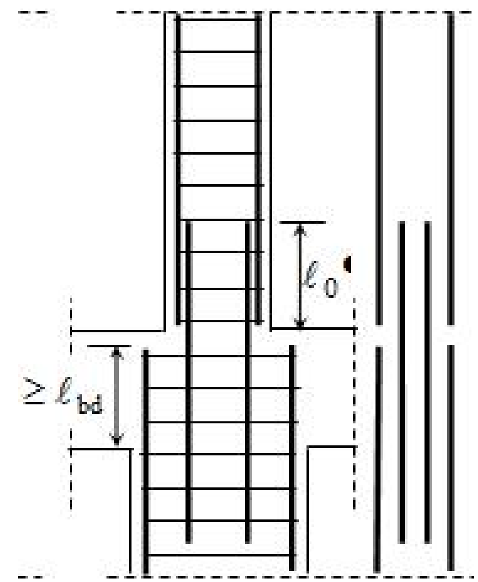
8.6. DETAILING OF COLUMNS

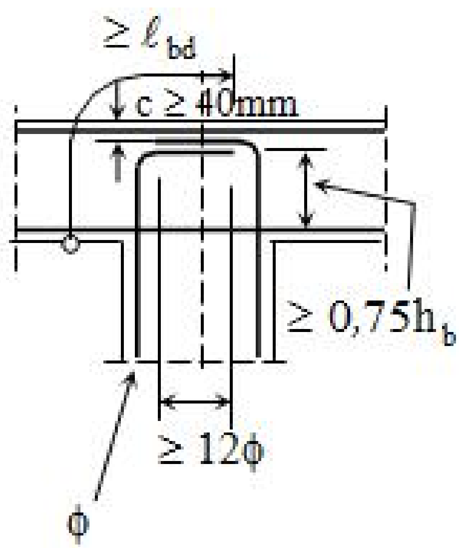
ARRANGEMENT OF BARS



Changes In Column Size



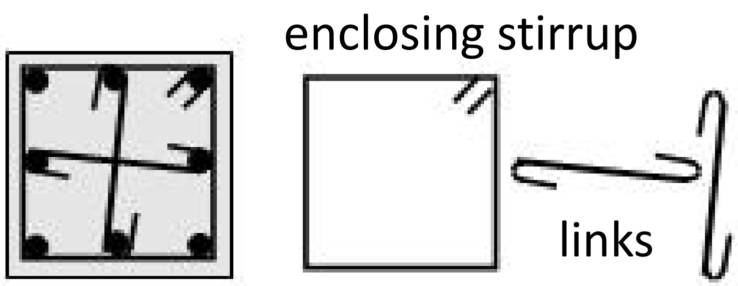


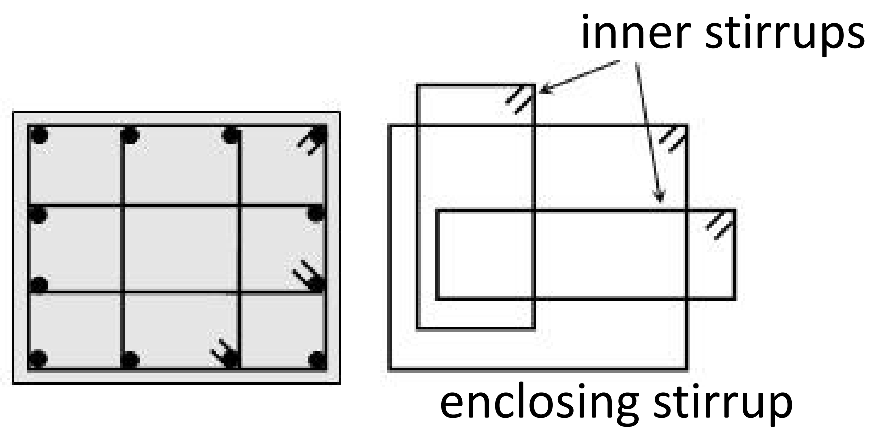


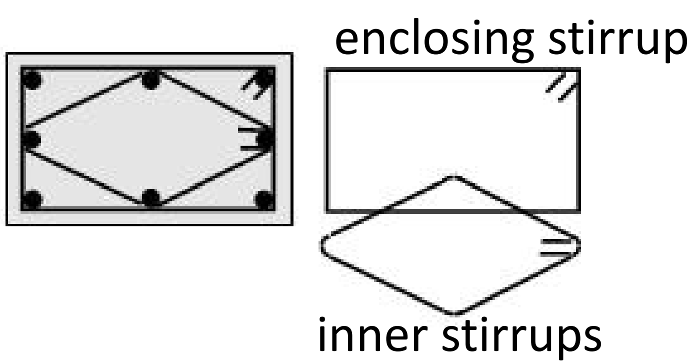
if tgα > 1/12

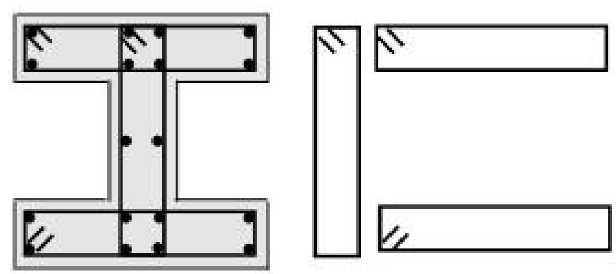
the spacing of transverse reinforcement should be calculated, taking account of the lateral forces involved

8.6. DETAILING OF COLUMNS





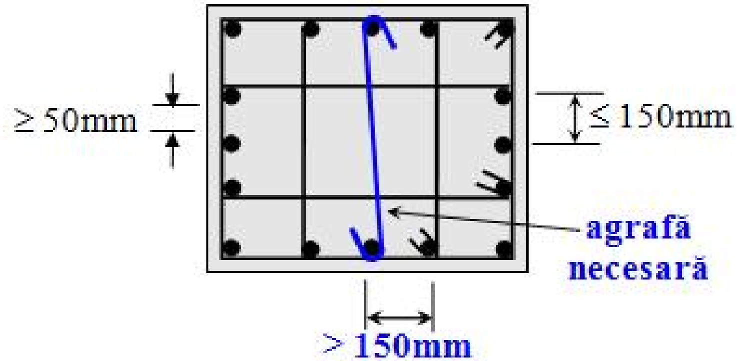


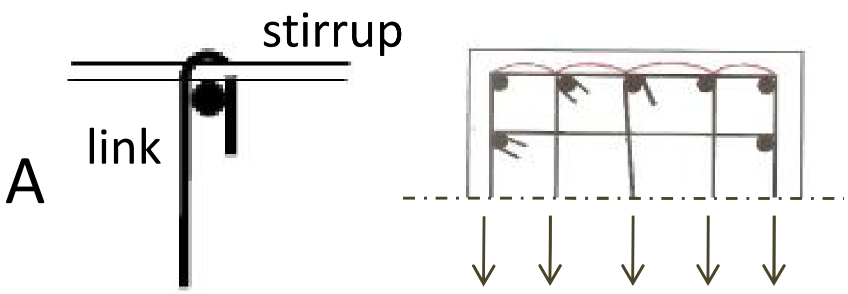


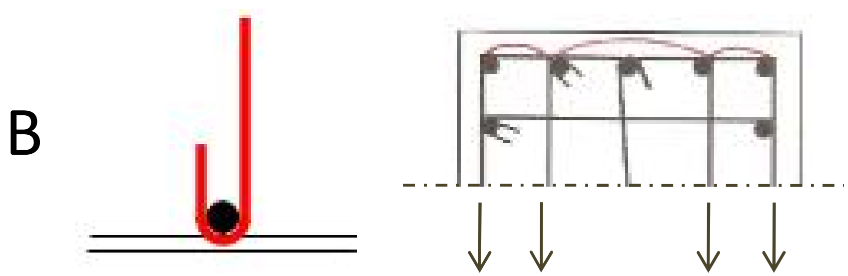
Every longitudinal bar placed in a corner of the section should be held by transverse reinforcements

8.6. DETAILING OF COLUMNS

No bar should be further than 150 mm from a restrained bar (in corner of stirrup; connected to a link)







Due to compressive force there is longitudinal shortening & transversal swelling of concrete

Red curves: deformed shape of the stirrup produced by swelling of concrete

Arrows show bars in tension due to swelling of concrete

Link in case **A** has contribution to confinement

Link in case **B** has no contribution to confinement