6. BENDING WITH AXIAL FORCE

6.1. BASIC ASSUMPTIONS

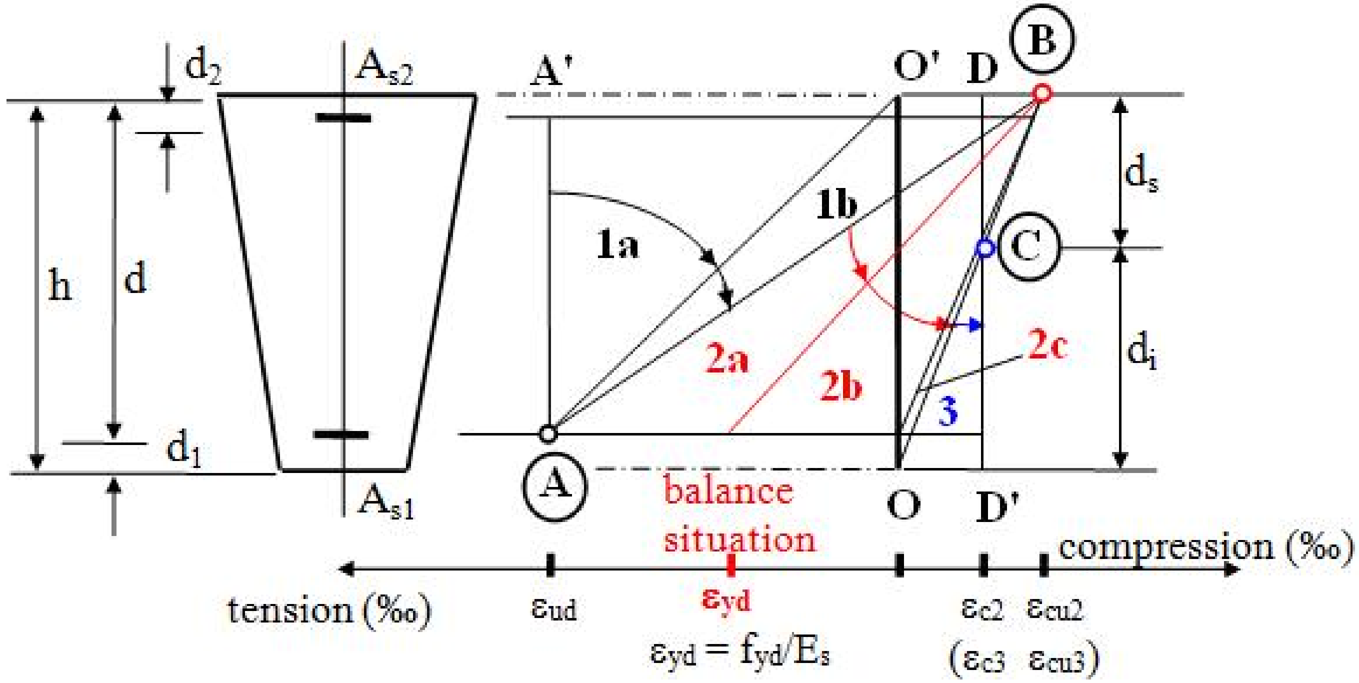
When determining the ultimate moment resistance $\mathbf{M_{Rd}}$ of reinforced concrete cross-sections, the following assumptions are made:

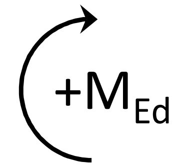
- plane sections remain plane;
- the strain in bonded reinforcement, whether in tension or in compression, is the same as that in the surrounding concrete;
- the tensile strength of the concrete is ignored;
- the stresses in the concrete in compression are derived from the design stress-strain relationship (chp. 4.3.6; slides 24…27);
- the stresses in the reinforcing steel are derived from the design curves.

For cross-sections with symmetrical reinforcement loaded by the compression force it is necessary to assume the minimum eccentricity, e_0 = h/30 but not less than 20 mm where h is the depth of the section. ← *this is about imperfections in execution*

6.2. STRAIN DIAGRAM IN ULTIMATE STAGE

The rule of 3 pivots





Triangles OO'B & CDB: $\frac{\varepsilon_{cu2}}{h} = \frac{\varepsilon_{cu2} - \varepsilon_{c2}}{d_s} \rightarrow d_s = \left(1 - \frac{\varepsilon_{c2}}{\varepsilon_{cu2}}\right)h$

$$\frac{\varepsilon_{cu3}}{h} = \frac{\varepsilon_{cu3} - \varepsilon_{c3}}{d_s} \rightarrow d_s = \left(1 - \frac{\varepsilon_{c3}}{\varepsilon_{cu3}}\right)h$$

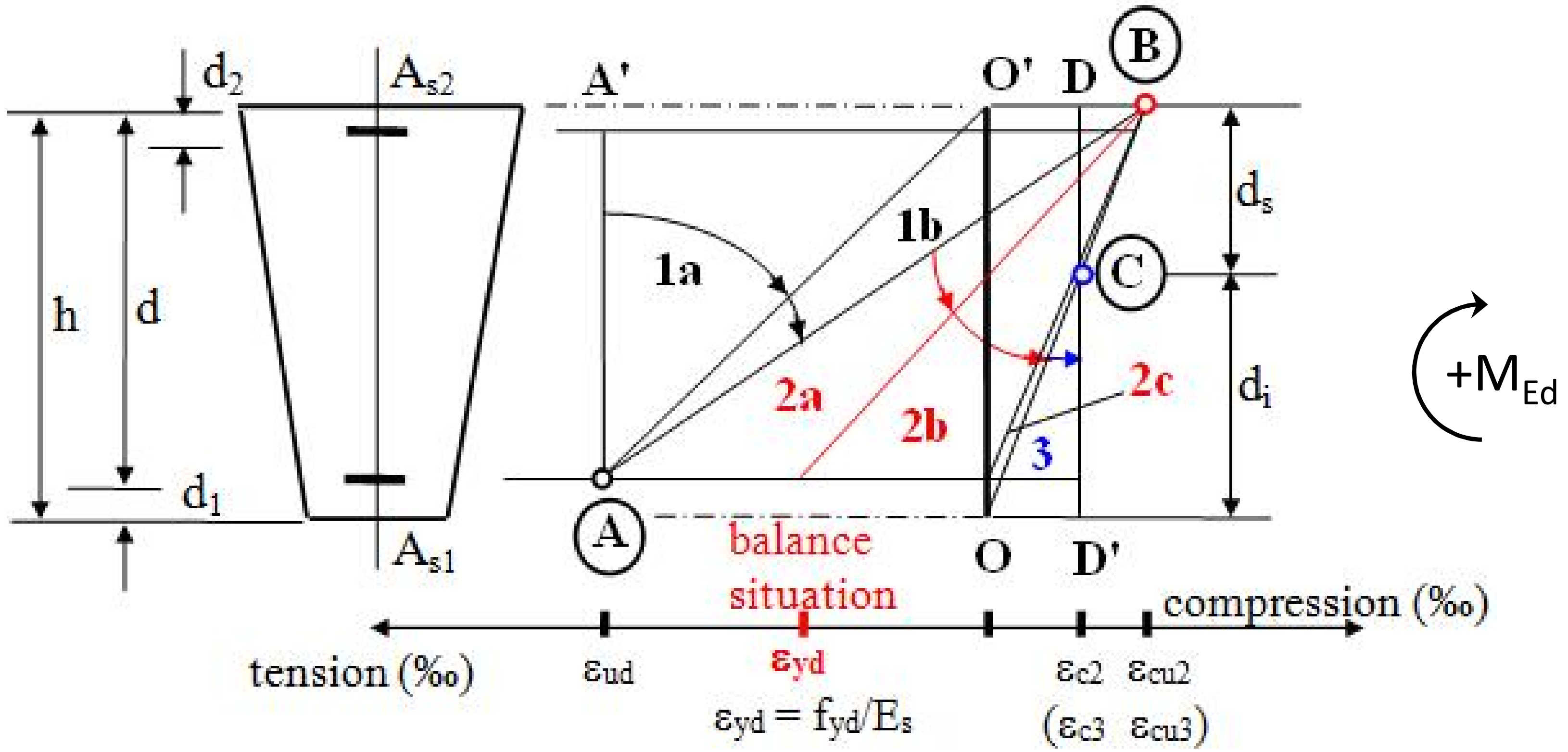
Line DD':

-N_{Ed} & M_{Ed} = 0: - section in concentric compression

- crushing of compressed concrete ($\varepsilon_c = \varepsilon_{c2}$ or $\varepsilon_c = \varepsilon_{c3}$)
- reinforcement yields: $\varepsilon_{yd} = f_{yd}/E_s$ (slide 18) < ε_c (ε_{c2} = 2‰; ε_{c3} = 1,75‰)

6.2. STRAIN DIAGRAM IN ULTIMATE STAGE

The rule of 3 pivots

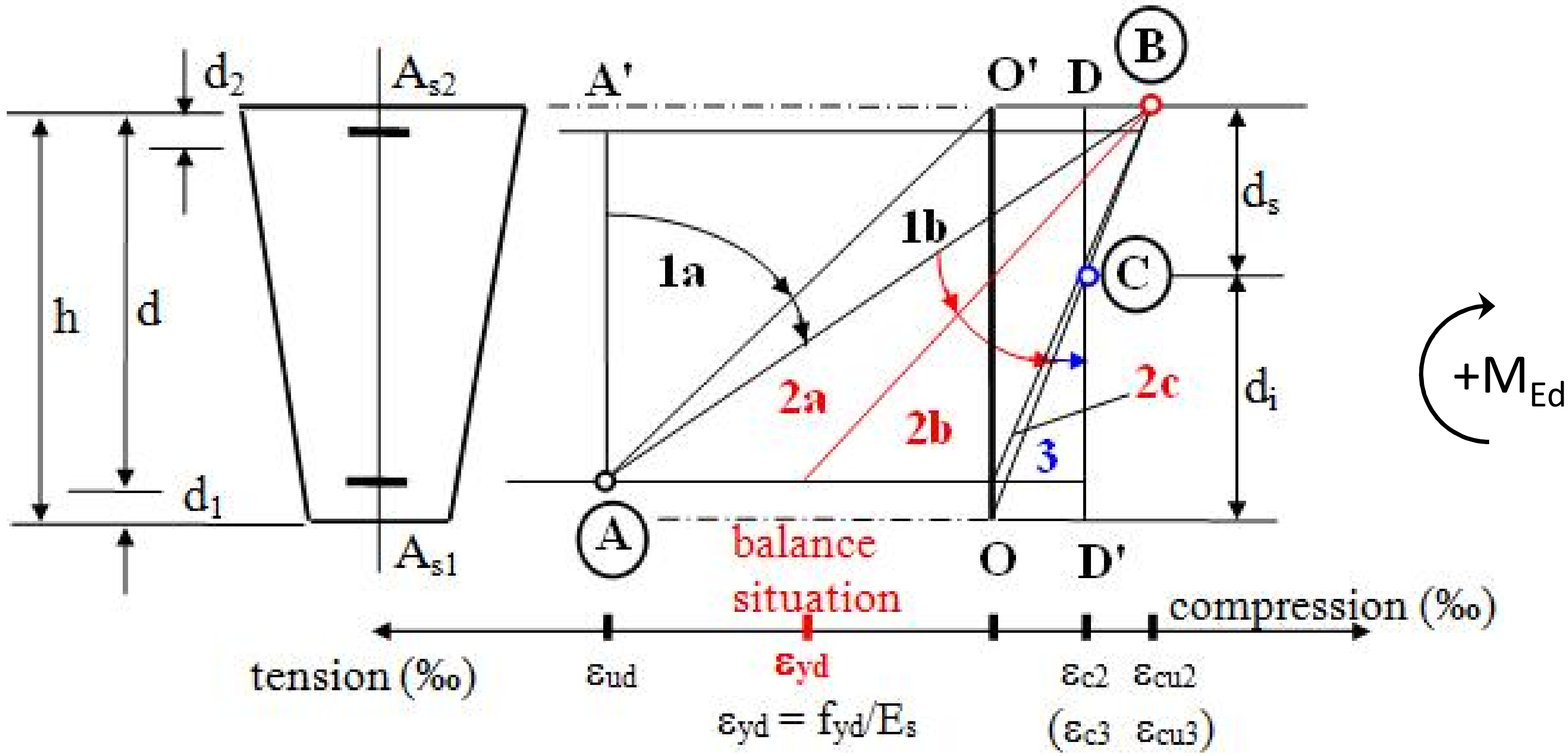


Increasing M_{Ed}: - section is rotated around pivot C until reaches the decompression state (line OB)

- from now rotation around pivot B
- the exceeding of decompression state leads to tension in concrete and afterwards cracking of tensioned concrete
- **balance situation**: yield of the tension reinforcement starts in the same time with crushing of compressed concrete

6.2. STRAIN DIAGRAM IN ULTIMATE STAGE

The rule of 3 pivots



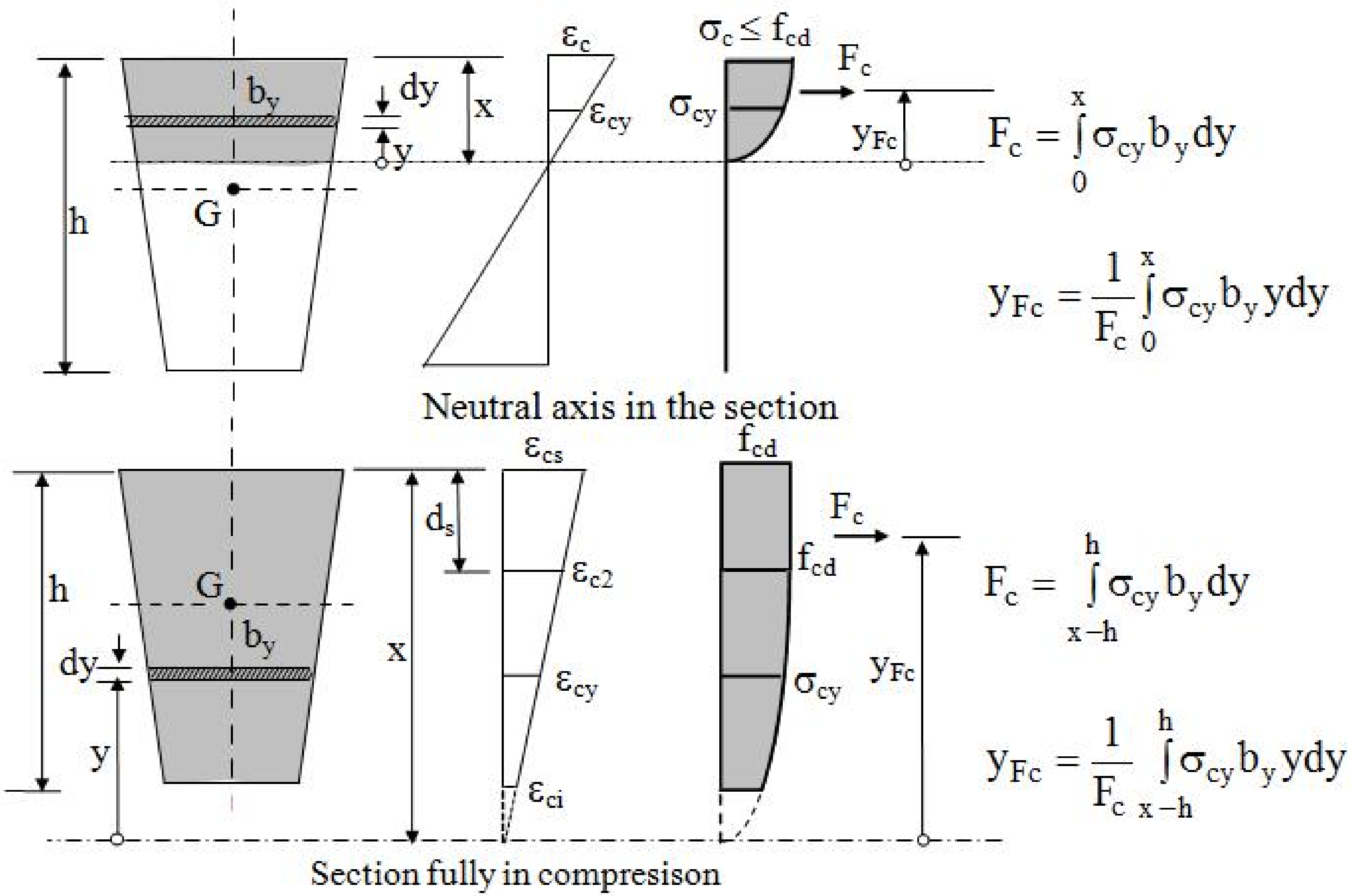
Line AA':

$+N_{Ed}$ & $M_{Ed} = 0$: - section in concentric tension
- fully cracked
- collapse of reinforcements

Increasing M_{Ed}: - section is rotated around pivot A
- after AO' line $\rightarrow$ compressed concrete

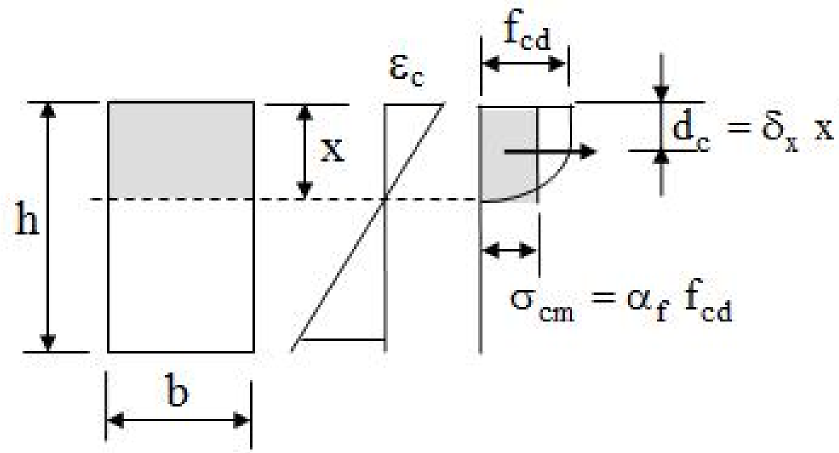
6.3. INTERNAL COMPRESIVE FORCE

MONO-SYMMETRIC SECTION

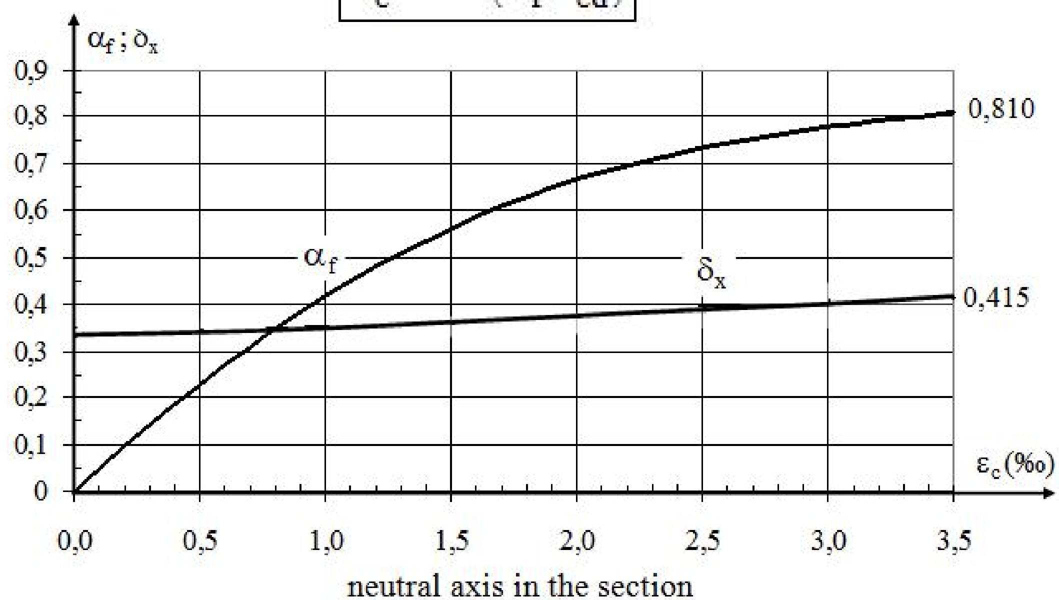


6.3. INTERNAL COMPRESIVE FORCE

RECTANGULAR SECTION

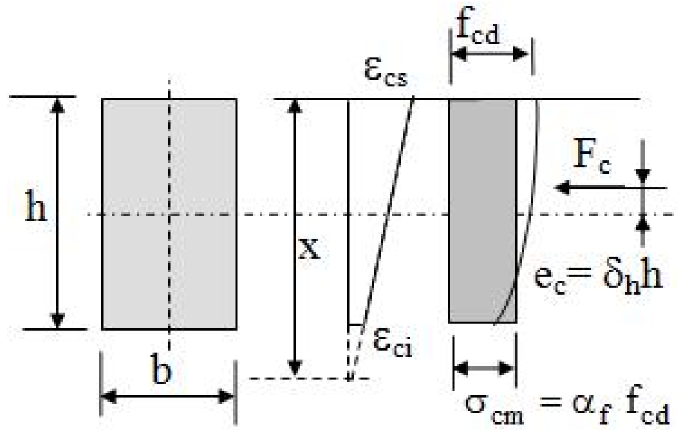


$$F_c = bx\left(\alpha_f\, f_{cd}\right)$$

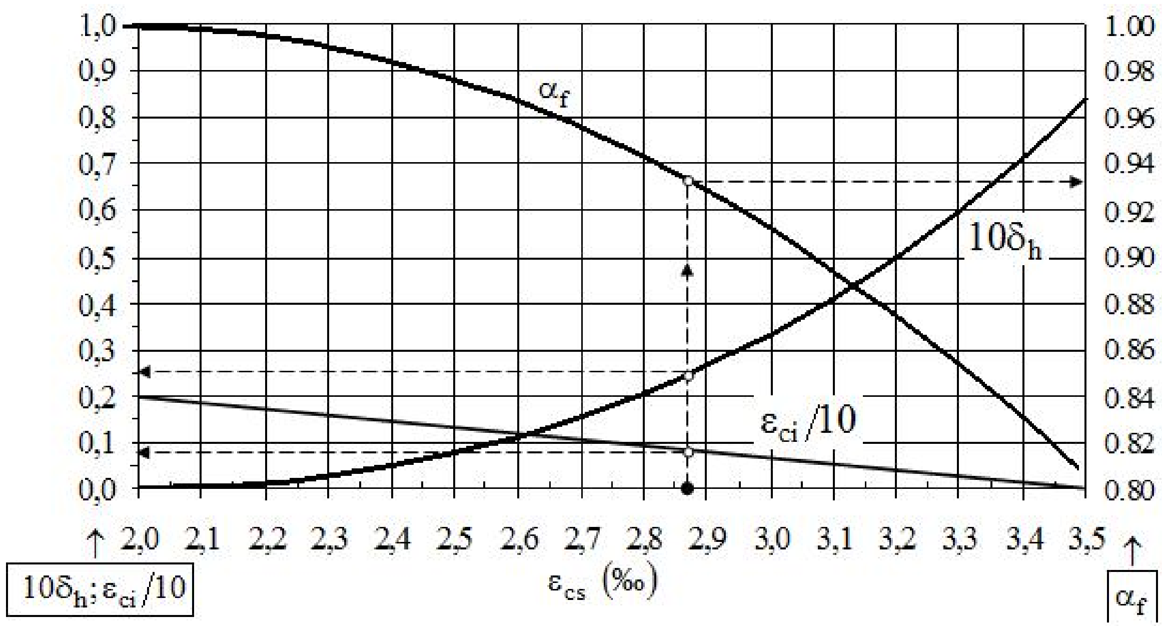


6.3. INTERNAL COMPRESIVE FORCE

RECTANGULAR SECTION



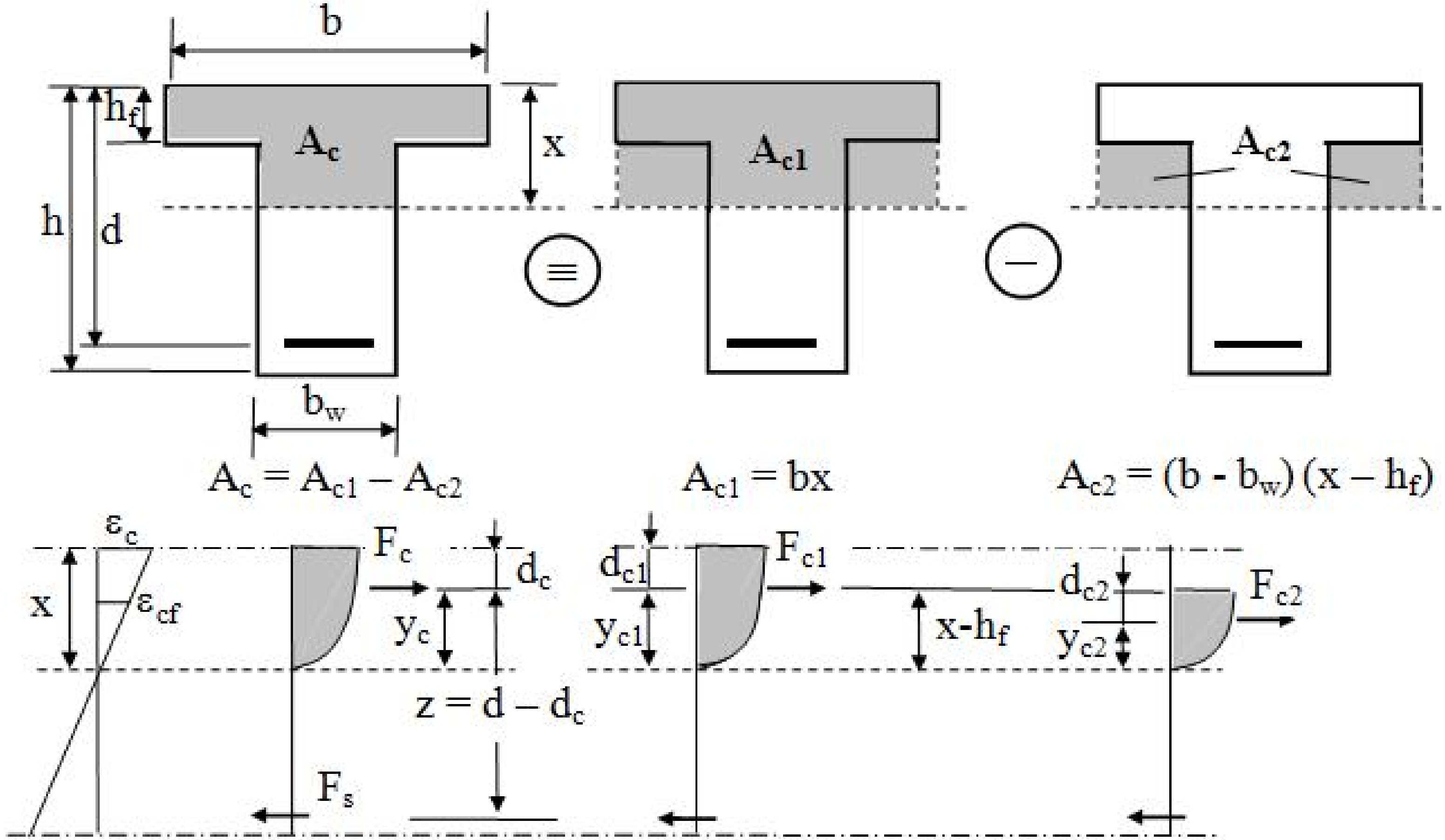
$$F_c = bh(\alpha_f\, f_{cd})$$



section fully in compression

6.3. INTERNAL COMPRESIVE FORCE

FLANGED SECTION



$$F_c = F_{c1} - F_{c2}$$
$$F_{c1} = A_{c1}(\alpha_{f1} f_{cd})$$
$$F_{c2} = A_{c2}(\alpha_{f2} f_{cd})$$

$$y_c = \frac{\sum_i F_{ci} y_{ci}}{\sum_i F_{ci}}$$

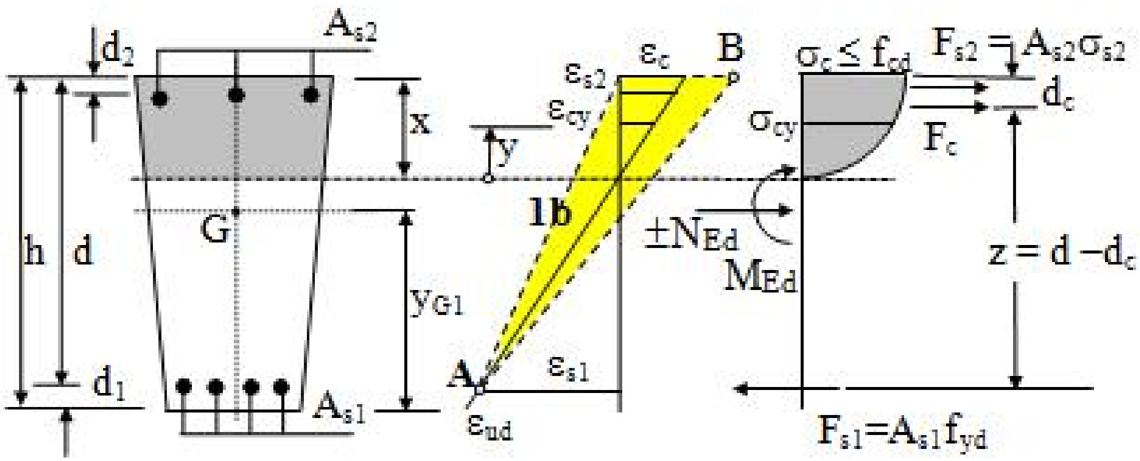
$$d_{c1} = \delta_{x,1}\, x$$
$$d_{c2} = \delta_{x,2}(x - h_f)$$

Slide 7 $\rightarrow$ $\left| \begin{array}{l} \alpha_{f1}; \delta_{x1} = f(\varepsilon_c) \\ \alpha_{f2}; \delta_{x2} = f(\varepsilon_{cf}) \end{array} \right.$

6.4. CASES OF DESIGN

NEUTRAL AXIS IN THE SECTION:

Eccentric tension with large eccentricity & Bending of slightly reinforced elements



- choose x
- strain diagram: $\dfrac{\varepsilon_{s2}}{x-d_2}=\dfrac{\varepsilon_{ud}}{d-x}$ $\rightarrow \varepsilon_{s2} \rightarrow \sigma_{s2} = \varepsilon_{s2} E_s < f_{yd}$
- if $\pm N_{Ed} = F_c + A_{s2}\sigma_{s2} - A_{s1}f_{yd}$ $\rightarrow$ x is correct
- moment equilibrium condition about F_{s1}

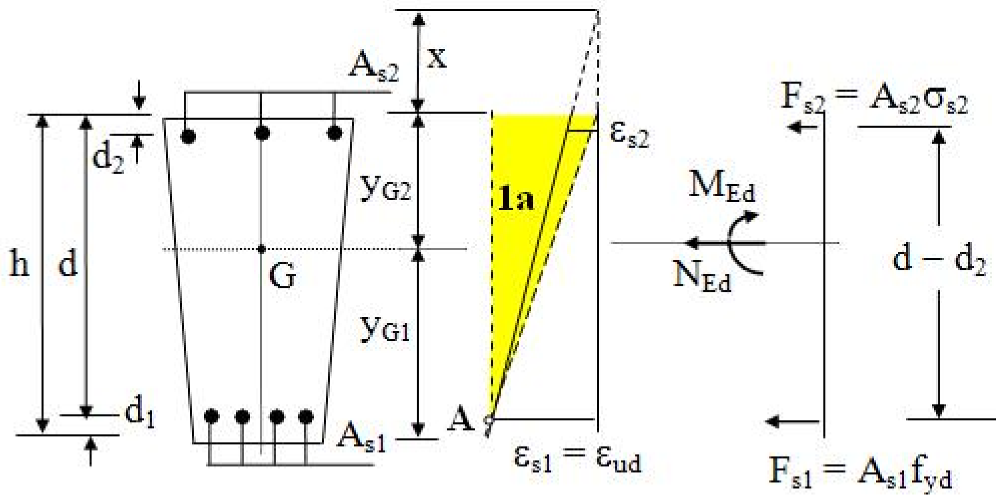
$$M_{Ed} \pm N_{Ed}(y_{G1}-d_1) = F_c(d-d_c)+A_{s2}\sigma_{s2}(d-d_2)$$

$$M_{Ed} = \underbrace{F_c(d-d_c)+A_{s2}\sigma_{s2}(d-d_2)\mp N_{Ed}(y_{G1}-d_1)}_{\text{resisting bending moment}}$$

$$M_{Ed} \le M_{Rd} = F_c(d-d_c)+A_{s2}\sigma_{s2}(d-d_2)\mp N_{Ed}(y_{G1}-d_1)$$

6.4. CASES OF DESIGN

NEUTRAL AXIS OUTSIDE THE SECTION - SECTION FULLY IN TENSION:
Concentric tension & eccentric tension with low eccentricity



- strain diagram: $\frac{\varepsilon_{s2}}{x + d_2} = \frac{\varepsilon_{ud}}{x + d} \rightarrow \varepsilon_{s2} \rightarrow \sigma_{s2} = \varepsilon_{s2} E_s < f_{yd}$
- if $N_{Ed} = A_{s1}f_{yd} + A_{s2}\sigma_{s2} \rightarrow$ x is correct
- moment equilibrium condition about F_{s2}

$$M_{Ed} + N_{Ed}(y_{G2} - d_2) = A_{s1}f_{yd}(d - d_2)$$

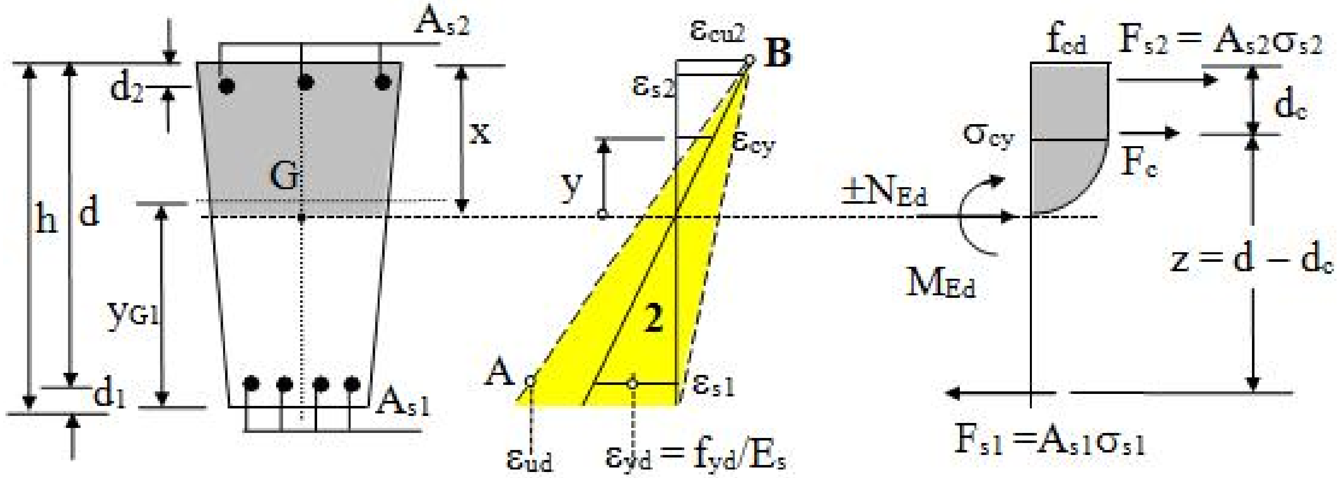
$$M_{Ed} = \underbrace{A_{s1}f_{yd}(d - d_2) - N_{Ed}(y_{G2} - d_2)}_{\text{resisting bending moment}}$$

$$M_{Ed} \leq M_{Rd} = A_{s1}f_{yd}(d - d_2) - N_{Ed}(y_{G2} - d_2)$$

6.4. CASES OF DESIGN

NEUTRAL AXIS IN THE SECTION:

Bending & Tensile/Compressive eccentric force with large eccentricity



- strain diagram: $\frac{\varepsilon_{s1}}{d-x} = \frac{\varepsilon_{cu2}}{x} \rightarrow \varepsilon_{s1} \rightarrow \sigma_{s1} = \varepsilon_{s1} E_s \le f_{yd}$
- strain diagram: $\frac{\varepsilon_{s2}}{x-d_2} = \frac{\varepsilon_{cu2}}{x} \rightarrow \varepsilon_{s2} \rightarrow \sigma_{s2} = \varepsilon_{s2} E_s \le f_{yd}$
- if $\pm N_{Ed} = F_c + A_{s2}\sigma_{s2} - A_{s1}f_{yd} \rightarrow$ x is correct
- moment equilibrium condition about F_{s1}

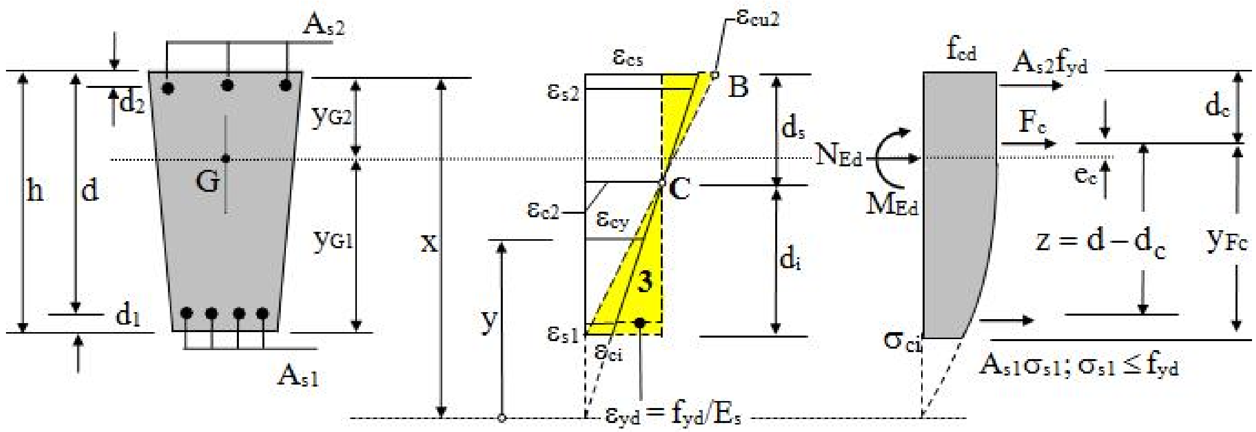
$$M_{Ed} \pm N_{Ed}(y_{G1} - d_1) = F_c(d - d_c) + A_{s2}\sigma_{s2}(d - d_2)$$

$$M_{Ed} = \underbrace{F_c(d - d_c) + A_{s2}\sigma_{s2}(d - d_2) \mp N_{Ed}(y_{G1} - d_1)}_{\text{resisting bending moment}}$$

$$M_{Ed} \le M_{Rd} = F_c(d - d_c) + A_{s2}\sigma_{s2}(d - d_2) \mp N_{Ed}(y_{G1} - d_1)$$

6.4. CASES OF DESIGN

NEUTRAL AXIS OUTSIDE THE SECTION - SECTION FULLY IN COMPRESSION:
Eccentric compression with low eccentricity & Concentric compression



- from corresponding σ - ε diagrams $\rightarrow \sigma_{cy}; \sigma_{s1}; \sigma_{s2}$
- F_c: for mono-symmetric section $\rightarrow$ slide 6
  for rectangular section $\rightarrow$ slide 8
  for flanged section $\rightarrow$ slide 8 & 9
- if $N_{Ed} = A_{s1}\sigma_{s1} + A_{s2}\sigma_{s2} + F_c$ $\rightarrow$ x is correct
- moment equilibrium condition about F_{s1}

$$M_{Ed} + N_{Ed}(y_{G1} - d_1) = F_c(d - d_c) + A_{s2}\sigma_{s2}(d - d_2)$$

$$M_{Ed} = \underbrace{F_c(d - d_c) + A_{s2}\sigma_{s2}(d - d_2) - N_{Ed}(y_{G1} - d_1)}_{\text{resisting bending moment}}$$

$$M_{Ed} \le M_{Rd} = F_c(d - d_c) + A_{s2}\sigma_{s2}(d - d_2) - N_{Ed}(y_{G1} - d_1)$$

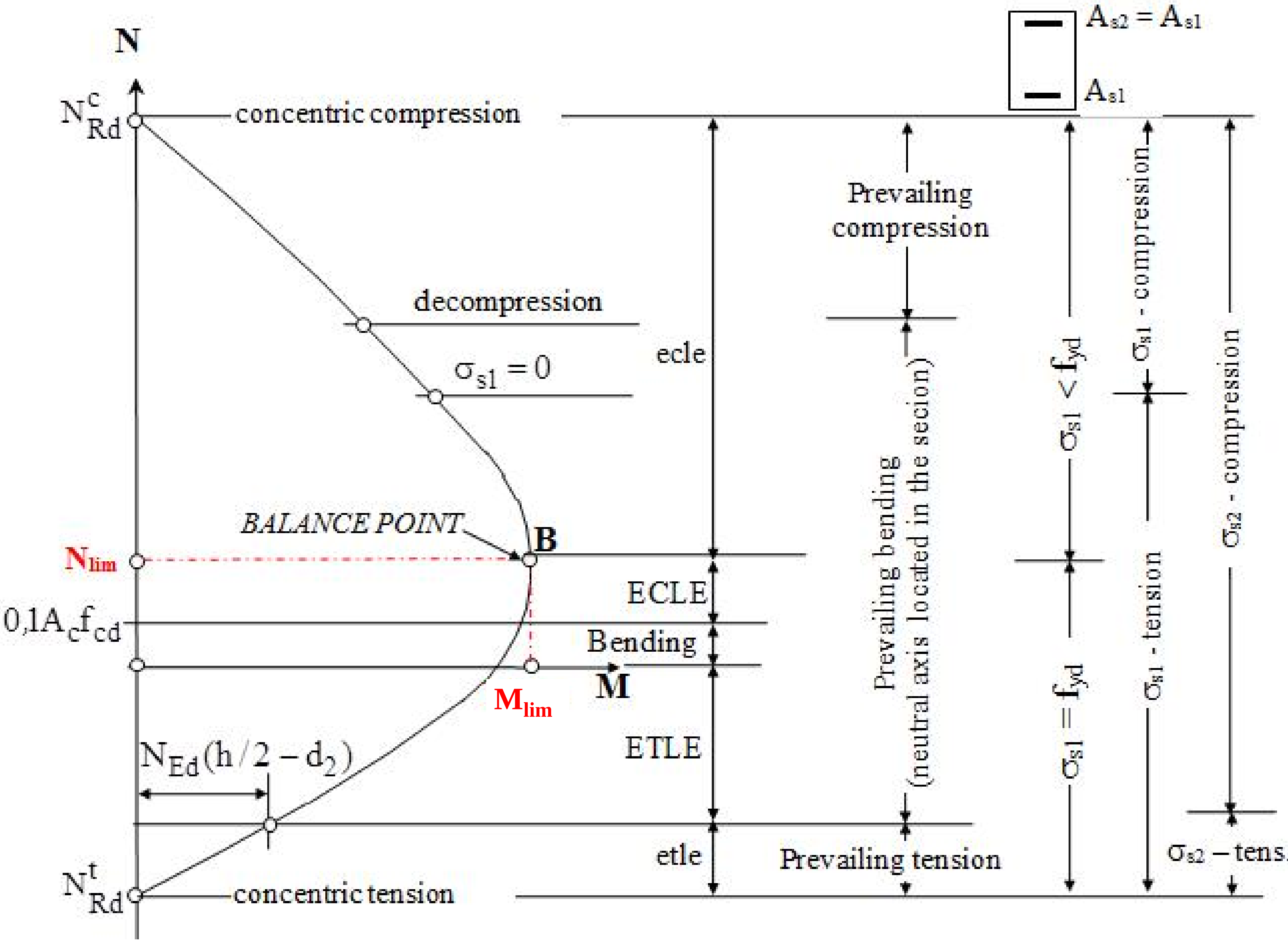
6.5. M-N INTERACTION DIAGRAM

For a given section, the way of failure depends on both M & N

Correlation between M & N may be transposed by M-N limit curve

M-N limit curve is obtained by reducing x between equilibrium conditions ($\Sigma N = 0$; $\Sigma M = 0$)

6.5. M-N INTERACTION DIAGRAM



6.5. M-N INTERACTION DIAGRAM

$N_{Rd}^{c} = A_c f_{cd} + (A_{s1} + A_{s2}) f_{yd}$ - resisting compressive force

$N_{Rd}^{t} = (A_{s1} + A_{s2}) f_{yd}$ - resisting tensile force

ECLE – **E**ccentric **C**ompression with **L**arge **E**ccentricity
(also named **Case I** of compression)

ecle – **e**ccentric **c**ompression with **l**ow **e**ccentricity
(also named **Case II** of compression)

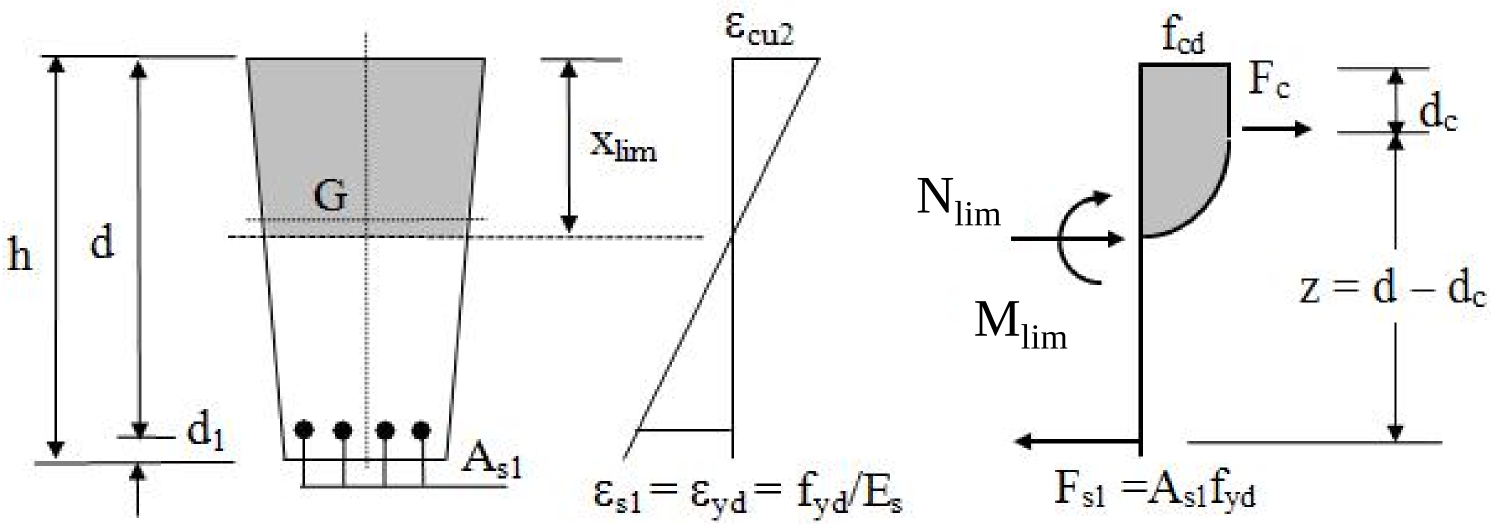
ETLE – **E**ccentric **T**ension with **L**arge **E**ccentricity

etle – **e**ccentric **t**ension with **l**ow **e**ccentricity

6.5. M-N INTERACTION DIAGRAM

BALANCE SITUATION

YIELDING OF THE TENSIONED REINFORCEMENT STARTS IN THE SAME TIME WITH CRUSHING OF COMPRESSED CONCRETE



$$\frac{\varepsilon_{cu2}}{x_{lim}} = \frac{\varepsilon_{yd}}{d - x_{lim}} \rightarrow x_{lim} = \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} + \varepsilon_{yd}} d$$

$\leq$ C50/60 $\rightarrow$

$$\xi_{lim} = \frac{x_{lim}}{d} = \frac{3{,}5}{3{,}5 + 1000 f_{yd} / E_s}$$

6.5. M-N INTERACTION DIAGRAM

ξ_{lim} values for strength class of concrete ≤ C50/60

Steel	f_{yd} (MPa)	E_s (MPa)	ε_{yd} (‰)	ξ_{lim}
Persistent design situations $\gamma_s = 1,15$				
S400	400/1,15 = 348	200000	1,74	0,668
S500	500/1,15 = 435		2,17	0,617
PC52	345/1,15 = 300	210000	1,43	0,710
PC60	405/1,15 = 352		1,68	0,676
Accidental design situations $\gamma_s = 1,0$				
S400	400	200000	2,00	0,636
S500	500		2,50	0,583
PC52	345	210000	1,64	0,681
PC60	405		1,93	0,645

P100: ¥$_s$ = 1,15

6.5. M-N INTERACTION DIAGRAM

TWO DIFFERENT WAYS OF FAILURE ARE SEPARATED BY THE BALANCE SITUATION

$N_{Ed} \leq N_{lim} \rightarrow$ - ductile failure due to yielding of tensioned reinforcement
- compulsory in case of seismic areas

$N_{Ed} > N_{lim} \rightarrow$ - brittle failure by crushing of concrete without yielding of reinforcement A_{s1} (whatever it is tension or compression)
- brittle character becomes stronger with the increasing the compressive force

6.6. FINAL REMARKS

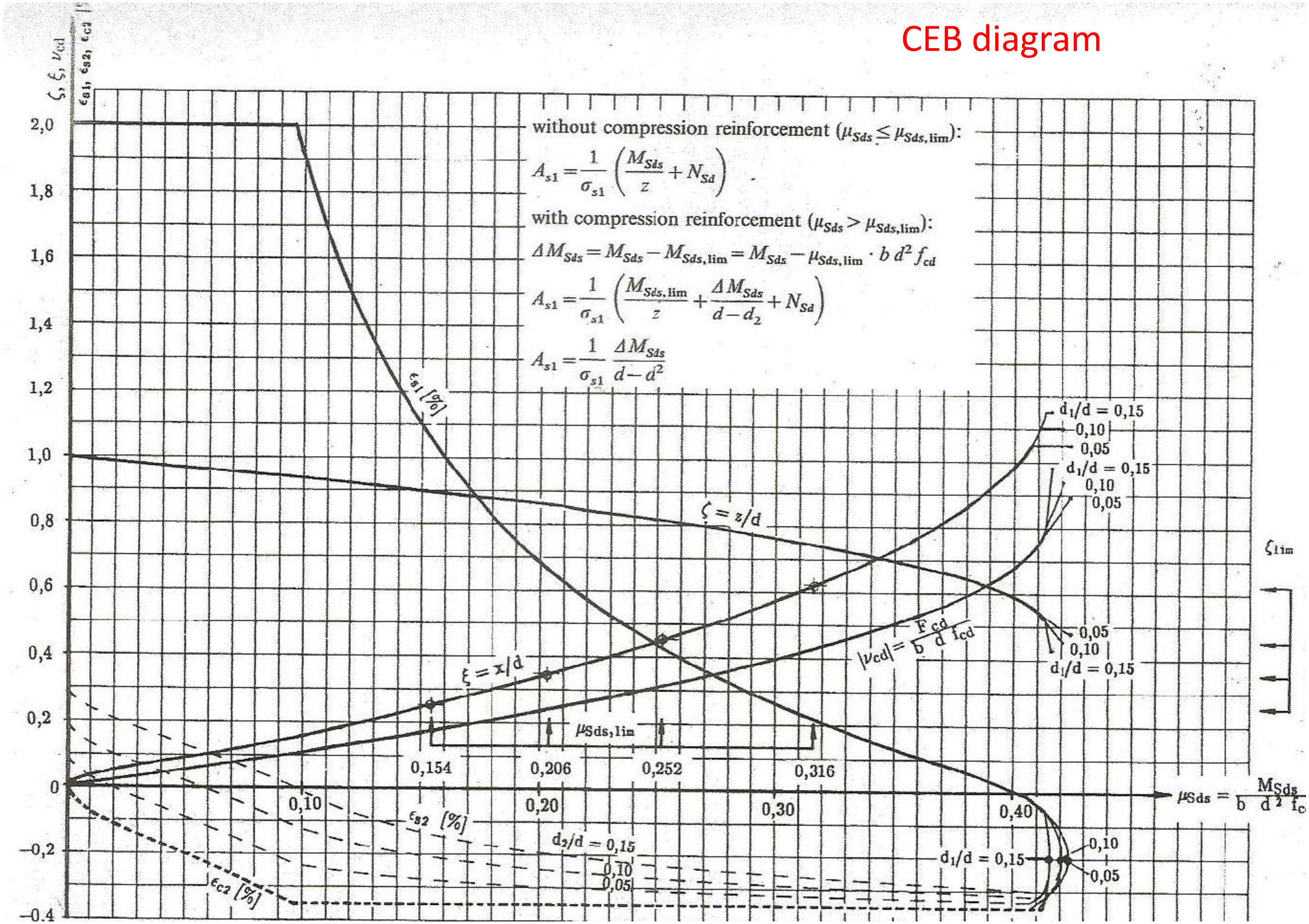
① **Approach presented in 6.4:**

- is not a practical tool for hand calculation
- is the basis for the CEB diagram for rectangular sections in bending (slide 21), also for M-N limit diagram (slide 22)
- was used to conceive tables for rectangular & flanged sections in bending
- may be used to write specific software
- is the only way for unusual cases (concrete section as well the reinforcement arrangement)

② **In the following:**

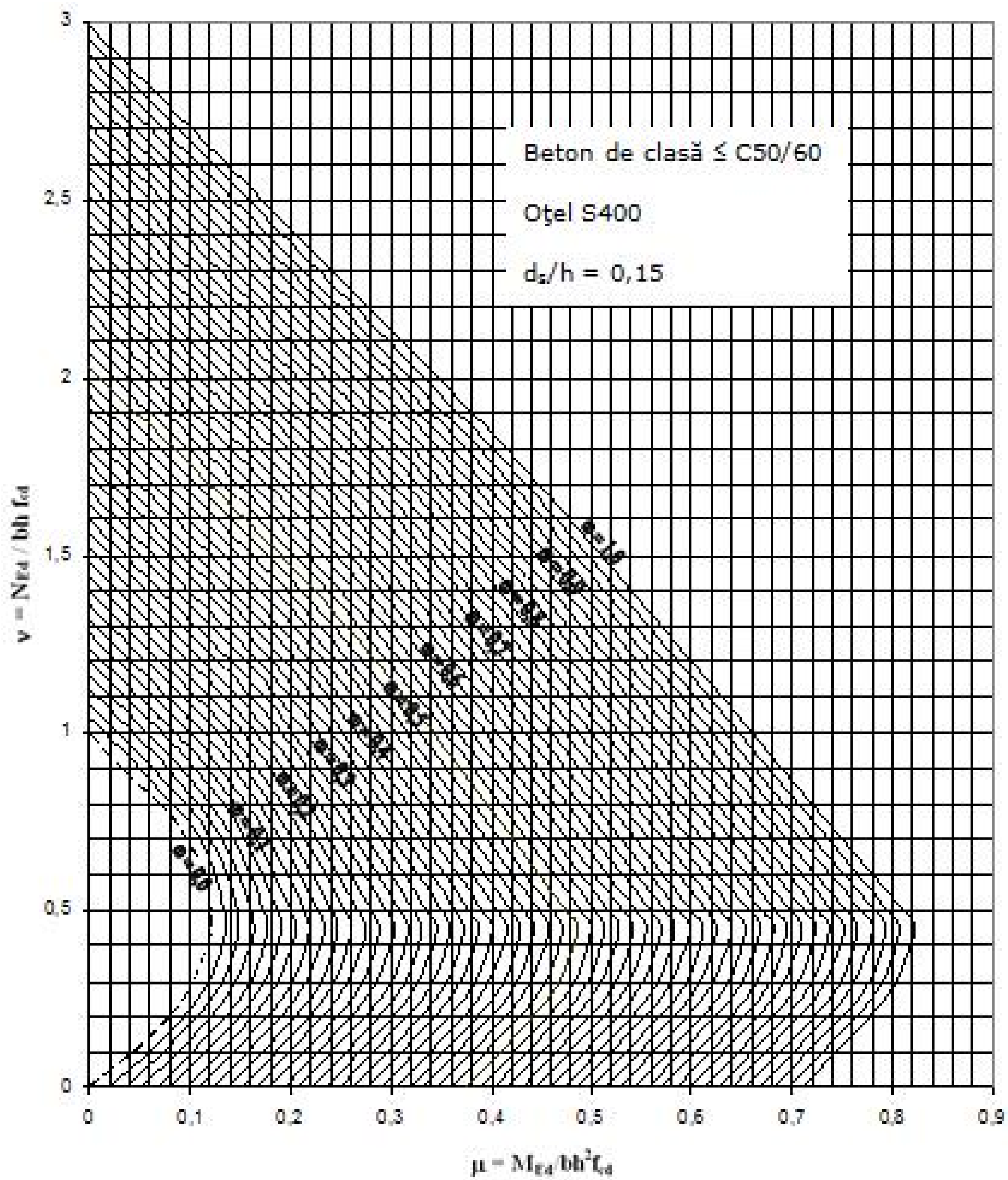
- strength class of concrete $\leq$C50/60
- steel without limit for ultimate strain (horizontal branch)
- calculations using stress block diagram in compressed concrete

6.6. FINAL REMARKS



6.6. FINAL REMARKS

	Calculation:	Design $A_{s1} = A_{s2}$	Check M_{Rd}
1	Input to diagram	μ_{Ed} și ν_{Ed}	ν_{Ed} și ω
2	Output from diagram	ω_{req}	μ_{Rd}
3	Result	$A_{s1} = A_{s2} = \omega_{req} b h f_{cd}/f_{yd}$	$M_{Rd} = \mu_{Rd} b h^2 f_{cd}$



M-N diagrams