8. COLUMNS

COLUMN = ELEMENT SUBJECTED TO:

BENDING MOMENT & COMPRESSIVE FORCE

ECCENTRIC COMPRESSIVE FORCE

\[ e = \frac{M_{Ed}}{N_{Ed}} \]
8. COLUMNS

RECTANGULAR SECTION

Eccentric compression

CIRCULAR SECTION

RING-SHAPED SECTION

Compression with biaxial bending
8. COLUMNS

DESIGN SITUATIONS

Ductility class DCH:
\[ N_{Ed} \leq 0,45A_c f_{cd} < N_{lim} \]
\[ N_{Ed} \leq 0,50A_c f_{cd} \approx N_{lim} \]

Ductility class DCM:
\[ N_{Ed} \leq 0,40A_c f_{cd} < N_{lim} \]
\[ N_{Ed} \leq 0,55A_c f_{cd} \approx N_{lim} \]

Ductility class DCL: only in areas with \( a_g \leq 0,10g \)

P100-1/2006

P100-1/2013
8. COLUMNS

COLUMNS + GIRDERS = FRAME

SENSITIVE TO LATERAL DISPLACEMENT

HIGH VALUES OF THE BENDING MOMENTS IN COLUMNS AND GIRDERS
8. COLUMNS

BRACING SYSTEMS ARE USED IN ORDER TO REDUCE THE LATERAL DISPLACEMENT

AS A RESULT OF THE ABOVE:

- BRACED COLUMNS
- UN-BRACED COLUMNS
8. COLUMNS

The ends of the columns can have different types of connections with neighboring elements:

• Restrained displacements & rotations (as foundations)

• Partially free displacements & rotations depending on:
  - stiffness of neighboring elements
  - with or without bracings

• Free displacements & rotations
8. COLUMNS

DEFINITIONS

First order effects - $M_{0 Ed}$: action effects calculated without consideration of the effect of structural deformations, but including geometric imperfections

Second order effects - $\Delta M$: additional action effects caused by structural deformations

Second order moment - $M_{Ed} = \eta M_{0 Ed}$ ($\eta > 1,0$): bending moment, taking into account the influence of structural deformations
The second order effects are produced by two types of deformations:

Lateral deformations of the story:
- depends on the structural stiffness,
- characteristic for unbraced structures

Individual deformations of the element:
- depends on slenderness & neighboring elements
- characteristic for braced structures
- may cause buckling
**Buckling**: failure due to instability of a member or structure under perfectly axial compression and without transverse load

**Buckling load**: the load at which buckling occurs; for isolated elastic members it is synonymous with the Euler load

**Effective length**: a length used to account for the shape of the deflection curve; it can also be defined as buckling length.

**Isolated members**: members that are isolated, or members in a structure that for design purposes may be treated as being isolated.
8. COLUMNS

8.1. GEOMETRIC IMPERFECTIONS

8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

8.5. CIRCULAR/RING-SHAPED COLUMNS

8.6. DETAILING OF COLUMNS
8.1. GEOMETRIC IMPERFECTIONS

The unfavorable effects of possible deviations shall be taken into account in the analysis of members and structures.

Deviations:
- cross section dimensions
- geometry of the structure
- position of loads

Deviations in cross section dimensions:
- are normally taken into account in the material safety factors
- these should not be included in structural analysis
- for cross section design it is necessary to assume the minimum eccentricity, $e_0 = h/30$ but not less than 20 mm where $h$ is the depth of the section

\[ h \]
8.1. GEOMETRIC IMPERFECTIONS

Deviations in the geometry of the structure:

• shall be taken into account in ultimate limit states in:
  - persistent design situations
  - accidental design situations

• need not be considered for serviceability limit states
8.1. GEOMETRIC IMPERFECTIONS

IMPERFECTIONS MAY BE REPRESENTED BY AN INCLINATION

\[ \theta_i = \theta_0 \alpha_h \alpha_m \]

\[ \theta_0 = 1/200 \ - \text{basic value} \]

- \( \alpha_h \) is the reduction factor for length or height:
- \( \alpha_h = 2/\sqrt{l} \); \( 2/3 \leq \alpha_h \leq 1 \)
- \( \alpha_m \) is the reduction factor for number of members:
- \( \alpha_m = \sqrt{0.5(1+1/m)} \)
- \( l \) is the length or height [m], see (4)
- \( m \) is the number of vertical members contributing to the total effect
8.1. GEOMETRIC IMPERFECTIONS

UNBRACED STRUCTURE

\[ H_i = \theta_i \sum F \]
8.1. GEOMETRIC IMPERFECTIONS

**BRACED STRUCTURE**

$$H_i = \theta_i \left( \Sigma N_b - \Sigma N_a \right)$$

**ACTION ON FLOOR**

$$H_i = \theta_i \left( \Sigma N_b + \Sigma N_a \right)/2$$
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.1. TOPIC OF SECOND ORDER EFFECTS

First order effects - $M_{0Ed}$: action effects calculated without consideration of the effect of structural deformations, but including geometric imperfections.

Second order effects - $\Delta M$: additional action effects caused by structural deformations.

Second order moment - $M_{Ed} = \eta M_{0Ed} \ (\eta > 1,0)$: bending moment, taking into account the influence of structural deformations.
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

ELEMENT SENSITIVITY TO SECOND ORDER EFFECTS DEPENDS ON SLENDERNESS RATIO

\[ \lambda = \frac{\ell_0}{i} \]

- \( \ell_0 \) - effective length
- \( i \) - radius of gyration

THERE ARE 3 CASES OF COLUMN FAILURE DEPENDING ON SLENDERNESS RATIO

Cantilevered column

Longitudinal force increases from zero till column failure

\[ M_{0Ed} = Ne \]

\[ \Delta M = N\delta \]
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Slender columns 35 < $\lambda$ ≤ 120
- important second order effects
- bending moment increases faster than longitudinal force → curve b
- element failure is produced by exhaustion of bearing capacity to a force equal to $N_{Rd}^{b} < N_{B}^{b}$
- $N_{B}^{b}$ - is the buckling force

Very slender columns $\lambda$ > 120
- buckling occurs at the force $N_{B}^{c}$
- deformations increase indefinitely under constant force
- in this case bearing capacity $N_{Rd}^{c} = N_{B}^{c}$

Short columns $\lambda$ ≤ 35
- negligible second order effects
- bending moment is proportional to the longitudinal force → line a
- element failure is produced by exhaustion of bearing capacity to a force equal to $N_{Rd}^{a}$
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

\[ N_B = \frac{\pi^2 EI}{l_0^2} \]

- Euler formula for buckling load of isolated columns

(1707 – 1783)
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

\[ N_B = \frac{\pi^2 EI}{l_0^2} \]

Euler formula does not correctly describe the correlation between bearing capacity and element slenderness.

EC2 defines \( \lambda_{\text{lim}} \)

Real correlation

2nd order effects may be ignored

<table>
<thead>
<tr>
<th>Column:</th>
<th>short</th>
<th>slender</th>
<th>very slender</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column failure:</td>
<td>ultimate limit state</td>
<td>buckling</td>
<td></td>
</tr>
</tbody>
</table>

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8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.2. SLENDERNESS AND EFFECTIVE LENGTH OF ISOLATED MEMBERS

a) double pined column in braced structures; not suitable in seismic areas
b) column in one level unbraced precast structure
c) column in one level braced precast structure
d) double fixed column in braced structure; bottom end = foundation !; top end = very stiff girder ?
e) case d in braced structure
f) column in braced structure; node rotation is possible
g) foundation rotation of case b
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

- Braced structure:  
  - no lateral deformations  
  - node rotations

- Double pinned column

- Double fix column

- Real column

- Extreme situations
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Unbraced structure:
- lateral deformations
- node rotations

Double fix column & free lateral deformations

Real column
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Regular frames

- Braced columns
- Unbraced columns

Alternative procedure for $k$ in case of braced frame

$$k = \frac{(EI/\ell)_c}{\sum 2(EI/\ell)_b} \geq 0.1$$

$c$ – considered column

$b$ – adjacent girders at the top & bottom column ends

$k_1, k_2$ are the relative flexibilities of rotational restraints at ends 1 and 2 respectively:

$$k = \left(\frac{\theta}{M}\right) \cdot \frac{(EI/\ell)}{l}$$

$\theta$ is the rotation of restraining members for bending moment $M$;

$EI$ is the bending stiffness of compression member, and

$l$ is the clear height of compression member between end restraints

Static analysis is required
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

PRELIMINARY ASSESSMENT: \( \ell_0 = \beta \cdot \ell \)

<table>
<thead>
<tr>
<th>Top end condition</th>
<th>Bottom end condition</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Braced frames</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.75</td>
<td>0.80</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
<td>0.85</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.90</td>
<td>0.95</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Unbraced frames</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.3</td>
<td>1.5</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.6</td>
<td>1.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

1 – fixed to foundation; monolithically connected to a beam \( h_b \geq h_c \)
2 – connected to a slab; monolithically connected to a beam \( h_b < h_c \)
3 - connected to simple supported beam
4 – unrestrained
For members with varying normal force and/or cross section

\[ \ell_0 = \pi \sqrt{\frac{E I_{\text{repr}}}{N_B}} \]

*EI_{\text{repr}}* – representative stiffness

*N_B* – buckling load calculated by appropriate software or numerical methods
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.2. CREEP INFLUENCE

1\textsuperscript{ST} order bending moment:
\[ M_{0Ed} = N_{Ed} e \]

2\textsuperscript{nd} order bending moment without creep influence:
\[ M_{Ed} = M_{0Ed} + N_{Ed} \delta \]

2\textsuperscript{nd} order bending moment with creep influence:
\[ M_{Ed\phi} = M_{0Ed} + N_{Ed} (1 + \varphi) \delta \]

The duration of loads may be taken into account by:
\[ \varphi_{ef} = \varphi(\infty, t_{0}) \frac{M_{0Edq_{p}}}{M_{0Ed}} \]
- calculated for section with maximum bending moment
- a representative mean value
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.3. SIMPLIFIED CRITERIA FOR SECOND ORDER EFFECTS

Second order effects may be ignored if they are less than 10% of the corresponding first order effects.

8.2.3.1. Slenderness criterion for isolated members

Second order effects may be ignored if $\lambda \leq \lambda_{\text{lim}}$

$\lambda_{\text{lim}} = \frac{20\text{ABC}}{\sqrt{n}}$
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

\[ A = \frac{1}{1 + 0.2 \varphi_{ef}} \quad \text{(if } \varphi_{ef} \text{ is not known, } A = 0.7 \text{ may be used)} \]

\[ B = \sqrt{1 + 2 \omega} \quad \text{(if } \omega \text{ is not known, } B = 1.1 \text{ may be used)} \]

\[ C = 1.7 - r_m \quad \text{(if } r_m \text{ is not known, } C = 0.7 \text{ may be used)} \]

\( \varphi_{ef} \) effective creep ratio;

\( \omega = \frac{A_s f_{yd}}{(A_c f_{cd})} \); mechanical reinforcement ratio;

\( A_s \) is the total area of longitudinal reinforcement

\( n = \frac{N_{Ed}}{(A_c f_{cd})} \); relative normal force

\( r_m = \frac{M_{01}}{M_{02}} \); moment ratio

\( M_{01}, M_{02} \) are the first order end moments, \( |M_{02}| \geq |M_{01}| \)

\[ r_m > 0 \quad r_m < 0 \]
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

$\lambda_{\text{lim}}$ based on accepted simplifications for coefficients A, B & C

<table>
<thead>
<tr>
<th>Column:</th>
<th>Unbraced</th>
<th>Braced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending moment diagram</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>$M_{01}$ &amp; $M_{02}$ predominant effect of geometric imperfections</td>
<td></td>
<td>$M_{01} = M_{02}$</td>
</tr>
<tr>
<td>$C$</td>
<td>0.7</td>
<td>1.7</td>
</tr>
<tr>
<td>$\lambda_{\text{lim}}$</td>
<td>$10.78 / \sqrt{n}$</td>
<td>$26.20 / \sqrt{n}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$41.60 / \sqrt{n}$</td>
</tr>
</tbody>
</table>
Global second order effects in buildings may be ignored if

\[ F_{V,Ed} \leq k_1 \cdot \frac{n_s}{n_s + 1,6} \cdot \frac{\sum E_{cd} I_c}{L^2} \]

- \( F_{V,Ed} \) is the total vertical load (on braced and bracing members)
- \( n_s \) is the number of storeys
- \( L \) is the total height of building above level of moment restraint
- \( E_{cd} \) is the design value of the modulus of elasticity of concrete, see 5.8.6 (3)
- \( I_c \) is the second moment of area (uncracked concrete section) of bracing member(s)

\( k_1 = 0.31 \)

\( k_1 = 0.62 \) if it can be verified that bracing members are uncracked in ultimate limit state
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Previous expression is valid only if all the following conditions are met:

- torsional instability is not governing, i.e. structure is reasonably symmetrical

- global shear deformations are negligible (as in a bracing system mainly consisting of shear walls without large openings)

- bracing members are rigidly fixed at the base, i.e. rotations are negligible

- the stiffness of bracing members is reasonably constant along the height

- the total vertical load increases by approximately the same amount per storey
8.2.4. Methods of analysis

General method based on nonlinear analysis
EC2 – 5.8.6

Method based on nominal curvature

Method based on nominal stiffness

Last two methods are simplified solutions.

There is the possibility of the second order static analysis (nonlinear static analysis) based on nominal stiffness. Efforts resulting from this calculation include second order effects.
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.4.1. Method based on nominal curvature

Method is suitable for isolated columns with constant $N_{Ed}$ and defined $l_0$

Second order effects depend on element deformed shape

Maximum deflection $e_2$ depends on curvature $1/r$ in the moment of failure

$1/r$ depends on $N_{Ed} \&$ creep
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

CURVATURE

For members with constant symmetrical cross sections, including reinforcement:

\[ \frac{1}{r} = K_r K_\varphi \cdot \frac{1}{r_0} \]

- \( K_r \) – correction factor for axial load
- \( K_\varphi \) – correction factor for creep

\( \frac{1}{r_0} \) - maximum curvature corresponds to balance situation (maximum bending moment)

\[ \frac{1}{r_0} = \frac{\varepsilon_{yd}}{d - x_{lim}} \]
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

**Correction factor** \( K_r \)

Higher \( N_{Ed} \) smaller curvature \( 1/r \)

\[
\frac{N_u - N_{bal}}{1/r_0} = \frac{N_u - N_{Ed}}{1/r}
\]

\[
l/r = 1/r_0 \frac{N_u - N_{Ed}}{N_u - N_{bal}} : A_c f_{cd}
\]

\[
n = \frac{N_{Ed}}{A_c f_{cd}}
\]

\[
n_u = 1 + \omega
\]

\[
\omega = \frac{A_{s,tot} f_{yd}}{A_c f_{cd}}
\]

\[
K_r = \frac{n_u - n}{n_u - n_{bal}} \leq 1,0
\]

\[
N_u = A_{s,tot} f_{yd} + A_c f_{cd} \leftrightarrow N_{Rd}^c
\]

\[
N_{bal} = \xi_{lim} b df_{cd} \approx 0,4 b df_{cd} \leftrightarrow N_{lim}
\]

Chp. 6
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Correction factor $K_\varphi$

$$K_\varphi = 1 + \beta \varphi_{ef} \geq 1,0$$

$$\beta = 0,35 + f_{ck}/200 - \lambda/150$$

$$\lambda = \frac{\ell_0}{i} \quad \leftarrow \text{slide 14}$$

$$\varphi_{ef} = \varphi(\infty, t_0) \frac{M_{0Edq_p}}{M_{0Ed}} \quad \leftarrow \text{slide 24}$$
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

BENDING MOMENTS

\[ M_{Ed} = M_{0Ed} + M_2 \quad \text{........... (*)} \]
\[ M_2 = N_{Ed} e_2 \]
\[ e_2 = \left(\frac{1}{r}\right)\ell_0^2 / c \]

\( c \) - factor depending on the curvature distribution; for constant cross section:
\( \pi^2 \approx 10 \) – for sine-shaped distribution of curvature
\( 8 \) – for constant curvature distribution (constant bending moment)

\( 1/r \) – curvature \( \leftarrow \) slide 32
\( l_0 \) – effective length \( \leftarrow \) slides 18 ... 23

The meaning of relation (*) is the summation of \( M_{0Ed} \) diagram with \( M_2 \) diagram.
The resulting diagram allows for the maximum bending moment.
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

1\textsuperscript{st} order bending moment $\rightarrow$ linear diagram; maximum value at the column ends

2\textsuperscript{nd} order bending moment $\rightarrow$ sine-shaped diagram between inflexion points

Braced column

Unbraced column

algebraic summation

arithmetic summation
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

**Braced columns**

Different first order end moments $M_{01}$ and $M_{02}$ may be replaced by an equivalent first order end moment $M_{0e}$

$$M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02}$$

$M_{01}$ and $M_{02}$ should have the same sign if they give tension on the same side, otherwise opposite signs. Furthermore, $|M_{02}| \geq |M_{01}|$. 
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Maximum 1\textsuperscript{st} order bending moments occur at the element ends.
The maximum 2\textsuperscript{nd} order bending moment occurs at about mid-length of column.
Therefore it is possible that the maximum bending moment is no longer at the element ends.

\[ M_{Ed} = \max(M_{02}; M_{0e} + M_2; M_{01} + 0.5M_2) \]
Lateral displacements may be generated by:
- asymmetry of the structure;
- horizontal seismic or wind forces.

All columns have the same mode of deformation due to high stiffness of reinforced concrete floors.

Therefore, it is reasonable to use an average curvature, even though the columns may have different rigidities.

Maximum 2\textsuperscript{nd} bending moment occurs at that end of the column which has the highest stiffness.
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Addition of 2nd bending moment to 1st bending moment

For the same rigidity at the both ends of column addition is done to the maximum 1st bending moment

For different rigidities of column ends the addition is done as follows:

- to the maximum 1st bending moment (which corresponds to highest rigidity)
- at the opposite end, the additional bending moment may be reduced in proportion to the ratio of the rigidities at the two ends of the column
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

8.2.4.2. Method based on nominal stiffness

In a second order analysis based on stiffness, nominal values of the flexural stiffness should be used, taking into account the effects of
• cracking,
• material non-linearity
• creep
on the overall behavior.

This also applies to adjacent members involved in the analysis:
• beams
• slabs.

Where relevant, soil-structure interaction should be taken into account.

The resulting design moment is used for the design of cross sections to bending moment and axial force
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

**NOMINAL STIFFNESS**

\[ EI = K_c E_{cd} I_c + K_s E_s I_s \]

- \( E_{cd} \) - Design value of the modulus of elasticity of concrete
- \( E_{cd} = E_{cm} / \gamma_{cE} \); \( \gamma_{cE} = 1.2 \)
- \( I_c \) - Moment of inertia of concrete cross section
- \( E_s \) - Design value of the modulus of elasticity of reinforcement
- \( I_s \) - Second moment of area of reinforcement, about the centroid of area of the concrete
- \( K_s = 1 \) - Factor for contribution of reinforcement
- \( K_c \) - Factor for effects of cracking, creep, etc.
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

\[ K_c = \frac{k_1 k_2}{(1 + \varphi_{ef})} \text{ if } \rho \geq 0.002 \]
\[ \rho = \frac{A_s}{A_c} \text{ - reinforcing ratio} \]
\[ A_s \text{ - total area of reinforcement} \]
\[ A_c \text{ - area of concrete section} \]
\[ \varphi_{ef} \text{ - effective creep ratio } \rightarrow \text{ slide 24} \]
\[ k_1 = \sqrt{f_{ck}}/20 \]
\[ k_2 = n \frac{\lambda}{170} \leq 0.20 \text{ with } \lambda \text{ - slenderness ratio} \]
\[ k_2 = n \cdot 0.3 \leq 0.20 \text{ if } \lambda \text{ is not defined} \]
\[ n = \frac{N_{Ed}}{A_c f_{cd}} \]
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

In statically indeterminate structures, unfavorable effects of cracking in adjacent members should be taken into account.

Expressions from slides 45 & 46 are not generally applicable to such members. Partial cracking and tension stiffening may be taken into account according chp. 16.3. *Simplified approach of deflection control*

However, as a simplification, fully cracked sections may be assumed. The stiffness should be based on an effective concrete modulus:

\[ E_{cd,ef} = \frac{E_{cd}}{1 + \phi_{ef}} \]

Note: Meaning of the text *Fully cracked section* is presented in chp. 16.3
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

MOMENT MAGNIFICATION FACTOR

The total design bending moment $M_{Ed}$, including second order effects, may be obtained by increasing $M_{0Ed}$ as follows:

$$M_{Ed} = M_{0Ed} \left[ 1 + \frac{\beta}{(N_B/N_{Ed}) - 1} \right] \quad \text{........ (**)}$$

$N_{Ed}$ – design value of axial force

$N_B$ – buckling load based on nominal stiffness

$\beta$ – factor depending on distribution of 1$^{\text{st}}$ and 2$^{\text{nd}}$ order moments

$$\beta = \pi^2/c_0$$ – for sine-shaped distribution of 2$^{\text{nd}}$ order moments of isolated columns

$c_0$ – factor depending on distribution of 1$^{\text{st}}$ order moment:

- $c_0 = 8$ for a constant bending moment
- $c_0 = 9,6$ for a parabolic distribution
- $c_0 = 12$ for symmetric triangular distribution
Where provision for $\beta$ or $c_0$ are not applicable, $\beta = 1$ is a reasonable simplification.

Consequently, relation (***) turns into:

\[
M_{Ed} = \frac{M_{0Ed}}{1 - N_{Ed}/N_B} = \eta M_{0Ed}
\]

\[
\eta = \frac{1}{1 - N_{Ed}/N_B}
\]
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Braced columns

For members without transverse load, different first order end moments $M_{01}$ and $M_{02}$ may be replaced by an equivalent constant first order moment $M_{0e}$ (see slide 37).

Depending on slenderness and axial force, the end bending moments can be greater than the magnified equivalent moment $\eta M_{0e}$

Therefore relation (**) from slide 45 is rewritten as follows:

$$M_{Ed} = M_{0e}\left[1 + \frac{\pi^2}{8\left(\frac{N_B}{N_{Ed}}\right)-1}\right] \geq M_{02}$$
8.2. SECOND ORDER EFFECTS WITH AXIAL FORCE

Unbraced columns

The same $l_0$ for all columns because they “work” together due to monolithic floor

Slide 27: $\varphi_{ef} = \varphi(\infty, t_0) \frac{M_{0Edqp}}{M_{0Ed}}$

Discussion on $M_{0Edqp}$ used for $\varphi_{ef}$: no horizontal variable loads (e.g. wind, bridge crane) are taken into account because do not induce creep
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.3.1. Balance situation

Chp. 6.5 – slide 17

\[ \xi_{lim} = \frac{x_{lim}}{d} = \frac{3,5}{3,5 + 1000f_{yd}/E_s} \]

\[ \Sigma F = 0 \]

\[ N_{lim} = F_c + F_{s2} - F_{s1} \]

\[ N_{lim} = F_c = 0,8b\xi_{lim}f_{cd} = 0,8\xi_{lim}bdf_{cd} \]
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

\[ \sum M = 0 \rightarrow \text{related to the } A_{s1} \text{ axis} \]

\[
M_{R, lim} + N_{lim} (0.5h - d_1) = F_c z_{lim} + F_{s2} (d - d_2) \\
M_{R, lim} + N_{lim} (0.5h - d_1) = 0.8b x_{lim} f_{cd} (d - 0.4x_{lim}) + F_{s2} (d - d_2) \\
M_{R, lim} + N_{lim} (0.5h - d_1) = 0.8 \xi_{lim} (1 - 0.4 \xi_{lim}) bd^2 f_{cd} + F_{s2} (d - d_2) \\
M_{R, lim} = \mu_{lim} bd^2 f_{cd} + A_{s2} f_{yd} (d - d_2) - N_{lim} (0.5h - d_1)
\]
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

TWO WAYS OF FAILURE

\[ N_{Ed} \leq N_{lim} \rightarrow \]
- compressive force with prevailing bending
  - ductile failure due to yield of
tensioned reinforcement
  - compulsory in case of seismic areas

\[ N_{Ed} > N_{lim} \rightarrow \]
- bending with prevailing compression
  - brittle failure by crushing of concrete
    without yielding of reinforcement \( A_{s1} \)
    (whether it is tensioned or compressed)
  - brittle character becomes stronger with the
    increasing of the compressive force
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.3.2. Section analysis

Stress diagram corresponds to yielding of $A_{s1}$ and $A_{s2}$
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

TENSION REINFORCEMENT YIELDING BEFORE CONCRETE CRUSHING
\[ \xi \leq \xi_{\text{lim}} \]

STRESS IN COMPRESSION REINFORCEMENT

There is yielding of compression reinforcement if \( \varepsilon_{s2} \geq \varepsilon_{yd} \)

\[ \varepsilon_{s2} = \varepsilon_{cu} \frac{x - d_{2s}}{x} \geq \varepsilon_{yd} \]

\[ x \geq \frac{\varepsilon_{cu} d_{s2}}{\varepsilon_{cu} - \varepsilon_{yd}} \]

\[ x \geq x_y \quad \sigma_{s2} = f_{yd} \]

\[ x < x_y \quad \sigma_{s2} < f_{yd} \]

- no yielding of compression reinforcement
- procedure in the chapter 6.4 (slide 12) applies
- simplified approach: \( F_c \) is acting at the level of \( F_{s2} \)

<table>
<thead>
<tr>
<th>Steel</th>
<th>PC52</th>
<th>PC60</th>
<th>S400</th>
<th>S500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_y )</td>
<td>1,69d_2</td>
<td>1,91d_2</td>
<td>1,98d_2</td>
<td>2,64d_2</td>
</tr>
<tr>
<td>STAS 10107/0-97</td>
<td></td>
<td></td>
<td></td>
<td>2,0d_2</td>
</tr>
</tbody>
</table>

\[ x \geq x_y \quad \sigma_{s2} = f_{yd} \]

\[ x < x_y \quad \sigma_{s2} < f_{yd} \]

- no yielding of compression reinforcement
- procedure in the chapter 6.4 (slide 12) applies
- simplified approach: \( F_c \) is acting at the level of \( F_{s2} \)
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

\[ \Sigma F = 0 \]

\[ N_{Ed} = F_c + F_{s2} - F_{s1} \]

(1) \( \xi = \frac{x}{d} \leq \xi_{lim} \) \( \Rightarrow \) the same as \( N_{Ed} \leq N_{lim} \) \( \rightarrow A_{s1} \) yields

Case I:

Let’s assume yielding

Case II:

\( \xi = \frac{x}{d} > \xi_{lim} \) \( \Rightarrow \) the same as \( N_{Ed} > N_{lim} \) \( \rightarrow A_{s1} \) does not yield
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

**Case I**: compression with prevailing bending - $A_{s1}$ yields (eccentric compression with large eccentricity)

$x \geq x_y \rightarrow A_{s2}$ yields

$\sum M = 0 \rightarrow$ related to the $A_{s1}$ axis

$M_{Ed} + N_{Ed} (0.5h - d_1) = F_c z + F_{s2} (d - d_2)$

slide 57: using relationship (1)

$M_{Ed} + N_{Ed} (0.5h - d_1) = N_{Ed} (d - 0.4x) + F_{s2} (d - d_2)$

$M_{Ed} = N_{Ed} (d - 0.4x) - N_{Ed} (0.5h - d_1) + F_{s2} (d - d_2)$

with $d = h - d_1$

$M_{Ed} = N_{Ed} (h - d_1 - 0.4x - 0.5h + d_1) + F_{s2} (d - d_2)$

(2) $\cdots$ $M_{Ed} = N_{Ed} (0.5h - 0.4x) + A_{s2} f_{yd} (d - d_2)$

resisting bending moment

$M_{Rd} = N_{Ed} (0.5h - 0.4x) + A_{s2} f_{yd} (d - d_2)$
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

\[ x < x_y \rightarrow A_{s2} \text{ does not yield} \]

simplified approach: \( F_c \) is located at the level of \( A_{s2} \)

\[ \sum M = 0 \rightarrow \text{related to the } A_{s2} \text{ axis:} \]

\[ M_{Ed} - N_{Ed}(0.5h - d_2) = F_{s1}(d - d_2) \]

\[ M_{Ed} = A_{s1} f_{yd} (d - d_2) + N_{Ed}(0.5h - d_2) \]

resisting bending moment

\[ M_{Rd} = A_{s1} f_{yd} (d - d_2) + N_{Ed}(0.5h - d_2) \]
Case II: bending with prevailing compression - $A_{s1}$ does not yield
(eccentric compression with low eccentricity)

$$x > x_{lim} \gg x_y \rightarrow A_{s2} \text{ yields}$$

Procedure described in cpt. 6.4 (slides 12, 13) should be applied using $\sigma_c - \varepsilon_c$ & $\sigma_s - \varepsilon_s$ diagrams

In what follows, relationships between the stress in reinforcement $A_{s1}$ and neutral axis position are used without the need for stress-strain diagram.
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

\[ x_{\text{lim}} < x \leq d \]

From triangles (red & black lines):

\[
\varepsilon_{\text{cu}} = \frac{x_{\lim}}{d - x_{\lim}} \frac{\varepsilon_{\text{yd}}}{d - x} = x \frac{\varepsilon_{s1}}{d - x} \quad \rightarrow \quad \varepsilon_{s1} = \frac{x_{\lim}}{x} \frac{d - x}{d - x_{\lim}} \varepsilon_{\text{yd}}
\]

\[
\sigma_{s1} = \frac{x_{\lim}}{x} \frac{d - x}{d - x_{\lim}} f_{\text{yd}} \quad \text{(tension)}
\]
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

It is accepted that $\sigma_{s1}$ is directly proportional to neutral axis depth

$$\sigma_{s1} = 4 \frac{x - d}{d} f_{yd} \text{ (compression)}$$
In view of the above, the stress in reinforcement $A_s$ is defined by the relationship:

$$\sigma_{s1} = f(x) \cdot f_{yd}$$

$$f(x) = \begin{cases} 
  x_{\text{lim}}(d - x)/x(d - x_{\text{lim}}) & \text{for } x_{\text{lim}} < x \leq d \\
  -4(x - d)/d & \text{for } d < x \leq h \\
  -1,0 & \text{for } x > h 
\end{cases}$$

**NOTE:**
Minus stands for compression
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

\[ \Sigma F = 0 \]

\[
N_{Ed} = F_c + F_{s2} - F_{s1} \\
N_{Ed} = 0,8bxf_{cd} + A_{s2}f_{yd} - A_{s1}\sigma_{s1} \\
N_{Ed} = 0,8bxf_{cd} + A_{s2}(f_{yd} - \sigma_{s1}) \\
N_{Ed} = 0,8bxf_{cd} + A_{s2}f_{yd}[1 - f(x)]
\]

\[ \Sigma M = 0 \rightarrow \text{related to the } A_{s1} \text{ axis:} \]

\[
M_{Ed} + N_{Ed}(0,5h - d_1) = F_c z + F_{s2}(d - d_2) \\
M_{Ed} = 0,8b(x(d - 0,4x)f_{cd} + A_{s2}f_{yd}(d - d_2) - N_{Ed}(0,5h - d_1))
\]

\[\text{resisting bending moment}\]

\[
M_{Rd} = 0,8b(x(d - 0,4x)f_{cd} + A_{s2}f_{yd}(d - d_2) - N_{Ed}(0,5h - d_1))
\]
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.3.3. Reinforcement design

<table>
<thead>
<tr>
<th>Input data</th>
<th>Output data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{Ed}$; $N_{Ed}$; $b$; $h$; $f_{cd}$; $f_{yd}$; $c_{nom}$</td>
<td>$A_{s1} = A_{s2}$; $x$; and eventually $\sigma_{s1}$</td>
</tr>
</tbody>
</table>

\[
x = \frac{N_{Ed}}{0.8bf_{cd}} \leq \xi_{\text{lim}} d \quad \text{- Case I}
\]

\[
x \geq x_y
\]

From relationship (2) – slide 58:

\[
A_{s1} = A_{s2} = \frac{M_{Ed} - N_{Ed}(0.5h - 0.4x)}{f_{yd}(d - d_2)}
\]

\[
x < x_y
\]

From relationship (3) slide 59:

\[
A_{s1} = A_{s2} = \frac{M_{Ed} - N_{Ed}(0.5h - d_2)}{f_{yd}(d - d_2)}
\]

\[
x = \frac{N_{Ed}}{0.8bf_{cd}} > \xi_{\text{lim}} d \quad \text{- Case II}
\]

Solve the system of equations to have $A_{s1} = A_{s2}$

\[
\begin{align*}
\Sigma F &= 0 \\
\Sigma M &= 0 \\
\sigma_{s1} &= f(x) \cdot f_{yd}
\end{align*}
\]

The system of equations is solved step by step, choosing $x$, because it is a non-linear system.
Approximate evaluation of reinforcement regardless of compression case \((d_1/d \equiv 0.1)\)

\[ A_{s1} = A_{s2} = A_{s,\text{tot}}/2 \quad \text{with} \quad A_{s,\text{tot}} = \omega_{\text{tot}} bh f_{cd}/f_{yd} \]

where:

\[ \omega_{\text{tot}} = (\mu - 0.55v\nu c)/\lambda \beta \quad \text{if} \quad 0 > \nu \geq -0.85 ; \]

\[ \omega_{\text{tot}} = \mu/\lambda \beta + \nu c \quad \text{if} \quad -0.85 > \nu ; \]

\[ \nu = N_{Ed}/bh f_{cd} \quad \text{(negativ for compression)} ; \]

\[ \nu c = -0.85 - \nu ; \]

\[ \lambda = 0.50 - d_s/h ; \]
8.3.4. Cross section check

Input data
- $M_{Ed}$; $N_{Ed}$; $b$; $h$; $f_{cd}$; $f_{yd}$; $A_{s1} = A_{s2}$; $c_{nom}$

Output data
- $M_{Rd}$; $x$; and eventually $\sigma_{s1}$

**Case I**

$$x = \frac{N_{Ed}}{0.8bf_{cd}} \leq \xi_{lim} d$$

- $x \geq x_y$
- $M_{Ed} \leq M_{Rd} = N_{Ed}(0.5h - 0.4x) + A_{s2}f_{yd}(d - d_2)$

- $x < x_y$
- $M_{Ed} \leq M_{Rd} = A_{s1}f_{yd}(d - d_2) + N_{Ed}(0.5h - d_2)$

**Case II**

$$x = \frac{N_{Ed}}{0.8bf_{cd}} > \xi_{lim} d$$

Solve the system of equations to have $M_{Rd}$

- $\Sigma F = 0$
- $\Sigma M = 0$
- $\sigma_{s1} = f(x) \cdot f_{yd}$

The system of equations is solved step by step, choosing $x$, because it is a non-linear system.
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

Simplified check for case II of compression accepting \( M-N \) curve in the form of a line where \( N_{Ed} > N_{lim} \)

\[
N_{Rd}^c = bh f_{cd} + (A_{s1} + A_{s2}) f_{yd}
\]

\[
\frac{M_{Rd}}{N_{Rd}^c - N_{Ed}} = \frac{M_{R \text{lim}}}{N_{Rd}^c - N_{lim}}
\]

\[
M_{Ed} \leq M_{Rd} = \frac{N_{Rd}^c - N_{Ed}}{N_{Rd}^c - N_{lim}} M_{R \text{lim}}
\]
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

8.3.5. Alternative calculation tools


Anexa 10.1 Nomogramme pentru calculul stâlpilor cu secțiune dreptunghiulară
8.3. COLUMNS WITH RECTANGULAR CROSS SECTION

Anexa 10.2 Tabele pentru calculul stâlpilor cu secțiune dreptunghiulară

\[ A_s = \frac{N_{Ed}}{bf_{cd}} \]
\[ \mu = \frac{M_{Ed}}{bh^2f_{cd}} \]
\[ \omega = \frac{A_s f_{yd}}{bh f_{cd}} \]
\[ A_s = \omega bh \frac{f_{cd}}{f_{yd}} \]

<table>
<thead>
<tr>
<th>Purposes of calculation</th>
<th>( A_{s1} = A_{s2} )</th>
<th>( M_{Rd} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input data</td>
<td>( \mu_{Ed} ) &amp; ( \nu_{Ed} )</td>
<td>( \nu_{Ed} ) &amp; ( \omega )</td>
</tr>
<tr>
<td>Output data</td>
<td>( \omega_{req} )</td>
<td>( \mu_{Rd} )</td>
</tr>
<tr>
<td>Result</td>
<td>( A_{s1} = A_{s2} = \omega_{req} bh f_{cd} / f_{yd} )</td>
<td>( M_{Rd} = \mu_{Rd} bh^2 f_{cd} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( 0 )</th>
<th>0.20</th>
<th>...</th>
<th>( \omega_{req} )</th>
<th>( \omega )</th>
<th>...</th>
<th>( \omega )</th>
<th>...</th>
<th>0.45</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.00 )</td>
<td>...</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_{Ed} )</td>
<td>reinforcement design</td>
<td>( 1000 \mu_{Ed} )</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \mu_{Ed} )</td>
<td></td>
<td></td>
<td></td>
<td>( 1000 \mu_{Ed} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_{Ed} )</td>
<td>M_{Rd} calculation</td>
<td>( 1000 \mu_{Rd} )</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( \mu_{Rd} )</td>
<td></td>
<td></td>
<td></td>
<td>( 1000 \mu_{Rd} )</td>
<td></td>
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<tr>
<td>...</td>
<td>0</td>
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</tr>
</tbody>
</table>
Independent design in each principal direction, disregarding biaxial bending, may be made as a first step.

Imperfections need to be taken into account only in the direction where they will have the most unfavourable effect.
No further check is necessary if the slenderness ratios satisfy the following condition:

(4a) \[ 0.5 \leq \frac{\lambda_y}{\lambda_z} \leq 2 \]

and if the eccentricities \( e_y \) and \( e_z \) satisfy one the following conditions:

(4b) \[ \frac{e_y}{h} \leq 0.2 \quad \text{or} \quad \frac{e_z}{b} \leq 0.2 \]

**Legend:**
- \( b, h \) are the width and depth of the section
- \( \lambda_y, \lambda_z \) are the slenderness ratios \( l_0/i \) with respect to y- and z-axis respectively
- \( i_y, i_z \) are the radii of gyration with respect to y- and z-axis respectively
- \( e_z = \frac{M_{Edy}}{N_{Ed}} \); eccentricity along z-axis
- \( e_y = \frac{M_{Edz}}{N_{Ed}} \); eccentricity along y-axis
- \( M_{Edy} \) is the design moment about y-axis, including second order moment
- \( M_{Edz} \) is the design moment about z-axis, including second order moment
- \( N_{Ed} \) is the design value of axial load in the respective load combination
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Definition of eccentricities $e_y$ and $e_z$

If the condition of Expression (4) is not fulfilled, biaxial bending should be taken into account including the $2^{nd}$ order effects in each direction

Graphical representation of the condition (4b)
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Procedure according to BS 8100, also accepted by IStructE

Column may be designed for a single axis bending but with an equivalent bending moment as follows:

- for: \( \frac{M_{Edz}}{d_z} \geq \frac{M_{Edy}}{d_y} \)

\[
M^\text{ech}_z = M_{Edz} + \beta_N \frac{d_z}{d_y} M_{Edy}
\]

- for: \( \frac{M_{Edz}}{d_z} < \frac{M_{Edy}}{d_y} \)

\[
M^\text{ech}_y = M_{Edy} + \beta_N \frac{d_y}{d_z} M_{Edz}
\]

\[
\beta_N = 1 - \frac{N_{Ed}}{bhf_{ck}} \geq 0.3
\]
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

8.4.1. Basics of calculation

Reinforcement is distributed on all sides of the section
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Force line is characterized by \( \tan \delta = \frac{M_{Edz}}{M_{Edy}} = \frac{e_y}{e_z} \)

Calculation is based on the assumptions from chp. 6.1

Position of the neutral axis is selected in such a way that internal forces (namely \( F_c + \sum F_{s2} \) and \( \sum F_{s1} \)) to be located on the line of forces

Failure is produced by:
- yielding of the most tensioned bars followed by crushing of compression concrete, according to pivot B;
- crushing of compression concrete without yielding of tension bars, according to pivot C;
- whatever is the way of failure, there are bars which are not yielding
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

INTERACTION SURFACE FOR COMPRESSION WITH BIAXIAL BENDING

Static analysis: \( N_{Ed}; M_{Edy}; M_{Edz} \)

By vectorial summation results:

\[
R_{Ed} = \sqrt{N_{Ed}^2 + M_{Edy}^2 + M_{Edz}^2} = \sqrt{N_{Ed}^2 + M_{Ed}^2}
\]

Bearing capacity is:

\[
R_{Rd} = \sqrt{N_{Rd}^2 + M_{Rdy}^2 + M_{Rdz}^2} = \sqrt{N_{Rd}^2 + M_{Rd}^2}
\]

The two vectors are in the same meridian plan \( P_\delta \)

The cross section resists to loads if point 2 (corresponding to the vector \( R_{Ed} \)) is inside the interaction surface or overlapped on the point 1:

\[
R_{Ed} \leq R_{Rd}
\]
8.4.2. Simplified procedure of calculation

**Load Contour Method**

Simplified procedure, taking into account the interaction of bending moments $M_{Edy}$ and $M_{Edz}$ for a constant axial force $N_{Ed}$, may be used for calculation by hand.

This method is suitable for structures located in seismic areas because the bending moments increase under constant gravitational load.

In this case, equation (5) becomes:

$$\sqrt{N_{Ed}^2 + M_{Ed}^2} \leq \sqrt{N_{Rd}^2 + M_{Rd}^2}$$

(6) ................................. $M_{Ed} \leq M_{Rd}$
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

The simplified procedure is based on the replacement of actual curve of interaction, dependent on angle $\delta$, with a simplified elliptic curve.

Calculation procedure is safe because simplified curve is located inside the real one.

$M_{Rdy0}$ – bearing capacity in uniaxial bending for $N_{Ed}$ when $M_{Edz} = 0$

$M_{Rdz0}$ – bearing capacity in uniaxial bending for $N_{Ed}$ when $M_{Edy} = 0$

Unfavorable conclusion: due to biaxial bending there is a decreasing in uniaxial resistance.
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Total area $A_{s,\text{tot}}$

Defining areas $A_{sy}$ and $A_{sz}$
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

\[
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1
\]

Checking relationship (6) becomes:

\[
\left(\frac{M_{\text{Edy}}}{M_{\text{Rdy}0}}\right)^a + \left(\frac{M_{\text{Edz}}}{M_{\text{Rdz}0}}\right)^a \leq 1
\]
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

EXPONENT $a$

SR EN 1992-1-1:2004

<table>
<thead>
<tr>
<th>$N_{Ed}/N_{Rd}$</th>
<th>0.1</th>
<th>0.7</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
</tbody>
</table>

$N_{Rd} = bh_{cd} + A_{s,tot} f_{yd}$

STAS 10107/0-90

<table>
<thead>
<tr>
<th>$N_{Ed}$ \ bhf_{cd}</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4 bars, in the corner</td>
<td>more than 4 bars $A_{sy} \approx A_{sz}$</td>
<td>more than 4 bars $A_{sy} = (1,5...2,0)A_{sz}$</td>
</tr>
<tr>
<td>0.1</td>
<td>1.60</td>
<td>1.70</td>
<td>1.75</td>
</tr>
<tr>
<td>0.2</td>
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<td>1.60</td>
<td>1.50</td>
</tr>
<tr>
<td>0.3</td>
<td>1.25</td>
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<td>1.40</td>
</tr>
<tr>
<td>0.4</td>
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</tr>
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<td>0.6</td>
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<td>1.40</td>
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<tr>
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<td>1.55</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>0.8</td>
<td>1.75</td>
<td>1.60</td>
<td>1.60</td>
</tr>
</tbody>
</table>

1. Exponent was evaluated on the basis of numerical analysis on the computer using general method (chp. 6.1).
2. The exponent was determined in such a way that, for diagonal of the section, the simplified method to give the same result as the general method (chp. 6.1).
Section verification involves the following steps:

- design axial resistance of section: \( N_{Rd} = A_c f_{cd} + A_{s,tot} f_{yd} \)
- determination of the coefficient \( a \) depending on the ratio \( N_{Rd}/N_{Ed} \)
- establishing reinforcements \( (A_{s1} = A_{s2})_y \) and \( (A_{s1} = A_{s2})_z \); bars in the corners are considered for every direction
- calculation of resisting bending moment \( M_{Rdy} \) for \( N_{Ed} \) and \( A_{sy} \)
- calculation of resisting bending moment \( M_{Rdz} \) for \( N_{Ed} \) and \( A_{sz} \)
- checking condition \( \left( \frac{M_{Edy}}{M_{Rdy0}} \right)^a + \left( \frac{M_{Edz}}{M_{Rdz0}} \right)^a \leq 1 \)
8.4.4. Reinforcement calculation

<table>
<thead>
<tr>
<th>Input data</th>
<th>Output data</th>
</tr>
</thead>
<tbody>
<tr>
<td>b; h; N_Ed; M_{Edy}; M_{Edz}; f_{cd}; f_{yd}; c_{nom}</td>
<td>A_{s,tot}</td>
</tr>
</tbody>
</table>

Reinforcement area is calculated for \( M_{Rd} = M_{Ed} \), namely:

\[
\left( \frac{M_{Edy}}{M_{Rdy0}} \right)^a + \left( \frac{M_{Edz}}{M_{Rdz0}} \right)^a = 1 \rightarrow \text{overlapping of points 1 and 2 (slide 79)}
\]

There is a problem: two unknowns & one equation

\( M_{Rdy} \); actually \((A_{s1} = A_{s2})_y\)
\( M_{Rdz} \); actually \((A_{s1} = A_{s2})_z\)
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

Consequently, reinforcement calculation involves an infinity of solutions.
Additional relationship is needed between $M_{Rdy}$ & $M_{Rdz}$

Between bearing capacities $M_{Rdy}$ & $M_{Rdz}$ to be the same ratio as between the bending moments $M_{Edy}$ & $M_{Edz}$:

$$\frac{M_{Rdy}}{M_{Rdz}} = \frac{M_{Edy}}{M_{Edz}}$$

$$\frac{M_{Edy}}{M_{Rdy}} = \frac{M_{Edz}}{M_{Rdz}}$$

In this case equation (7) becomes:

$$(8) \quad \left( \frac{M_{Edy}}{M_{Rdy}} \right)^a = \left( \frac{M_{Edz}}{M_{Rdz}} \right)^a \leq 0.5$$
8.4. BIAXIAL BENDING OF COLUMNS WITH RECTANGULAR CROSS SECTION

The calculation procedure is as follows:

- it is estimated $A_{s,tot}$
- $N_{Rd} = A_c f_{cd} + A_{s,tot}$
- choose exponent $a$ depending on $N_{Ed}/N_{Rd}$
- according to (8), choose $\Omega = \left(\frac{M_{Edy}}{M_{Rdy}}\right)^a = \left(\frac{M_{Edz}}{M_{Rdz}}\right)^a \leq 0.5$
- $\frac{M_{Edy}}{M_{Rdy}} = \frac{M_{Edz}}{M_{Rdz}} = a\sqrt{\Omega}$
- required bearing capacity for $y$ axis: $M_{Rdy} = M_{Edy}/a\sqrt{\Omega}$
- required bearing capacity for $z$ axis: $M_{Rdz} = M_{Edz}/a\sqrt{\Omega}$
8.4. BIAXIAL BENDING OF COLUMNS WITH
RECTANGULAR CROSS SECTION

- calculation of reinforcement \((A_{s1} = A_{s2})_y\) shall be made for
  \(N_{Ed}\) and \(M_{Edy}/\sqrt[2]{\Omega}\) in order to achieve required \(M_{Rdy}\)

- calculation of reinforcement \((A_{s1} = A_{s2})_z\) shall be made for
  \(N_{Ed}\) and \(M_{Edz}/\sqrt[2]{\Omega}\) in order to achieve required \(M_{Rdz}\)

- bar detailing

- if \((A_{s1} = A_{s2})_y\) is rounded up then \((A_{s1} = A_{s2})_z\) is rounded down

- with \(A_{s,\text{tot eff}}\) compute the new \(N_{Rd}\); if necessary calculation
  is made again

Advantage: biaxial bending is divided in two uniaxial bending with increased moments

Note: using exponent from former romanian code no recalculation is required because
exponent \(a\) depends only on \(N_{Ed}/bhf_{cd}\)
8.5. CIRCULAR/RING-SHAPED COLUMNS

Bars are evenly distributed along the section contour

Reinforcement is considered evenly distributed on the contour if in the section there are at least six bars

Calculation is based on the assumptions from chp. 6.1

In case of ring-shaped (annular) section it is recommended that between the inner radius and the outer radius to have the following relation:

\[ r_i \geq 0.5r_e \]
8.5. CIRCULAR/RINGED-SHAPED COLUMNS

Failure is produced by:

- yielding of the most tensioned bars followed by crushing of compression concrete;
- crushing of compression concrete without yielding of tension bars;
- whatever is the way of failure, there are bars which are not yielding.
Approximate evaluation of reinforcement for \(0.15 \leq \omega_{\text{tot}} \leq 1.0\)

\[
A_{s,\text{tot}} = \omega_{\text{tot}} A_c \frac{f_{\text{cd}}}{f_{\text{yd}}}
\]

with:

\[
\omega_{\text{tot}} = \beta_1 \mu + \beta_2
\]

\[
\mu = \frac{M_{\text{Ed}}}{A_c D f_{\text{cd}}}
\]

\[
A_c = 0.25 \pi D^2
\]

\(\beta_1, \beta_2\) - coefficients depending on \(\nu = \frac{N_{\text{Ed}}}{A_c f_{\text{cd}}}\)
8.5. CIRCULAR/RINGED-SHAPED COLUMNS

Tools for current calculations

Anexa 10.4. Tabele pentru calculul stâlpilor cu secţiune circulară

\[
\begin{align*}
\nu &= \frac{N_{Ed}}{A_c f_{cd}} \\
\mu &= \frac{M_{Ed}}{A_c D f_{cd}} \\
\omega_\text{tot} &= \frac{A_{s,\text{tot}} f_{yd}}{A_c f_{cd}} \\
A_{s,\text{tot}} &= \omega_\text{tot} A_c \left( \frac{f_{cd}}{f_{yd}} \right)
\end{align*}
\]

<table>
<thead>
<tr>
<th>Purpose of calculation</th>
<th>( A_{s,\text{tot}} )</th>
<th>( M_{Rd} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1  Input data</td>
<td>( \mu_{Ed} &amp; \nu_{Ed} )</td>
<td>( \nu_{Ed} &amp; \omega )</td>
</tr>
<tr>
<td>2  Output data</td>
<td>( \omega_{req} )</td>
<td>( \mu_{Rd} )</td>
</tr>
<tr>
<td>3  Result</td>
<td>( A_{s,\text{tot}} = \omega_{req} A_c f_{cd}/f_{yd} )</td>
<td>( M_{Rd} = \mu_{Rd} A_c D f_{cd} )</td>
</tr>
</tbody>
</table>

Values 1000\( \mu \) for \( \omega_\text{tot} = \)

<table>
<thead>
<tr>
<th>0</th>
<th>0.20</th>
<th>...</th>
<th>( \omega_{req} )</th>
<th>( \omega )</th>
<th>...</th>
<th>( \omega )</th>
<th>...</th>
<th>0.45</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,00</td>
<td>...</td>
<td>( \nu_{Ed} )</td>
<td>reinforcement design</td>
<td>1000( \mu_{Ed} )</td>
<td>...</td>
<td>( \omega_{req} )</td>
<td>( \omega )</td>
<td>...</td>
<td>1000( \mu_{Rd} )</td>
</tr>
</tbody>
</table>
8.5. CIRCULAR/RINGED-SHAPED COLUMNS

Anexa 10.5 Tabele pentru calculul stâlpilor cu secțiune inelară
8.6. DETAILING OF COLUMNS

EN 1992-1-1:2004
SR EN 1992-1-1:2006
P100-1/2013 → very specific provisions & highly severe

ANCHORAGE & BAR LAPS → CHP. 2.2

CROSS SECTION DIMENSIONS
Usually h/b ≤ 2,5, maximum value being 4
The minimum size of the rectangular cross section is 300 mm
The minimum diameter of circular cross section is 300 mm
Usually sizes are multiples of 50 mm

LONGITUDINAL REINFORCEMENTS
\[ \phi_{\text{min}} = 8 \text{ mm}; \quad \text{NA: } 12 \text{ mm}; \quad \text{.... in romanian practice } \phi \geq 14 \text{ mm} \]
\[ A_{s,\text{min}} = \max \begin{cases} 0,1N_{Ed}/f_{yd} & \\ 0,2A_c & \text{(NA: } 0,4A_c) \end{cases} \]
\[ A_{s,\text{max}} = 4\%A_c \]
8.6. DETAILING OF COLUMNS

TRANSVERSAL REINFORCEMENTS

\[ \phi \geq \max \left\{ \frac{\phi_{\text{long}}}{4}, 6 \text{ mm} \right\} \]

- shear force;
- compressed concrete confinement;
- no buckling of longitudinal bars between stirrups

The spacing of the transverse reinforcement \( s_{cl,t} \) should be limited to
\[ s_{cl,t} \leq s_{cl,t_{\text{max}}} = \max \left\{ \frac{20\phi_{\text{min_long}}}{\min (b; h)}, 400 \text{ mm} \right\} \]

\( s_{cl,t_{\text{max}}} \) should be reduced by a factor 0.6:
- above or below a beam or slab
- near lapped joints if \( \phi_{\text{max}} > 14 \text{ mm} \)
8.6. DETAILING OF COLUMNS

TRANSVERSAL REINFORCEMENTS

- shear force;
- compressed concrete confinement;
- no buckling of longitudinal bars between stirrups

Weak stirrup = small $\phi$ & large distance between stirrups

Weak stirrups:
- buckling of longitudinal bars between stirrups
- no confinement of compressed concrete

Buckling in lap zone with weak stirrups

High $V_{Ed}$ with weak stirrups (0.6 m)
8.6. DETAILING OF COLUMNS

ARRANGEMENT OF BARS

Changes In Column Size

If \( \tan \alpha > 1/12 \), the spacing of transverse reinforcement should be calculated, taking account of the lateral forces involved.
Every longitudinal bar placed in a corner of the section should be held by transverse reinforcements.
8.6. DETAILING OF COLUMNS

No bar should be further than 150 mm from a restrained bar (in corner of stirrup; connected to a link).

Due to compressive force there is longitudinal shortening & transversal swelling of concrete.

Red curves: deformed shape of the stirrup produced by swelling of concrete.

Arrows show bars in tension due to swelling of concrete.

Link in case A has contribution to confinement.

Link in case B has no contribution to confinement.