Universitatea Politehnica Timişoara
Departamentul de Construcţii Metalice şi
Mecanica Construcţiilor

Metal constructions

6th Semester

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Cuprins

1. MAIN OBJECTIVES

The structural design should provide steel structures with the following attributes:

a) Overall strength and stability under the design loads

b) Serviceability under all normal loads and imposed deformations

c) Integrity, ductility and robustness against abnormal loads from extreme events

d) Adequate fire resistance

e) Durability in various natural environments (durability = the ability of a product to maintain its required performance over a given or long time, under the influence of foreseeable actions)

f) Buildability (the extent to which design of the buildings facilitates ease of construction, subject to the overall requirements of completed building)

g) Operability during the design working life

h) Economy: The structure should fulfill the above requirements at economic costs

i) Low environmental impact over the entire life cycle

2. CONTENT & LIST OF REFERENCES

- Elements subjected to axial force (tension, compression):
    - Configuration, type of sections
    - Strength verifications
    - Concept of instability, stability verifications
    - Eurocode provisions
- Restrained beams in bending:
    - Configuration, type of sections
    - Strength, deformation (serviceability)
- Unrestrained beams: elastic buckling, design approach
- Columns
    - Short columns, slender columns, slenderness
    - In plane behavior (axial force and uniaxial bending)
    - In and out of plane behavior (axial force and biaxial bending);
    - Buckling (lateral torsional, flexural)
    - Eurocode design provisions
- Plated girders;
    - Types of plated girders, applications
    - Behavior of plated structural elements with slender web
    - Design of plated structural elements in bending
    - Actions induced by cranes machinery, runway beams
- Cold-formed members:
    - Steel sections, fabrication
    - Cold-formed steel design
    - Connections
    - Types of applications
- Plastic behavior:
    - Generalities
    - Conditions of application
    - Type of analysis vs. cross section class
    - Cyclic behavior in bending
    - Factors affecting the plastic resistance
- Fatigue in steel elements:
    - High cycle fatigue
    - Low cycle fatigue
    - Fatigue curves
    - Connection details

<u>References:</u>

- Eurocode 3:
    - EN 1993-1-1: General rules and rules for buildings
    - EN 1993-1-3: Cold-formed thin gauge members and sheeting
    - EN 1993-1-5: Plated structural elements
    - EN 1993-1-6: Strength and stability of shell structures
    - EN 1993-1-8: Design of joints
    - EN 1993-1-9: Fatigue strength of steel structures
    - EN 1993-1-10: Selection of steel for fracture toughness and through-thickness properties.

  - EN 1993-1-11: Design of structures with tension components made of steel
  - EN 1993-1-12: Supplementary rules for high strength steel.

- Calculul si proiectarea constructiilor din profile metalice din profile metalice cu pereti subtiri formate la rece (D.Dubina, V.Ungureanu) Vol.I, Colectia LINDAB, 2004.
- Access steel: (www.access-steel.com)
- Calculul structural global al structurilor metalice în conformitate cu SR EN 1993-1-1 şi SR EN 1998-1: recomandări, comentarii şi exemple de aplicare (Dan Dubina, Dinu Florea, Aurel Stratan, Norin Filip vacarescu)
- Verificarea la stabilitate a elementelor din oţel în conformitate cu SR EN 1993-1.1. Recomandări de calcul, comentarii şi exemple de aplicare
- Elastic Design of Single-Span Steel Portal Frame Buildings to Eurocode 3, D M Koschmidder and D G Brown, The Steel Construction Institute, Publication Number SCI P397.
- Design of portal frames to Eurocode 3, D G Brown, The Steel Construction Institute, Publication Number SCI P400.
- Rules for Member Stability in EN 1993-1-1, ECCS, 2006.
- Simões da Silva L., Simões R., Gervásio, H.: Design of steel structures. ECCS Eurocode Design Manuals, ECCS and Ernst & Sohn, 2010, 438 p.
- Design of Cold-formed Steel Structures, ECCS and Ernst & Sohn, 2012
- Design of steel structures for buildings in seismic areas, ECCS and Ernst & Sohn, 2017.
- Design of plated structures, ECCS and Ernst & Sohn, 2011.
- The Behaviour and Design of Steel Structures to EC3, N.S. Trahair, M.A. Bradford, D.A. Nethercot, and L. Gardner, Taylor & Francis, 490 pg, 2008.

3. STEEL SECTIONS, MEMBERS, STRUCTURES

Bars	• Single • Compound	Single sections (beams, columns, …)
		Battened member, laced member, trusses
Wires		Built-up members (beams, columns, …)
		Individual elements (simple supported, double hinged, cantilevers, …)
Plates		Straight, curved, …
		Portal frames, braced, …
		Plated structural elements, shells

4. MEMBER DESIGN

4.1. General

- Design of members should be done according to EN 1993 (relevant chapters).
- The overall process of member design includes:
  - Classification of cross sections
  - Cross-section resistance
  - Member buckling
  - Combined effects (interaction): axial force + shear force + bending, where applicable.

4.2. Partial factors for resistance

- The partial factors γ_M that are applied to the various characteristic values of resistance in member design are:

γ_{M0} partial factor for resistance of cross-sections (whatever the class is)

γ_{M1} partial factor for resistance of members to instability assessed by member checks

γ_{M2} partial factor for resistance of cross-sections in tension to fracture

Partial factors γ_{Mi} for buildings may be defined in the National Annex. The following numerical values are recommended for buildings:

$\gamma_{M0} = 1.00$

$\gamma_{M1} = 1.00$

$\gamma_{M2} = 1.25$

4.3. Resistance of cross-sections (see en1993-1-1, &6.2)

The design value of an action effect in each cross section should not exceed the corresponding design resistance and, if several action effects act simultaneously, the combined effect should not exceed the resistance for that combination.
The design values of resistance should depend on the classification of the cross-section.

Elastic verification according to the elastic resistance may be carried out for all cross-sectional classes using the following yield criterion for a critical point of the cross section (unless other interaction formulae apply):

$$\left(\frac{\sigma_{x,Ed}}{f_y/\gamma_{M0}}\right)^2+\left(\frac{\sigma_{z,Ed}}{f_y/\gamma_{M0}}\right)^2-\left(\frac{\sigma_{x,Ed}}{f_y/\gamma_{M0}}\right)\left(\frac{\sigma_{z,Ed}}{f_y/\gamma_{M0}}\right)+3\left(\frac{\tau_{Ed}}{f_y/\gamma_{M0}}\right)^2\leq 1 \qquad (1)$$

where
$\sigma_{x,Ed}$ is the design value of the local longitudinal stress at the point of consideration
$\sigma_{z,Ed}$ is the design value of the local transverse stress at the point of consideration
τ_{Ed} is the design value of the local shear stress at the point of consideration.

Note:
- the effective cross-sectional properties are used for the verification of class 4 cross sections
- The verification according to (**1**) can be conservative as it excludes partial plastic stress distribution, which is permitted in elastic design. Therefore, it should only be performed where the interaction on the basis of resistances N_{Rd}, M_{Rd}, V_{Rd} cannot be performed.

The plastic resistance of cross sections should be verified by finding a stress distribution which is in equilibrium with the internal forces and moments without exceeding the yield strength. This stress distribution should be compatible with the associated plastic deformations

As a conservative approximation for all cross-section classes, a linear summation of the utilization ratios for each stress resultant may be used. For class 1, class 2 or class 3 cross sections subjected to the combination of N_{Ed} , $M_{y,Ed}$ and $M_{z,Ed}$ this method may be applied by using the following criteria:

$$\frac{N_{Ed}}{N_{Rd}}+\frac{M_{y,Ed}}{M_{y,Rd}}+\frac{M_{z,Ed}}{M_{z,Rd}}\leq 1 \qquad (2)$$

where N_{Rd}, $M_{y,Rd}$ and $M_{z,Rd}$ are the design values of the resistance depending on the cross sectional classification and including any reduction that may be caused by shear effects.

Where all the compression parts of a cross-section are at least Class 2, the cross-section may be taken as capable of developing its full plastic resistance in bending.

Where all the compression parts of a cross-section are Class 3, its resistance should be based on an elastic distribution of strains across the cross-section. Compressive stresses should be limited to the yield strength at the extreme fibres.

Note:

Where yielding first occurs on the tension side of the cross section, the plastic reserves of the tension zone may be utilized by accounting for partial plasticization when determining the resistance of a Class 3 cross-section.

4.4. Class of cross-section

When parts of the cross sections are in compression, strength and stability design is related to the element class: **1, 2, 3, 4**

- Rolled or welded sections may be considered as an assembly of individual plate elements (see Figure 1):
    - Outstand elements: flanges of I beams, legs of angles and T-s
    - Internal elements: web of open beam, webs and flanges of box section
- As the plate elements are relatively thin (compare two the other two dimensions), they may buckle locally when loaded in compression
- The tendency of any steel plate element within the cross section to buckle may limit the axial load carrying capacity, or the bending resistance of the section, by preventing the attainment of yield resistance.
- Avoidance of premature failure arising from the effects of local buckling may be achieved by limiting the **width-to-thickness ratio** for individual elements within the cross section.

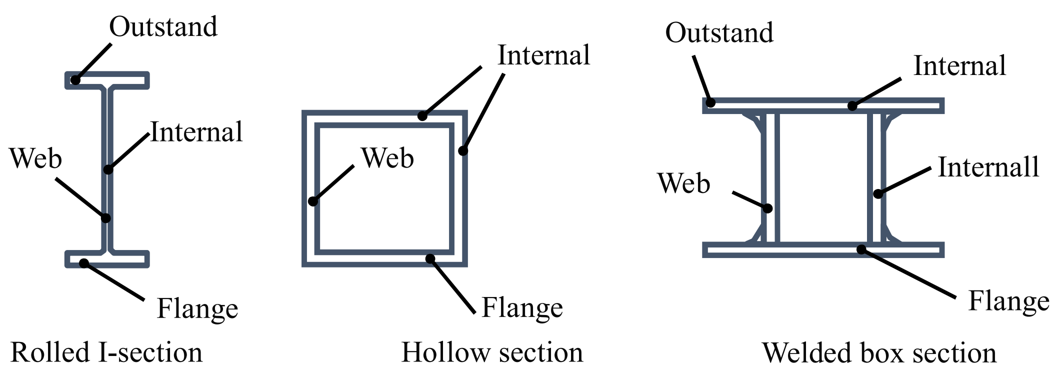


Figure 1: Cross sections – constitutive parts

- EN1993-1-1 defines four classes of cross section:
    - Class 1: “plastic”, cross-sections can form a plastic hinge with the required rotational capacity for plastic analysis.
    - Class 2: “plastic”, cross-sections although can develop a plastic moment, have limited rotational capacity and are therefore unsuitable for structures designed by plastic analysis
    - Class 3: “elastic”, the calculated stress in the extreme compression fiber can reach yield but local buckling prevents the development of the plastic moment resistance.
    - Class 4: “elastic” with reduced section (effective section)

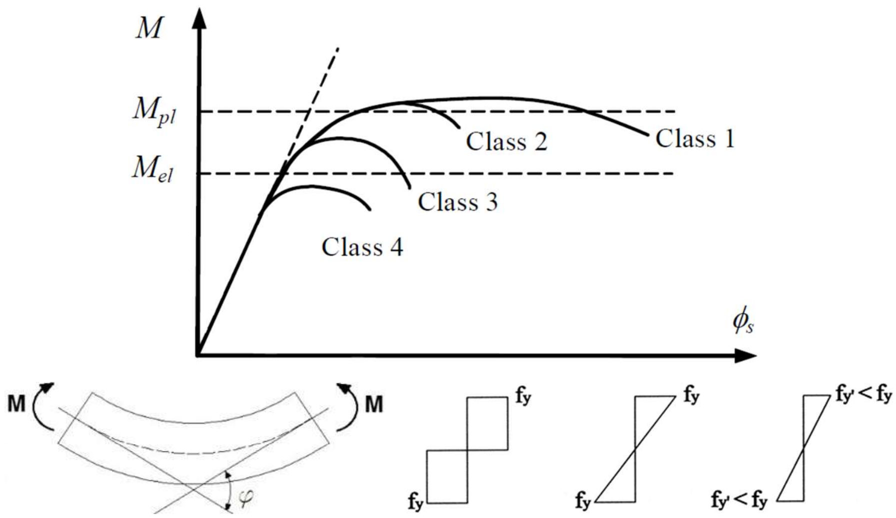


Figure 2. Classification of cross sections

Cross section classification is required for:
- Selection of the global frame analysis:
  - Elastic frame analysis
  - Plastic frame analysis
- Decision about the type of cross-section verification:
  - Plastic verification
  - Elastic verification
  - Elastic verification with effective cross-section properties
- Decision on the member buckling formulae with respect to the degree of local plastic capacity:
  - Plastic interaction: class 1, 2
  - Elastic interaction

Classification must be made:
- for loading states including all internal forces/moments $\mathbf{N_{Ed}}$ + $\mathbf{M_{y,Ed}}$ + $\mathbf{M_{z,Ed}}$ (internal forces which induce compressive stresses);
  for each load combination.

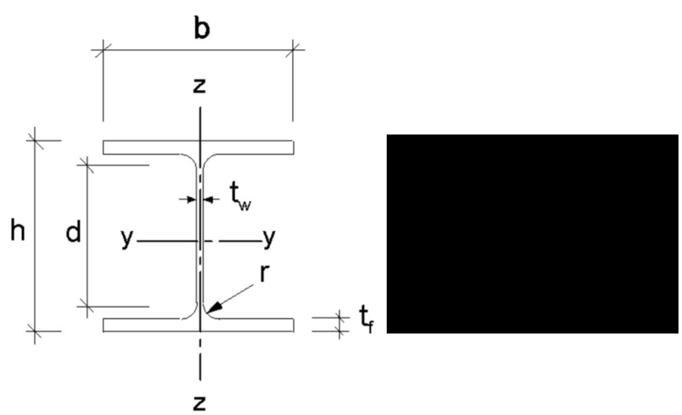


Figure 3. Dimensions, axes and internal forces/moment

- The class into which a cross section falls depends upon:
- slenderness of each element (defined by a width-to-thickness ratio).
- steel grade: the parameter is $\varepsilon = \sqrt{235/f_y}$, where f_y is the nominal yield strength
- the compressive stress distribution.

The classification of a cross section is based on its maximum resistance to the type of applied internal forces, independent from their values. This procedure is straightforward to apply for cross sections to compression forces or bending moment, acting separately. However, in the case of bending and axial force, there is a range of *M-N* values that correspond to the ultimate resistance of the cross section. Consequently, there are several values of the parameter α (limit for classes 1 and 2) or the parameter ψ (limit for class 3), both being dependent on the position of the neutral axis.
Bearing in mind this additional complexity, simplified procedures are often adopted, such as:

i) to consider the cross section subjected to compression only, being the most unfavorable situation (too conservative in some cases);
ii) to classify the cross section based on an estimate of the position of the neutral axis based on the applied internal forces.

4.5. Behavior of plate elements in compression

A thin flat rectangular plate subjected to compressive forces along its short edges (see Figure 4.a) has an elastic critical buckling stress (σ_{cr}) given by:

$$\sigma_{cr} = \frac{k_\sigma \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \qquad (3)$$

where:
k_σ is the plate buckling parameter which accounts for edge support conditions, stress distribution and aspect ratio of the plate;
v= Poisson's coefficient
E = Young's modulus

Open structural sections comprise several plates that are free along one longitudinal edge (Figure 4.b) and tend to be very long compared with their width. These plates buckled shape is seen in figure Figure 4.c.

The relationship between aspect ratio and buckling parameter for a long thin outstand element of this type is shown in Figure 4.d. As seen, the buckling parameter tends towards a limiting value of 0.425 as the plate aspect ratio increases. Figure 5 presents also the relationship between aspect ratio and buckling parameter but considering different edge support conditions.

For a section to be classified as class 3 (or better, i.e. 1 and 2), the elastic critical buckling stress (σ_{cr}) must attain or exceed the yield stress, f_y.
From equation (3) this condition is fulfilled if:

$$\frac{\boldsymbol{b}}{\boldsymbol{t}} < \mathbf{0.92}\left(\frac{\boldsymbol{k_\sigma E}}{\boldsymbol{f_y}}\right)^{\mathbf{0.5}} \qquad (4)$$

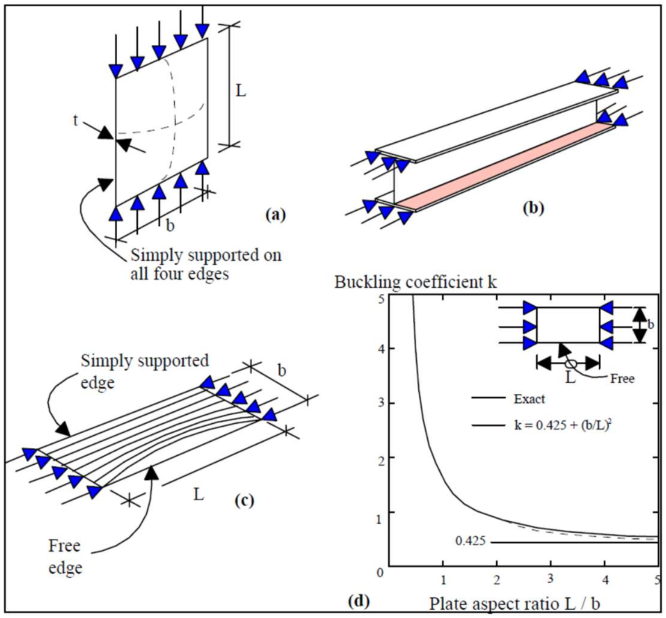
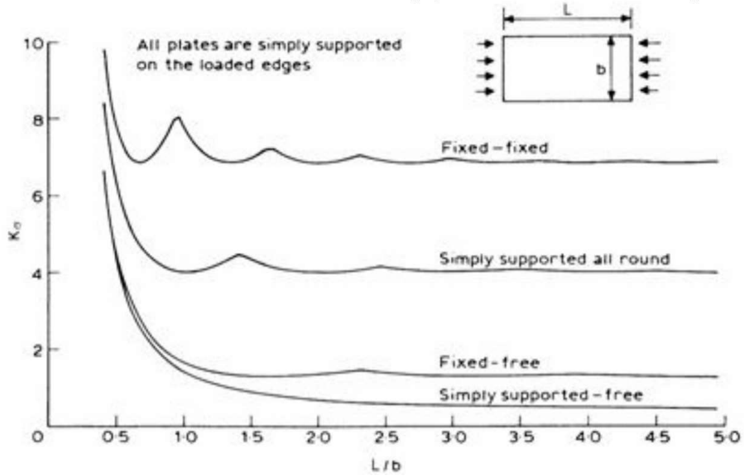


Figure 4. Behavior of plate elements in compression: a) short plate supported on four edges; b) I section with flanges (outstand element) as long plates; c) deformed shape of a long plate supported along a longitudinal edge; d) relationship between aspect ratio L/b and buckling parameter for a long thin outstand element



Support conditions at long edges	Buckling factor k_σ
Clamped + clamped	6,97
Clamped + simply supported	5,40
Simply supported + simply supported	4,00
Clamped + free	1,28
Simply supported + free	0,43
Free + free	$(b/a)_2$

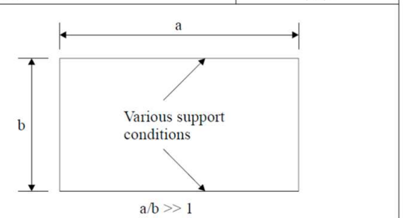


Figure 5. The relationship between aspect ratio and buckling parameter for different edge support conditions

The elastic-plastic behavior of a perfect plate element subjected to uniform compression may be represented by a normalized load-slenderness diagram where normalized ultimate load, $\overline{N_p}$, and normalised plate slenderness, $\overline{\lambda_p}$, are given by:

$$\overline{\boldsymbol{N_p}} = \frac{\boldsymbol{\sigma_{ult}}}{\boldsymbol{f_y}} \qquad (5)$$

$$\overline{\boldsymbol{\lambda_p}} = \left(\frac{\boldsymbol{f_y}}{\boldsymbol{\sigma_{cr}}}\right)^{\mathbf{0.5}} \qquad (6)$$

Using (3) and (6), and replacing f_y with $235/\varepsilon^2$ (to extend the formula for other steel graded, e.g. S275, S355), the normalized plate slenderness, $\overline{\lambda_p}$, may be written as:

$$\overline{\lambda_p} = \left(\frac{f_y}{\sigma_{cr}}\right)^{0.5} = \left(\frac{b/t}{28.4\varepsilon\sqrt{k_\sigma}}\right) \tag{7}$$

Figure 6 shows the relationship between $\overline{N_p}$ and $\overline{\lambda_p}$. For large values of the normalized slenderness, the ultimate load capacity is limited due to attainment of elastic critical buckling stress, σ_{cr}, before reaching the yielding. For normalized plate slenderness less than one, the normalized ultimate load can reach its squash load.
The actual behavior is somewhat different from the ideal elastic-plastic behavior due to:

i. initial geometrical and material imperfections
ii. strain-hardening of the material
iii. the post-buckling behavior.

These factors require $\overline{\lambda_p}$ values to be reduced. This is made to delay the onset of local buckling until the requisite strain distribution through the section (yield at the extreme fiber or fully plastic distribution) has been attained.

EN1993-1-1 uses the following normalized plate slenderness as limits for classifications:

- Class 1 $\overline{\lambda_p} < 0{,}5$
- Class 2 $\overline{\lambda_p} < 0{,}6$
- Class 3 $\overline{\lambda_p} < 0{,}9$ for elements under a stress gradient; this is further reduced to 0,74 for elements in compression throughout.

By substituting the appropriate values of k_σ into equation (7) and noting the $\overline{\lambda_p}$ to be used for each class, limiting b/t ratios can be calculated.
Rolled sections of usual dimensions (HEA, HEB, IPE, etc.) belong, in general, to classes 1, 2 or 3. Class 4 cross sections are typical of plate girders and cold-formed sections. Class 4 cross sections are characterized by local buckling phenomena, preventing the cross section from reaching its elastic resistance. The limiting proportions for Class 1, 2, and 3 compression parts are given in next tables (see EN1993-1-1, Table 5.2). A part which fails to satisfy the limits for Class 3 should be taken as Class 4.

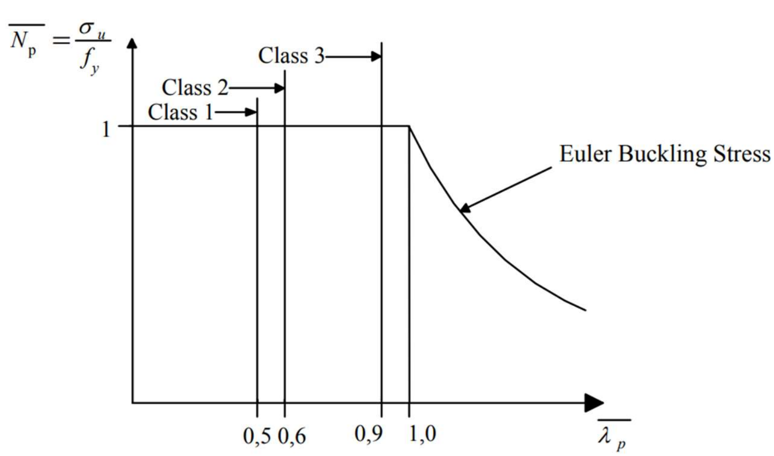
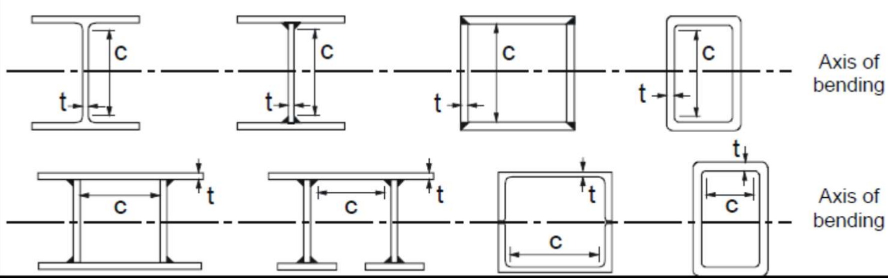
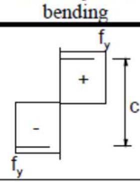
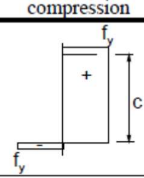
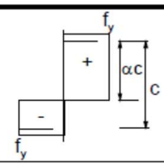
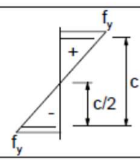
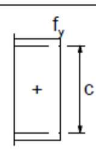
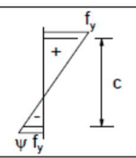


Figure 6. Normalized load-slenderness

Table 1. Maximum width-to-thickness ratios for compression parts – internal elements

Internal compression parts



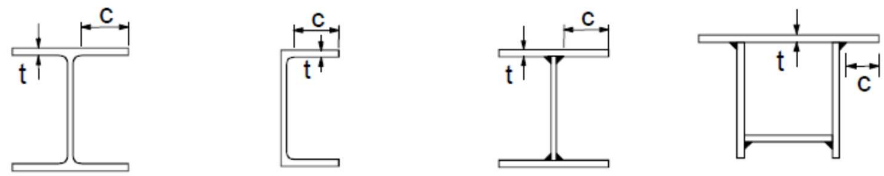
Class	Part subject to bending	Part subject to compression	Part subject to bending and compression
Stress distribution in parts (compression positive)			
1	$c/t \le 72\varepsilon$	$c/t \le 33\varepsilon$	when $\alpha > 0{,}5$: $c/t \le \frac{396\varepsilon}{13\alpha - 1}$ when $\alpha \le 0{,}5$: $c/t \le \frac{36\varepsilon}{\alpha}$
2	$c/t \le 83\varepsilon$	$c/t \le 38\varepsilon$	when $\alpha > 0{,}5$: $c/t \le \frac{456\varepsilon}{13\alpha - 1}$ when $\alpha \le 0{,}5$: $c/t \le \frac{41{,}5\varepsilon}{\alpha}$
Stress distribution in parts (compression positive)			
3	$c/t \le 124\varepsilon$	$c/t \le 42\varepsilon$	when $\psi > -1$: $c/t \le \frac{42\varepsilon}{0{,}67 + 0{,}33\psi}$ when $\psi \le -1^{*)}$: $c/t \le 62\varepsilon(1-\psi)\sqrt{(-\psi)}$

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

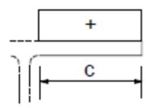
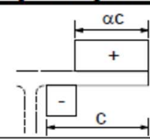
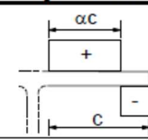
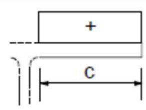
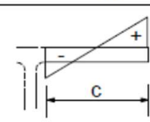
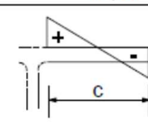
*) $\psi \le -1$ applies where either the compression stress $\sigma \le f_y$ or the tensile strain $\varepsilon_y > f_y/E$

Table 2. Maximum width-to-thickness ratios for compression parts – outstand elements

Outstand flanges



Rolled sections | Welded sections

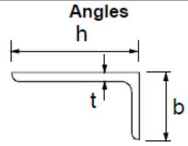
Class	Part subject to compression	Part subject to bending and compression: Tip in compression	Part subject to bending and compression: Tip in tension
Stress distribution in parts (compression positive)			
1	$c/t \le 9\varepsilon$	$c/t \le \frac{9\varepsilon}{\alpha}$	$c/t \le \frac{9\varepsilon}{\alpha\sqrt{\alpha}}$
2	$c/t \le 10\varepsilon$	$c/t \le \frac{10\varepsilon}{\alpha}$	$c/t \le \frac{10\varepsilon}{\alpha\sqrt{\alpha}}$
Stress distribution in parts (compression positive)			
3	$c/t \le 14\varepsilon$	$c/t \le 21\varepsilon\sqrt{k_\sigma}$ For k_σ see EN 1993-1-5	

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

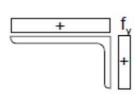
Table 3. Maximum width-to-thickness ratios for compression parts – angles and tubular sections

Angles

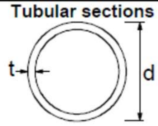
Refer also to "Outstand flanges" (see sheet 2 of 3)



Does not apply to angles in continuous contact with other components

Class	Section in compression
Stress distribution across section (compression positive)	
3	$h/t \le 15\varepsilon: \ \frac{b+h}{2t} \le 11{,}5\varepsilon$

Tubular sections



Class	Section in bending and/or compression
1	$d/t \le 50\varepsilon^2$
2	$d/t \le 70\varepsilon^2$
3	$d/t \le 90\varepsilon^2$ **NOTE** For $d/t > 90\varepsilon^2$ see EN 1993-1-6.

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71
	ε^2	1,00	0,85	0,66	0,56	0,51

4.6. Elastic buckling of thin plates. Effective width approach used for class 4 sections in compression

Class 4 cross-sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross-section.
In Class 4 cross sections, effective widths can be used to make the necessary allowances for reductions in resistance (for more details, see EN 1993-1-5). An additional moment $\Delta M = N \times e_N$ due to the possible shift of the centroid should be also considered.
To allow for the reduction in strength, the actual nonlinear distribution of stress is considered by a linear distribution of stress acting on a reduced "effective plate width" leaving an "effective hole" where the buckle occurs.

The reason for the reduction in strength is that local buckling occurs at an early stage in parts of the compression elements of the member; the stiffness of these parts in compression is thereby reduced and the stresses are distributed to the stiffer edges, see Figure 7.

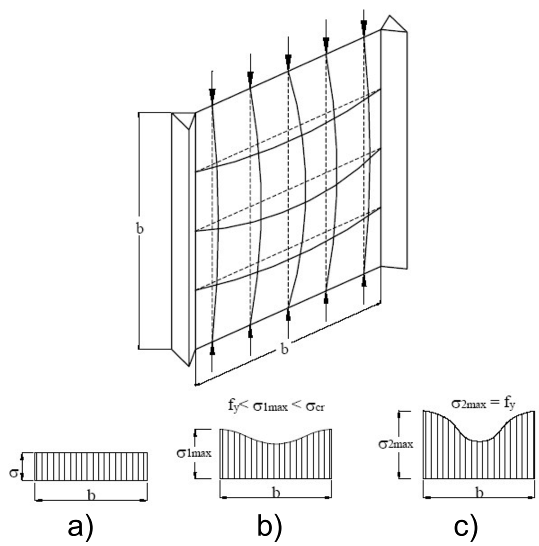
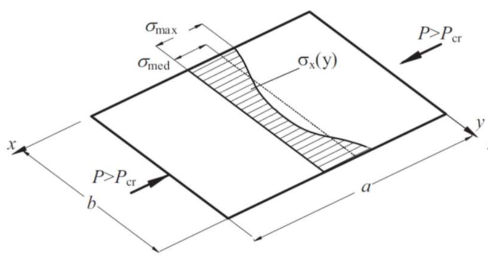


Figure 7. Consecutive stress distribution in stiffened (internal) compression elements: (a) pre-critical stage; (b) intermediate post-critical stage; (c) ultimate post-critical stage

The elastic post-buckling behavior of a plate can be analyzed by using the large deflection theory.

However, it has been found that the solution of the differential equation for large deflections has little application in practical design because of its complexity. For this reason, the concept of "effective width" was introduced by von Karman et al. in 1932. In this approach, instead of considering the non-uniform distribution of stress, $\sigma_x(y)$, over the entire width of the plate b_f, it was assumed that the total load, P, is carried by a fictitious effective width, b_{eff}, subjected to a uniformly distributed stress equal to the edge stress, σ_{max}, as shown in Figure 8.



a) b)

Figure 8. Stress distribution in simply supported plate, uniaxially compressed: (a) actual stress distribution; (b) equivalent stress distribution based on the "effective width" approach

The width b_{eff} is selected so that the area under the curve of the actual non-uniform stress distribution is equal to the sum of the two parts of the equivalent rectangular shaded area with a total width b_{eff} and an intensity of stress equal to the edge stress σ_{max}, that is:

$$\boldsymbol{P = \sigma_{med} \times b \times t = \int_0^b \sigma_x(y) \times t \times dy = \sigma_{max} \times b_{eff} \times t} \tag{8}$$

The magnitude of effective width, b_{eff}, changes as the magnitude of σ_{max} changes (see Figure 9). Therefore the minimum effective width results when σ_{max} equals to f_y (see Figure 7.c).

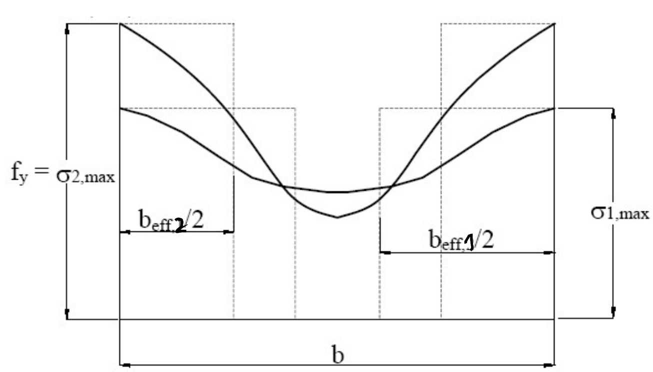


Figure 9. Change of effective width in terms of maximum edge stress

In the limit $\sigma_{max}=f_y$, it may also be considered that the effective width b_{eff} represents a particular width of the plate for which the plate strength is achieved when the applied stress ($\sigma_{max}=f_y$) causes buckling. Therefore, for a long plate, the value of b_{eff} to be used for strength design may be determined from Eqn. (3) as follows:

$$\boldsymbol{\sigma_{max} = f_y = \frac{k_\sigma \cdot \pi^2 \cdot E}{12 \cdot (1-\vartheta^2) \cdot (b_{eff}/t)^2} = \sigma_{cr,eff}} \tag{9}$$

or:

$$b_{eff} = \frac{\sqrt{k_\sigma} \cdot \pi}{\sqrt{12(1-\vartheta^2)}} \cdot t \cdot \sqrt{\frac{E}{f_y}} \tag{10}$$

or:

$$b_{eff} = C \cdot t \sqrt{\frac{E}{f_y}} \tag{11}$$

where:

$$C = \frac{\sqrt{k_\sigma \cdot \pi^2}}{\sqrt{12(1-\vartheta^2)}} \tag{12}$$

is a constant for a given type of plate element, depending of the value of buckling coefficient, k_σ.

If $k_\sigma = 4$ and v=0.3, ➔ C = 1.9, and eq. (9) becomes:

$$b_{eff} = 1.9 \cdot t \sqrt{\frac{E}{f_y}} \tag{13}$$

which represents the ***von Karman*** formula for the design of stiffened elements (web type) as derived in 1932.

Since the critical elastic buckling stress of the complete plate is given by eq. (3), then by substitution:

$$\frac{b_{eff}}{b} = \sqrt{\frac{\sigma_{cr}}{f_y}} \tag{14}$$

➔ the effective width, b_{eff}, of a thin wall of width *b* in compression is calculated with the following formula:

$$b_{eff} = b \times \rho \tag{15}$$

where:

$$\rho = \frac{b_{eff}}{b} = \frac{1}{\lambda_p} \tag{16}$$

is the reduction factor of the plate within the post-buckling range.

In the intermediate post-buckling stage, when $\sigma_{cr} < \sigma_{max} < f_y$, the effective width can be obtained from:

$$b_{eff} = C \cdot t \sqrt{\frac{E}{\sigma_{max}}} \quad (17)$$

or

$$\frac{b_{eff}}{b} = \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \quad (18)$$

with the corresponding relative slenderness of the plate defined as:

$$\overline{\lambda_p} = \sqrt{\frac{\sigma_{max}}{\sigma_{cr}}} \quad (19)$$

Eqn. (10) for *C*, which for plates that are simply supported on both longitudinal edges (i.e. k_σ=4) leads to the value of 1.9, was confirmed by test of plates with large b/t ratios.

However, for plates with intermediate b/t ratios, Winter proposed in 1946 to replace the expression for *C* with:

$$C = 1.9\left[1 - 0.415\frac{t}{b}\sqrt{\frac{E}{f_y}}\right] \quad (20)$$

which leads to the effective width equation:

$$\rho = \frac{b_{eff}}{b} = \sqrt{\frac{\sigma_{cr}}{f_y}}\left(1 - 0.22\sqrt{\frac{\sigma_{cr}}{f_y}}\right) \leq 1 \quad (21)$$

or, in terms of relative plate slenderness:

$$\rho = \frac{1}{\overline{\lambda_p}}\left(1 - \frac{0.22}{\overline{\lambda_p}}\right) \quad (22)$$

The effective width depends on both edge stress σ_{max} and b/t ratio. The plate is fully effective when ρ = 1, i.e. b = b_{eff}. It is easy to show that this happens when $\overline{\lambda_p} \leq$ 0,673 or:

$$\frac{b}{t} < \left(\frac{b}{t}\right)_{lim} = 19.11 \cdot \varepsilon \cdot \sqrt{k_\sigma} \quad (23)$$

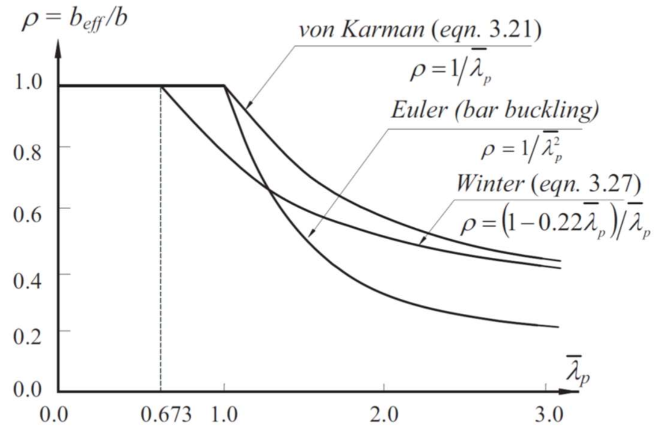


Figure 10. Reduction factor $\boldsymbol{\rho}$ vs. relative plate slenderness, $\overline{\boldsymbol{\lambda_p}}$

If $\boldsymbol{k_\sigma}$= 4 and $\boldsymbol{k_\sigma}$= 0.425 are substituted into Eqn. (21) for simply supported edge stiffened (web type plate), and for unstiffened plate elements (flange type), respectively, the following limiting *b/t* ratios are obtained:

- Web type

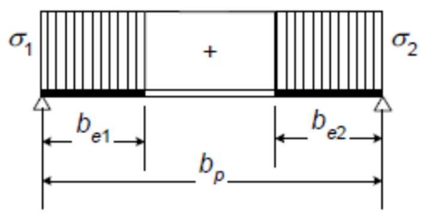
$$\left(\frac{\boldsymbol{b}}{\boldsymbol{t}}\right)_{\boldsymbol{lim}} = \mathbf{38.3} \cdot \boldsymbol{\varepsilon} \tag{24}$$

- Flange type

$$\left(\frac{\boldsymbol{b}}{\boldsymbol{t}}\right)_{\boldsymbol{lim}} = \mathbf{12.5} \cdot \boldsymbol{\varepsilon} \tag{25}$$

Table 4 and Table 5 show the effective widths for internal elements (web type, doubly supported) and outstand elements (flange type) in compression.

Table 4. Effective width for internal elements (doubly supported) in compression

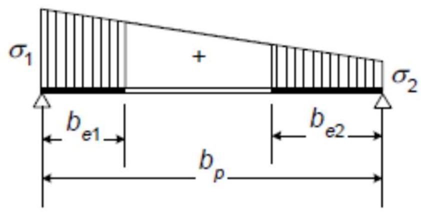
<table>
<tr><th>Stress distribution [compression positive]</th><th>Effective width b_{eff}</th></tr>
<tr><td>
</td><td>

$\psi = +1$:

$b_{eff} = \rho b_p$

$b_{e1} = 0{,}5\, b_{eff}$

$b_{e2} = 0{,}5\, b_{eff}$

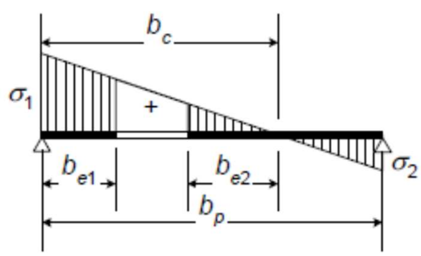
</td></tr>
<tr><td>
</td><td>

$0 \leq \psi < +1$:

$b_{eff} = \rho b_p$

$b_{e1} = \dfrac{2 b_{eff}}{5-\psi}$

$b_{e2} = b_{eff} - b_{e1}$

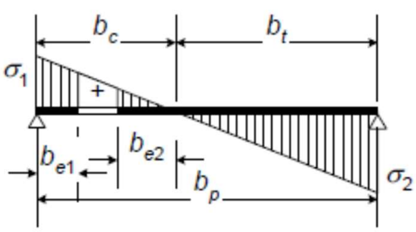
</td></tr>
<tr><td>
</td><td>

$-1 \leq \psi < 0$:

$b_{eff} = \rho b_c$

$b_{e1} = 0{,}4\, b_{eff}$

$b_{e2} = 0{,}6\, b_{eff}$

</td></tr>
<tr><td>
</td><td>

$\psi < -1$:

$b_{eff} = \rho b_c$

$b_{e1} = 0{,}4\, b_{eff}$

$b_{e2} = 0{,}6\, b_{eff}$

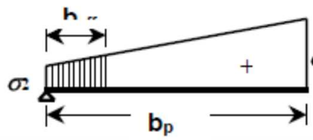
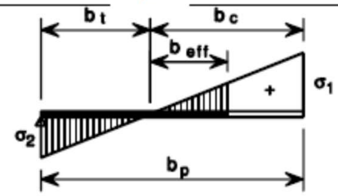
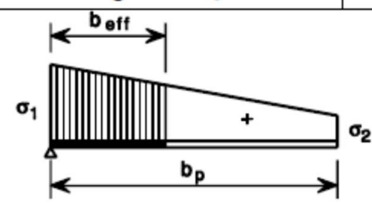
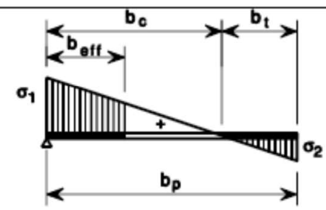
</td></tr>
</table>

$\psi = \sigma_2 / \sigma_1$	+1	$+1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling Factor k_σ	4,0	$\dfrac{8{,}2}{1{,}05+\psi}$	7,81	$7{,}81 - 6{,}29\psi + 9{,}78\psi^2$	23,9	$5{,}98(1-\psi)^2$

Alternatively, for $+1 \geq \psi \geq -1$:

$$k_\sigma = \frac{16}{\left[(1+\psi)^2 + 0{,}112(1-\psi)^2\right]^{0,5} + (1+\psi)}$$

Table 5. Effective width for outstand elements in compression

<table>
<tr><td colspan="4">Stress distribution
[compression positive]</td><td colspan="4">Effective width b_{eff}</td></tr>
<tr><td colspan="4"></td><td colspan="4">$0 \leq \psi < +1$:
$b_{eff} = \rho b_p$</td></tr>
<tr><td colspan="4"></td><td colspan="4">$\psi < 0$:
$b_{eff} = \rho b_c$</td></tr>
<tr><td colspan="3">$\psi = \sigma_2 / \sigma_1$</td><td>+1</td><td>0</td><td>-1</td><td colspan="2">$+1 \geq \psi \geq -1$</td></tr>
<tr><td colspan="3">Buckling factor k_σ</td><td>0,43</td><td>0,57</td><td>0,85</td><td colspan="2">$0,57 - 0,21\psi + 0,07\psi^2$</td></tr>
<tr><td colspan="4"></td><td colspan="4">$0 \leq \psi < +1$:
$b_{eff} = \rho b_p$</td></tr>
<tr><td colspan="4"></td><td colspan="4">$\psi < 0$:
$b_{eff} = \rho b_c$</td></tr>
<tr><td>$\psi = \sigma_2 / \sigma_1$</td><td>+1</td><td colspan="2">$+1 > \psi > 0$</td><td>0</td><td colspan="2">$0 > \psi > -1$</td><td>-1</td></tr>
<tr><td>Buckling Factor k_σ</td><td>0,43</td><td colspan="2">$\frac{0,578}{\psi + 0,34}$</td><td>1,70</td><td colspan="2">$1,70 - 5\psi + 17,1\psi^2$</td><td>23,8</td></tr>
</table>

The shift of the centroid axis of the effective cross-section relative to the centroid axis of the gross cross-section shall be considered (Figure 11, Figure 12). For members in compression, the shift of the centroid axis will give rise to a moment that should be accounted for in member design. For bending members this will be considered when calculating the effective section properties (shift of neutral axis).

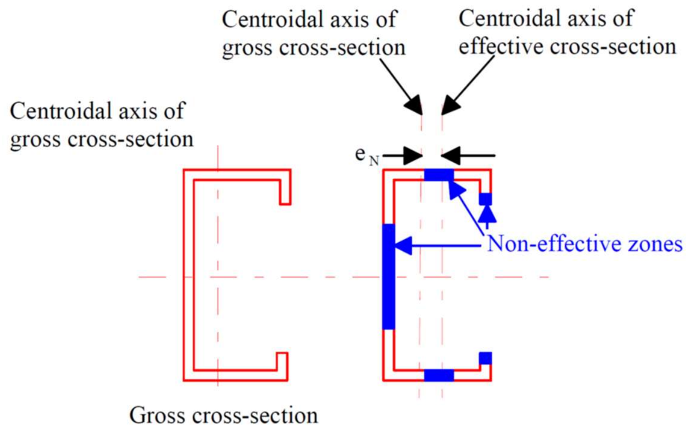


Figure 11. Shift of neutral axis for a class 4 cross-section in compression

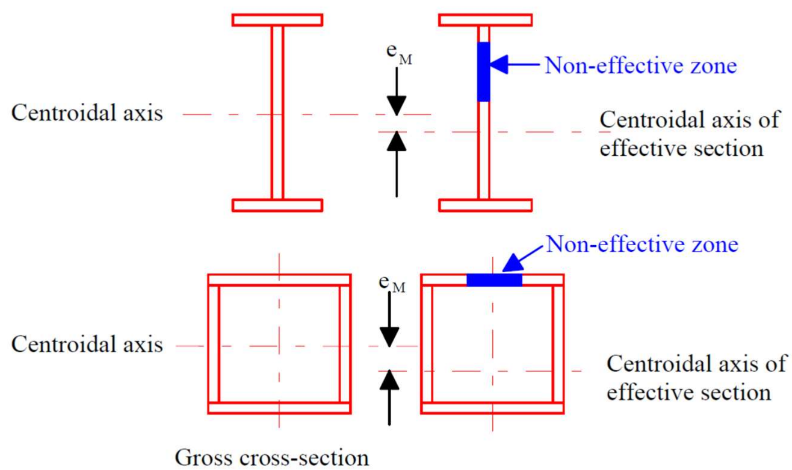


Figure 12. Shift of neutral axis for a class 4 cross-section in bending

5. ELEMENTS UNDER AXIAL LOADS

There are different kinds of structural elements that can be used to carry axial loads (tension, compression), but the most common are the pin ended struts that are found in trusses, lattice girders or bracing members. Columns under special loading conditions can also enter in this category, but in most cases, they are loaded by significant bending (due to eccentrical application of axial loads, transverse forces or bending moments) thus becoming beam-column elements.

Under compressive forces, slender elements tend to buckle, thus adding bending moments to the axial load. In such cases, member should be verified also against buckling.

Examples of application and recommended section shapes are presented below.

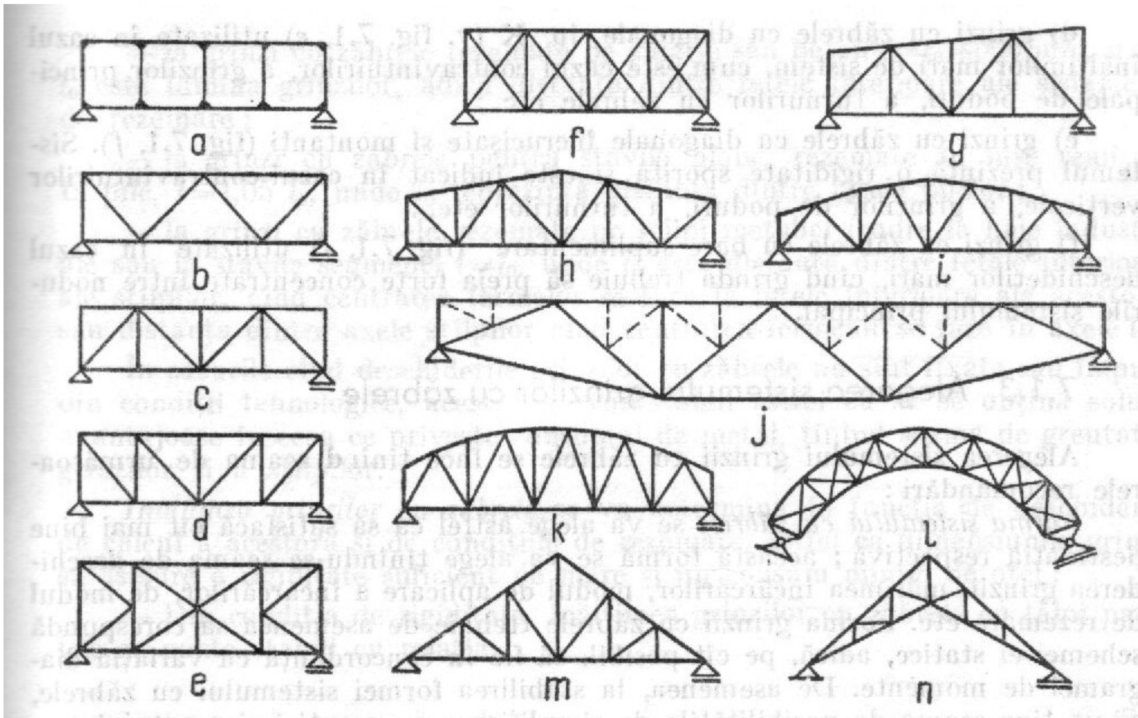


Figure 13. Truss girders – typical applications for elements under axial loads (except case (a) - Vierendeel truss with rigid connections where the elements also resist substantial bending forces).

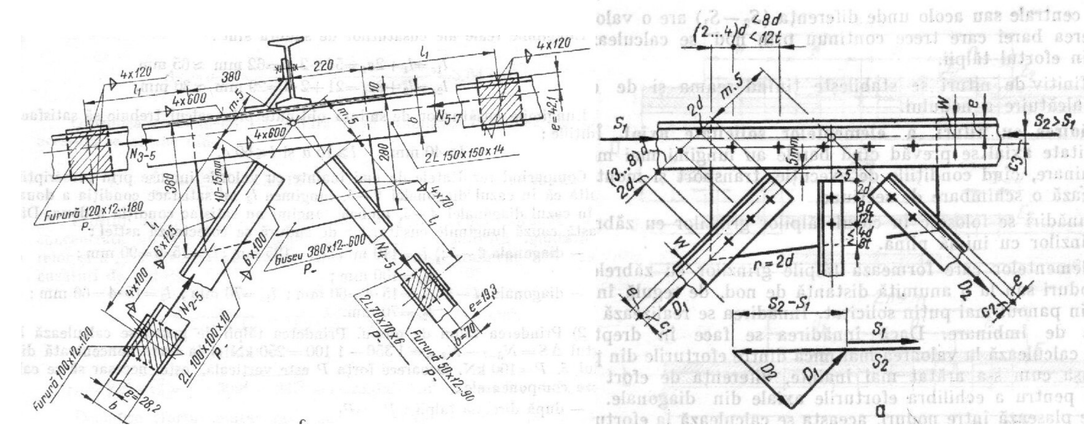
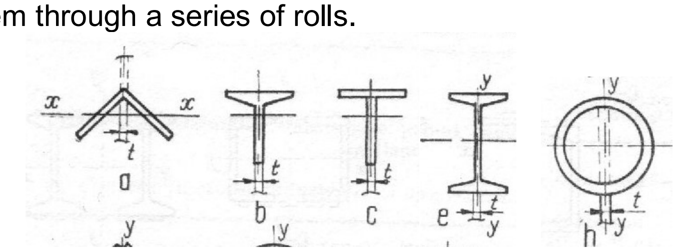
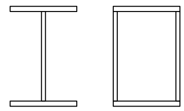


Figure 14. Typical joint details for truss girders: welded (left) and bolted (right)

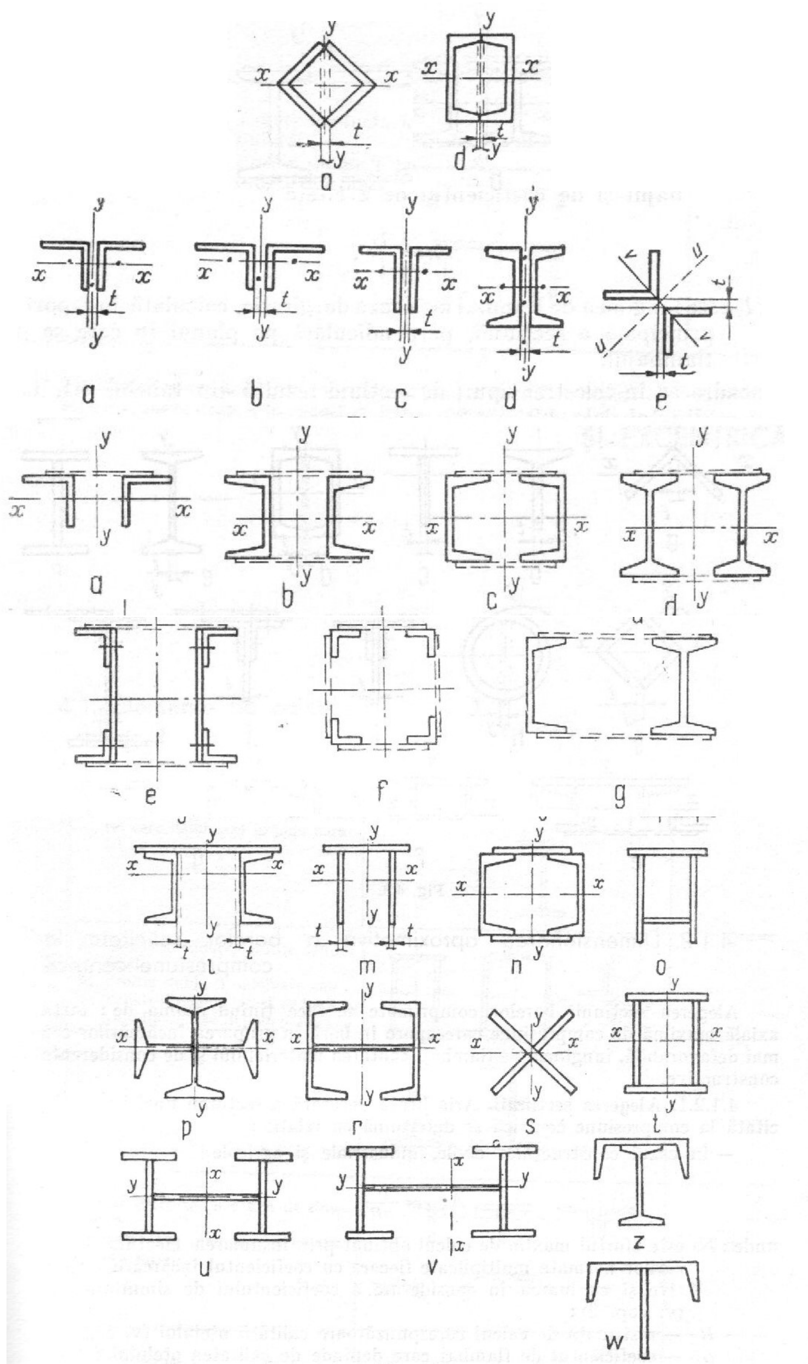
- Rolled and formed sections: are produced in steel mills from steel billets by passing them through a series of rolls.



- Built-up sections – are made by welding plates together

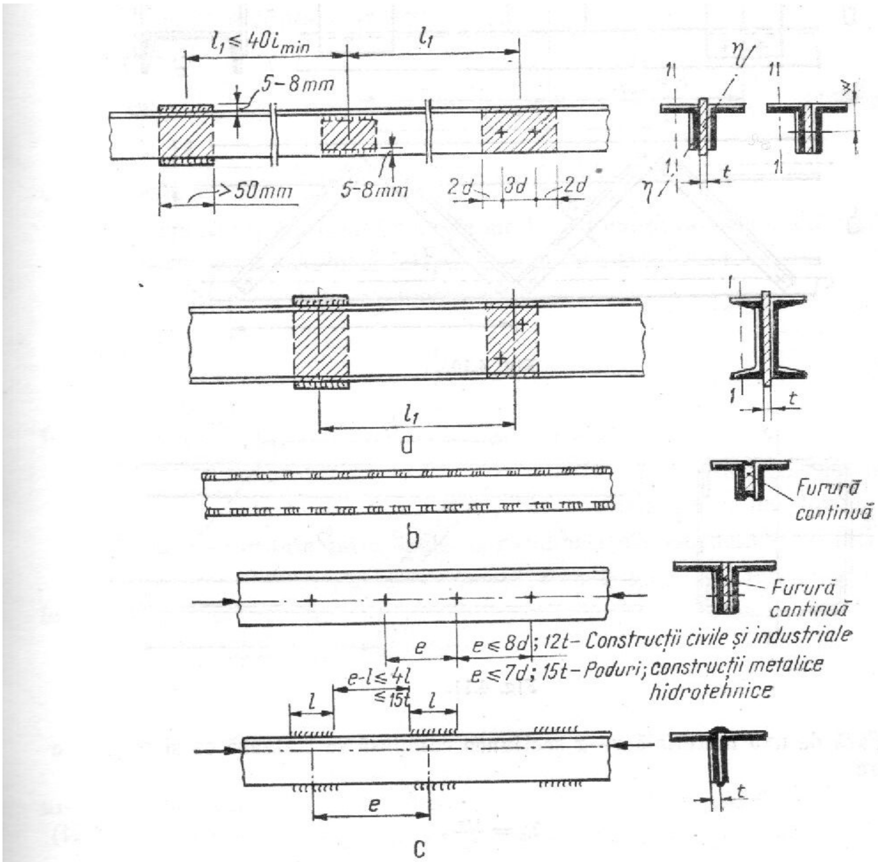


- Compound sections - combining two or more separate sections (rolled profiles, plates)



- Closely spaced built-up members

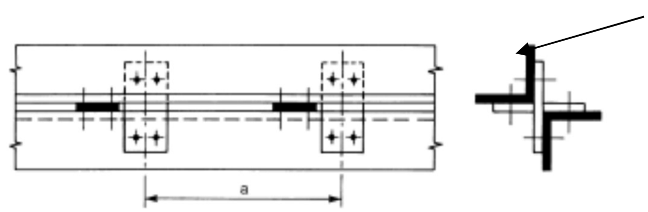
- Built-up members with double angles or channels, with chords in contact or closely spaced and connected through packing plates



Distance between gussets:

- $l_1 \leq 40 - 50i_1$ – compression
- $l_1 \leq 80i_1$ – tension

- Star-battened angle members



5.1. Resistance of elements in tension

For the elements in tension, the design value of the tension force N_{Ed} at each cross section shall satisfy:

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1.0 \tag{26}$$

where

N_{Ed} : design value of the tension force N_{Ed}

$N_{t,Rd}$: design tension resistance of the cross section

- For members connected by welding, design tension resistance $N_{t.Rd}$ is

$$N_{pl.Rd} = \frac{Af_y}{\gamma_{M0}},\ \gamma_{M_0} = 1.0 \quad (27)$$

- For members connected by bolting, design tension resistance $N_{t.Rd}$ is reduced due to presence of holes and is the lesser of:

$$N_{t,Rd} = \min(N_{pl,Rd}, N_{u,Rd}) \quad (28)$$

- in gross cross-section:

$$N_{pl,Rd} = \frac{Af_y}{\gamma_{M_0}} \quad (29)$$

where A is the gross area of the cross-section.

- in net cross-section

$$N_{u,Rd} = \frac{0.9 A_{net} f_u}{\gamma_{M_2}},\ \gamma_{M_2} = 1.25 \quad (30)$$

where A_{net} is the net area of the cross-section, 0,9 is a reduction factor for eccentricity, stress concentration etc, f_u is the ultimate tensile strength.

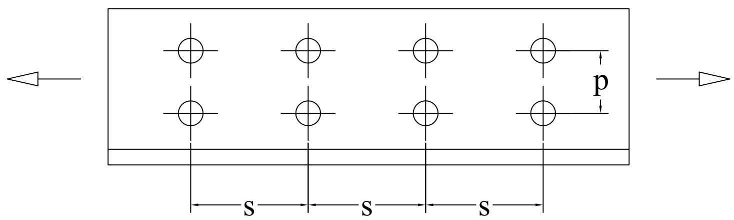
For category “C” connections (EN 1993-1.8) – slip resistance at ultimate limit state:

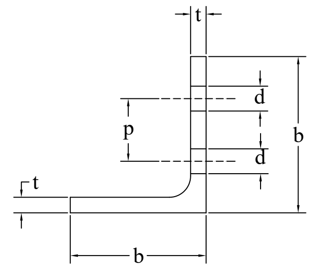
$$N_{u,Rd} = N_{net,Rd} = \frac{A_{net} f_u}{\gamma_{M0}} \quad (31)$$

The properties of the gross cross-section should be determined using the nominal dimensions. Holes for fasteners need not be deducted, but allowance should be made for larger openings. Splice materials should not be included.

The net area of a cross-section should be taken as its gross area less appropriate deductions for all holes and other openings. For calculating net section properties, the deduction for a single fastener hole should be the gross cross-sectional area of the hole in the plane of its axis. For countersunk holes, appropriate allowance should be made for the countersunk portion.

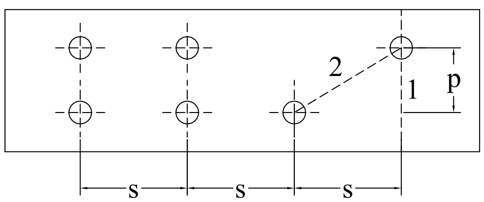
- Fasteners are not staggered

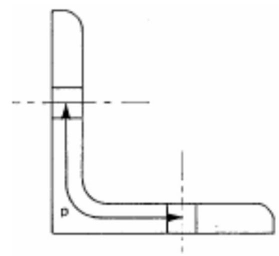




$A_{net} = A_{gross} - 2(d \times t)$
$2(d \times t)$ = deduction

- Fasteners are staggered





The total area to be deducted is the greater of:

- the area for holes crossing a perpendicular cross section

or

- the sum of the areas of all holes in any diagonal or zig-zag line across the member less $s^2t/4p$ for each gauge space in the chain of holes.

$$A_{net,1} = A_{gross,1} - (d \times t)$$

$$A_{net,2} = A_{gross,2} - 2(d \times t)$$

$$\Rightarrow A_{net,2} = \left[2(b - p/2) + \sqrt{p^2 + s^2}\right] \times t - 2 \times d \times t$$

5.2. Angles connected by one leg and other unsymmetrically connected members in tension

The effects of the spacing and edge distances of the bolts (and the eccentricity in joints) should be taken into account in determining the design resistance of:

- Unsymmetrical members;
- Symmetrical members that are connected unsymmetrically, such as angles connected by one leg.

For an angle connected by a single row of bolts in one leg, the member may be treated as concentrically loaded and the design ultimate resistance should be determined as follows:

- with 1 bolt:

$$N_{u,Rd} = \frac{2.0(e_2 - 0.5d_0)tf_u}{\gamma_{M_2}}, \tag{32}$$

- with 2 bolts:

$$N_{u,Rd} = \frac{\beta_2 A_{net} f_u}{\gamma_{M_2}}, \tag{33}$$

- with 3 or more bolts:

$$N_{u,Rd} = \frac{\beta_3 A_{net} f_u}{\gamma_{M_2}}, \tag{34}$$

where:

- β_2 and β_3 are reduction factors dependent on the pitch p_1 as given in Table 6. For intermediate values of p_1 the value of β may be determined by linear interpolation;
- A_{net} is the net area of the angle. For an unequal-leg angle connected by its smaller leg, A_{net} should be taken as equal to the net section area of an equivalent equal-leg angle of leg size equal to that of the smaller leg.

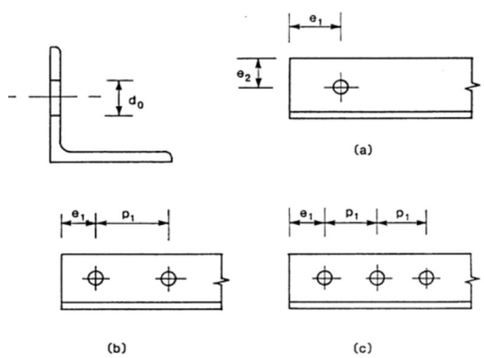


Figure 15. Angles connected by one leg: a) 1 bolt; b) 2 bolts; c) 3 bolts

Table 6. Reduction factors β_2 and β_3

Pitch	p_1	≤ 2,5 d_o	≥ 5,0 d_o
2 bolts	β_2	0,4	0,7
3 bolts or more	β_3	0,5	0,7

For angles connected by welding, the eccentricity of welded lap joint end connections may be allowed for by adopting an effective cross-sectional area and then treating the member as concentrically loaded.

For an equal-leg angle, or an unequal-leg angle connected by its larger leg, the effective area may be taken as equal to the gross area.

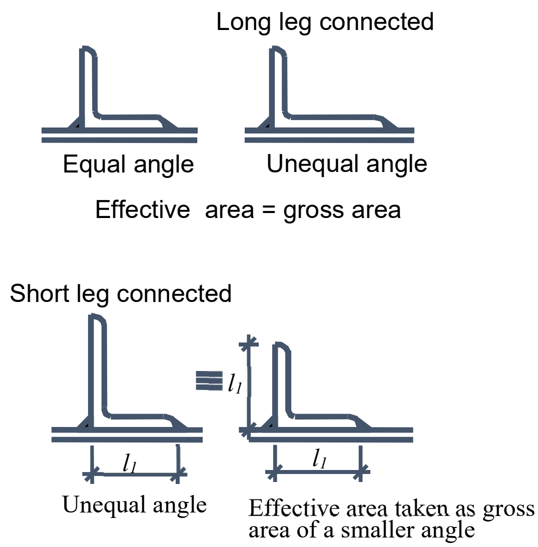


Figure 16. Angles connected by welding

For an unequal-leg angle connected by its smaller leg, the effective area should be taken as equal to the gross cross-sectional area of an equivalent equal-leg angle of leg size equal to that of the smaller leg, when determining the design resistance of the cross-section. However, when determining the design buckling resistance of a compression member, the actual gross cross-sectional area should be used.

5.3. Resistance of elements in compression

For the elements in compression, the design value of the compression force N_{Ed} at each cross section shall satisfy:

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1.0 \qquad (35)$$

where

N_{Ed} : design value of the axial force N_{Ed}

$N_{c,Rd}$: design resistance of the section for uniform compression

The design resistance of the section for uniform compression, $N_{c,Rd}$, should be determined as follows:

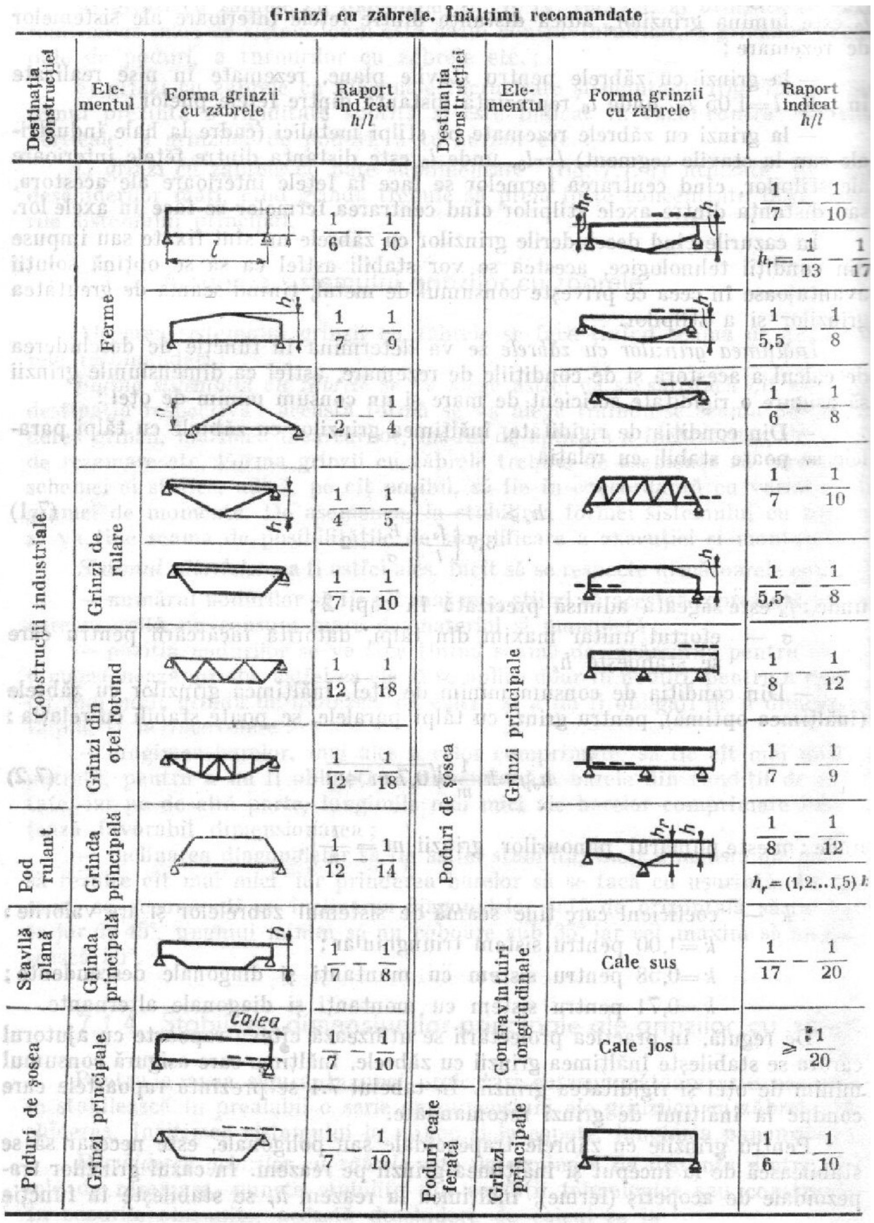
- for class 1, 2 or 3 cross-sections:

$$N_{c,Rd} = \frac{Af_y}{\gamma_{M_0}} \qquad (36)$$

- for class 4 cross-sections:

$$N_{c,Rd} = \frac{A_{eff} f_y}{\gamma_{M_0}} \qquad (37)$$

Fastener holes except for oversize and slotted holes (as defined in EN 1090) need not be allowed for in compression members, if they are filled by fasteners.



h_r height at the support

h_r = (1/15....1/17)l for pinned connection

h_r = (1/13....1/17)l for rigid connection

Figure 17. Recommended ratios h/L for typical trusses

6. STABILITY OF ELEMENTS IN COMPRESSION

6.1. Basic concepts of structural stability

Stability is a property of structures in their extremes of geometry, for example a long slender strut. In classical buckling problems, the element (or the system) is stable if the external load N is small enough and becomes unstable when N is large. The value of N for which the structural system ceases to be stable is called the critical value, N_{cr}. The issues that are important and should be determined are the following:

- the equilibrium configurations for the element (or structure) under prescribed loadings.
- which configurations are stable.
- the critical value of the loadings and what are the consequences on the behavior of these load levels.

In general terms, stability can be defined as the ability of a physical system – e.g. a pin ended strut - to return to equilibrium when slightly disturbed.

For a mechanical system, one can adopt the definition given by Dirichlet:

"The equilibrium of a mechanical system is stable if, in displacing the points of the system from their equilibrium positions by an infinitesimal amount and giving each one a small initial velocity, the displacements of different points of the system remain, throughout the course of the motion, contained within small prescribed limits".

This definition shows that the stability implies the existence of a single solution – the equilibrium solution - of the system, and that the problem of ascertaining the stability of a solution is concerned with the "neighborhood" of this particular solution.

If one considers an elastic conservative system, which is initially in a state of equilibrium under the action of a set of forces, the system will depart from this equilibrium state only if acted upon by some transient disturbing force.

If the energy imparted to the system by the disturbing force is W, then:

$$\boldsymbol{W = T + V = constant} \tag{38}$$

by means of the principle of conservation of energy.

where:

- T is the kinetic energy of the system

- V is the potential energy.

The eq. **38** shows that a small increase in T, is accompanied by an equally small decrease in V, or vice versa. If the system is initially in an equilibrium configuration of minimum potential energy, then the kinetic energy T during subsequent free motion decreases since V must increase. Hence the displacement from the initial state will remain small and the equilibrium state is a stable one.

For rigid bodies, the stability can be illustrated by the well-known example of a ball on a curved plane (Figure 18). Resting on a concave surface, the equilibrium is stable; if one gives the ball a small initial velocity, it will begin to oscillate but will remain in the close neighbourhood of its equilibrium state. On the other hand, if the system is not in a configuration of minimum V (potential energy), then an impulse leads to large deflections and velocities which develop very quickly, and the system is said to be unstable. This is the case where the ball rests on the crest of a convex surface. If the ball rests on a horizontal plane, the equilibrium is called "neutral".

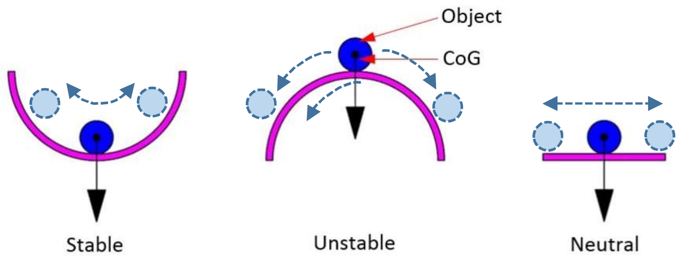


Figure 18. Equilibrium conditions of an object

Same principle is well lustrated also in Figure 19, where the conical shaped body is in a stable configuration (left), unstable (middle) and neutral (or indifferent) – right.

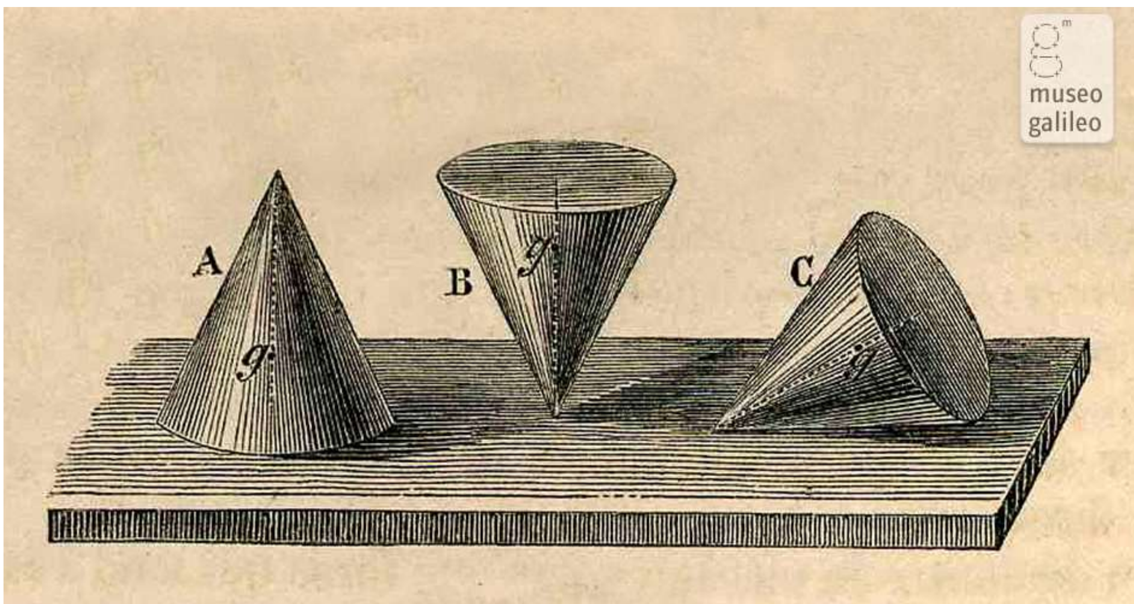


Figure 19. Stable / unstable / indifferent equilibrium (Museo Galileo - Institute and Museum of the History of Science, Florence, Italy)

6.2. Stability of elements in compression - bifurcation of equilibrium

The stability concept is related to potential energy of the system. However, stability of a static elastic system, or structure, may also be explained by stiffness considerations. From Figure 18 (left) it can be seen that the derivative of the potential energy with respect to displacement gives the stiffness (in the figure, the slope of the surface) of the system.

Thus, positive stiffness implies a stable state, whereas at a stability limit the stiffness vanishes. For a structure, the stiffness is given in matrix form, which if it has both a positive and definite condition, guarantees a stable state for the structure. The point at which the state of a system changes from stable equilibrium into neutral equilibrium is called "the stability limit".

The system of a ball can be compared to a structure such as a compressed strut, or column. In this case, the strut may be stable or unstable, depending of the magnitude of the axial load, which is the controlling parameter of the system (Figure 20). Since the member is initially straight and the load is axial, the structure will be in stable equilibrium for small values of N ($N<N_{cr}$); if a disturbing force produces deflections, the column will return to its straight position. When the load reaches a certain level, called "critical load", the stable equilibrium reaches a limit ($N=N_{cr}$). At this load N_{cr}, there is another equilibrium position in a slightly deflected configuration of the column; if, at this load, the member is deflected by some small disturbance, it will not return to the straight configuration. If the load exceeds the critical value ($N>N_{cr}$), the straight position is unstable and a slight disturbance leads to large displacements of the member and, finally, to the failure of the column by buckling. The critical point, after which the deflections of the member become very large, is called the "bifurcation point" of the system (Figure 21). In case of symmetric bifurcation, if the load capacity increases after the bifurcation point (buckling), then there is a stable symmetric bifurcation. If the load capacity decreases after buckling, then there is an unstable symmetric bifurcation.

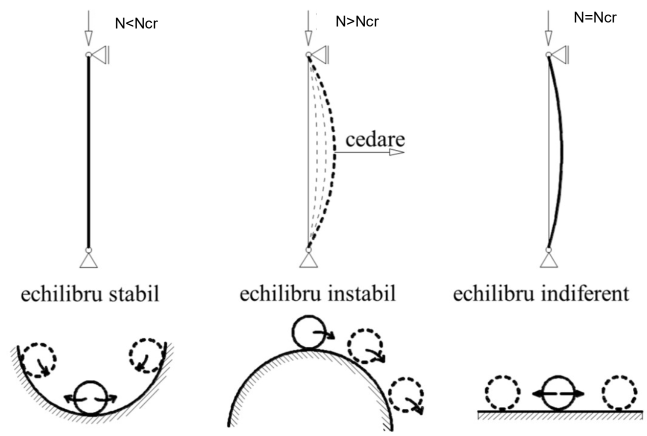


Figure 20. Stability of a compressed column vs. equilibrium conditions of a rigid body

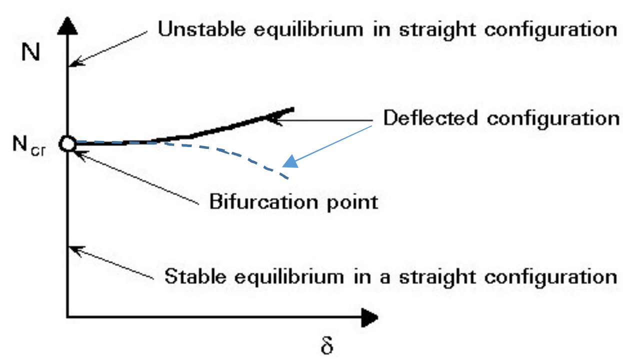


Figure 21. Stability of a compressed column – bifurcation of equilibrium

For evaluating the critical buckling load, N_{cr}, let us consider a simply supported prismatic member under axial compression, N, without any initial imperfection. The length is L, the cross section second moment of area is I and E is the modulus of elasticity.

For small values of N, if the system is pulled sideways and released, it will spring back to the straight initial position. However, if a large axial compressive load is applied, any side sway displacement applied to the system will be held in position by the compressive load, resulting in a neutral configuration. This is called critical buckling load, N_{cr}.

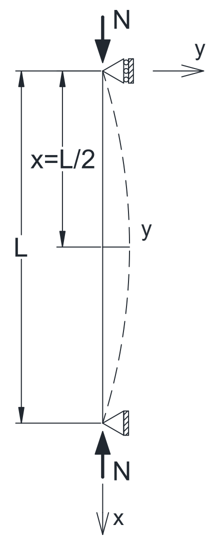


Figure 22. Simply supported prismatic member under axial compression

The bending moment at distance x from origin will be equal to:

$$\boldsymbol{M(x) = N_{cr} \times y} \tag{39}$$

Substituting in the moment-curvature relationship (deflection theory):

$$\boldsymbol{M(x) = -EIy''} \tag{40}$$

➔ we obtain:

$$\boldsymbol{N_{cr} \times y = -EIy''} \tag{41}$$

➔ the governing equation is:

$$\boldsymbol{EIy'' + N_{cr} \times y = 0} \tag{42}$$

Dividing by EI and noting $N_{cr}/EI = \boldsymbol{\alpha^2}$, we obtain:

$$\boldsymbol{y'' + \alpha^2 \times y = 0} \tag{43}$$

This is a differential linear equation, homogeneous and with constant coefficients, therefore the solution is:

$$\boldsymbol{y(x) = c_1 sin\alpha x + c_2 cos\alpha x} \tag{44}$$

Using the boundary conditions:

$x = 0, y = 0$ ➔ $c_2 = 0$
$x = L, y = 0$ ➔ $\boldsymbol{y}(L) = c_1 sin\boldsymbol{\alpha}l = 0$

which shows that the shape of the buckling mode is a sine curve.

The two solutions of the equation are:
- $c_1 = 0$
- $sin\boldsymbol{\alpha}l = 0$

As $c_1 \neq 0$ (otherwise no deflections occur), it results in:

$sin\boldsymbol{\alpha}l = 0$, with the solutions: $\boldsymbol{\alpha}l = 0, \pi, 2\pi, 3\pi, ...$

➔ $\boldsymbol{\alpha} = \frac{n\pi}{L}$, where n is any integer 0, 1, 2, 3, …

➔

$$\boldsymbol{N_{cr}} = \frac{\boldsymbol{n^2\pi^2EI}}{\boldsymbol{L^2}} \quad (45)$$

In most engineering problems the lowest value (different than 0) is of critical importance, as for large values we obtain increasing values of N_{cr}. Therefore n=1 end eq. (**45**) becomes:

$$\boldsymbol{N_{cr}} = \frac{\boldsymbol{\pi^2EI}}{\boldsymbol{L^2}} \quad (46)$$

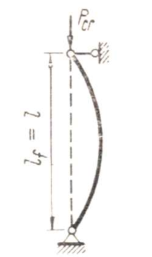
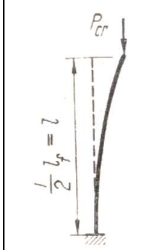
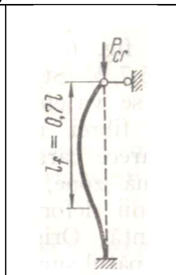
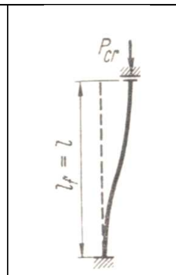
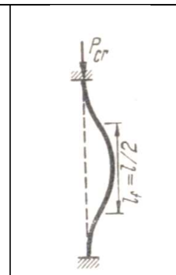
Eq. (**46**) is the expression of critical buckling load or Euler (critical) buckling load (first developed around the year 1750).

The type of supports is important because the Euler formula is established for a pin-ended element with axial compressive end load.
Therefore, in Eq. (**46**) we need to replace the length of the system, L, by the effective length, L_{eff} (or buckling length L_{cr}, L_f), defined as the distance between two consecutive points of inflexion on the deformed shape of the element.

$$\boldsymbol{N_{cr}} = \frac{\boldsymbol{\pi^2EI}}{\boldsymbol{L_{eff}^2}} \quad (47)$$

The most common cases are shown in Table 7.

Table 7. Buckling length for common cases

Deformed shape					
Characteristic equations	$\sin \alpha l = 0$	$\cos \alpha l = 0$	$tg\alpha l - \alpha l = 0$	$\sin \alpha l = 0$	$\cos \alpha l - 1 = 0$
Critical force N_{cr}	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{4l^2}$	$\frac{20{,}19EI}{l^2}$	$\frac{\pi^2 EI}{l^2}$	$\frac{4\pi^2 EI}{l^2}$
Effective length, L_{eff} (or buckling length, l_f)	*l*	*2l*	*0,7l*	*l*	*0,5l*
Buckling length coefficient, μ $L_{eff} = \mu L$	1	2	0,7	1	0,5

Depending on their slenderness, elements in compression (pin-ended struts, columns) exhibit two different types of behavior:

- those with high slenderness present a quasi-elastic buckling behavior
- those of medium slenderness are very sensitive to the effects of imperfections.

Considering the eq. (**47**), it is possible to define the Euler critical stress σ_{cr} as:

$$\boldsymbol{\sigma_{cr} = \frac{N_{cr}}{A} = \frac{\pi^2 EI}{L_{eff}^2 A}} \quad (48)$$

Introducing the radius of gyration, i, and the slenderness, λ:

$$i = \sqrt{\frac{I}{A}};\ \lambda = L_{eff}/i\ ;$$

Then Eq. (48) becomes:

$$\boldsymbol{\sigma_{cr} = \frac{\pi^2 EI}{L_{eff}^2 A} = \frac{\pi^2 E}{\lambda^2}} \quad (49)$$

If we plot the curve σ_{cr} as a function of λ on a graph, with the horizontal line representing perfect plasticity, $\sigma = f_y$, we can distinguish between the idealized zones representing failure by buckling, failure by yielding and safety (Figure 23).

The point N represents the maximum theoretical slenderness of an element in compression that reaches the yield strength.
We note:

$$N_{pl} = A \times f_y \tag{50}$$

$$\overline{N} = \frac{N_{cr}}{N_{pl}} \tag{51}$$

➔

$$\overline{N} = \frac{N_{cr}}{N_{pl}} = \frac{\boldsymbol{\pi^2 EI}}{\boldsymbol{L_{eff}^2 A f_y}} = \frac{\boldsymbol{\pi^2 E}}{\lambda^2 f_y} \tag{52}$$

By replacing:

$$\boldsymbol{\lambda_1 = \pi \sqrt{\frac{E}{f_y}}} \tag{53}$$

➔

$$\overline{N} = \frac{N_{cr}}{N_{pl}} = \left(\frac{\boldsymbol{\lambda_1}}{\lambda}\right)^2 = \left(\frac{1}{\bar{\lambda}}\right)^2 \tag{54}$$

where

$$\bar{\lambda} = \frac{\lambda}{\boldsymbol{\lambda_1}} \tag{55}$$

$\overline{N}$ - normalized buckling force
$\bar{\lambda}$ - non-dimensional slenderness
$\boldsymbol{\lambda_1}$ - slenderness of perfect bar (for which $N_{cr} = N_{pl}$)

The limiting slenderness when σ_{cr} is equal to the yield strength of the steel is given by:

$$\boldsymbol{\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = 93.9\varepsilon} \tag{56}$$

where

$$\boldsymbol{\varepsilon = \sqrt{\frac{235}{f_y}}} \tag{57}$$

For a steel S235 (f_y = 235 N/mm^2), $\boldsymbol{\lambda_1}$ is equal to 93.9, while for S275 and S355 $\boldsymbol{\lambda_1}$ is equal to 86 and 76.4 respectivelly.

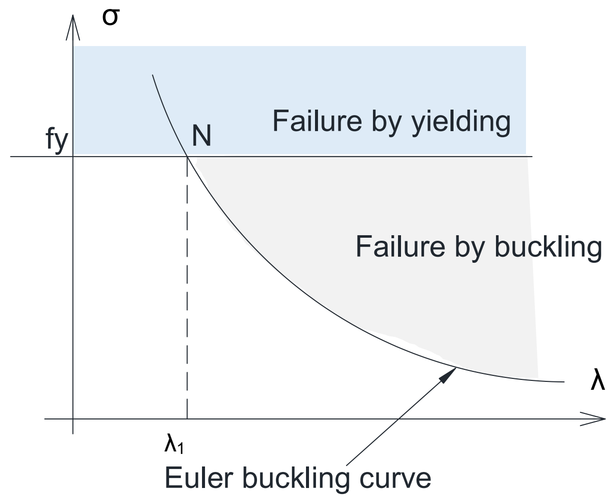


Figure 23. Euler buckling curve and modes of failure

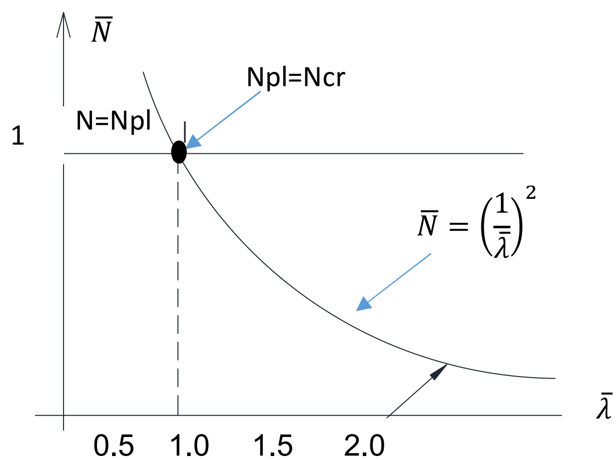


Figure 24. Non-dimensional buckling curve

6.3. Stability of elements in compression – divergence of equilibrium

The stability concept presented in the previous section is rather idealized, as it overlooks several important features or imperfections that will be present in most practical columns (struts), such as:

• initial lack of straightness
• material that is not perfectly linearly elastic
• eccentricity of axial applied loads and material strain-hardening
• residual stresses

Compared to the theoretical curves, the real behavior shows greater differences in the range of medium slenderness than in the range of large slenderness, with the greatest reduction in the region of limiting slenderness λ_1.

Also, in the zone of the medium values of λ (representing most practical columns), the effect of structural imperfections is significant and must be carefully considered.

When buckling occurs, some fibers have already reached the yield strength and the ultimate load is not simply a function of slenderness; the more numerous the imperfections, the larger the difference between the actual and theoretical behavior.
From the main factors listed above, the out-of-straightness and residual stresses plays a fundamental role on the behavior of these columns.
The load-carrying capacity of an element in compression reaches its divergency with the loading (the moment of external loadings) which tends to increase as the element reached the maximum load-carrying capacity. Model of instability by divergence of equilibrium was used at the elaboration of the European buckling curves.

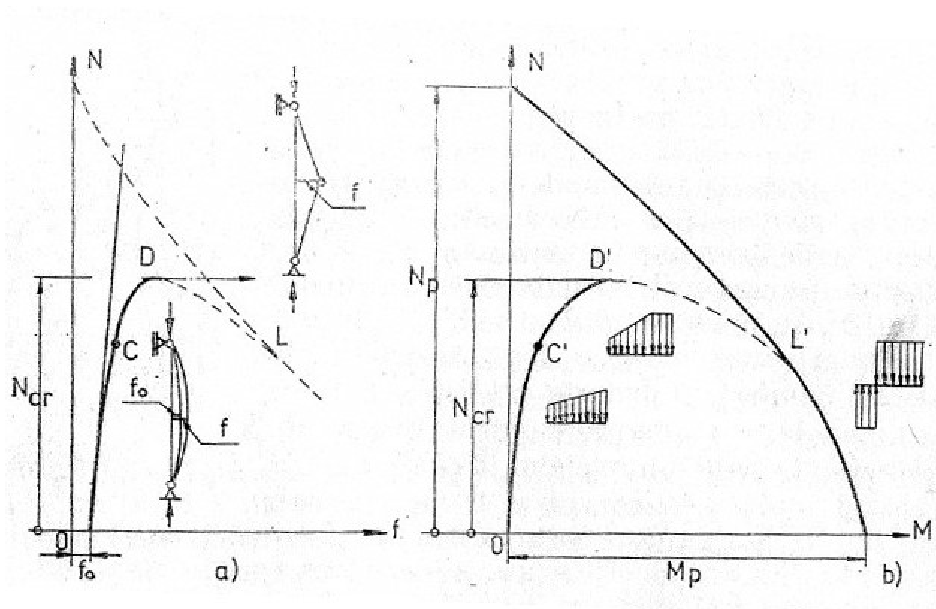


Figure 25. Behavior of real columns affected by imperfections

Let us consider a simply supported prismatic member under axial compression N_{Ed}. The member has an initial geometrical imperfection $v_0(x)$ that is assumed to be sinusoidal with a maximum value of $e_{0,d}$ at mid-span:

$$v_0(x) = e_{0,d} sin\frac{\pi x}{L} \tag{58}$$

This is a common method for representation of imperfections. When the axial force N_{Ed} is applied, an additional deflection $v(x)$ develops on the member length. Because of the support conditions, this additional deflection that is associated with instability can be written as:

$$v(x) = A sin\frac{\pi x}{L} \tag{59}$$

where A represents the maximum value of the additional deflection (at mid-span).

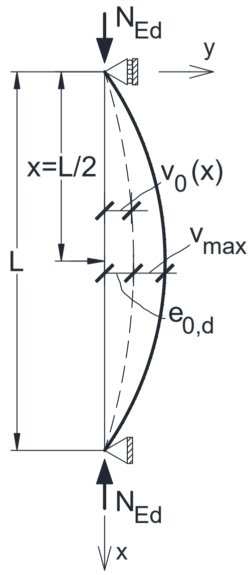


Figure 26. Simply supported member with initial imperfection

In this case, the classical elastic flexural equilibrium equation, accounting for the initial imperfection, becomes:

$$\boldsymbol{v}'' + \frac{\boldsymbol{N_{Ed}}}{\boldsymbol{EI}}(\boldsymbol{v_0} + \boldsymbol{v}) = \boldsymbol{0} \tag{60}$$

where I represents the second moment of area in the plane of bending. Replacing Eqs. (**58**) and (**59**) in Eq. (**60**) allows the determination of the expression of A as follows:

$$\boldsymbol{A} = \frac{\boldsymbol{N_{Ed}}}{\boldsymbol{N_{cr}} - \boldsymbol{N_{Ed}}}\boldsymbol{e_{0,d}} \tag{61}$$

where N_{cr} is the critical flexural buckling load given in eq. (**46**).

The total deflection at mid-span may be expressed as:

$$\boldsymbol{v_{max}} = \frac{\boldsymbol{N_{cr}}}{\boldsymbol{N_{cr}} - \boldsymbol{N_{Ed}}}\boldsymbol{e_{0,d}} = \frac{\boldsymbol{1}}{\boldsymbol{1} - \boldsymbol{N_{Ed}}/\boldsymbol{N_{cr}}}\boldsymbol{e_{0,d}} \tag{62}$$

As can be seen, the maximum deflection $\boldsymbol{v_{max}}$ depends on the value of the axial compression through the amplification factor K:

$$\boldsymbol{K} = \frac{\boldsymbol{1}}{\boldsymbol{1} - \boldsymbol{N_{Ed}}/\boldsymbol{N_{cr}}} \tag{63}$$

A second-order in-plane elastic check of the most heavily loaded cross-section on the member (at mid-span) then becomes:

$$\frac{N_{Ed}}{N_{Rd}} + \frac{1}{1 - N_{Ed}/N_{cr}} \frac{N_{Ed} e_{0,d}}{M_{Rd}} \leq 1 \tag{64}$$

When the axial force increases until collapse, N_{Ed} is just equal to $N_{b,Rd} = \chi N_{Rd}$, and the lefthand side of Eq. (**64**) is equal to one. Using the reduced slenderness definition:

$$\bar{\lambda}^2 = \frac{N_{Rd}}{N_{cr}} \tag{65}$$

Eq. (**64**) can be rearranged into the so-called Ayrton-Perry format:

$$(1 - \chi)(1 - \chi\bar{\lambda}^2) = e_{0,d} \frac{A}{W_{el}} \chi = \eta\chi \tag{66}$$

In Eq. (**66**), χ is the flexural buckling reduction factor, and η represents the generalised initial imperfection, that can be used to estimate the effects on the buckling phenomenon of initial imperfections such as residual stresses, initial out of straightness or eccentrically applied forces. Because the influence of some of these initial imperfections is linked with the length of the member, it has been chosen to express η as follows:

$$\eta = \alpha(\bar{\lambda} - 0.2) \tag{67}$$

where the imperfection factor α depends on the shape of the cross-section, buckling plane, etc, and 0,2 defines the length of the plateau.

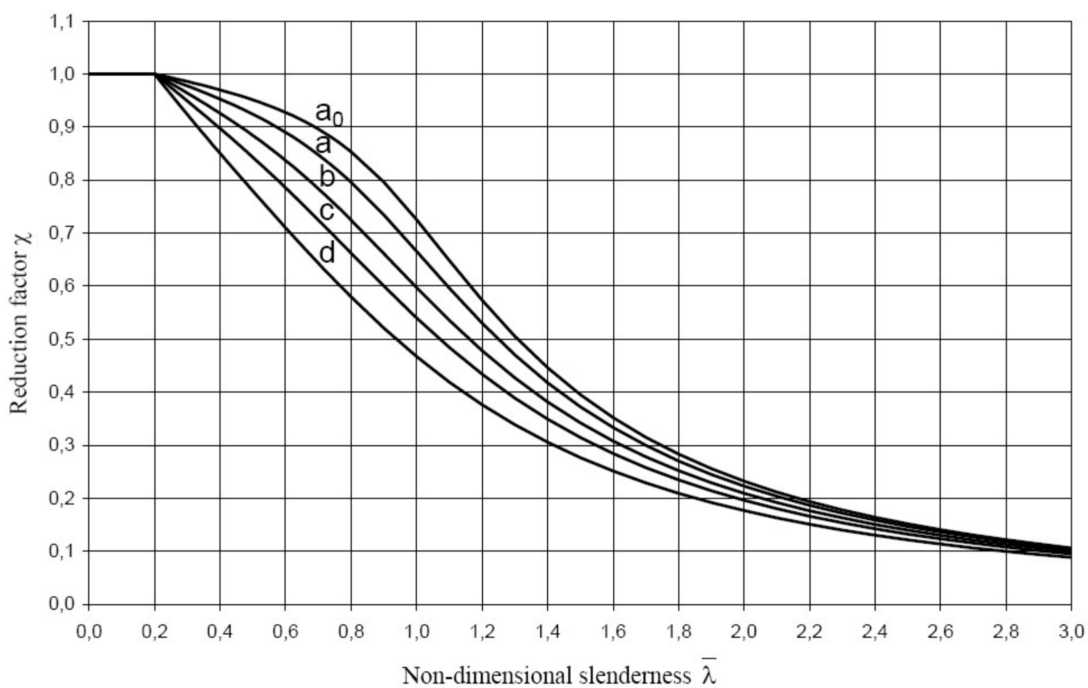
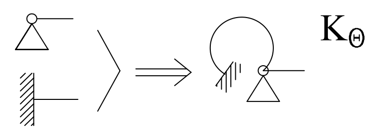


Figure 27. Buckling curves

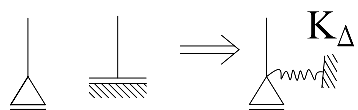
6.4. Buckling of uniform members in compression - influence of supporting conditions

In many cases, fundamental buckling cases are not met in real structures. The main factors are the supporting conditions (rotational rigidity) and flexibility of the frame (sway, non-sway).

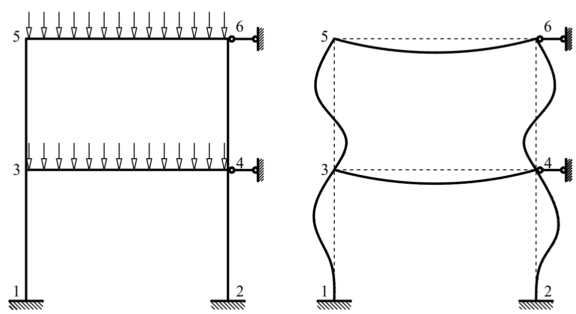
Rotation

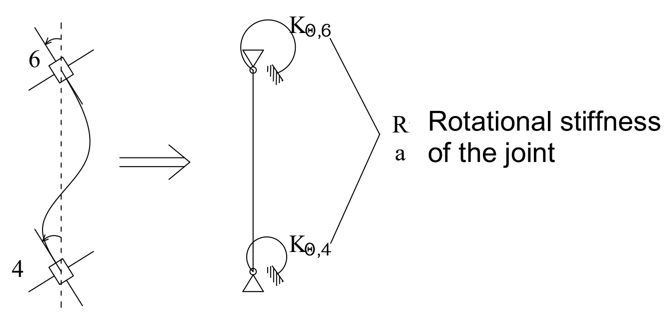


Translation

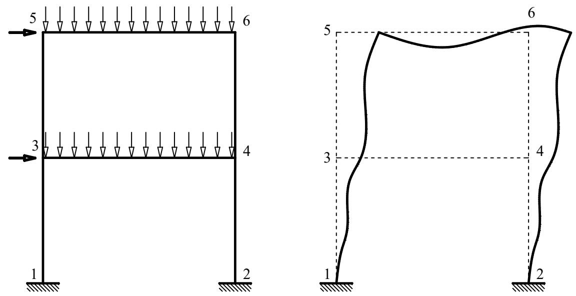


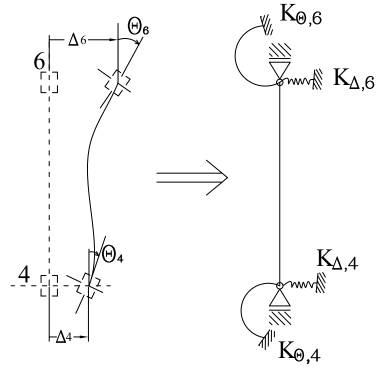
Non-sway frames





Sway frames





Buckling length can be calculated based on the so called distribution factors, η.

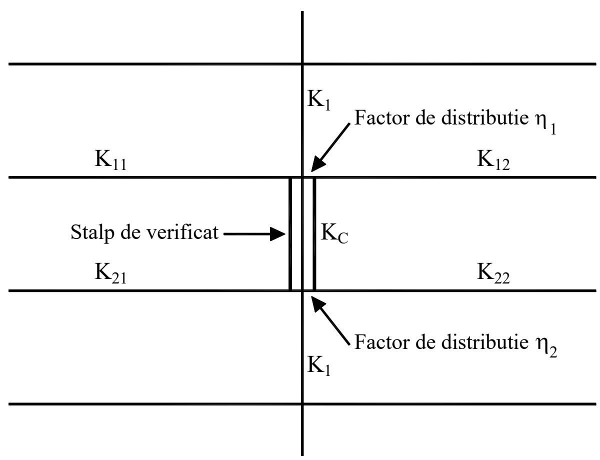


Figure 28. Distributions factors for continuous columns (P100-1/2013, Annex F)

$$\eta_1 = \frac{K_C + K_1}{K_C + K_1 + K_{11} + K_{12}} \qquad \text{(F.1)}$$

$$\eta_2 = \frac{K_C + K_2}{K_C + K_2 + K_{21} + K_{22}} \qquad \text{(F.2)}$$

<u>Non-sway frames:</u>

$$\frac{l_f}{L} = \left[\frac{1 + 0{,}145(\eta_1 + \eta_2) - 0{,}265\eta_1\eta_2}{2 - 0{,}364(\eta_1 + \eta_2) - 0{,}247\eta_1\eta_2} \right] \qquad \text{(F.3)}$$

<u>Sway frames:</u>

$$\frac{l_f}{L} = \left[\frac{1 - 0,2(\eta_1 + \eta_2) - 0,12\eta_1\eta_2}{1 - 0,8(\eta_1 + \eta_2) + 0,60\eta_1\eta_2} \right]^{0,5} \qquad \text{(F.4)}$$

Braced Frames vs. Unbraced Frames

According to EN1993-1-1, a frame may be classified as braced if its sway resistance is supplied by a bracing system in which its response to lateral loads is sufficiently stiff for it to be acceptably accurate to assume all horizontal loads are resisted by the bracing system. The frame can be considered as braced, when bracing system reduces the horizontal displacements by at least 80%.

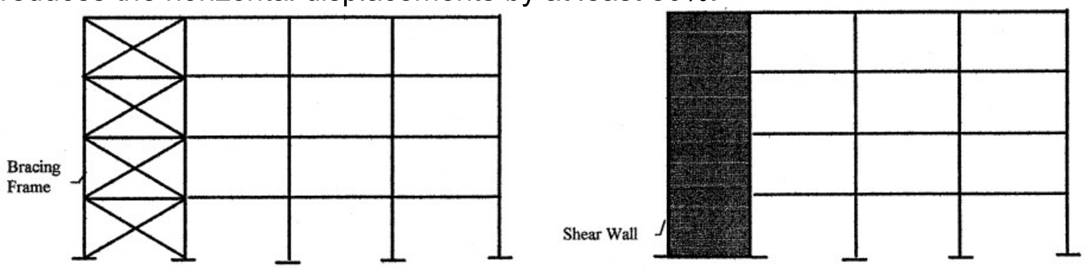


Figure 29. Vertical braces in multi-story frames

Sway Frames vs. Non-Sway Frames

(1) The identification of sway frames and non-sway frames in a building is useful for two reasons:

- to see how the global stability of the frame affects the column behavior (ex. For non-sway frame, the column effective length may be evaluated based on the column end restraint conditions).
- the need to adopt conventional analysis in which all the internal forces are computed on the basis of the undeformed geometry of the structure, which is valid only if second-order effects are negligible.

(2) A frame can be classified as non-sway if its response to in-plane horizontal forces is sufficiently stiff for it to be acceptably accurate to neglect any additional internal forces or moments arising from horizontal displacements of its nodes.

(3) A frame may be considered as non-sway, if the following criterion is satisfied:

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 10 \quad \text{for elastic analysis}$$

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 15 \quad \text{for plastic analysis}$$

where:

α_{cr} is the factor by which the design loading would have to be increased to cause elastic instability in a global mode

F_{Ed} is the design loading on the structure

F_{cr} is the elastic critical buckling load for global instability mode based on initial elastic stiffness

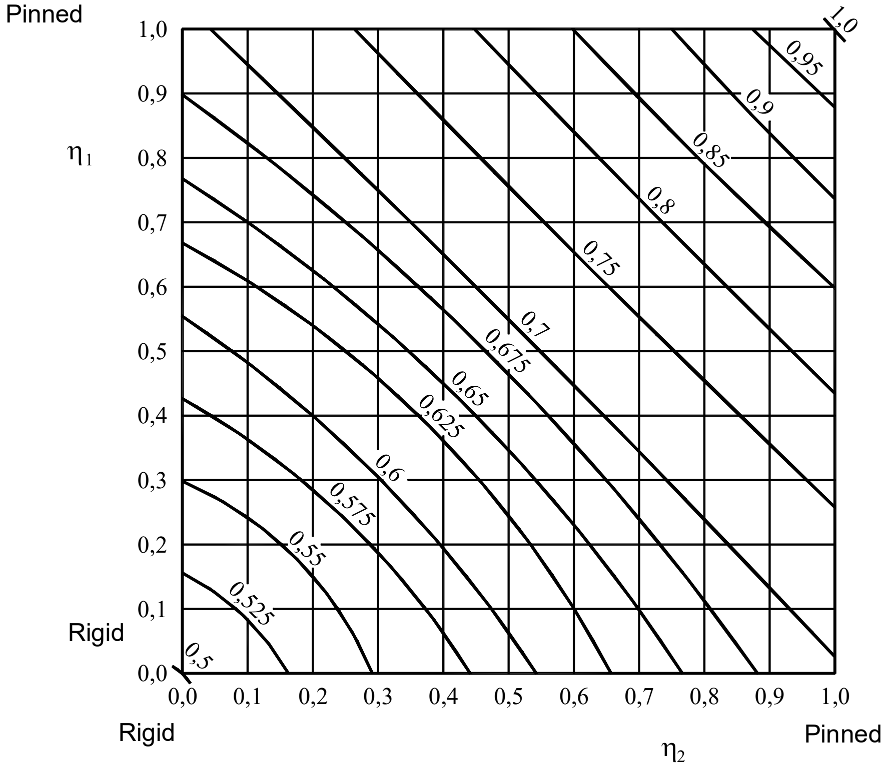


Figure 30. Ratio l_f/L between buckling length and theoretical length of a column for a non-sway frame

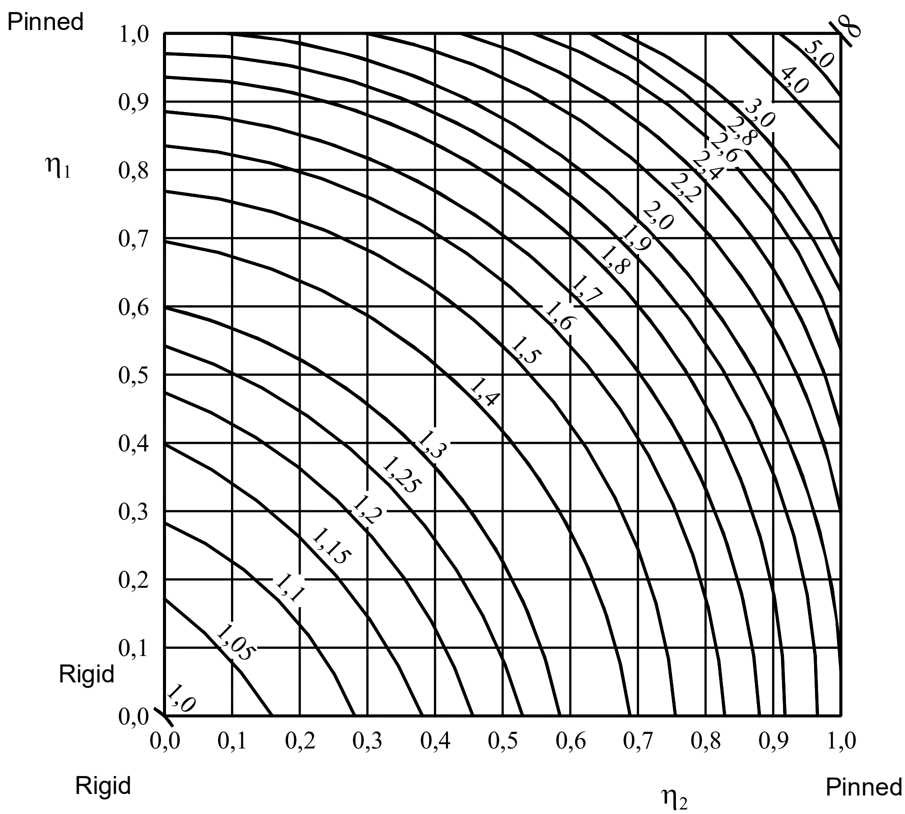


Figure 31. Ratio l_f/L between buckling length and theoretical length of a column for a sway frame

6.5. Buckling resistance of members (EN1993-1-1)

Notes:

- Material reproduced from EN1993-1-1.
- For ease of understanding, numbering is done according to EN1993-1-1.

6.3.1 Uniform members in compression

6.3.1.1 Buckling resistance

(1) A compression member should be verified against buckling as follows:

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1,0 \qquad (6.46)$$

where N_{Ed} is the design value of the compression force;

$N_{b,Rd}$ is the design buckling resistance of the compression member.

(2) For members with non-symmetric Class 4 sections allowance should be made for the additional moment ΔM_{Ed} due to the eccentricity of the centroidal axis of the effective section, see also 6.2.2.5(4), and the interaction should be carried out to 6.3.4 or 6.3.3.

(3) The design buckling resistance of a compression member should be taken as:

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \quad \text{for Class 1, 2 and 3 cross-sections} \qquad (6.47)$$

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} \quad \text{for Class 4 cross-sections} \qquad (6.48)$$

where χ is the reduction factor for the relevant buckling mode.

NOTE For determining the buckling resistance of members with tapered sections along the member or for non-uniform distribution of the compression force second order analysis according to 5.3.4(2) may be performed. For out-of-plane buckling see also 6.3.4.

(4) In determining A and A_{eff} holes for fasteners at the column ends need not to be taken into account.

6.3.1.2 Buckling curves

(1) For axial compression in members the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ should be determined from the relevant buckling curve according to:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi \leq 1,0 \qquad (6.49)$$

where $\Phi = 0,5\left[1 + \alpha\left(\bar{\lambda} - 0,2\right) + \bar{\lambda}^2\right]$

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 cross-sections}$$

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} \quad \text{for Class 4 cross-sections}$$

α is an imperfection factor

N_{cr} is the elastic critical force for the relevant buckling mode based on the gross cross sectional properties.

(2) The imperfection factor α corresponding to the appropriate buckling curve should be obtained from Table 6.1 and Table 6.2.

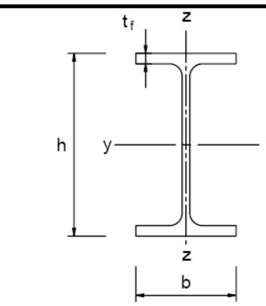
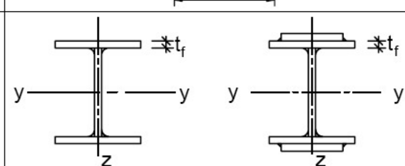
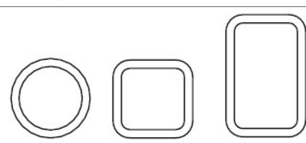
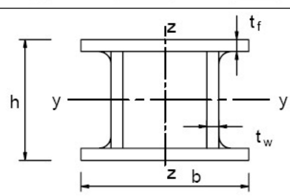
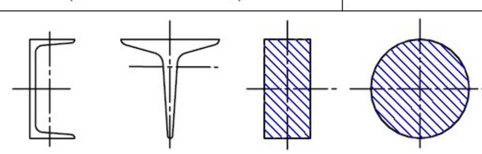
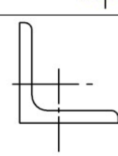
Table 6.1: Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

(3) Values of the reduction factor χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ may be obtained from Figure 6.4.

(4) For slenderness $\bar{\lambda} \leq 0,2$ or for $\frac{N_{Ed}}{N_{cr}} \leq 0,04$ the buckling effects may be ignored and only cross sectional checks apply.

Table 6.2: Selection of buckling curve for a cross-section

	Cross section	Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled sections		h/b > 1,2	$t_f \leq 40$ mm	y – y z – z	a b	a_0 a_0
			40 mm < $t_f \leq 100$	y – y z – z	b c	a a
		h/b ≤ 1,2	$t_f \leq 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections		$t_f \leq 40$ mm		y – y z – z	b c	b c
		$t_f > 40$ mm		y – y z – z	c d	c d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c
Welded box sections		generally (except as below)		any	b	b
		thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$		any	c	c
U-, T- and solid sections				any	c	c
L-sections				any	b	b

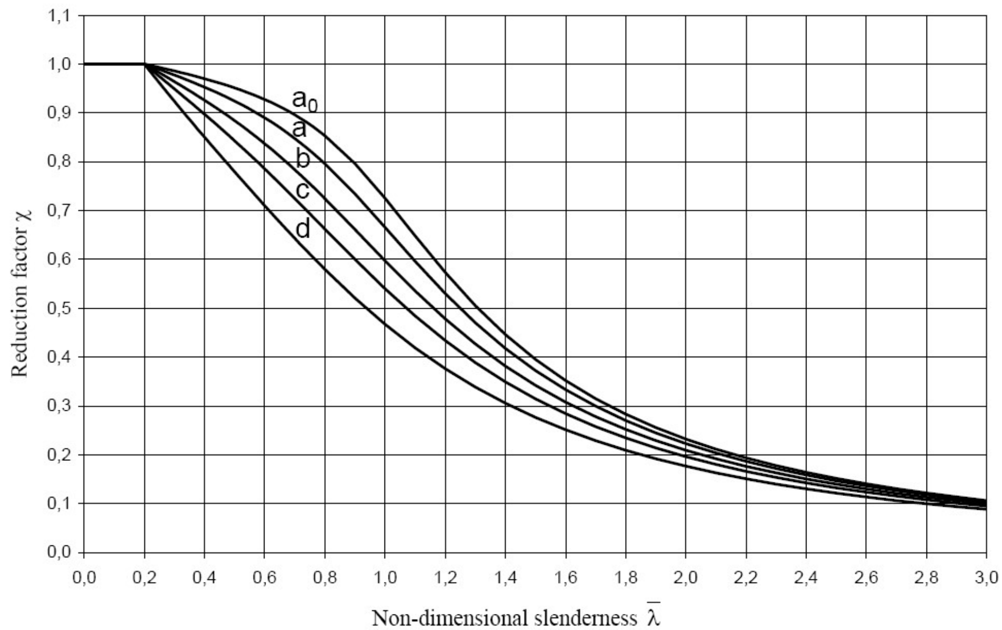


Figure 6.4: Buckling curves

6.3.1.3 Slenderness for flexural buckling

(1) The non-dimensional slenderness $\bar{\lambda}$ is given by:

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{1}{\lambda_1} \qquad \text{for Class 1, 2 and 3 cross-sections} \qquad (6.50)$$

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{\sqrt{\frac{A_{eff}}{A}}}{\lambda_1} \qquad \text{for Class 4 cross-sections} \qquad (6.51)$$

where L_{cr} is the buckling length in the buckling plane considered

i is the radius of gyration about the relevant axis, determined using the properties of the gross cross-section

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93{,}9\varepsilon$$

$$\varepsilon = \sqrt{\frac{235}{f_y}} \qquad (f_y \text{ in N/mm}^2)$$

NOTE B For elastic buckling of components of building structures see Annex BB.

(2) For flexural buckling the appropriate buckling curve should be determined from Table 6.2.

6.3.1.4 Slenderness for torsional and torsional-flexural buckling

(1) For members with open cross-sections account should be taken of the possibility that the resistance of the member to either torsional or torsional-flexural buckling could be less than its resistance to flexural buckling.

(2) The non-dimensional slenderness $\bar{\lambda}_T$ for torsional or torsional-flexural buckling should be taken as:

$$\bar{\lambda}_T = \sqrt{\frac{Af_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 cross-sections} \tag{6.52}$$

$$\bar{\lambda}_T = \sqrt{\frac{A_{eff}f_y}{N_{cr}}} \quad \text{for Class 4 cross-sections} \tag{6.53}$$

where $N_{cr} = N_{cr,TF}$ but $N_{cr} < N_{cr,T}$

$N_{cr,TF}$ is the elastic torsional-flexural buckling force;

$N_{cr,T}$ is the elastic torsional buckling force.

(3) For torsional or torsional-flexural buckling the appropriate buckling curve may be determined from Table 6.2 considering the one related to the z-axis.

6.6. Buckling of uniform built-up members in compression

Closely spaced built-up members are uniform built-up compression members with hinged ends that are laterally supported. The main types of uniform built-up members are:

- Laced built-up members
- Battened built-up members

The model applies when the lacings or battens consist of equal modules with parallel chords the minimum numbers of modules in a member is three.

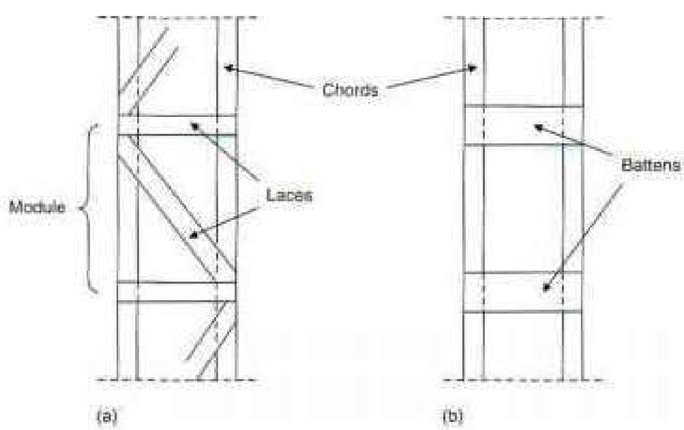


Figure 32. Types of built-up members: a) laced column; b) battened column

a)

b)

Figure 33. Example of built-up members in an industrial steel frame building: a) lower part of columns made of laced built-up sections; b) battened columns

In case of battened or laced members (ex. columns), shear deformation of the battens or lacings are important and therefore cannot be neglected, because:

- Reduces the flexural stiffness
- Influences (reduces) the elastic critical force of the built-up member, $N_{cr,comp}$.

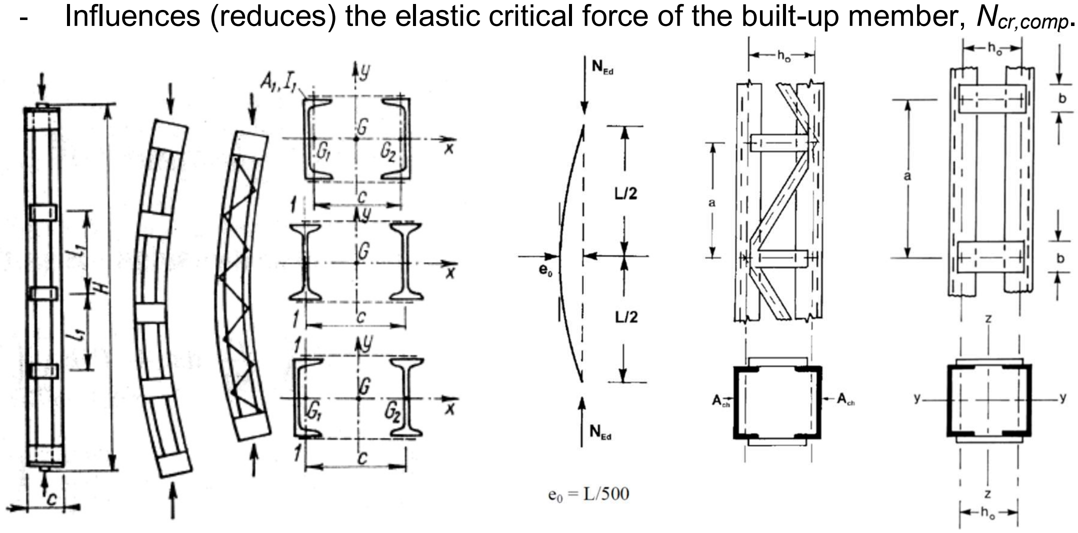


Figure 34. Notations for uniform built-up columns with laces and battens

The elastic critical force of the built-up member, $N_{cr,comp}$, can be expressed as:

$$N_{cr,comp} = \frac{1}{\frac{1}{N_{cr}} + \frac{1}{S_v}} = N_{cr} \frac{1}{1 + \frac{N_{cr}}{S_v}} \qquad (68)$$

where:

$N_{cr} =$ Elastic critical force (Euler), calculated neglecting shear force

$$N_{cr} = \frac{\pi^2 * E * I_{eff}}{L^2}$$

$I_{eff} =$ Effective moment of inertia calculated in a first approximation.

$$I_{eff} = 0.5 A_{ch} h_0^2 \qquad (69)$$

S_v = shear stiffness of the lacing or battened panel

$$S_v = G \times A_{ech} \qquad (70)$$

G = transversal stiffness modulus

A_{ech}= the cross-sectional area of the equivalent column web.

Observation:

In general, Sv >> Ncr ➔ Ncr/Sv <<1 ➔ Ncr,comp ≈ Ncr.

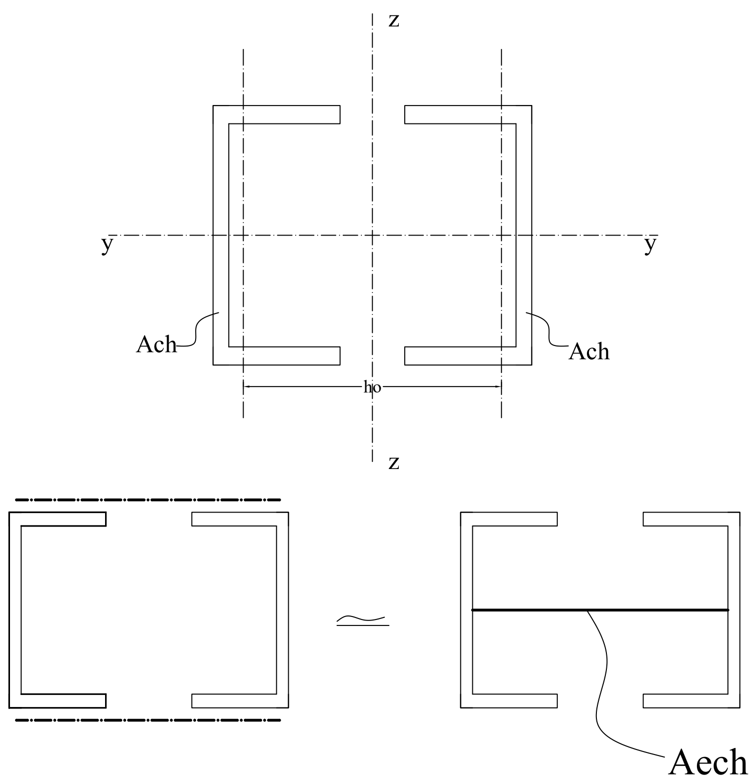


Figure 35. Representation of A_{ech} and A_{ch}

6.6.1. Design of uniform built-up members (EN1993-1-1)

Notes:

- Material reproduced from EN1993-1-1.
- For ease of understanding, numbering is done according to EN1993-1-1.

6.4 Uniform built-up compression members

6.4.1 General

(1) Uniform built-up compression members with hinged ends that are laterally supported should be designed with the following model, see Figure 6.7.

1. The member may be considered as a column with a bow imperfection $e_0 = \frac{L}{500}$

2. The elastic deformations of lacings or battenings, see Figure 6.7, may be considered by a continuous (smeared) shear stiffness S_V of the column.

NOTE For other end conditions appropriate modifications may be performed.

(2) The model of a uniform built-up compression member applies when

1. the lacings or battenings consist of equal modules with parallel chords
2. the minimum numbers of modules in a member is three.

NOTE This assumption allows the structure to be regular and smearing the discrete structure to a continuum.

(3) The design procedure is applicable to built-up members with lacings in two planes, see Figure 6.8.

(4) The chords may be solid members or may themselves be laced or battened in the perpendicular plane.

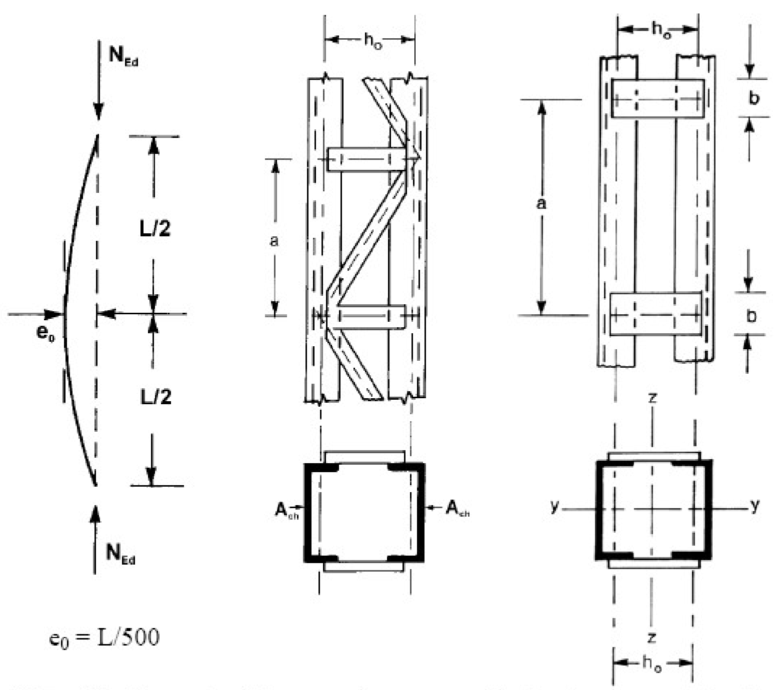


Figure 6.7: Uniform built-up columns with lacings and battenings

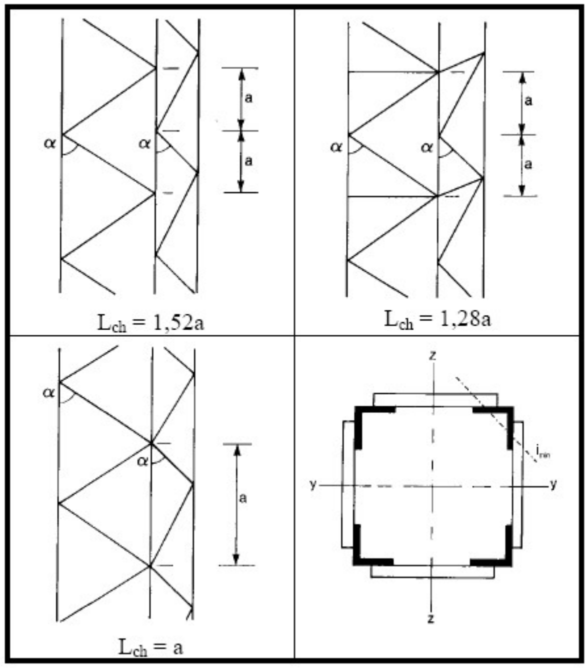


Figure 6.8: Lacings on four sides and buckling length L_{ch} of chords

(5) Checks should be performed for chords using the design chord forces $N_{ch,Ed}$ from compression forces N_{Ed} and moments M_{Ed} at mid span of the built-up member.

(6) For a member with two identical chords the design force $N_{ch,Ed}$ should be determined from:

$$N_{ch,Ed} = 0{,}5N_{Ed} + \frac{M_{Ed} h_0 A_{ch}}{2I_{eff}} \tag{6.69}$$

where $M_{Ed} = \dfrac{N_{Ed} e_0 + M_{Ed}^{I}}{1 - \dfrac{N_{Ed}}{N_{cr}} - \dfrac{N_{Ed}}{S_v}}$

$N_{cr} = \dfrac{\pi^2 E I_{eff}}{L^2}$ is the effective critical force of the built-up member

N_{Ed} is the design value of the compression force to the built-up member

M_{Ed} is the design value of the maximum moment in the middle of the built-up member considering second order effects

M_{Ed}^{I} is the design value of the maximum moment in the middle of the built-up member without second order effects

h_0 is the distance between the centroids of chords

A_{ch} is the cross-sectional area of one chord

I_{eff} is the effective second moment of area of the built-up member, see 6.4.2 and 6.4.3

S_v is the shear stiffness of the lacings or battened panel, see 6.4.2 and 6.4.3.

(7) The checks for the lacings of laced built-up members or for the frame moments and shear forces of the battened panels of battened built-up members should be performed for the end panel taking account of the shear force in the built-up member:

$$V_{Ed} = \pi \frac{M_{Ed}}{L} \qquad (6.70)$$

6.4.2 Laced compression members

6.4.2.1 Resistance of components of laced compression members

(1) The chords and diagonal lacings subject to compression should be designed for buckling.

NOTE Secondary moments may be neglected.

(2) For chords the buckling verification should be performed as follows:

$$\frac{N_{ch,Ed}}{N_{b,Rd}} \leq 1{,}0 \qquad (6.71)$$

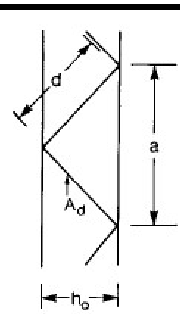
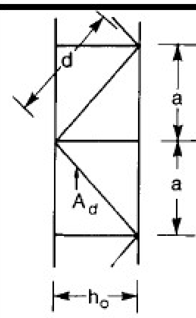
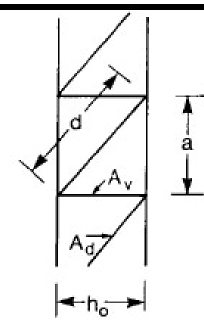
where $N_{ch,Ed}$ is the design compression force in the chord at mid-length of the built-up member according to 6.4.1(6)

and $N_{b,Rd}$ is the design value of the buckling resistance of the chord taking the buckling length L_{ch} from Figure 6.8.

(3) The shear stiffness S_V of the lacings should be taken from Figure 6.9.

(4) The effective second order moment of area of laced built-up members may be taken as:

$$I_{eff} = 0{,}5 h_0^2 A_{ch} \qquad (6.72)$$

System			
S_V	$\frac{nEA_d a h_0^2}{2d^3}$	$\frac{nEA_d a h_0^2}{d^3}$	$\frac{nEA_d a h_0^2}{d^3\left[1+\frac{A_d h_0^3}{A_V d^3}\right]}$

n is the number of planes of lacings
A_d and A_V refer to the cross sectional area of the bracings

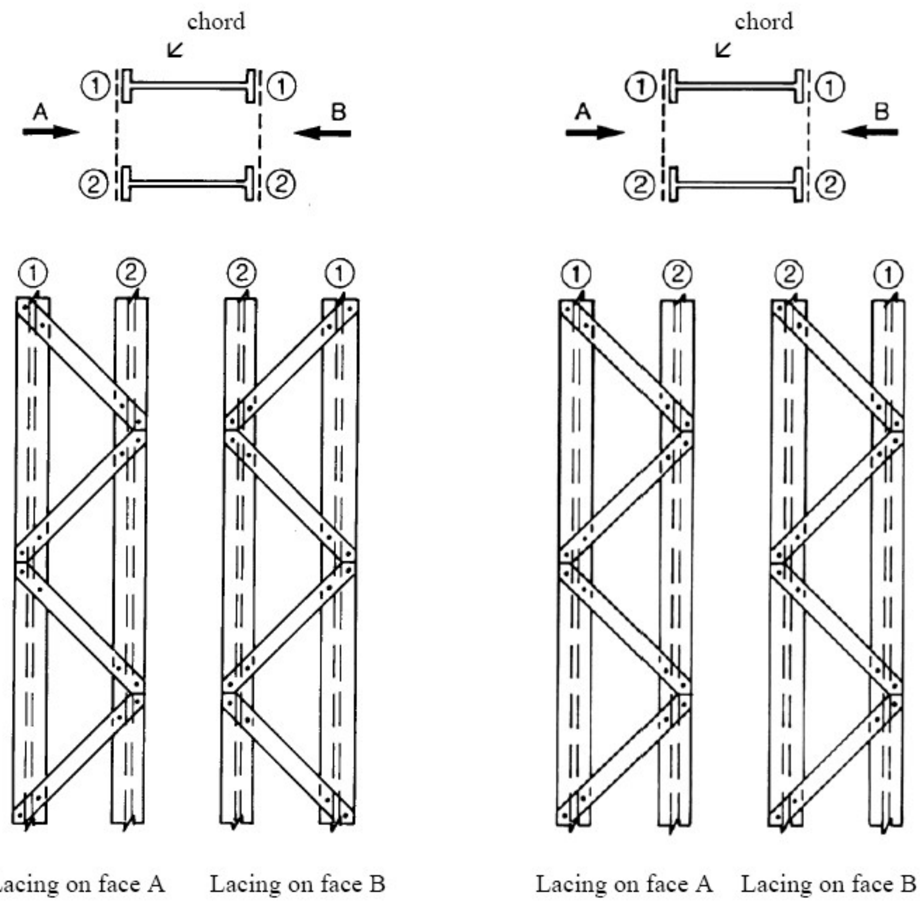
Figure 6.9: Shear stiffness of lacings of built-up members

6.4.2.2 Constructional details

(1) Single lacing systems in opposite faces of the built-up member with two parallel laced planes should be corresponding systems as shown in Figure 6.10(a), arranged so that one is the shadow of the other.

(2) When the single lacing systems on opposite faces of a built-up member with two parallel laced planes are mutually opposed in direction as shown in Figure 6.10(b), the resulting torsional effects in the member should be taken into account.

(3) Tie panels should be provided at the ends of lacing systems, at points where the lacing is interrupted and at joints with other members.



a) Corresponding lacing system (Recommended system)

b) Mutually opposed lacing system (Not recommended)

Figure 6.10: Single lacing system on opposite faces of a built-up member with two parallel laced planes

6.4.3 Battened compression members

6.4.3.1 Resistance of components of battened compression members

(1) The chords and the battens and their joints to the chords should be checked for the actual moments and forces in an end panel and at mid-span as indicated in Figure 6.11.

NOTE For simplicity the maximum chord forces $N_{ch,Ed}$ may be combined with the maximum shear force V_{Ed}.

6.4.3.2 Design details

(1) Battens should be provided at each end of a member.

(2) Where parallel planes of battens are provided, the battens in each plane should be arranged opposite each other.

(3) Battens should also be provided at intermediate points where loads are applied or lateral restraint is supplied.

6.4.4 Closely spaced built-up members

(1) Built-up compression members with chords in contact or closely spaced and connected through packing plates, see Figure 6.12, or star battened angle members connected by pairs of battens in two perpendicular planes, see Figure 6.13 should be checked for buckling as a single integral member ignoring the effect of shear stiffness ($S_V = \infty$), when the conditions in Table 6.9 are met.

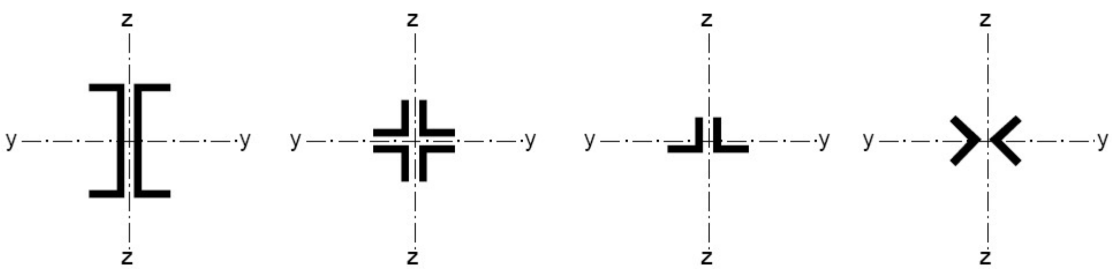


Figure 6.12: Closely spaced built-up members

Table 6.9: Maximum spacings for interconnections in closely spaced built-up or star battened angle members

Type of built-up member	Maximum spacing between interconnections *)
Members according to Figure 6.12 connected by bolts or welds	15 i_{min}
Members according to Figure 6.13 connected by pair of battens	70 i_{min}
*) centre-to-centre distance of interconnections i_{min} is the minimum radius of gyration of one chord or one angle	

(2) The shear forces to be transmitted by the battens should be determined from 6.4.3.1(1).

(3) In the case of unequal-leg angles, see Figure 6.13, buckling about the y-y axis may be verified with:

$$i_y = \frac{i_0}{1{,}15} \qquad (6.75)$$

where i_0 is the minimum radius of gyration of the built-up member.

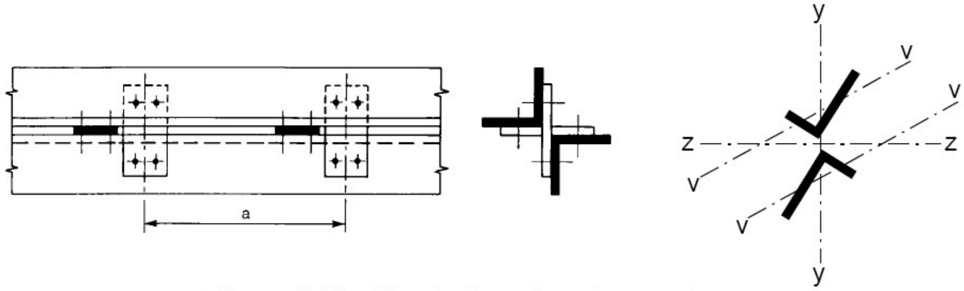
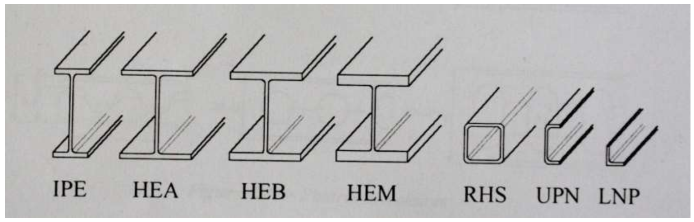


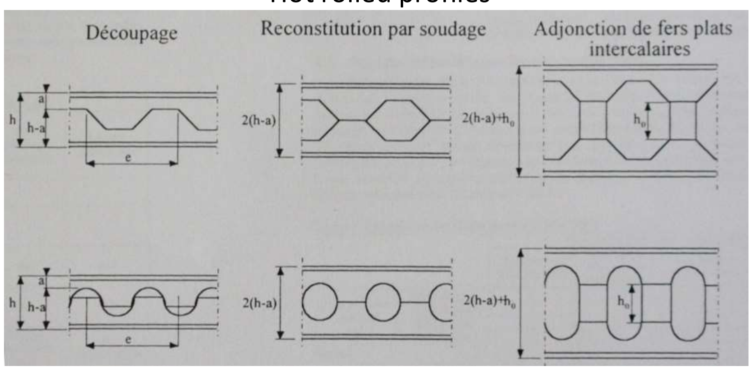
Figure 6.13: Star-battened angle members

7. RESTRAINED BEAMS

Beams are very common structural components in steel frame structures - perhaps the most basic structural component. A variety of section shapes and beams types may be used depending on the magnitude of loading, span, or other conditions.



Hot rolled profiles



Castellated beams

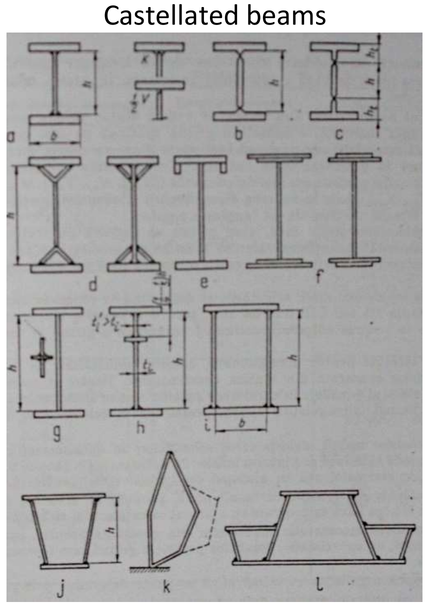
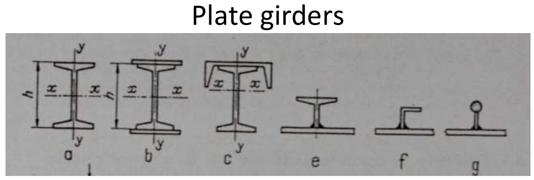


Plate girders



Compound beams

Figure 36. Typical beam types

Steel beams can often be designed simply based on:

- bending moment resistance (ensuring the design moment resistance of the selected cross-section exceeds the maximum applied moment)
- stiffness - the beam does not deflect so much that it affects serviceability considerations.

Beams which are unable to move laterally are termed "restrained" and are unaffected by out-of-plane buckling (lateral-torsional instability).

Beams may be considered laterally restrained if:

- full lateral restraint is provided by for example positive attachment of a floor system to the top flange of a simply supported beam.
- adequate torsional restraint of the compression flange is provided, i.e. by profiled roof sheeting
- closely spaced bracing elements are provided such that the minor axis slenderness is low.

Additionally, sections bent about their minor axis cannot fail by lateral torsional instability and it is unlikely that high torsional and lateral stiffness sections (rectangular hollow sections) will fail in this way.

7.1. Moment Resistance

In a single span, failure occurs when design value of the bending moment M_{Ed} exceeds design moment resistance of the cross-section $M_{c.Rd}$.

Magnitude depends on section shape, material strength and section classification. Where shear force on cross-section is small its effect on the resistance moment may be neglected. EN1993-1-1 sets a shear force value of 50% of the plastic shear resistance. The design moment resistance, $M_{c.Rd}$, may be taken as:

For class 1 and 2 cross-sections, the design plastic resistance moment of the gross section:

$$M_{c.Rd} = M_{pl.Rd} = \frac{W_{pl} f_y}{\gamma_{M0}} \quad (71)$$

For a class 3 cross-section, the design elastic resistance moment of the gross section:

$$M_{c.Rd} = M_{el.Rd} = \frac{W_{el} f_y}{\gamma_{M0}} \quad (72)$$

For a class 4 cross-section, the design local buckling resistance

$$M_{c.Rd} = M_{eff.Rd} = \frac{W_{eff} f_y}{\gamma_{M0}} \quad (73)$$

Where γ_{M0} is the partial safety factor (recommended value = 1,0), and W_{eff} is effective section modulus.

For beams with holes located in the tension flange at the critical cross-section it is required to check that the ratio of the net area to gross area of the flange is not so small that the section would rupture on the net section before the gross section yielded. This check is the same as that given for ductile tension members (section 5: ELEMENTS UNDER AXIAL LOADS) and will be satisfied provided that:

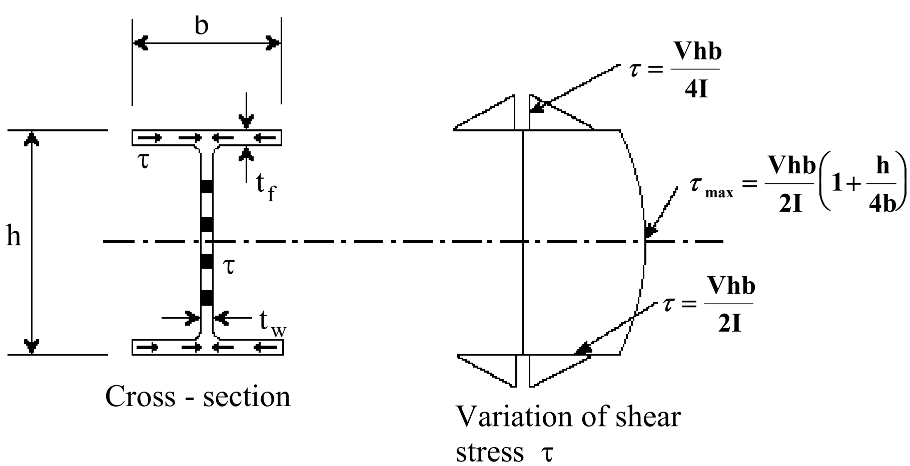
$$\frac{A_{f,net} 0.9 f_u}{\gamma_{M_2}} \geq \frac{A_f f_y}{\gamma_{M_0}} \tag{74}$$

where A_f is the area of the tension flange.

Fastener holes in tension zone of the web need not be allowed for, provided that the limit given in (eq. 74) is satisfied for the complete tension zone comprising the tension flange plus the tension zone of the web. Also, fastener holes except for oversize and slotted holes in compression zone of the cross-section need not be allowed for, if they are filled by fasteners.

7.2. Shear resistance

In general, bending governs the design of many steel beams but shear resistance can be significant as well, especially for short beams with heavy concentrated loads. As almost all the shear force is carried by the web and since the variation in shear stress through the web is quite small, it is sufficiently accurate for design to assume an average shear stress over the web. Figure 37 shows the pattern of shear stress in an I section assuming an elastic behavior.



$$\tau_{Rd} = \frac{V_z S_y}{t_i I_y} > \tau_{Ed}$$

V_z = shear force in z direction
I_y = moment of inertia
t_i = web thickness
S_y = statical moment of area

Figure 37. The pattern of shear stress in an I section assuming an elastic behavior

The design value of the shear force (V_{Ed}) at each cross section should be compared with the plastic shear resistance, $V_{pl.Rd}$ of the shear area (A_v):

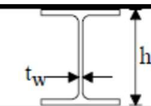
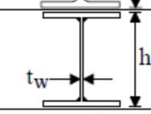
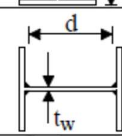
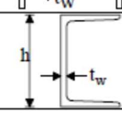
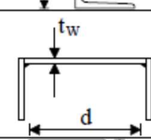
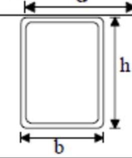
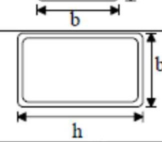
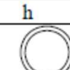
$$V_{pl.Rd} = A_v \frac{f_y/\sqrt{3}}{\gamma_{M_0}} \qquad (75)$$

For a rolled I beam, the shear area is given by:

$$A_v = \text{A} - 2\text{bt}_\text{f} + (\text{t}_\text{w} + 2\text{r})\,\text{t}_\text{f} \cong 1.04\text{t}_\text{w} \qquad (76)$$

Shear areas for a range of section types are shown in Table 8.

Table 8: Shear areas A_v for typical steel sections

I and H sections	Rolled	Load parallel to web	$1{,}04\ h\ t_w$ *	
	Fabricated	Load parallel to web	$(h - 2t_f)\ t_w$	
		Load parallel to flanges	$A - (h - 2t_f)\ t_w$ *	
Rolled channel sections		Load parallel to web	$1{,}04\ h\ t_w$ *	
Rolled angle sections		Load parallel to longer leg	h t	
Rolled rectangular hollow		Load parallel to depth	$Ah/(b + h)$ **	
sections of uniform thickness		Load parallel to breadth	$Ah/(b + h)$ **	
Circular hollow sections and tubes of uniform thickness			$0{,}6\ A$ **	
Plates and solid bars			A **	

* This is an approximate formula. More accurate values of A_v for rolled sections can be determined from:

- for I and H sections: $A_v = A - 2bt_f + (t_w + 2r)\ t_f$
- for channel sections: $A_v = A - 2bt_f + (t_w + 2r)\ t_f$

It is convenient to note that $1{,}04 / \sqrt{3} = 0{,}60$ and so for a rolled I, H or channel section:

$V_{pl.Rd} = 0{,}60\ h\ t_w\ f_y / \gamma_{M0}$

** A is the total cross-sectional area

The shear buckling for webs without stiffeners should be also verified (in accordance with EN1993-1-5), if:

$$\frac{h_w}{t_w} > \frac{72}{\eta}\varepsilon \qquad (77)$$

where:

- $\varepsilon = \sqrt{235/f_y}$
- The National Annex will define η. The value η = 1,20 is recommended. For steel grades higher than S460, η = 1,00 is recommended.

7.3. Moment resistance with high shear

If the design shear force exceeds by 50% the plastic shear resistance, the design moment resistance of the cross-section needs to be reduced. Thus, it is assumed that under a combination of direct and shear stress steel beam yields in accordance with the interaction formula:

$$\left(\frac{\sigma}{f_y}\right)^2 + \left(\frac{\tau}{\tau_y}\right)^2 = 1 \qquad (78)$$

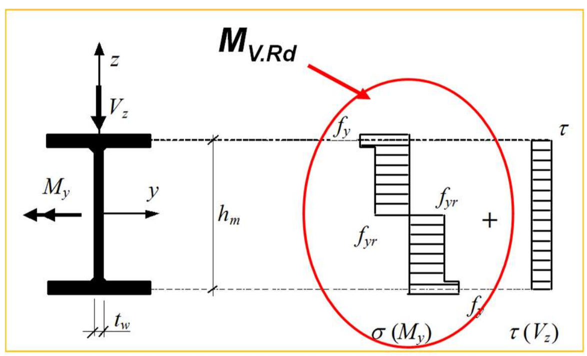


Figure 38. Beam with high shear

The design plastic moment can be calculated using a reduced strength for the shear area. This reduced strength can be expressed as $f_{yr}=(1-\rho) \times f_y$ and is dependent on the ratio of the shear load to shear capacity by the relationship:

$$\rho = \left(\frac{2V_{Ed}}{V_{pl.Rd}} - 1\right)^2 \qquad (79)$$

For an I beam loaded about its major axis, the reduced design plastic resistance moment in the presence of shear, $M_{V.Rd}$, is given by:

$$M_{y.V.Rd} = \left[W_{pl.y} - \frac{\rho A_v^2}{4t_w} \right] \frac{f_y}{\gamma_{M_0}} \quad (80)$$

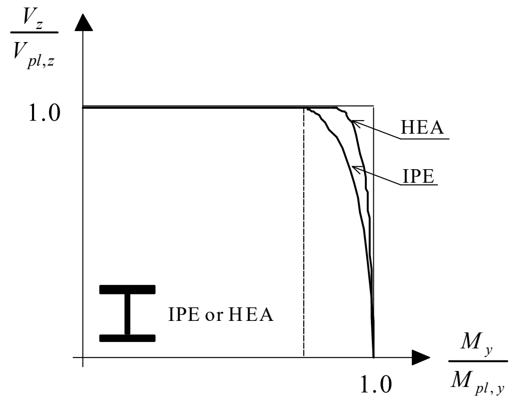


Figure 39. Bending-shear interaction

7.4. Design of restrained beams (EN1993-1-1)

Notes:

- Material reproduced from EN1993-1-1.
- For ease of understanding, numbering is done according to EN1993-1-1.

6.2.5 Bending moment

(1) The design value of the bending moment M_{Ed} at each cross-section should satisfy:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1,0 \quad (6.12)$$

where $M_{c,Rd}$ is determined considering fastener holes, see (4) to (6).

(2) The design resistance for bending about one principal axis of a cross-section is determined as follows:

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl}\ f_y}{\gamma_{M0}} \quad \text{for class 1 or 2 cross sections} \quad (6.13)$$

$$M_{c,Rd} = M_{el,Rd} = \frac{W_{el,min}\ f_y}{\gamma_{M0}} \quad \text{for class 3 cross sections} \quad (6.14)$$

$$M_{c,Rd} = \frac{W_{eff,min}\ f_y}{\gamma_{M0}} \quad \text{for class 4 cross sections} \quad (6.15)$$

where $W_{el,min}$ and $W_{eff,min}$ corresponds to the fibre with the maximum elastic stress.

(3) For bending about both axes, the methods given in 6.2.9 should be used.

(4) Fastener holes in the tension flange may be ignored provided that for the tension flange:

$$\frac{A_{f,net}\,0{,}9\,f_u}{\gamma_{M2}} \geq \frac{A_f\,f_y}{\gamma_{M0}} \qquad (6.16)$$

where A_f is the area of the tension flange.

NOTE The criterion in (4) provides capacity design (see 1.5.8) in the region of plastic hinges.

(5) Fastener holes in tension zone of the web need not be allowed for, provided that the limit given in (4) is satisfied for the complete tension zone comprising the tension flange plus the tension zone of the web.

(6) Fastener holes except for oversize and slotted holes in compression zone of the cross-section need not be allowed for, provided that they are filled by fasteners.

6.2.6 Shear

(1) The design value of the shear force V_{Ed} at each cross section should satisfy:

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1{,}0 \qquad (6.17)$$

where $V_{c,Rd}$ is the design shear resistance. For plastic design $V_{c,Rd}$ is the design plastic shear resistance $V_{pl,Rd}$ as given in (2). For elastic design $V_{c,Rd}$ is the design elastic shear resistance calculated using (4) and (5).

(2) In the absence of torsion the design plastic shear resistance is given by:

$$V_{pl,Rd} = \frac{A_v\left(f_y/\sqrt{3}\right)}{\gamma_{M0}} \qquad (6.18)$$

where A_v is the shear area.

(3) The shear area A_v may be taken as follows:

a) rolled I and H sections, load parallel to web $A - 2bt_f + (t_w + 2r)\,t_f$ but not less than $\eta h_w t_w$

b) rolled channel sections, load parallel to web $A - 2bt_f + (t_w + r)\,t_f$

c) rolled T-section, load parallel to web $0{,}9\,(A - bt_f)$

d) welded I, H and box sections, load parallel to web $\eta\sum(h_w t_w)$

e) welded I, H, channel and box sections, load parallel to flanges $A-\sum(h_w t_w)$

f) rolled rectangular hollow sections of uniform thickness:

load parallel to depth $Ah/(b+h)$

load parallel to width $Ab/(b+h)$

g) circular hollow sections and tubes of uniform thickness $2A/\pi$

where A is the crosssectional area;

b is the overall breadth;

h is the overall depth;

h_w is the depth of the web;

r is the root radius;

t_f is the flange thickness;

t_w is the web thickness (If the web thickness in not constant, t_w should be taken as the minimum thickness.).

η see EN 1993-1-5.

NOTE η may be conservatively taken equal 1,0.

(4) For verifying the design elastic shear resistance $V_{c,Rd}$ the following criterion for a critical point of the cross section may be used unless the buckling verification in section 5 of EN 1993-1-5 applies:

$$\frac{\tau_{Ed}}{f_y / \left(\sqrt{3}\,\gamma_{M0}\right)} \leq 1,0 \qquad (6.19)$$

where τ_{Ed} may be obtained from: $\tau_{Ed} = \frac{V_{Ed}\,S}{I\,t}$ (6.20)

where V_{Ed} is the design value of the shear force

S is the first moment of area about the centroidal axis of that portion of the cross-section between the point at which the shear is required and the boundary of the cross-section

I is second moment of area of the whole cross section

t is the thickness at the examined point

NOTE The verification according to (4) is conservative as it excludes partial plastic shear distribution, which is permitted in elastic design, see (5). Therefore it should only be carried out where the verification on the basis of $V_{c,Rd}$ according to equation (6.17) cannot be performed.

(5) For I- or H-sections the shear stress in the web may be taken as:

$$\tau_{Ed} = \frac{V_{Ed}}{A_w} \quad \text{if } A_f / A_w \geq 0,6 \qquad (6.21)$$

where A_f is the area of one flange;

A_w is the area of the web: $A_w = h_w\, t_w$.

(6) In addition the shear buckling resistance for webs without intermediate stiffeners should be according to section 5 of EN 1993-1-5, if

$$\frac{h_w}{t_w} > 72\frac{\varepsilon}{\eta} \qquad (6.22)$$

For η see section 5 of EN 1993-1-5.

NOTE η may be conservatively taken equal to 1,0.

(7) Fastener holes need not be allowed for in the shear verification except in verifying the design shear resistance at connection zones as given in EN 1993-1-8.

(8) Where the shear force is combined with a torsional moment, the plastic shear resistance $V_{pl,Rd}$ should be reduced as specified in 6.2.7(9).

6.2.7 Torsion

- See section 8: ELEMENTS IN TORSION

6.2.8 Bending and shear

(1) Where the shear force is present allowance should be made for its effect on the moment resistance.

(2) Where the shear force is less than half the plastic shear resistance its effect on the moment resistance may be neglected except where shear buckling reduces the section resistance, see EN 1993-1-5.

(3) Otherwise the reduced moment resistance should be taken as the design resistance of the cross-section, calculated using a reduced yield strength

$$(1 - \rho)\, f_y \qquad (6.29)$$

for the shear area,

where $\rho = \left(\frac{2 V_{Ed}}{V_{pl,Rd}} - 1 \right)^2$ and $V_{pl,Rd}$ is obtained from 6.2.6(2).

NOTE See also 6.2.10(3).

(4) When torsion is present ρ should be obtained from $\rho = \left(\frac{2 V_{Ed}}{V_{pl,T,Rd}} - 1 \right)^2$, see 6.2.7, but should be taken as 0 for $V_{Ed} \leq 0{,}5 V_{pl,T,Rd}$.

(5) The reduced design plastic resistance moment allowing for the shear force may alternatively be obtained for I-cross-sections with equal flanges and bending about the major axis as follows:

$$M_{y,V,Rd} = \frac{\left[W_{pl,y} - \frac{\rho A_w^2}{4 t_w} \right] f_y}{\gamma_{M0}} \quad \text{but } M_{y,V,Rd} \leq M_{y,c,Rd} \qquad (6.30)$$

where $M_{y,c,Rd}$ is obtained from 6.2.5(2)

and $A_w = h_w t_w$

(6) For the interaction of bending, shear and transverse loads see section 7 of EN 1993-1-5.

6.2.9 Bending and axial force

6.2.9.1 Class 1 and 2 cross-sections

(1) Where an axial force is present, allowance should be made for its effect on the plastic moment resistance.

(2) For class 1 and 2 cross sections, the following criterion should be satisfied:

$$M_{Ed} \leq M_{N,Rd} \qquad (6.31)$$

where $M_{N,Rd}$ is the design plastic moment resistance reduced due to the axial force N_{Ed}.

(3) For a rectangular solid section without fastener holes $M_{N,Rd}$ should be taken as:

$$M_{N,Rd} = M_{pl,Rd}\left[1-\left(N_{Ed}/N_{pl,Rd}\right)^2\right] \qquad (6.32)$$

(4) For doubly symmetrical I- and H-sections or other flanges sections, allowance need not be made for the effect of the axial force on the plastic resistance moment about the y-y axis when both the following criteria are satisfied:

$$N_{Ed} \leq 0{,}25\, N_{pl,Rd} \text{ and} \qquad (6.33)$$

$$N_{Ed} \leq \frac{0{,}5\, h_w\, t_w\, f_y}{\gamma_{M0}} \qquad (6.34)$$

For doubly symmetrical I- and H-sections, allowance need not be made for the effect of the axial force on the plastic resistance moment about the z-z axis when:

$$N_{Ed} \leq \frac{h_w\, t_w\, f_y}{\gamma_{M0}} \qquad (6.35)$$

(5) For cross-sections where fastener holes are not to be accounted for, the following approximations may be used for standard rolled I or H sections and for welded I or H sections with equal flanges:

$$M_{N,y,Rd} = M_{pl,y,Rd}\,(1-n)/(1-0{,}5a) \quad \text{but } M_{N,y,Rd} \leq M_{pl,y,Rd} \qquad (6.36)$$

$$\text{for } n \leq a: \quad M_{N,z,Rd} = M_{pl,z,Rd} \qquad (6.37)$$

$$\text{for } n > a: \quad M_{N,z,Rd} = M_{pl,z,Rd}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] \qquad (6.38)$$

where $n = N_{Ed} / N_{pl,Rd}$

$a = (A-2bt_f)/A$ but $a \leq 0{,}5$

For cross-sections where fastener holes are not to be accounted for, the following approximations may be used for rectangular structural hollow sections of uniform thickness and for welded box sections with equal flanges and equal webs:

$$M_{N,y,Rd} = M_{pl,y,Rd}\,(1-n)/(1-0{,}5a_w) \quad \text{but } M_{N,y,Rd} \leq M_{pl,y,Rd} \qquad (6.39)$$

$$M_{N,z,Rd} = M_{pl,z,Rd}\,(1-n)/(1-0{,}5a_f) \quad \text{but } M_{N,z,Rd} \leq M_{pl,z,Rd} \qquad (6.40)$$

where $a_w = (A-2bt)/A$ but $a_w \leq 0{,}5$ for hollow sections

$a_w = (A-2bt_f)/A$ but $a_w \leq 0{,}5$ for welded box sections

$a_f = (A-2ht)/A$ but $a_f \leq 0{,}5$ for hollow sections

$a_f = (A-2ht_w)/A$ but $a_f \leq 0{,}5$ for welded box sections

(6) For bi-axial bending the following criterion may be used:

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} \leq 1 \qquad (6.41)$$

in which α and β are constants, which may conservatively be taken as unity, otherwise as follows:

– I and H sections:

$\alpha = 2 \; ; \; \beta = 5n \quad \text{but} \; \beta \geq 1$

– circular hollow sections:

$\alpha = 2 \; ; \; \beta = 2$

– rectangular hollow sections:

$$\alpha = \beta = \frac{1{,}66}{1 - 1{,}13\,n^2} \qquad \text{but } \alpha = \beta \leq 6$$

where $n = N_{Ed} / N_{pl,Rd}$.

6.2.9.2 Class 3 cross-sections

(1) In the absence of shear force, for Class 3 cross-sections the maximum longitudinal stress should satisfy the criterion:

$$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}} \qquad (6.42)$$

where $\sigma_{x,Ed}$ is the design value of the local longitudinal stress due to moment and axial force taking account of fastener holes where relevant, see 6.2.3, 6.2.4 and 6.2.5

6.2.9.3 Class 4 cross-sections

(1) In the absence of shear force, for Class 4 cross-sections the maximum longitudinal stress $\sigma_{x,Ed}$ calculated using the effective cross sections (see 5.5.2(2)) should satisfy the criterion:

$$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}} \qquad (6.43)$$

where $\sigma_{x,Ed}$ is the design value of the local longitudinal stress due to moment and axial force taking account of fastener holes where relevant, see 6.2.3, 6.2.4 and 6.2.5

(2) The following criterion should be met:

$$\frac{N_{Ed}}{A_{eff}\, f_y / \gamma_{M0}} + \frac{M_{y,Ed} + N_{Ed}\, e_{Ny}}{W_{eff,y,min}\, f_y / \gamma_{M0}} + \frac{M_{z,Ed} + N_{Ed}\, e_{Nz}}{W_{eff,z,min}\, f_y / \gamma_{M0}} \leq 1 \qquad (6.44)$$

where A_{eff} is the effective area of the cross-section when subjected to uniform compression

$W_{eff,min}$ is the effective section modulus (corresponding to the fibre with the maximum elastic stress) of the cross-section when subjected only to moment about the relevant axis

e_N is the shift of the relevant centroidal axis when the cross-section is subjected to compression only, see 6.2.2.5(4)

NOTE The signs of N_{Ed} , $M_{y,Ed}$, $M_{z,Ed}$ and $\Delta M_i = N_{Ed}\, e_{Ni}$ depend on the combination of the respective direct stresses.

6.2.10 Bending, shear and axial force

(1) Where shear and axial force are present, allowance should be made for the effect of both shear force and axial force on the resistance moment.

(2) Provided that the design value of the shear force V_{Ed} does not exceed 50% of the design plastic shear resistance $V_{pl.Rd}$ no reduction of the resistances defined for bending and axial force in 6.2.9 need be made, except where shear buckling reduces the section resistance, see EN 1993-1-5.

(3) Where V_{Ed} exceeds 50% of $V_{pl.Rd}$ the design resistance of the cross-section to combinations of moment and axial force should be calculated using a reduced yield strength

$$(1-\rho)f_y \qquad (6.45)$$

for the shear area

where $\rho = (2V_{Ed} / V_{pl.Rd} - 1)^2$ and $V_{pl,Rd}$ is obtained from 6.2.6(2).

NOTE Instead of reducing the yield strength also the plate thickness of the relevant part of the cross section may be reduced.

7.5. Serviceability requirements for beams

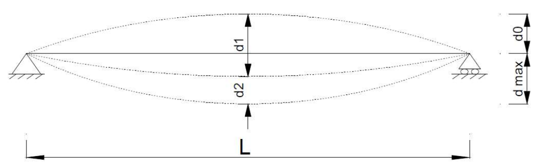
Serviceability limit states are conditions in which the functions of a building are disrupted because of local minor damage, deterioration of building components or because of occupant discomfort (e.g. vibrations).
Actual codes and standards for steel constructions (and for constructions in general) deal with the complex problems of serviceability limit-states using simple rules. The most common of this rule, which remained essentially unchanged for years, requires only a check of deflections of the frame (e.g. beam) under service load conditions. This approach presumes that a broad spectrum of building structure performance issues can be dealt with simply by means of static deflection checks.
The point at which structural deformations become sufficient to cause serviceability problems depends on the nature of the structure and its detailing, as well as on the perceptions of the occupants. Thus, deflection or drift limits in the range of 1/200 to 1/500 of the floor span appear to be useful as general indices of nonstructural damage or unsightliness. These limits indeed seem to have protected against such serviceability problems in most instances.
Apart from limiting the static deflections, structural motions which arise from normal activities of the building occupants (operation of mechanical equipment within the building, traffic, etc.), or from windstorms and earthquakes need to be checked. Structural vibrations of floors or of the building can cause occupant discomfort and alarm. Reductions in mass, stiffness and damping that result from using lighter structural systems have led to an increasing number of complaints about vibrations. Occupant activities of a rhythmic nature, such as dancing, jumping, exercising and cheering, cause essentially steady-state excitations with frequencies in the range of 2 to 6 Hz. Many modern long-span floor systems have fundamental frequencies in this range.
EN1993 gives no recommendations for limiting the values for serviceability conditions (e.g. vertical deflections). As a result, acceptable limits for deflections should be agreed between the client, designer and competent authorities. For guidance, Table 9 gives some recommended limiting values for vertical deflections.
Apart from increasing the cross-section size, deflection can be controlled for example by beam cambering (curve of the beam in the vertical plane), see Figure 40.

Figure 40. Serviceability condition of a floor system can be controlled through camber (curve of the beam in the vertical plane) at the time of construction

Table 9: Limits on deflections

Conditions	Limits	
	δ_{max}	δ_2
Roofs generally	L/200	L/250
Roofs frequently carrying personnel other than for maintenance	L/250	L/300
Floors generally	L/250	L/300
Floors and roofs supporting plaster or other brittle finish or nonflexible partitions	L/250	L/350
Floors supporting columns (unless the deflection has been included in the global analysis for the ultimate limit state)	L/400	L/500
Where δ_{max} can impair the appearance of the building	L/250	-

δ_{max} - total final deflection relative to a straight line between supports
δ_0 - is the precamber of the beam in the unloaded state (state 0)
δ_1 - is the variation of the deflection of the beam due to the permanent loads, immediately after loading (state 1)
δ_2 – deflection of the beam due to the variable loads, increased with the deflection of the beam due to the permanent loads (state 2).



Vertical deflections of a simple supported beam

The limitation of vibrations has been mainly controlled by limiting the ration L/d to 20. This limitation was important, because the oscillation and vibrations can cause discomfort to users. If for short beams these limitations were proved to be adequate, for long beams (L > 9 meters), the current limitations of deflections lead to vibrations in excess. Verification of the suitability of a design may be done by means of a dynamic analysis but in many cases limiting the deflection is sufficient:

- Floors in dwellings and offices should have the lowest natural frequency > 3 cycles/second. This is satisfied if d_{max} < 28 mm (independent of the beam span).
- Floors in gymnasia/discos should have the lowest natural frequency > 5 cycles/second. This is satisfied if d_{max} < 10 mm (independent of the beam span).

8. ELEMENTS IN TORSION

When a steel element (e.g. beam) is transversely loaded such that the resultant force passes through the longitudinal shear center axis, the beam only bends, and no torsion will occur. However, when the resultant acts away from the shear center axis, then the beam will not only bend but also twist (Figure 41). Torsion may be very unfavorable especially for open sections or thin walled simple section members.

Shear center is defined as the point on the beam section where load is applied, and no twisting is produced.
The shear center and the centroid of the cross section coincide when section has two axes of symmetry (Figure 42).

The shear center is on the axis of symmetry when the cross section has one axis of symmetry (Figure 42).

In practice, torsion (twisting) does not typically appears alone, but in combination with other stress states (ex. bending + torsion, compression + bending + torsion).
There are two main situations that involve consideration of torsion in design:

- Function of member include transmission of torque (combined or not with bending or axial load);
- Members in which torsion is a secondary undesirable side effect tending to cause excessive deformation or premature failure.

When torsion is applied to a structural member, its cross section may warp in addition to twisting:

- If the member is allowed to warp freely (the ends of the member are free), then the applied torque is resisted entirely by torsional shear stresses τ_t (called St. Venant's torsional shear stress) – called Uniform or Free torsion (see Figure 43);
- If the member is not allowed to warp freely (the end plane of the member is not free), the applied torque is resisted by St. Venant's torsional shear stress (τ_t) and warping torsion (resulting in normal stresses, σ_w, and shear stresses, τ_w), - see Figure 44. This behavior is called nonuniform torsion.

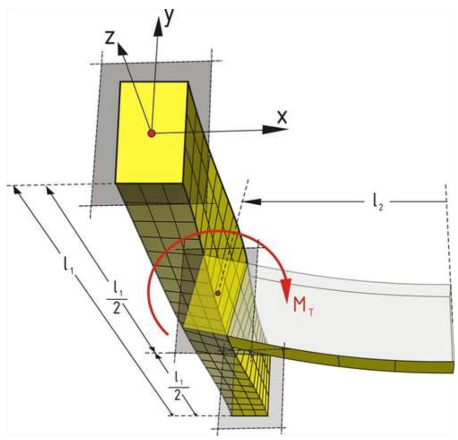


Figure 41. Moment in the secondary beam transferred to the main beam as a torsional moment

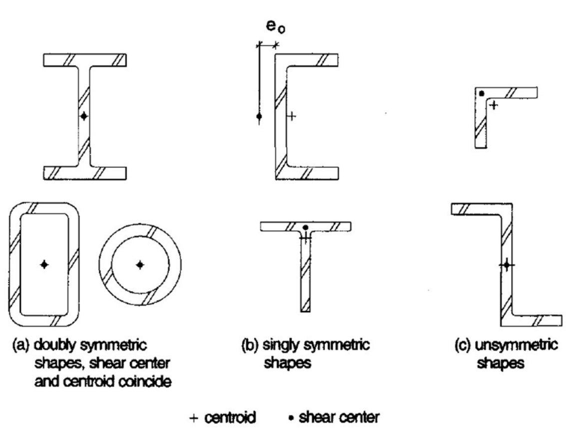
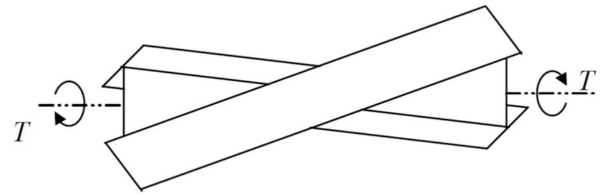
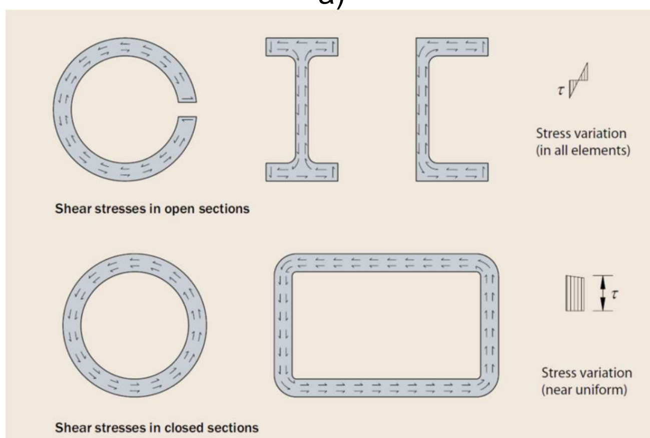


Figure 42. Shear center locations for some typical structural sections



a)



b)

Figure 43. Free torsion (St Venant torsion): a) deformed shape, ends are free to rotate; b) St Venant shear streses τ_t

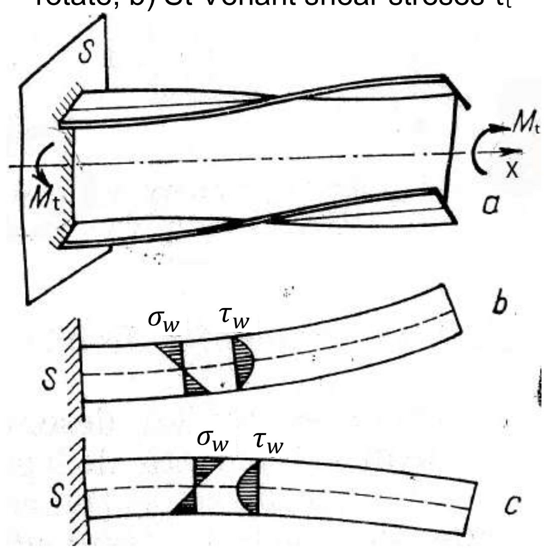


Figure 44. Warping torsion: a) deformed shape, one end is not free to rotate; b) distribution of normal stresses, σ_w, and shear stresses, τ_w

Free torsion

Shear stress and angle of rotation for a rectangular section can be calculated as:

$$\tau_t = \frac{M_t}{I_t} t \tag{81}$$

$$\theta = \frac{d_\varphi}{d_z} = \frac{M_t}{G \bullet I_t} \tag{82}$$

where

$$I_t \cong \frac{1}{3} h \bullet t^3 \tag{83}$$

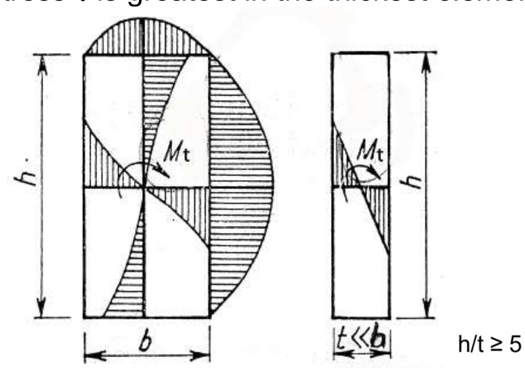
<u>For open sections</u>, shear stress can be calculated as:

$$\tau = \frac{M_t \bullet t}{\frac{1}{3}\sum h_i \bullet t_i^3} \tag{84}$$

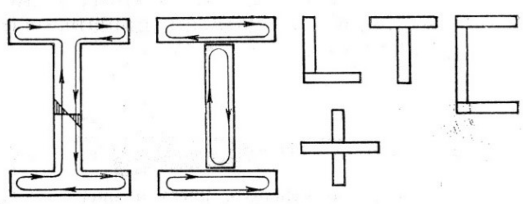
where

$$I_t = \frac{\alpha}{3}\sum h_i \bullet t_i^3 \tag{85}$$

For the resistance, it may be conservatively adopted α = 1.
The pure torsional shear stress τ is greatest in the thickest element.



a)



b)

Figure 45. Shear stress distribution: simple rectangular sections; b) open sections

<u>For closed cross sections</u>: shear stress can be calculated as:

$$\begin{aligned} M_t &= T_a \cdot b + T_b \cdot a \\ T_a &= \tau_a (t_a \cdot a) \\ T_b &= \tau_b (t_b \cdot b) \\ \tau_a \cdot t_a &= \tau_b \cdot t_b = const \end{aligned} \tag{86}$$

$$\tau_a = \frac{M_t}{2a \cdot b \cdot t_a}; \tau_b = \frac{M_t}{2a \cdot b \cdot t_b} \text{ (General formula (Bredt)} \tag{87}$$

$$I_t = \frac{4A_m^2}{\oint \frac{ds}{t}} \text{ (General formula (Bredt)} \tag{88}$$

The resultant of the shear stresses on the element is called shearing flux (constant !!) and has the value:

$$\tau \times t = \frac{M_t}{2A_m} \text{ (General formula (Bredt)} \tag{89}$$

where A_m is the area described by median axis of the section.

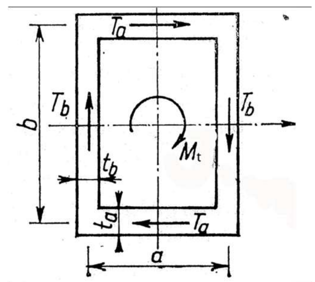


Figure 46. Shear stress in closed sections

Warping (Theory of Vlasov)

- Material is isotropic, homogeneous, perfect elastic.
- Longitudinal stresses from warping vary linear on the element thickness.
- Transversal cross section of the element maintains its shape

The normal stresses, σ_ω, and shear stresses, τ_ω, can be calculated as:

$$\sigma_w = \frac{B \cdot \omega}{I_w}; \tag{90}$$

$$\tau_w = \frac{M_w S_w}{t \cdot I_w}$$

where:

M_w	warping moment
B	bimoment
S_w	warping statical moment
ω	sectorial coordinate

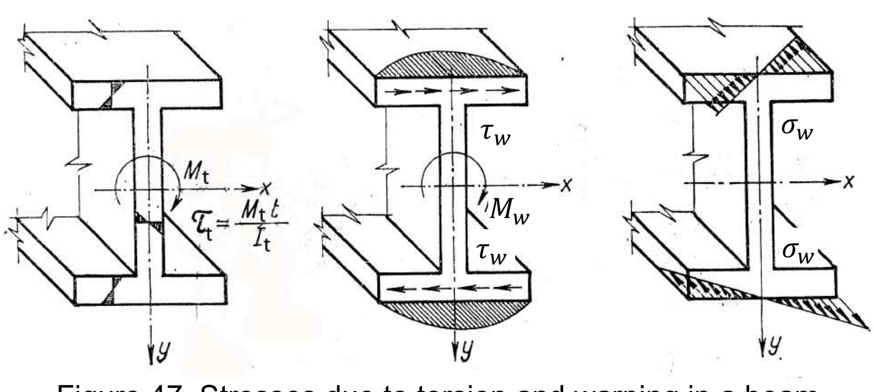


Figure 47. Stresses due to torsion and warping in a beam

8.1. Design of uniform members in torsion (EN1993-1-1)

Notes:

- Material reproduced from EN1993-1-1.
- For ease of understanding, numbering is done according to EN1993-1-1.

6.2.7 Torsion

(1) For members subject to torsion for which distortional deformations may be disregarded the design value of the torsional moment T_{Ed} at each cross-section should satisfy:

$$\frac{T_{Ed}}{T_{Rd}} \leq 1,0 \qquad (6.23)$$

where T_{Rd} is the design torsional resistance of the cross section.

(2) The total torsional moment T_{Ed} at any cross- section should be considered as the sum of two internal effects:

$$T_{Ed} = T_{t,Ed} + T_{w,Ed} \qquad (6.24)$$

where $T_{t,Ed}$ is the internal St. Venant torsion;

$T_{w,Ed}$ is the internal warping torsion.

(3) The values of $T_{t,Ed}$ and $T_{w,Ed}$ at any cross-section may be determined from T_{Ed} by elastic analysis, taking account of the section properties of the member, the conditions of restraint at the supports and the distribution of the actions along the member.

(4) The following stresses due to torsion should be taken into account:

- the shear stresses $\tau_{t,Ed}$ due to St. Venant torsion $T_{t,Ed}$
- the direct stresses $\sigma_{w,Ed}$ due to the bimoment B_{Ed} and shear stresses $\tau_{w,Ed}$ due to warping torsion $T_{w,Ed}$

(5) For the elastic verification the yield criterion in 6.2.1(5) may be applied.

(6) For determining the plastic moment resistance of a cross section due to bending and torsion only torsion effects B_{Ed} should be derived from elastic analysis, see (3).

(7) As a simplification, in the case of a member with a closed hollow cross-section, such as a structural hollow section, it may be assumed that the effects of torsional warping can be neglected. Also as a simplification, in the case of a member with open cross section, such as I or H, it may be assumed that the effects of St. Venant torsion can be neglected.

(8) For the calculation of the resistance T_{Rd} of closed hollow sections the design shear strength of the individual parts of the cross section according to EN 1993-1-5 should be taken into account.

(9) For combined shear force and torsional moment the plastic shear resistance accounting for torsional effects should be reduced from $V_{pl,Rd}$ to $V_{pl,T,Rd}$ and the design shear force should satisfy:

$$\frac{V_{Ed}}{V_{pl,T,Rd}} \leq 1{,}0 \tag{6.25}$$

in which $V_{pl,T,Rd}$ may be derived as follows:

- for an I or H section:

$$V_{pl,T,Rd} = \sqrt{1 - \frac{\tau_{t,Ed}}{1{,}25\,\left(f_y/\sqrt{3}\right)/\gamma_{M0}}}\; V_{pl,Rd} \tag{6.26}$$

- for a channel section:

$$V_{pl,T,Rd} = \left[\sqrt{1 - \frac{\tau_{t,Ed}}{1{,}25\,\left(f_y/\sqrt{3}\right)/\gamma_{M0}}} - \frac{\tau_{w,Ed}}{\left(f_y/\sqrt{3}\right)/\gamma_{M0}}\right] V_{pl,Rd} \tag{6.27}$$

- for a structural hollow section:

$$V_{pl,T,Rd} = \left[1 - \frac{\tau_{t,Ed}}{\left(f_y/\sqrt{3}\right)/\gamma_{M0}}\right] V_{pl,Rd} \tag{6.28}$$

where $V_{pl,Rd}$ is given in 6.2.6.

9. UNRESTRAINED BEAMS IN BENDING

Slender structural elements loaded in a stiff plane tend to fail by buckling in a more flexible plane. In the case of a beam bent about its major axis, failure may occur by a form of buckling which involves both lateral deflection (about z axis) and twisting – called lateral torsional buckling LTB.

Beams with sufficient restraint to the compression flange are not susceptible to lateral-torsional buckling.
Beams with certain types of cross-sections, such as square or circular hollow sections, are not susceptible to lateral-torsional buckling.

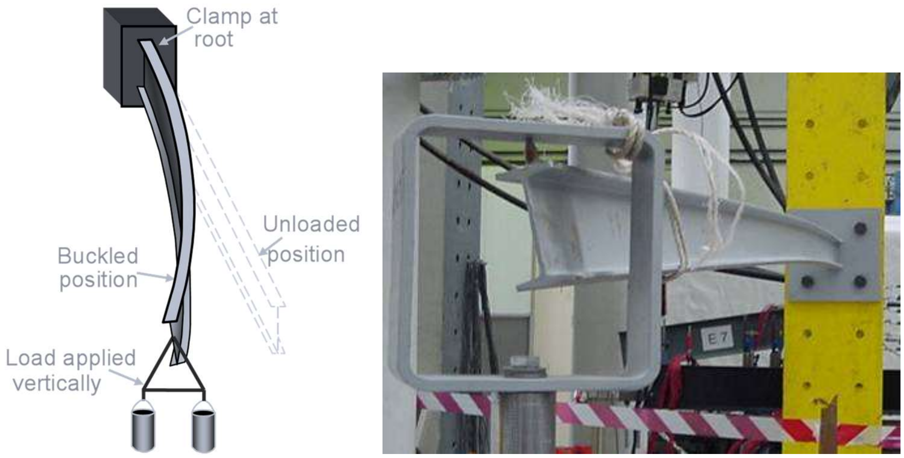


Figure 48. Unrestrained beams in bending – lateral-torsional buckling

Let's consider a beam, perfectly elastic, initially straight, and loaded by equal and opposite end moments about its major axis, see Figure 49. The beam is unrestrained along its length. At the end supports, twisting and lateral deflection are prevented, while are free to rotate both in the plane of the web and on plan.

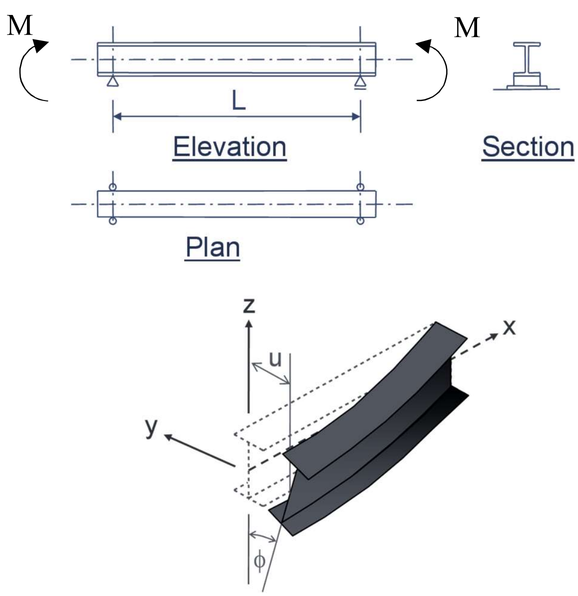
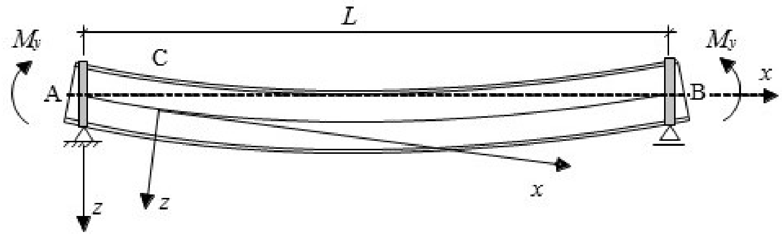


Figure 49. Deformed shape of an unrestrained beam in bending (only twisting and lateral deflection are prevented at both ends)

The Elastic critical moment M_{cr}^{E} is the maximum value of bending moment supported by a beam without imperfections – see eq. **91**; this quantity plays a fundamental role on the analysis of this type of phenomena.

$$M_{cr}^{E} = \frac{\pi}{L}\sqrt{G\,I_T\,E\,I_z\left(1+\frac{\pi^2\,E\,I_W}{L^2 G\,I_T}\right)} \tag{91}$$



M_{cr}^{E} depends mainly of:

- Loading and support conditions;
- Length between lateral braced sections (L);
- Lateral flexural stiffness EI_z
- Torsional stiffness GI_t and Warping stiffness EI_w

For other distributions of bending moment, the Elastic critical moment needs to be corrected:

$$M_{cr} = \alpha_m\,M_{cr}^{E} \tag{92}$$

Table 10. Factors α and β for different bending moment diagrams

Member	Diagram of moments	α_m	Validity limits
M, βM, L	M, +, -, βM	$1.75+1.05\beta+0.3\beta^2 \le 2.5$	$-1\le\beta\le1$
P, P, d, d	+, $\frac{PL}{2}\left(1-\frac{2d}{L}\right)$	$1.0+0.35(1-2d/L)^2$	$0\le\frac{2d}{L}\le1$
P, d	+, $\frac{PL}{4}\left(1-\frac{4d^2}{L^2}\right)$	$1.35+0.4\,(2d/L)^2$	$0\le\frac{2d}{L}\le1$
P, $\frac{\beta 3PL}{16}$, L/2, L/2	$\frac{\beta 3PL}{16}$, -, +, $\frac{PL}{4}\left(1-\frac{3\beta}{8}\right)$	$1.35+0.15\beta$	$0\le\beta\le0.89$
		$-1.2+3\beta$	$0.89\le\beta\le1$
$\frac{\beta PL}{8}$, P, $\frac{\beta PL}{8}$, L/2, L/2	$\frac{\beta PL}{8}$, $\frac{\beta PL}{8}$, -, +, -, $\frac{PL}{4}\left(1-\frac{\beta}{2}\right)$	$1.35+0.36\beta$	$0\le\beta\le1$
p, $\frac{\beta pL^2}{8}$	$\frac{\beta pL^2}{8}$, -, +, $\frac{pL^2}{8}\left(1-\frac{\beta}{4}\right)^2$	$1.13+0.10\beta$	$0\le\beta\le0.7$
		$-1.25+3.5\beta$	$0.7\le\beta\le1$
$\frac{\beta pL^2}{12}$, p, $\frac{\beta pL^2}{12}$	$\frac{\beta pL^2}{12}$, $\frac{\beta pL^2}{12}$, -, +, -, $\frac{pL^2}{8}\left(1-\frac{2\beta}{3}\right)$	$1.13+0.12\beta$	$0\le\beta\le0.75$
		$-2.38+4.8\beta$	$0.75\le\beta\le1$

For the general case, the Elastic critical moment can be calculated using the general formulae:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j \right)^2 \right]^{0.5} - \left(C_2 z_g - C_3 z_j \right) \right\} \quad (93)$$

$$z_g = (z_a - z_s)$$

$$z_j = z_s - \left(0.5 \int_A (y^2 + z^2) z \, dA \right) \Big/ I_y$$

The eq. (**Eroare! Fără sursă de referință.**):

- Is applicable to symmetrical and mono symmetric sections,
- Includes the favourable or unfavourable effects of the loading applied below or above the shear centre and several degrees of restriction to lateral bending (k_z) and warping (k_w);
- Can be applied to several shapes of bending moment diagram (C1, C2 and C3).

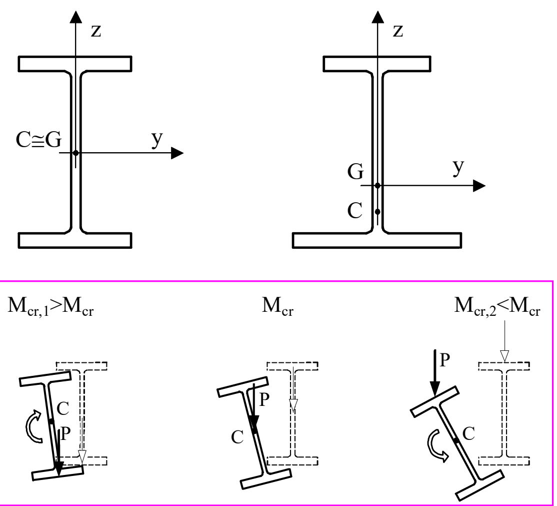


Figure 50. Factors affecting the elastic critical moment

9.1. Buckling resistance of uniform members in bending

Notes:

- Material reproduced from EN1993-1-1.
- For ease of understanding, numbering is done according to EN1993-1-1.

6.3.2.1 Buckling resistance

(1) A laterally unrestrained member subject to major axis bending should be verified against lateral-torsional buckling as follows:

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1{,}0 \quad (6.54)$$

where M_{Ed} is the design value of the moment

$M_{b,Rd}$ is the design buckling resistance moment.

(2) Beams with sufficient restraint to the compression flange are not susceptible to lateral-torsional buckling. In addition, beams with certain types of cross-sections, such as square or circular hollow sections, fabricated circular tubes or square box sections are not susceptible to lateral-torsional buckling.

(3) The design buckling resistance moment of a laterally unrestrained beam should be taken as:

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}} \qquad (6.55)$$

where W_y is the appropriate section modulus as follows:

- $W_y = W_{pl,y}$ for Class 1 or 2 cross-sections
- $W_y = W_{el,y}$ for Class 3 cross-sections
- $W_y = W_{eff,y}$ for Class 4 cross-sections

χ_{LT} is the reduction factor for lateral-torsional buckling.

NOTE 1 For determining the buckling resistance of beams with tapered sections second order analysis according to 5.3.4(3) may be performed. For out-of-plane buckling see also 6.3.4.

NOTE 2B For buckling of components of building structures see also Annex BB.

(4) In determining W_y holes for fasteners at the beam end need not to be taken into account.

6.3.2.2 Lateral torsional buckling curves – General case

(1) Unless otherwise specified, see 6.3.2.3, for bending members of constant cross-section, the value of χ_{LT} for the appropriate non-dimensional slenderness $\bar{\lambda}_{LT}$, should be determined from:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but } \chi_{LT} \leq 1{,}0 \qquad (6.56)$$

where $\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0{,}2\right) + \bar{\lambda}_{LT}^2\right]$

α_{LT} is an imperfection factor

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

M_{cr} is the elastic critical moment for lateral-torsional buckling

(2) M_{cr} is based on gross cross sectional properties and takes into account the loading conditions, the real moment distribution and the lateral restraints.

NOTE The imperfection factor α_{LT} corresponding to the appropriate buckling curve may be obtained from the National Annex. The recommended values α_{LT} are given in Table 6.3.

Table 6.3: Recommended values for imperfection factors for lateral torsional buckling curves

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

The recommendations for buckling curves are given in Table 6.4.

Table 6.4: Recommended values for lateral torsional buckling curves for cross-sections using equation (6.56)

Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$ $h/b > 2$	**a** **b**
Welded I-sections	$h/b \leq 2$ $h/b > 2$	**c** **d**
Other cross-sections	-	**d**

(3) Values of the reduction factor χ_{LT} for the appropriate non-dimensional slenderness $\bar{\lambda}_{LT}$ may be obtained from Figure 6.4.

(4) For slendernesses $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,0}$ (see 6.3.2.3) or for $\frac{M_{Ed}}{M_{cr}} \leq \bar{\lambda}_{LT,0}^{\ 2}$ (see 6.3.2.3) lateral torsional buckling effects may be ignored and only cross sectional checks apply.

6.3.2.3 Lateral torsional buckling curves for rolled sections or equivalent welded sections

(1) For rolled or equivalent welded sections in bending the values of χ_{LT} for the appropriate non-dimensional slenderness may be determined from

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} \quad \text{but} \quad \begin{cases} \chi_{LT} \leq 1{,}0 \\ \chi_{LT} \leq \dfrac{1}{\bar{\lambda}_{LT}^2} \end{cases} \tag{6.57}$$

$$\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta\bar{\lambda}_{LT}^2\right]$$

NOTE The parameters $\bar{\lambda}_{LT,0}$ and β and any limitation of validity concerning the beam depth or h/b ratio may be given in the National Annex. The following values are recommended for rolled sections or equivalent welded sections:

$\bar{\lambda}_{LT,0} = 0{,}4$ (maximum value)

$\beta = 0{,}75$ (minimum value)

The recommendations for buckling curves are given in Table 6.5.

Table 6.5: Recommendation for the selection of lateral torsional buckling curve for cross sections using equation (6.57)

Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$ $h/b > 2$	**b** **c**
Welded I-sections	$h/b \leq 2$ $h/b > 2$	**c** **d**

(2) For taking into account the moment distribution between the lateral restraints of members the reduction factor χ_{LT} may be modified as follows:

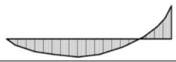
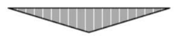
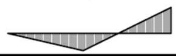
$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} \quad \text{but} \quad \chi_{LT,mod} \leq 1 \tag{6.58}$$

NOTE The values f may be defined in the National Annex. The following minimum values are recommended:

$f = 1 - 0{,}5(1 - k_c)[1 - 2{,}0(\bar{\lambda}_{LT} - 0{,}8)^2]$ but $f \leq 1{,}0$

k_c is a correction factor according to Table 6.6

Table 6.6: Correction factors k_c

Moment distribution	k_c
 $\psi = 1$	1,0
 $-1 \leq \psi \leq 1$	$\frac{1}{1,33 - 0,33\psi}$
	0,94
	0,90
	0,91
	0,86
	0,77
	0,82

10. ELEMENTS SUBJECTED TO BENDING AND AXIAL FORCE. BEAM-COLUMNS

Structural members subjected to axial force and bending are known as beam-columns. In practice, most members in framed structures are beam-columns (Figure 51). In principle, all members in frame structures are actually beam-columns, with the particular cases of beams (N = 0) and columns (M = 0) simply being the two extremes. The behavior and design of beam-columns are presented within the context of members subjected to uniaxial bending, whose response is such that deformation takes place only in the plane of the applied moments.
In the case of beam-columns which are susceptible to lateral-torsional buckling, the out-of-plane flexural buckling of the "column" has to be combined with the lateral-torsional buckling of the "beam" using the relevant interaction formulae.
For beam-columns with biaxial bending, the interaction formula is expanded by the addition of an additional term.

Figure 51. View of a steel frame structure, with cruciform columns and I shape beams

10.1. Types of loadings

1. Tension and bending
    a) This is a particular case (e.g. columns in structures loaded predominantly to lateral loads – earthquake, wind, …) – see Figure 52.a

2. Compression and bending
    b) **Eccentrically compression** – see Figure 52.b
    c) **Compression and uniaxial bending** – see Figure 52.c
    d) **Compression and biaxial bending** – see Figure 52.d

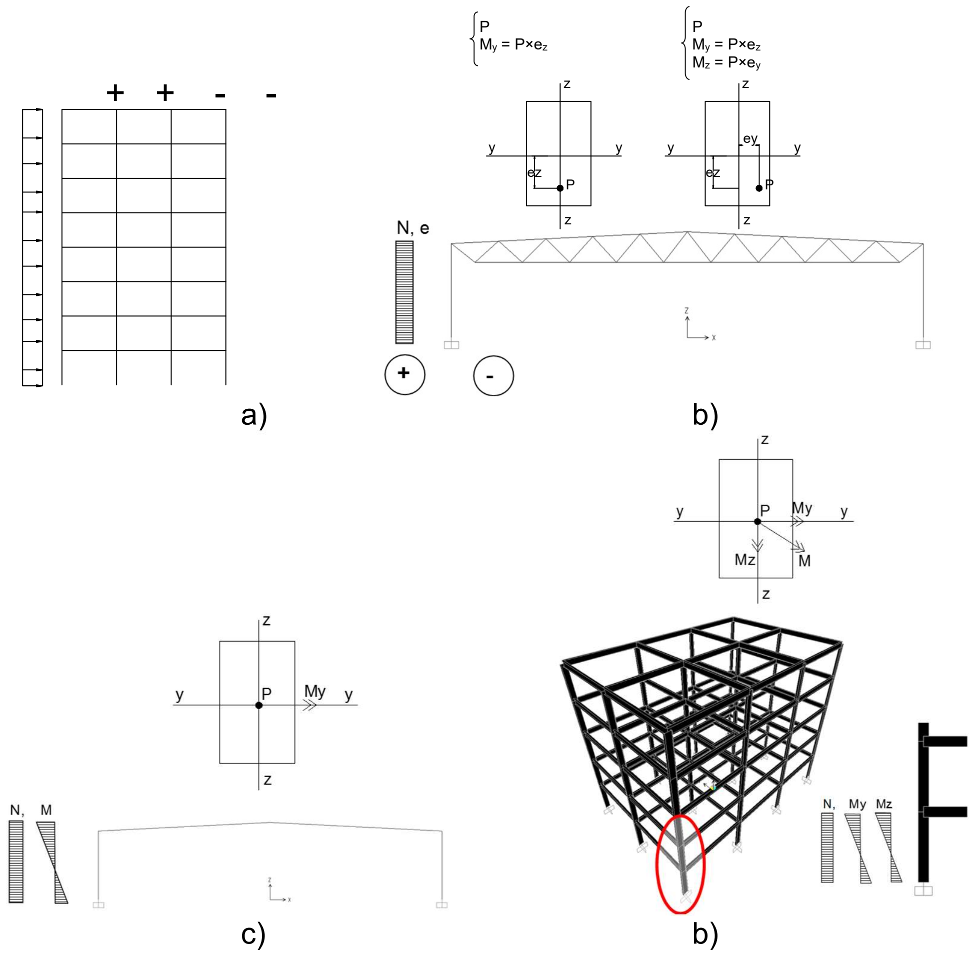
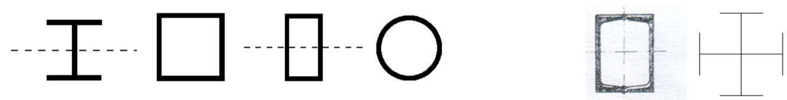


Figure 52. Types of loadings

Main issues:

- Strength of beam-columns
- Stability of beam-columns: buckling may occur in bending (flexural buckling) or in torsion and out of plane bending (lateral-torsional), depending on:
    - ratio of the two types of loadings (bending moment – axial force) transferred to the member
    - member's cross-sectional shape
    - the form of support provided
- The interaction of normal force and bending moment may be treated elastically or plastically using equilibrium for the classification of cross-section.

There are several options for selecting the types of cross sections for beam-column members:



Hot rolled profiles, simple sections, hollow sections

Compound sections

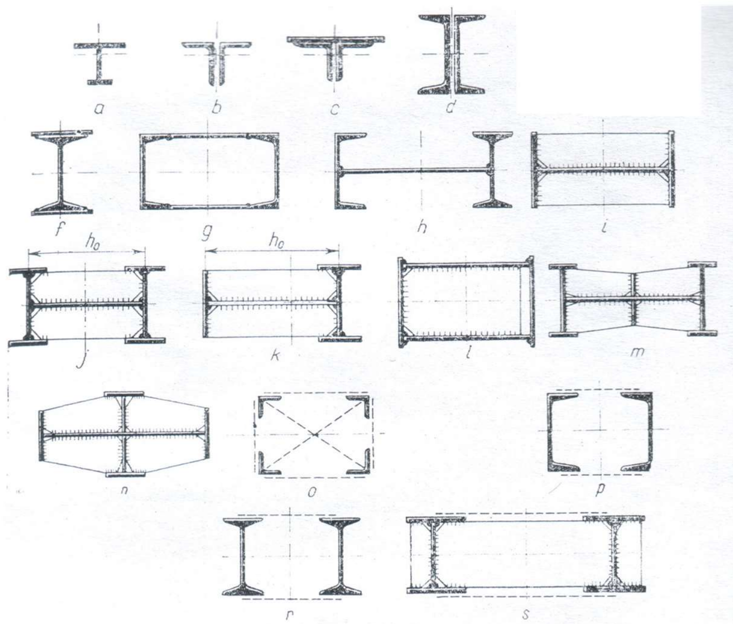
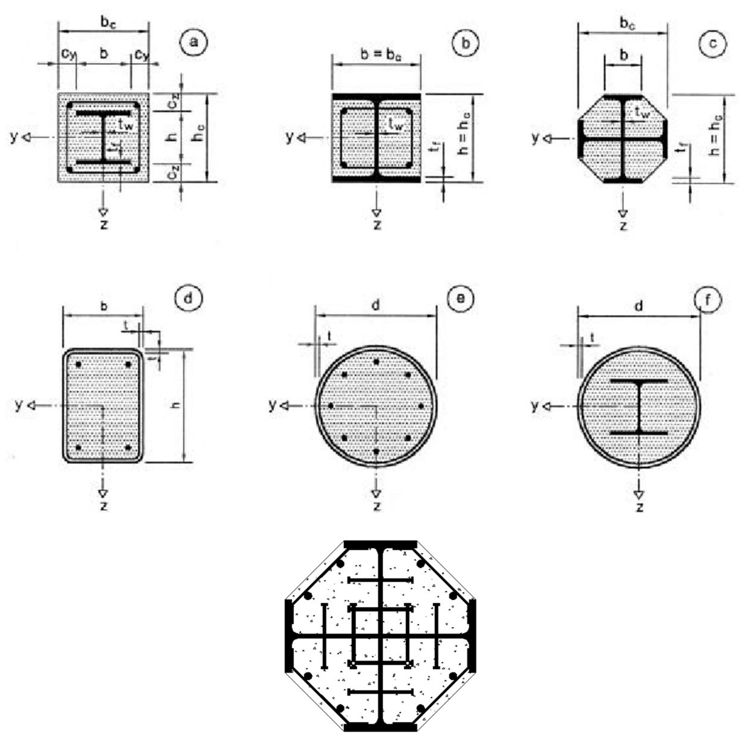


Plate sections: simple sections and box sections



Composite sections (partially or totally encased in concrete)

Figure 53. Cross sections for beam-columns

Consider a plastic (compact) H section column. The behaviour depends on the:

- Column length
- How the moments are applied
- Lateral support provided (if any)

The behaviour can be classified in 5 classes (see Figure 54):

Case 1: Short column subjected to axial load and uniaxial bending about either axis or biaxial bending.
→ Failure – generally occurs when the plastic capacity of the section is reached.

Case 2: A slender column subjected to axial load and uniaxial bending about the major axis YY.
→ Failure: If the column is supported laterally against buckling about minor axis ZZ out of the plane of bending, the column fails by buckling about the YY axis. At low axial force or if the column is not very slender a plastic hinge forms at the end or point of maximum moment

Case 3: A slender column subjected to axial load and uniaxial bending about the minor axis ZZ. The column does not require lateral support and there is no buckling out of the plane of bending.
→ Failure – buckling about the ZZ axis. At very low axial loads it will reach the bending capacity for the ZZ axis

Case 4: A slender column subjected to axial load and uniaxial bending about the major axis YY. The column has no lateral support.
→ Failure – The column fails due to a combination of column buckling about the ZZ axis and lateral torsional buckling where the column section twists as well as deflecting in the YY and ZZ planes.

Case 5: A slender column subjected to axial load and biaxial bending. The column has no lateral support
→ Failure – similar to case 4 but minor axis buckling will usually have the greatest effect. This is the general loading case

Slender columns subjected to axial load and moment

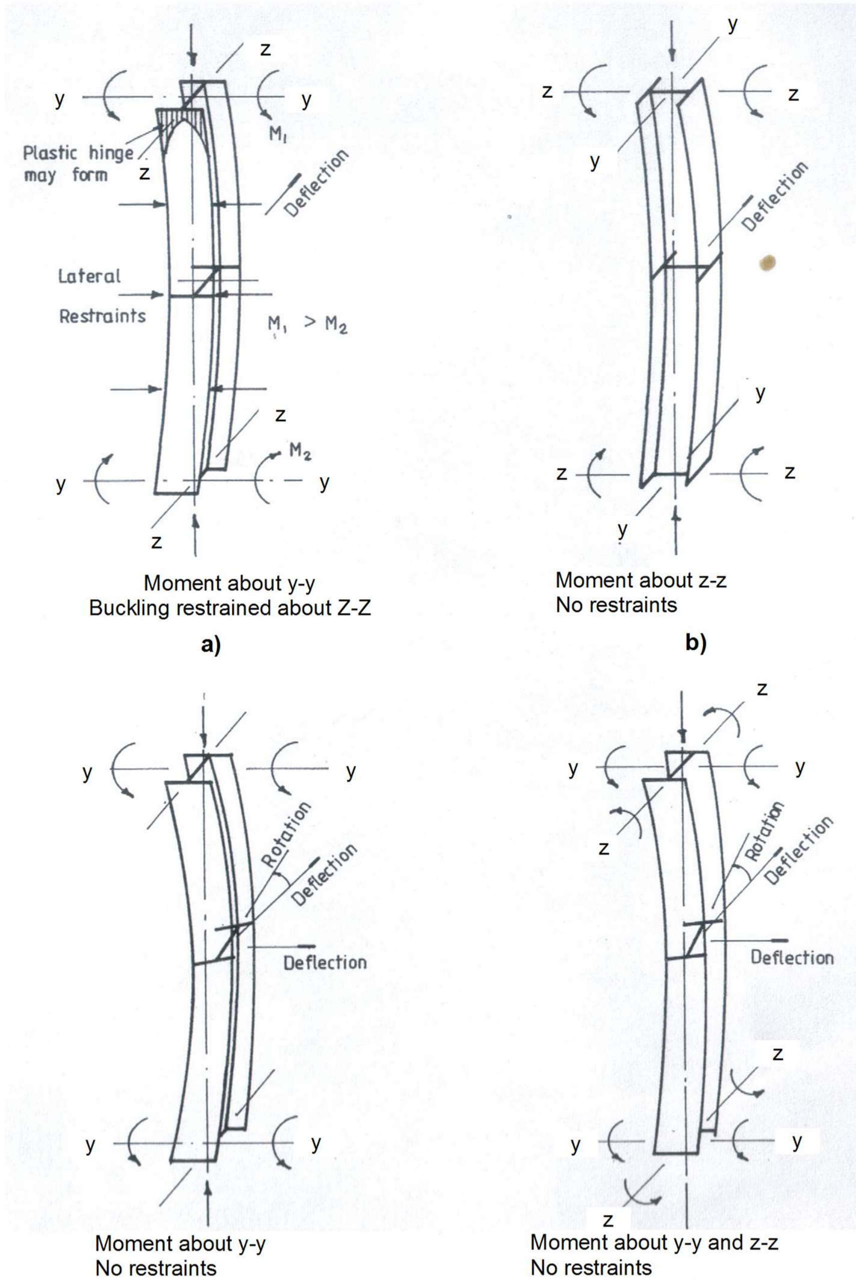


Figure 54. Slender columns subjected to axial load and moment

10.2. Cross-sectional behavior

In absence of buckling, if full plasticity is allowed to occur, then the failure condition will be as shown in Figure 55 and the combination of axial load and moment giving this condition will be:

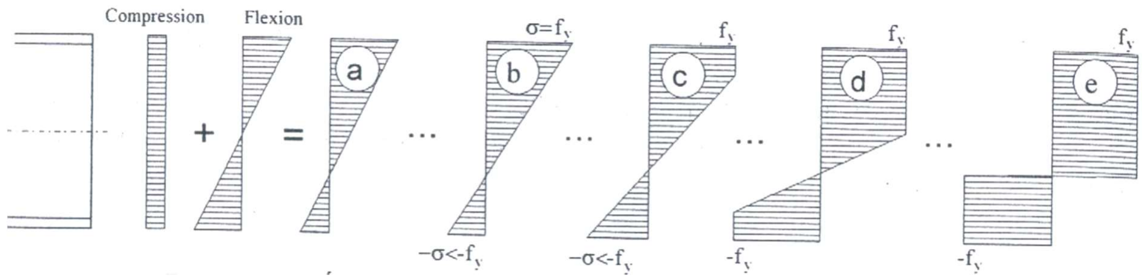


Figure 55. Evolution of combined stresses from axial load and moment

Class 1 and 2 cross-sections

- Resistance of a class 1 or 2 cross-section may be evaluated by comparing the design moment M_{Ed} with the design plastic moment reduced in the presence of the axial force, denoted as $M_{N,Rd}$.
- If full plasticity is allowed to occur, then the failure condition will be as shown in figure:

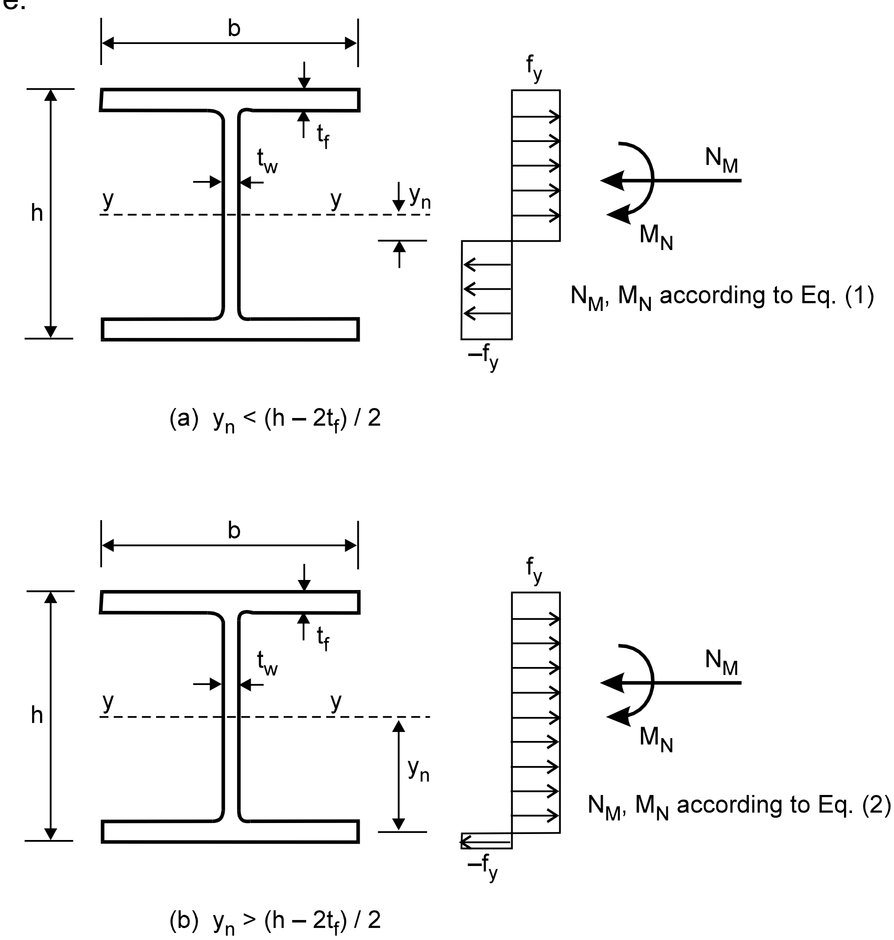


Figure 56. Full plasticity under axial load and moment M-N

- Neutral axis in web:

$$N_M = 2 f_y t_w y_n \quad M_N = f_y b t_f (h - t_f) + f_y \left[\left(\frac{h - 2t_f}{2} \right)^2 - y_n^2 \right] t_w \tag{94}$$

- Neutral axis in flange:

$$N_M = f_y \left[t_w (h - 2t_f) + 2b \left(t_f - \frac{h}{2} + y_n \right) \right] \quad M_N = f_y b \left(\frac{h}{2} - y_n \right) (h - y_n) t_f \tag{95}$$

Approximation used in EC3:

$$M_{y.Ed} \leq M_{Ny.Rd} = M_{pl.y.Rd} (1 - n) / (1 - 0,5a) \tag{96}$$

but

$M_{Ny.Rd} \leq M_{pl.y.Rd}$

where:

$n = N_{Ed} / N_{pl.Rd}$; $a = (A - 2bt_f) / A \leq 0,5$

In Figure 57, the M-N interaction curves for HEB450 section are presented (Eqs. (94) and (95) are compared with the approximation used in EC3 -Eq. (**96**)):

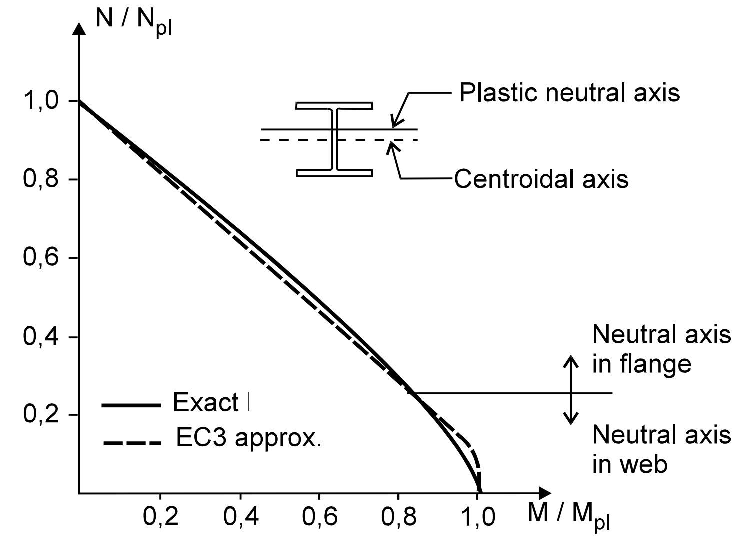
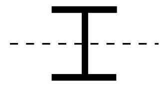
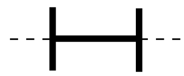
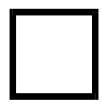
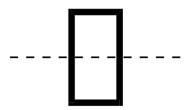
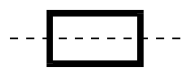
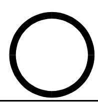


Figure 57. M-N interaction, major axis y-y, section HEB450

Table 11: Expressions for reduced plastic moment resistance M-N for different cross-sections
Notation: $n = N_{Ed} / N_{pl.Rd}$

Cross-section	Shape	Expression for M_N
Rolled I or H		$M_{N,y} = 1{,}11 M_{pl.y}(1-n)$
		$M_{N,z} = 1{,}56 M_{pl.z}(1-n)(0{,}6+n)$
Square hollow section		$M_{N,y} = 1{,}26 M_{pl}(1-n)$
Rectangular hollow section		$M_{N,y} = 1{,}33 M_{pl.y}(1-n)$
		$M_{N,y} = M_{pl.z} \dfrac{1-n}{0{,}5+\dfrac{ht}{A}}$
Circular hollow section		$M_{N,y} = 1{,}04 M_{pl}(1-n^{1,7})$

ng and axial force for Class 3 cross-sections:

Figure 58 shows a point along the length of an H-shape column where the applied compression and bending moment about the *y* axis produce the uniform and varying stress distribution:

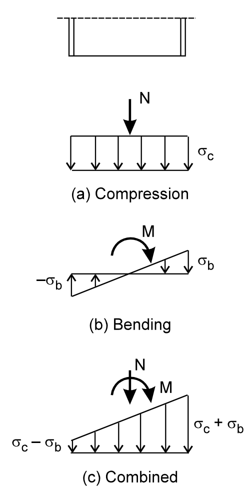


Figure 58. Elastic behaviour of a class 3 cross-section under compression and bending

- Yielding will develop at the edge where the maximum compressive bending stress occurs and will correspond to the condition:

$$f_y = \sigma_c + \sigma_b \qquad (97)$$

where:

- f_y is the material yield stress
- $\sigma_c = N / A$ is the stress due to the compressive load N
- $\sigma_b = \dfrac{Mh/2}{I}$ is the maximum compressive stress due to the moment M, h is the overall depth of section, and I is the second moment of area about the y axis.

In the absence of shear force, resistance of a class 3 cross-section will be satisfactory if the maximum longitudinal stress $\sigma_{x.Ed}$ satisfies the criterion:

$$\sigma_{x,Ed} \le f_y / \gamma_{M0} \qquad (98)$$

Previous equation may be rewritten as follows:

$$\frac{N_{Ed}}{Af_y/\gamma_{M0}} + \frac{M_{y.Ed}}{W_{el,y}f_y/\gamma_{M0}} < 1 \qquad (99)$$

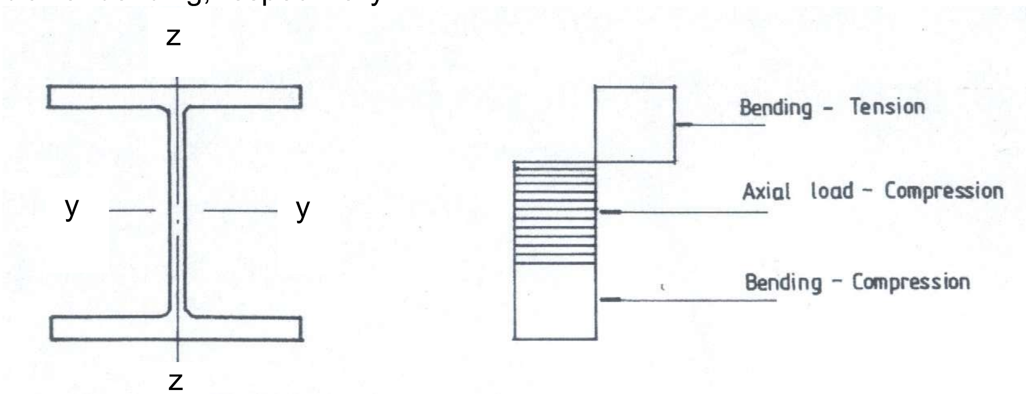
<u>Class 4 cross-sections</u>
In the absence of shear force, resistance of a class 4 cross-section will be satisfactory if the maximum longitudinal stress $\sigma_{x.Ed}$ (calculated using the effective widths of the compression elements) satisfies the criterion:

$$\sigma_{x,Ed} \le f_y / \gamma_{M0} \qquad (100)$$

<u>Short column failure</u>

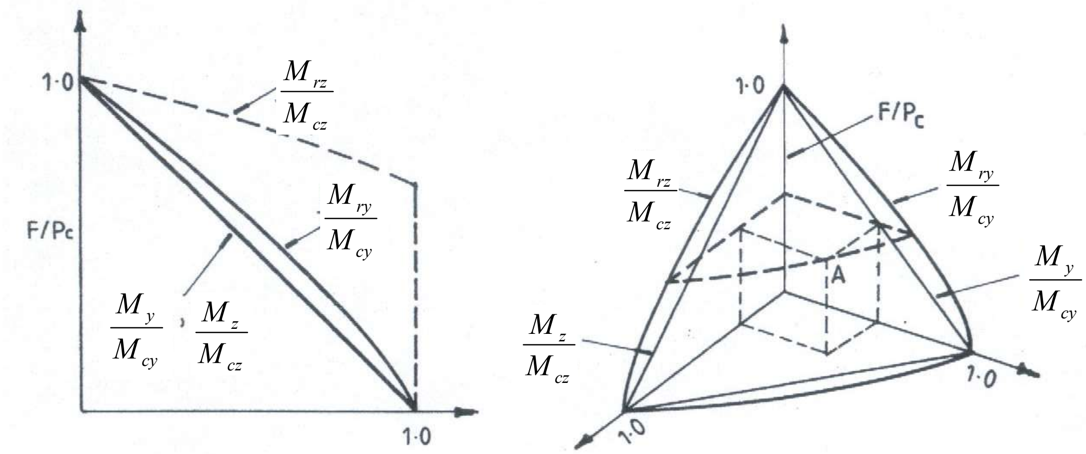
- Consider a plastic H section column (see Figure 59.a).
- The plastic stress distribution for uniaxial bending is shown in Figure 59.b.

Figure 60 (a-b) show the interaction curves and interaction surfaces for uniaxial and biaxial bending, respectivelly.



a) H cross-section column b) Plastic stress distribution Bending about y-y

Figure 59. Plastic stress distribution in a H cross-section short column



Interaction curves for bending about y-y and z-z

Interaction surface
Biaxial bending
Full plasticity

Note:
My (Mz): applied moments on Y (Z) axis
Mcy (Mcz): moment capacity about Z (Z) axis in the absence of axial load
Mry (Mrz): reduced moment capacity about Z (Z) axis in the presence of axial load

Figure 60. Interaction curves

10.3. Design of members for bending and axial force (EN1993-1-1)

Notes:

- Material reproduced from EN1993-1-1.
- For ease of understanding, numbering is done according to EN1993-1-1.

6.2.9 Bending and axial force

6.2.9.1 Class 1 and 2 cross-sections

(1) Where an axial force is present, allowance should be made for its effect on the plastic moment resistance.

(2) For class 1 and 2 cross sections, the following criterion should be satisfied:

$$M_{Ed} \leq M_{N,Rd} \qquad (6.31)$$

where $M_{N,Rd}$ is the design plastic moment resistance reduced due to the axial force N_{Ed}.

(3) For a rectangular solid section without fastener holes $M_{N,Rd}$ should be taken as:

$$M_{N,Rd} = M_{pl,Rd}\left[1-\left(N_{Ed}/N_{pl,Rd}\right)^2\right] \qquad (6.32)$$

(4) For doubly symmetrical I- and H-sections or other flanges sections, allowance need not be made for the effect of the axial force on the plastic resistance moment about the y-y axis when both the following criteria are satisfied:

$$N_{Ed} \leq 0{,}25\,N_{pl,Rd} \text{ and} \qquad (6.33)$$

$$N_{Ed} \leq \frac{0{,}5\,h_w\,t_w\,f_y}{\gamma_{M0}} \qquad (6.34)$$

For doubly symmetrical I- and H-sections, allowance need not be made for the effect of the axial force on the plastic resistance moment about the z-z axis when:

$$N_{Ed} \leq \frac{h_w t_w f_y}{\gamma_{M0}} \qquad (6.35)$$

(5) For cross-sections where fastener holes are not to be accounted for, the following approximations may be used for standard rolled I or H sections and for welded I or H sections with equal flanges:

$$M_{N,y,Rd} = M_{pl,y,Rd}(1\text{-}n)/(1\text{-}0{,}5a) \quad \text{but } M_{N,y,Rd} \leq M_{pl,y,Rd} \qquad (6.36)$$

$$\text{for } n \leq a: \quad M_{N,z,Rd} = M_{pl,z,Rd} \qquad (6.37)$$

$$\text{for } n > a: \quad M_{N,z,Rd} = M_{pl,z,Rd}\left[1 - \left(\frac{n-a}{1-a}\right)^2\right] \qquad (6.38)$$

where $n = N_{Ed} / N_{pl.Rd}$

$a = (A\text{-}2bt_f)/A$ but $a \leq 0{,}5$

For cross-sections where fastener holes are not to be accounted for, the following approximations may be used for rectangular structural hollow sections of uniform thickness and for welded box sections with equal flanges and equal webs:

$$M_{N,y,Rd} = M_{pl,y,Rd}(1 - n)/(1 - 0{,}5a_w) \quad \text{but } M_{N,y,Rd} \leq M_{pl,y,Rd} \qquad (6.39)$$

$$M_{N,z,Rd} = M_{pl,z,Rd}(1 - n)/(1 - 0{,}5a_f) \quad \text{but } M_{N,z,Rd} \leq M_{pl,z,Rd} \qquad (6.40)$$

where $a_w = (A - 2bt)/A$ but $a_w \leq 0{,}5$ for hollow sections

$a_w = (A\text{-}2bt_f)/A$ but $a_w \leq 0{,}5$ for welded box sections

$a_f = (A - 2ht)/A$ but $a_f \leq 0{,}5$ for hollow sections

$a_f = (A\text{-}2ht_w)/A$ but $a_f \leq 0{,}5$ for welded box sections

(6) For bi-axial bending the following criterion may be used:

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} \leq 1 \qquad (6.41)$$

in which α and β are constants, which may conservatively be taken as unity, otherwise as follows:

- I and H sections:

  $\alpha = 2$; $\beta = 5n$ but $\beta \geq 1$

- circular hollow sections:

  $\alpha = 2$; $\beta = 2$

- rectangular hollow sections:

$$\alpha = \beta = \frac{1{,}66}{1 - 1{,}13\, n^2} \qquad \text{but } \alpha = \beta \leq 6$$

where $n = N_{Ed} / N_{pl,Rd}$.

6.2.9.2 Class 3 cross-sections

(1) In the absence of shear force, for Class 3 cross-sections the maximum longitudinal stress should satisfy the criterion:

$$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}} \tag{6.42}$$

where $\sigma_{x,Ed}$ is the design value of the local longitudinal stress due to moment and axial force taking account of fastener holes where relevant, see 6.2.3, 6.2.4 and 6.2.5

6.2.9.3 Class 4 cross-sections

(1) In the absence of shear force, for Class 4 cross-sections the maximum longitudinal stress $\sigma_{x,Ed}$ calculated using the effective cross sections (see 5.5.2(2)) should satisfy the criterion:

$$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}} \tag{6.43}$$

where $\sigma_{x,Ed}$ is the design value of the local longitudinal stress due to moment and axial force taking account of fastener holes where relevant, see 6.2.3, 6.2.4 and 6.2.5

(2) The following criterion should be met:

$$\frac{N_{Ed}}{A_{eff}\, f_y / \gamma_{M0}} + \frac{M_{y,Ed} + N_{Ed}\, e_{Ny}}{W_{eff,y,min}\, f_y / \gamma_{M0}} + \frac{M_{z,Ed} + N_{Ed}\, e_{Nz}}{W_{eff,z,min}\, f_y / \gamma_{M0}} \leq 1 \tag{6.44}$$

where A_{eff} is the effective area of the cross-section when subjected to uniform compression

$W_{eff,min}$ is the effective section modulus (corresponding to the fibre with the maximum elastic stress) of the cross-section when subjected only to moment about the relevant axis

e_N is the shift of the relevant centroidal axis when the cross-section is subjected to compression only, see 6.2.2.5(4)

NOTE The signs of N_{Ed} , $M_{y,Ed}$, $M_{z,Ed}$ and $\Delta M_i = N_{Ed}\, e_{Ni}$ depend on the combination of the respective direct stresses.

6.2.10 Bending, shear and axial force

(1) Where shear and axial force are present, allowance should be made for the effect of both shear force and axial force on the resistance moment.

(2) Provided that the design value of the shear force V_{Ed} does not exceed 50% of the design plastic shear resistance $V_{pl.Rd}$ no reduction of the resistances defined for bending and axial force in 6.2.9 need be made, except where shear buckling reduces the section resistance, see EN 1993-1-5.

(3) Where V_{Ed} exceeds 50% of $V_{pl.Rd}$ the design resistance of the cross-section to combinations of moment and axial force should be calculated using a reduced yield strength

$(1-\rho)f_y$ (6.45)

for the shear area

where $\rho = (2V_{Ed} / V_{pl.Rd} - 1)^2$ and $V_{pl,Rd}$ is obtained from 6.2.6(2).

NOTE Instead of reducing the yield strength also the plate thickness of the relevant part of the cross section may be reduced.

10.4. Overall stability of members under compression and bending

Members under axial compression and bending develop specific load-carrying behaviour in the different ranges of slenderness.
At very low slenderness, the cross-sectional resistance dominates, and this is described by the well-known interaction formulae for elastic or plastic limit states (see sections 10.2 and 10.3).
With increasing slenderness, a pronounced second-order effect appears, which is significantly influenced by both **geometrical imperfections and residual stresses**.

In the high slenderness range, member buckling is dominated by elastic behaviour; the larger the slenderness the greater the dominance.

In the present approach of EN 1993-1-1, the effects of the axial force and the bending moments are linearly summed, and the non-linear effects are accounted for by specific **interaction factors**.

Two different formats of the interaction formulae are provided in EN 1993-1-1, called Method 1 and Method 2.
The main difference between them is the kind of presentation of the different structural effects, either by specific coefficients (Method 1) or by one compact interaction factor (Method 2).

The design concept differentiates between the two cases of buckling behaviour of members:

- Members susceptible to torsional deformations (fail in lateral-torsional buckling, such as slender I-sections or similar open sections)
- Members not susceptible to torsional deformations (fail in flexural buckling, by in-plane or spatial deflection – e.g. closed sections RHS, or open sections appropriately restrained against torsional deformations)

10.4.1. Elastic flexural buckling

10.4.1.1. Member under $N_{Ed} + M_{Ed}$

The stability of a member with an initial imperfection $v_o(x)$ subject to axial compression N_{ed} can be expressed by the eq.:

$$\frac{N_{Ed}}{N_{Rd}} + \frac{1}{1 - N_{Ed}/N_{cr}} \frac{N_{Ed} e_{0,d}}{M_{Rd}} \leq 1 \quad (101)$$

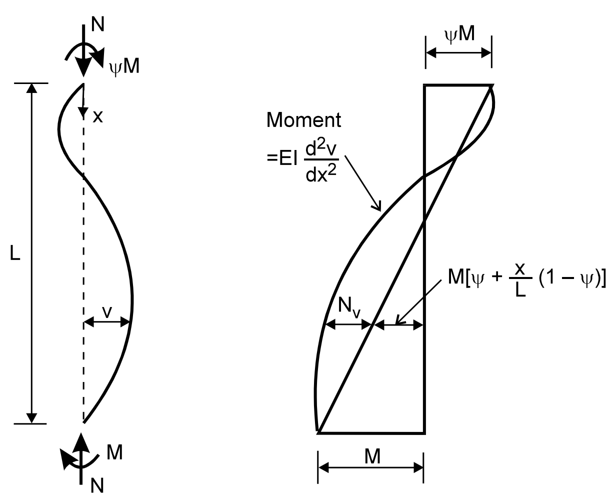
where $e_{0,d}$ is denoted as the equivalent geometrical imperfection and is given by:

$$\boldsymbol{e_{0,d}} = \frac{(\mathbf{1}-\chi)\left(\mathbf{1}-\chi\bar{\lambda}^2\right)}{\chi}\frac{\boldsymbol{M_{el,Rd}}}{\boldsymbol{N_{pl,Rd}}} \tag{102}$$

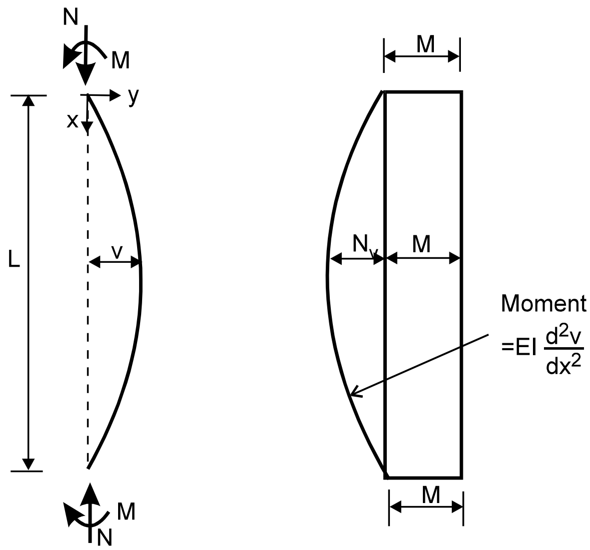
When the member is subject to additional first order moments M_{Ed}, the format of Eq. (101) can be extended as follows:

$$\frac{\boldsymbol{N_{Ed}}}{\boldsymbol{N_{Rd}}} + \frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{N_{Ed}}/\boldsymbol{N_{cr}}}\frac{\boldsymbol{N_{Ed}e_{0,d}}}{\boldsymbol{M_{Rd}}} + \frac{\boldsymbol{M^{II}}_{\boldsymbol{Ed,max}}}{\boldsymbol{M_{Rd}}} \leq \mathbf{1} \tag{103}$$

where $M^{II}_{Ed,max}$ represents the second-order maximum bending moment induced by the additional first order bending moment. Indeed, because Eq. (103) represents a second-order cross-sectional check of the most heavily loaded section, it becomes necessary to determine its location, in order to evaluate M^{II}_{Edmax}. When a first order bending moment M_{Ed} exists, an additional lever arm to the axial force N_{Ed} arises, causing an amplification of the deflection and of the bending moment, in the same way as for the initial imperfection $e_{0,d}$.
In order to avoid the determination of the location of the cross-section most heavily loaded by second-order effects (see Figure 61.a), the equivalent moment concept is used. This consists of replacing the actual first order bending system on the member already subjected to the same axial force by a sinusoidal first order bending moment (the equivalent one, see Figure 61.b), that produces the same amplified bending moment. The latter is usually expressed as C_mM_{Ed}.



a) actual 2nd order bending moment



b) equivalent sinusoidal first order moment

Figure 61. A beam-column undergoing lateral deflection due to compression and bending

The second-order maximum bending moment $M^{II}_{Ed,max}$ can then expressed as:

$$M^{II}{}_{Ed,\max} = \frac{C_m M_{Ed,\max}}{1 - N_{Ed} / N_{cr}} \tag{104}$$

using the same amplification factor as for buckling (see eq. 105).

$$\boldsymbol{K = \frac{1}{1 - N_{Ed}/N_{cr}}} \tag{105}$$

Then, an elastic second-order check of the most loaded cross-section on the member can be expressed as:

$$\boldsymbol{\frac{N_{Ed}}{N_{Rd}} + \frac{1}{1 - N_{Ed}/N_{cr}} \frac{N_{Ed} e_{0,d}}{M_{Rd}} \frac{1}{1 - N_{Ed}/N_{cr}} \frac{C_m M_{Ed}}{M_{Rd}} \leq 1} \tag{106}$$

or

$$\boldsymbol{\frac{N_{Ed}}{\chi N_{Rd}} + \mu \frac{1}{1 - N_{Ed}/N_{cr}} \frac{C_m M_{Ed}}{M_{Rd}} \leq 1} \tag{107}$$

with:

$$\boldsymbol{\mu = \frac{1 - N_{Ed}/N_{cr}}{1 - \chi N_{Ed}/N_{cr}} \leq 1} \tag{108}$$

10.4.1.2. Member under N_{Ed} + $M_{y,Ed}$ + $M_{z,Ed}$

When biaxial bending occurs, the axial force amplifies the moments about both of principal axes y - y and z - z of the cross-section. This results in a complex coupling instability in both principal planes. However, this coupling is generally disregarded in codes for practical reasons, and because unsafe cases are very rare. At this point, the proposed formulae diverge slightly from a strict theoretical approach.

Accordingly, biaxial bending is accounted for using a doublet of formulae, obtained by adding a second bending term in an Eq. (107) to give:

$$\frac{N_{Ed}}{\chi_y N_{Rd}} + \mu_y \left[\frac{C_{my} M_{y,Ed}}{\left(1 - N_{Ed}/N_{cr,y}\right) M_{el,y,Rd}} + \frac{C_{mz} M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,z}\right) M_{el,z,Rd}} \right] \leq 1 \tag{109}$$

$$\frac{N_{Ed}}{\chi_z N_{Rd}} + \mu_z \left[\frac{C_{my} M_{y,Ed}}{\left(1 - N_{Ed}/N_{cr,y}\right) M_{el,y,Rd}} + \frac{C_{mz} M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,z}\right) M_{el,z,Rd}} \right] \leq 1 \tag{110}$$

with:

$$\mu_y = \frac{1 - N_{Ed}/N_{cr,y}}{1 - \chi_y N_{Ed}/N_{cr,y}} \leq 1 \tag{111}$$

$$\mu_z = \frac{1 - N_{Ed}/N_{cr,z}}{1 - \chi_z N_{Ed}/N_{cr,z}} \leq 1 \tag{112}$$

Figure 62. Failure of a slender column

10.4.2. Elastic-plastic flexural buckling without lateral torsional buckling

10.4.2.1. General

In principle, steel members show linear behaviour in the elastic range and non-linear behaviour in the plastic range. The higher the slenderness, the lower the capacity and the less pronounced is the plastic behaviour.
This ideal material behaviour is significantly affected by the presence of residual stresses in rolled and welded sections, which result in non-linearities even at low load-levels. In this respect a large range of members behave inelastically in principle. Presently, the EN 1993-1-1 differentiates between the interaction formulae for "elastic-plastic" (Class 1 and 2 sections) and "elastic" (Class 3 and 4 sections). For the latter, plastic capacity is not, or is only partly, taken into account. Accordingly, the interaction formulae for Class 3 and 4 sections follow the analytically derived equations for flexural buckling.

10.4.2.2. Member under N_{Ed} + M_{Ed}

To allow yielding along the member, the general format of Eq. (107) is kept, but with C × $M_{pl,Rd}$ replacing $M_{el,Rd}$:

$$\frac{N_{Ed}}{\chi N_{pl,Rd}} + \mu \frac{1}{1 - N_{Ed}/N_{cr}} \frac{C_m M_{Ed}}{C M_{pl,Rd}} \leq 1 \tag{113}$$

Indeed, because of instability effects, the full plastic bending resistance $M_{pl,Rd}$ may not reached but only an intermediate elastic-plastic value given by C×$M_{pl,Rd}$. Then, factor C to take account of the plasticity effects in the interaction between mono-axial bending axial force along the member.

This C factor must obviously depend on the axial force, member slenderness and moment distribution, because all these factors play a role on the extent of yielding.

C should however tend to unity when the axial force is small enough for pure bending to the member resistance.

10.4.2.3. Member under N_{Ed} + $M_{y,Ed}$+ $M_{z,Ed}$

When a member is subjected to both biaxial bending and axial compression, the general format given in section 10.4.1.2, based on elastic second-order theory, is only valid for Class 3 cross sections. When sections of Class 1 or 2 are of concern, the inelastic effects on the member resistance can be accounted for by means of the following expressions:

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + \mu_y \left[\frac{C_{my} M_{y,Ed}}{\left(1 - N_{Ed}/N_{cr,y}\right) C_{yy} M_{pl,y,Rd}} + \alpha * \frac{C_{mz} M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,z}\right) C_{yz} M_{pl,z,Rd}} \right] \leq 1 \tag{114}$$

$$\frac{N_{Ed}}{\chi_z N_{pl,Rd}} + \mu_z \left[\beta * \frac{C_{my} M_{y,Ed}}{\left(1 - N_{Ed}/N_{cr,y}\right) C_{zy} M_{pl,y,Rd}} + \frac{C_{mz} M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,z}\right) C_{zz} M_{pl,z,Rd}} \right] \leq 1 \tag{115}$$

Eqs. (114) and (115) are then able to deal with inelastic effects for N - M elastic-plastic member interaction. It is however necessary to introduce indices in these C factors, C_{yy} and C_{zz} dealing with the N - M plasticity effects when the bending plane is the same as the buckling plane, while the factors C_{yz} and C_{zy} concern the plane perpendicular to the plane of buckling.

General expressions for C_{ii} and C_{ij} factors must necessarily be different, because of the smaller effects of plasticity on the M - N interaction when the plane of bending and the plane considered for buckling are not coincident.

10.4.2.4. Design formulae (Method 2)

The format of the interaction formula is based on the interaction factor k_y. the dependencies on the physical parameters influencing the interaction factor can be adopted from the theoretical approach. They are:

- The relative slenderness $\bar{\lambda}$;
- The value $\boldsymbol{n} = N_{Ed}/\boldsymbol{\chi} N_{pl,Rd}$;
- The moment diagram;
- The cross-section shape.

10.4.2.5. Members with Class 1 and 2 cross-sections

10.4.2.5.1.1. Axial Compression and strong-axis bending (N + My)

$$\frac{\boldsymbol{N_{Ed}}}{\boldsymbol{\chi_y N_{pl,Rd}}} + \boldsymbol{k_y} \frac{\boldsymbol{C_{my} M_{y,Ed}}}{\boldsymbol{M_{pl,y,Rd}}} \leq \mathbf{1} \qquad (116)$$

$$\frac{\boldsymbol{N_{Ed}}}{\boldsymbol{\chi_z N_{pl,Rd}}} + \mathbf{0.6}\boldsymbol{k_y} \frac{\boldsymbol{C_{my} M_{y,Ed}}}{\boldsymbol{M_{pl,y,Rd}}} \leq \mathbf{1} \qquad (117)$$

where:

χ_y, χ_z - is the reduction factor for column buckling,

k_y - a modification factor

$$k_y = 1 + \left(\bar{\lambda}_y - 0.2\right) n_y \leq 1 + 0.8 n_y$$

$$n_y = \frac{N}{\chi_y N_{pl}}$$

and

$$C_{my} = 0.6 + 0.4\psi \geq 0.4$$

10.4.2.5.1.2. Axial Compression and weak-axis bending (N + M_z)

$$\frac{\boldsymbol{N_{Ed}}}{\boldsymbol{\chi_z N_{pl,Rd}}} + \boldsymbol{k_z} \frac{\boldsymbol{C_{mz} M_{z,Ed}}}{\boldsymbol{M_{pl,z,Rd}}} \leq \mathbf{1} \qquad (118)$$

where:

$k_z = 1 + \left(2\bar{\lambda}_z - 0.6\right) n_z \leq 1 + 1.4 n_z$ I sections

$k_z = 1 + \left(\bar{\lambda}_z - 0.2\right) n_z \leq 1 + 0.8 n_z$ RHS sections

$$n_z = \frac{N_{Ed}}{\chi_z N_{pl,Rd}}$$

and

$$C_{mz} = 0.6 + 0.4\psi \geq 0.4$$

10.4.2.5.1.3. Axial Compression and biaxial bending ($N + M_y + M_z$)

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + k_y \frac{C_{my} M_{y,Ed}}{M_{pl,y,Rd}} + 0.6 k_z \frac{C_{mz} M_{z,Ed}}{M_{pl,z,Rd}} \leq 1 \quad (119)$$

$$\frac{N_{Ed}}{\chi_z N_{pl,Rd}} + 0.6 k_y \frac{C_{my} M_{y,Ed}}{M_{pl,y,Rd}} + k_z \frac{C_{mz} M_{z,Ed}}{M_{pl,z,Rd}} \leq 1 \quad (120)$$

10.4.2.6. Members with Class 3 and 4 cross-sections

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + k_y \frac{C_{my} M_{y,Ed}}{M_{el,y,Rd}} + k_z \frac{C_{mz} M_{z,Ed}}{M_{el,z,Rd}} \leq 1 \quad (121)$$

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + 0.8 k_y \frac{C_{my} M_{y,Ed}}{M_{el,y,Rd}} + k_z \frac{C_{mz} M_{z,Ed}}{M_{el,z,Rd}} \leq 1 \quad (122)$$

where:

$k_y = 1 + 0.6\bar{\lambda}_y n_y \leq 1 + 0.6 n_y$

$k_z = 1 + 0.6\bar{\lambda}_z n_z \leq 1 + 0.6 n_z$

- The coefficients $\bar{\lambda}_y$, $\bar{\lambda}$, n_y, n_z, C_{my}, C_{mz} are identical with those in section above.
- For class 4 sections, the section properties $N_{pl,Rd}$ and $M_{el,y,Rd}$, $M_{el,z,Rd}$ need to be replaced by the resistances calculated from the properties of the effective section A_{eff} and W_{eff}.

10.4.3. Elastic-plastic flexural buckling without lateral torsional buckling

10.4.3.1. General

When an unrestrained beam-column is bent about its major axis (see Figure 63), it may buckle by deflecting laterally and twisting at a load which is significantly less than the maximum load predicted by an in-plane analysis. This lateral-torsional buckling may occur while the member is still elastic (see curve 1), or after some yielding takes place (curve 2) due to in-plane bending and compression.

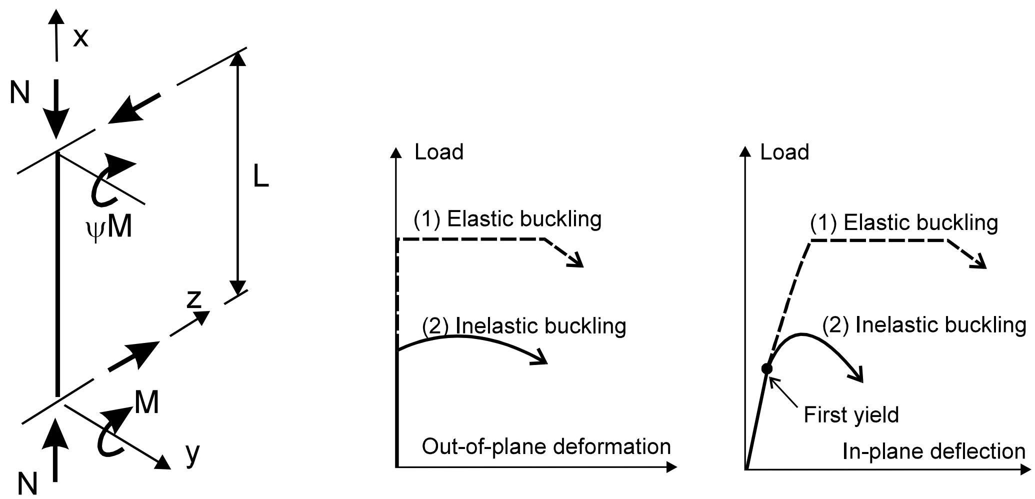


Figure 63. Lateral-torsional behaviour

Let's consider the lateral-torsional behaviour of an unrestrained I section beam-column bent about its major axis. Assuming elastic behaviour and the arrangement of applied loading and support conditions given in next figure, the critical combinations of N and M may be obtained from the solution of (Chen and Atsuta, 1976):

$$\frac{M^2}{i_0^2 N_{cr,z} N_{E,0}} = \left(1 - \frac{N}{N_{cr,z}}\right)\left(1 - \frac{N}{N_{cr,T}}\right) \tag{123}$$

where:

- $i_0 = \sqrt{\frac{I_y + I_z}{A}}$ the polar radius of gyration
- $N_{cr,z} = \frac{\pi^2 E I_z}{L^2}$ the minor axis critical load
- $N_{cr,T} = \frac{G I_t}{i_0^2}\left(1 + \frac{\pi^2 E I_w}{G I_t L^2}\right)$ the torsional buckling load

Previous eq. reduces to the buckling of a beam when N → 0 and to the buckling of a column in either flexure ($N_{cr,z}$) or torsion ($N_{cr,T}$) as M → 0. In the first case the critical value of M will be given by:

$$M_{cr} = \frac{\pi}{L}\sqrt{E I_z G I_t}\sqrt{1 + \frac{\pi^2 E I_w}{L^2 G I_t}}$$

where:

- EI_z is the minor axis flexural rigidity
- GI_t is the torsional rigidity
- EI_w is the warping rigidity.

- If we take into account the influence of axial force compression on the behaviour of the member, this may be approximated as:

$$\frac{M}{1 - N / N_{cr,y}}$$

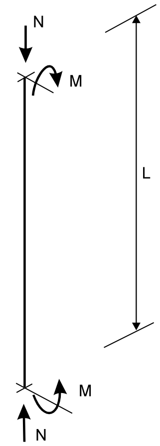


Figure 64. Lateral-torsional behaviour

Eq. (123) may be modified as follows:

$$\frac{M^2}{i_0^2 N_{cr,z} N_{cr,T}} = \left(1 - \frac{N}{N_{cr,y}}\right)\left(1 - \frac{N}{N_{cr,z}}\right)\left(1 - \frac{N}{N_{cr,T}}\right) \tag{124}$$

➔

$$\frac{N}{N_{cr,z}} + \frac{1}{1 - N / N_{cr,y}} \frac{M}{i_0 \sqrt{N_{cr,z} N_{cr,T}}} = 1 \tag{125}$$

or

$$\frac{N}{N_{cr,z}} + \frac{1}{1 - N / N_{cr,y}} \frac{M}{M_{cr}} = 1 \tag{126}$$

Next figure presents a diagrammatic version of the design requirement N – M_y – M_z
The N-M_z and N-M_y axes correspond to the two axial cases already examined.
Interaction between M_z and M_y corresponds to the horizontal plane.
Any point falling within the boundary corresponds to a safe combination of loads.

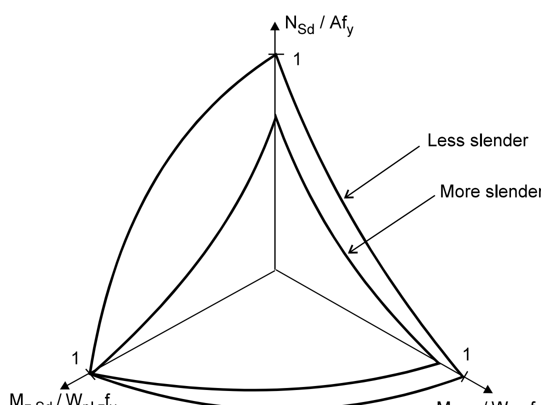


Figure 65. Interaction diagram for biaxial bending N – $M_{y,Ed}$ – $M_{z,Ed}$

10.4.3.2. Design formulae (Method 2)

Following the same approach as described in section 10.4.2, the design formulae of Method 2 were developed as interaction formulae with as few factors as possible. The terms for axial compression and bending moment are added by applying an interaction factor to each.
Thereby, the moment resistance is taken as the reduced LT-buckling resistance. Also, different formulae are given for class 1 and 2 sections and for class 3 and 4 sections.

10.4.3.2.1. Members with Class 1 and 2 cross-sections

Axial Compression and strong-axis bending (N + M_y)

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + k_y \frac{C_{my} M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \leq 1 \quad (127)$$

$$\frac{N_{Ed}}{\chi_z N_{pl,Rd}} + k_{LT} \frac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \leq 1 \quad (128)$$

where:

χ_y, χ_z k_y, n_y, C_{my} – defined in the previous sections.

$$k_{LT} = 1 - \frac{0.1\bar{\lambda}_z n_z}{C_{m,LT} - 0.25} \geq 1 - \frac{0.1 n_z}{C_{m,LT} - 0.25}$$

$$n_z = \frac{N_{Ed}}{\chi_z N_{pl,Rd}}$$

and

$$C_{mLT} = 0.6 + 0.4\psi \geq 0.4$$

Axial compression and biaxial bending (N + M_y + M_z)

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + k_y \frac{C_{my} M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} + 0.6 k_z \frac{C_{mz} M_{z,Ed}}{M_{pl,z,Rd}} \leq 1 \quad (129)$$

$$\frac{N_{Ed}}{\chi_z N_{pl,Rd}} + k_{LT} \frac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} + k_z \frac{C_{mz} M_{z,Ed}}{M_{pl,z,Rd}} \leq 1 \quad (130)$$

10.4.3.2.2. Members with Class 3 and 4 cross-sections

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + k_y \frac{C_{my} M_{y,Ed}}{\chi_{LT} M_{el,y,Rd}} + k_z \frac{C_{mz} M_{z,Ed}}{M_{el,z,Rd}} \leq 1 \quad (131)$$

$$\frac{N_{Ed}}{\chi_z N_{pl,Rd}} + k_{LT}\frac{M_{y,Ed}}{\chi_{LT} M_{el,y,Rd}} + k_z\frac{C_{mz} M_{z,Ed}}{M_{el,z,Rd}} \leq 1 \qquad (132)$$

$$k_{LT} = 1 - \frac{0.05\bar{\lambda}_z n_z}{C_{m,LT} - 0.25} \geq 1 - \frac{0.05 n_z}{C_{m,LT} - 0.25}$$

- The coefficients $\bar{\lambda}_z$, n_z, C_{my}, k_y, k_z are identical with those in section above.
- For class 4 sections, the section properties $N_{pl,Rd}$ and $M_{el,y,Rd}$, $M_{el,z,Rd}$ need to be replaced by the resistances calculated from the properties of the effective section A_{eff} and W_{eff}.

10.5. Buckling resistance of uniform members in bending and axial compression

Notes:

- Material reproduced from EN1993-1-1.
- For ease of understanding, numbering is done according to EN1993-1-1.

(1) Unless second order analysis is carried out using the imperfections as given in 5.3.2, the stability of uniform members with double symmetric cross sections for sections not susceptible to distortional deformations should be checked as given in the following clauses, where a distinction is made for:

- members that are not susceptible to torsional deformations, e.g. circular hollow sections or sections restraint from torsion
- members that are susceptible to torsional deformations, e.g. members with open cross-sections and not restraint from torsion.

(2) In addition, the resistance of the cross-sections at each end of the member should satisfy the requirements given in 6.2.

NOTE 1 The interaction formulae are based on the modelling of simply supported single span members with end fork conditions and with or without continuous lateral restraints, which are subjected to compression forces, end moments and/or transverse loads.

NOTE 2 In case the conditions of application expressed in (1) and (2) are not fulfilled, see 6.3.4.

(3) For members of structural systems the resistance check may be carried out on the basis of the individual single span members regarded as cut out of the system. Second order effects of the sway system (P-Δ-effects) have to be taken into account, either by the end moments of the member or by means of appropriate buckling lengths respectively, see 5.2.2(3)c) and 5.2.2(8).

(4) Members which are subjected to combined bending and axial compression should satisfy:

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \qquad (6.61)$$

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \qquad (6.62)$$

where N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ are the design values of the compression force and the maximum moments about the y-y and z-z axis along the member, respectively

$\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$ are the moments due to the shift of the centroidal axis according to 6.2.9.3 for class 4 sections, see Table 6.7,

χ_y and χ_z are the reduction factors due to flexural buckling from 6.3.1

χ_{LT} is the reduction factor due to lateral torsional buckling from 6.3.2

k_{yy}, k_{yz}, k_{zy}, k_{zz} are the interaction factors

Table 6.7: Values for $N_{Rk} = f_y A_i$, $M_{i,Rk} = f_y W_i$ and $\Delta M_{i,Ed}$

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	0	0	0	$e_{N,y} N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z} N_{Ed}$

NOTE For members not susceptible to torsional deformation χ_{LT} would be $\chi_{LT} = 1,0$.

(5) The interaction factors k_{yy} , k_{yz} , k_{zy} , k_{zz} depend on the method which is chosen.

NOTE 1 The interaction factors k_{yy}, k_{yz}, k_{zy} and k_{zz} have been derived from two alternative approaches. Values of these factors may be obtained from Annex A (alternative method 1) or from Annex B (alternative method 2).

NOTE 2 The National Annex may give a choice from alternative method 1 or alternative method 2.

NOTE 3 For simplicity verifications may be performed in the elastic range only.

6.3.4 General method for lateral and lateral torsional buckling of structural components

(1) The following method may be used where the methods given in 6.3.1, 6.3.2 and 6.3.3 do not apply. It allows the verification of the resistance to lateral and lateral torsional buckling for structural components such as

- single members, built-up or not, uniform or not, with complex support conditions or not, or
- plane frames or subframes composed of such members,

which are subject to compression and/or mono-axial bending in the plane, but which do not contain rotative plastic hinges.

NOTE The National Annex may specify the field and limits of application of this method.

(2) Overall resistance to out-of-plane buckling for any structural component conforming to the scope in (1) can be verified by ensuring that:

$$\frac{\chi_{op}\alpha_{ult,k}}{\gamma_{M1}} \geq 1,0 \tag{6.63}$$

where $\alpha_{ult,k}$ is the minimum load amplifier of the design loads to reach the characteristic resistance of the most critical cross section of the structural component considering its in plane behaviour without taking lateral or lateral torsional buckling into account however accounting for all effects due to in plane geometrical deformation and imperfections, global and local, where relevant;

χ_{op} is the reduction factor for the non-dimensional slenderness $\bar{\lambda}_{op}$, see (3), to take account of lateral and lateral torsional buckling.

(3) The global non dimensional slenderness $\bar{\lambda}_{op}$ for the structural component should be determined from

$$\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \tag{6.64}$$

where $\alpha_{ult,k}$ is defined in (2)

$\alpha_{cr,op}$ is the minimum amplifier for the in plane design loads to reach the elastic critical resistance of the structural component with regards to lateral or lateral torsional buckling without accounting for in plane flexural buckling

NOTE In determining $\alpha_{cr,op}$ and $\alpha_{ult,k}$ Finite Element analysis may be used.

(4) The reduction factor χ_{op} may be determined from either of the following methods:

a) the minimum value of

χ for lateral buckling according to 6.3.1

χ_{LT} for lateral torsional buckling according to 6.3.2

each calculated for the global non dimensional slenderness $\bar{\lambda}_{op}$.

NOTE For example where $\alpha_{ult,k}$ is determined by the cross section check $\frac{1}{\alpha_{ult,k}} = \frac{N_{Ed}}{N_{Rk}} + \frac{M_{y,Ed}}{M_{y,Rk}}$ this method leads to:

$$\frac{N_{Ed}}{N_{Rk}/\gamma_{M1}} + \frac{M_{y,Ed}}{M_{y,Rk}/\gamma_{M1}} \leq \chi_{op} \tag{6.65}$$

b) a value interpolated between the values χ and χ_{LT} as determined in a) by using the formula for $\alpha_{ult,k}$ corresponding to the critical cross section

NOTE For example where $\alpha_{ult,k}$ is determined by the cross section check $\frac{1}{\alpha_{ult,k}} = \frac{N_{Ed}}{N_{Rk}} + \frac{M_{y,Ed}}{M_{y,Rk}}$ this method leads to:

$$\frac{N_{Ed}}{\chi N_{Rk}/\gamma_{M1}} + \frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} \leq 1 \tag{6.66}$$

Annex A [informative] – Method 1: Interaction factors k_{ij} for interaction formula in 6.3.3(4)

Table A.1: Interaction factors k_{ij} (6.3.3(4))

Interaction factors	Design assumptions	
	elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	$C_{my} C_{mLT} \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{cr,y}}} \frac{1}{C_{yy}}$
k_{yz}	$C_{mz} \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{cr,z}}} \frac{1}{C_{yz}} 0{,}6 \sqrt{\frac{w_z}{w_y}}$
k_{zy}	$C_{my} C_{mLT} \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{cr,y}}} \frac{1}{C_{zy}} 0{,}6 \sqrt{\frac{w_y}{w_z}}$
k_{zz}	$C_{mz} \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{cr,z}}} \frac{1}{C_{zz}}$

Auxiliary terms:

$\mu_y = \frac{1 - \frac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \frac{N_{Ed}}{N_{cr,y}}}$	$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1{,}6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \frac{1{,}6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{W_{el,y}}{W_{pl,y}}$
	with $b_{LT} = 0{,}5\, a_{LT}\, \bar{\lambda}_0^2 \frac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{M_{pl,z,Rd}}$
$\mu_z = \frac{1 - \frac{N_{Ed}}{N_{cr,z}}}{1 - \chi_z \frac{N_{Ed}}{N_{cr,z}}}$	$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq 0{,}6 \sqrt{\frac{w_z}{w_y}} \frac{W_{el,z}}{W_{pl,z}}$
$w_y = \frac{W_{pl,y}}{W_{el,y}} \leq 1{,}5$	with $c_{LT} = 10\, a_{LT} \frac{\bar{\lambda}_0^2}{5 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$
$w_z = \frac{W_{pl,z}}{W_{el,z}} \leq 1{,}5$	$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0{,}6 \sqrt{\frac{w_y}{w_z}} \frac{W_{el,y}}{W_{pl,y}}$
$n_{pl} = \frac{N_{Ed}}{N_{Rk} / \gamma_{M1}}$	with $d_{LT} = 2\, a_{LT} \frac{\bar{\lambda}_0}{0{,}1 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{C_{mz} M_{pl,z,Rd}}$
C_{my} see Table A.2	
$a_{LT} = 1 - \frac{I_T}{I_y} \geq 0$	$C_{zz} = 1 + (w_z - 1)\left[\left(2 - \frac{1{,}6}{w_z} C_{mz}^2 \bar{\lambda}_{max} - \frac{1{,}6}{w_z} C_{mz}^2 \bar{\lambda}_{max}^2\right) n_{pl} - e_{LT}\right] \geq \frac{W_{el,z}}{W_{pl,z}}$
	with $e_{LT} = 1{,}7\, a_{LT} \frac{\bar{\lambda}_0}{0{,}1 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$

Table A.1 (continued)

$$\bar{\lambda}_{max} = \max\begin{cases}\bar{\lambda}_y \\ \bar{\lambda}_z\end{cases}$$

$\bar{\lambda}_0$ = non-dimensional slenderness for lateral-torsional buckling due to uniform bending moment, i.e. $\psi_y = 1{,}0$ in Table A.2

$\bar{\lambda}_{LT}$ = non-dimensional slenderness for lateral-torsional buckling

If $\bar{\lambda}_0 \le 0{,}2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,TF}}\right)}$:

$$C_{my} = C_{my,0}$$

$$C_{mz} = C_{mz,0}$$

$$C_{mLT} = 1{,}0$$

If $\bar{\lambda}_0 > 0{,}2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,TF}}\right)}$:

$$C_{my} = C_{my,0} + \left(1 - C_{my,0}\right)\frac{\sqrt{\varepsilon_y}\,a_{LT}}{1+\sqrt{\varepsilon_y}\,a_{LT}}$$

$$C_{mz} = C_{mz,0}$$

$$C_{mLT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}} \ge 1$$

$$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A}{W_{el,y}} \quad \text{for class 1, 2 and 3 cross-sections}$$

$$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A_{eff}}{W_{eff,y}} \quad \text{for class 4 cross-sections}$$

$N_{cr,y}$ = elastic flexural buckling force about the y-y axis

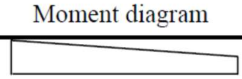
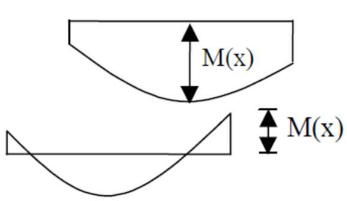
$N_{cr,z}$ = elastic flexural buckling force about the z-z axis

$N_{cr,T}$ = elastic torsional buckling force

I_T = St. Venant torsional constant

I_y = second moment of area about y-y axis

Table A.2: Equivalent uniform moment factors $C_{mi,0}$

Moment diagram	$C_{mi,0}$
 M_1 … ψM_1; $-1 \le \psi \le 1$	$C_{mi,0} = 0{,}79 + 0{,}21\psi_i + 0{,}36(\psi_i - 0{,}33)\frac{N_{Ed}}{N_{cr,i}}$
 M(x); M(x)	$C_{mi,0} = 1 + \left(\frac{\pi^2 EI_i \lvert\delta_x\rvert}{L^2 \lvert M_{i,Ed}(x)\rvert} - 1\right)\frac{N_{Ed}}{N_{cr,i}}$ $M_{i,Ed}(x)$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ $\lvert\delta_x\rvert$ is the maximum member displacement along the member
	$C_{mi,0} = 1 - 0{,}18\frac{N_{Ed}}{N_{cr,i}}$
	$C_{mi,0} = 1 + 0{,}03\frac{N_{Ed}}{N_{cr,i}}$

Annex B [informative] – Method 2: Interaction factors k_{ij} for interaction formula in 6.3.3(4)

Table B.1: Interaction factors k_{ij} for members not susceptible to torsional deformations

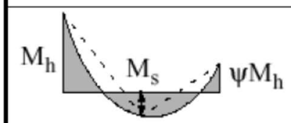
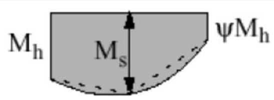
Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	I-sections RHS-sections	$C_{my}\left(1+0{,}6\bar{\lambda}_y\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0{,}6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+\left(\bar{\lambda}_y-0{,}2\right)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0{,}8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I-sections RHS-sections	k_{zz}	$0{,}6\ k_{zz}$
k_{zy}	I-sections RHS-sections	$0{,}8\ k_{yy}$	$0{,}6\ k_{yy}$
k_{zz}	I-sections	$C_{mz}\left(1+0{,}6\bar{\lambda}_z\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+0{,}6\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(2\bar{\lambda}_z-0{,}6\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+1{,}4\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
	RHS-sections		$C_{mz}\left(1+\left(\bar{\lambda}_z-0{,}2\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+0{,}8\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy} = 0$.			

Table B.2: Interaction factors k_{ij} for members susceptible to torsional deformations

Interaction factors	Design assumptions	
	elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	k_{yy} from Table B.1	k_{yy} from Table B.1
k_{yz}	k_{yz} from Table B.1	k_{yz} from Table B.1
k_{zy}	$\left[1-\frac{0{,}05\bar{\lambda}_z}{\left(C_{mLT}-0{,}25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ $\geq\left[1-\frac{0{,}05}{\left(C_{mLT}-0{,}25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$	$\left[1-\frac{0{,}1\bar{\lambda}_z}{\left(C_{mLT}-0{,}25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ $\geq\left[1-\frac{0{,}1}{\left(C_{mLT}-0{,}25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ for $\bar{\lambda}_z<0{,}4$: $k_{zy}=0{,}6+\bar{\lambda}_z\leq 1-\frac{0{,}1\bar{\lambda}_z}{\left(C_{mLT}-0{,}25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}$

k_{zz}	k_{zz} from Table B.1	k_{zz} from Table B.1

Table B.3: Equivalent uniform moment factors C_m in Tables B.1 and B.2

<table>
<tr><th rowspan="2">Moment diagram</th><th rowspan="2" colspan="2">range</th><th colspan="2">C_{my} and C_{mz} and C_{mLT}</th></tr>
<tr><th>uniform loading</th><th>concentrated load</th></tr>
<tr><td></td><td colspan="2">$-1 \le \psi \le 1$</td><td colspan="2">$0,6 + 0,4\psi \ge 0,4$</td></tr>
<tr><td rowspan="3">$\alpha_s = M_s/M_h$</td><td>$0 \le \alpha_s \le 1$</td><td>$-1 \le \psi \le 1$</td><td>$0,2 + 0,8\alpha_s \ge 0,4$</td><td>$0,2 + 0,8\alpha_s \ge 0,4$</td></tr>
<tr><td rowspan="2">$-1 \le \alpha_s < 0$</td><td>$0 \le \psi \le 1$</td><td>$0,1 - 0,8\alpha_s \ge 0,4$</td><td>$-0,8\alpha_s \ge 0,4$</td></tr>
<tr><td>$-1 \le \psi < 0$</td><td>$0,1(1-\psi) - 0,8\alpha_s \ge 0,4$</td><td>$0,2(-\psi) - 0,8\alpha_s \ge 0,4$</td></tr>
<tr><td rowspan="3">$\alpha_h = M_h/M_s$</td><td>$0 \le \alpha_h \le 1$</td><td>$-1 \le \psi \le 1$</td><td>$0,95 + 0,05\alpha_h$</td><td>$0,90 + 0,10\alpha_h$</td></tr>
<tr><td rowspan="2">$-1 \le \alpha_h < 0$</td><td>$0 \le \psi \le 1$</td><td>$0,95 + 0,05\alpha_h$</td><td>$0,90 + 0,10\alpha_h$</td></tr>
<tr><td>$-1 \le \psi < 0$</td><td>$0,95 + 0,05\alpha_h(1+2\psi)$</td><td>$0,90 - 0,10\alpha_h(1+2\psi)$</td></tr>
<tr><td colspan="5">For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{my} = 0,9$ or $C_{Mz} = 0,9$ respectively.</td></tr>
<tr><td colspan="5">C_{my} , C_{mz} and C_{mLT} should be obtained according to the bending moment diagram between the relevant braced points as follows:

moment factor | bending axis | points braced in direction
C_{my} | y-y | z-z
C_{mz} | z-z | y-y
C_{mLT} | y-y | y-y</td></tr>
</table>

11. PLATED STRUCTURAL ELEMENTS

11.1. Introduction, configurations

Plated structural elements are large steel elements - commonly made from welded steel plates. Large (deep) webs provide more optimal structural solution than rolled or compound sections to resist bending.
Typical use:

- Bridge girders
- Girders for heavy overhead cranes
- Columns (e.g. in industrial buildings with heavy cranes)
- Portal frames (large span industrial buildings)

Compared to steel structures of rolled profiles, plated structures are more prone to local buckling and therefore require design rules to cover such phenomena. In Eurocode 3, such rules are collected in Part 1-5 "Plated structures", EN 1993-1-5.

- Because bending is mainly taken up by the flanges => tensile/compressive stress => Class 1 section
- Web => as thin as possible for weight control => Class 4 section
- Web subjected to direct bending stress and shear stress
- Stiffeners and end posts: to prevent buckling due to bending and shear as well as local failure under patch loads.

Optimum shape for a beam:

- rectangular section:

$$W_{\square} = \frac{bh^2}{6}; \; A_{\square} = bh$$

- two flanges section (no web):

$$W = \frac{I}{h/2} \cong 2\frac{2\frac{hb}{2}\left(\frac{h}{2}\right)^2}{h} = \frac{bh^2}{2}$$

$$A = 2h\frac{b}{2} = bh$$

$$W / W_{\square} = 3$$

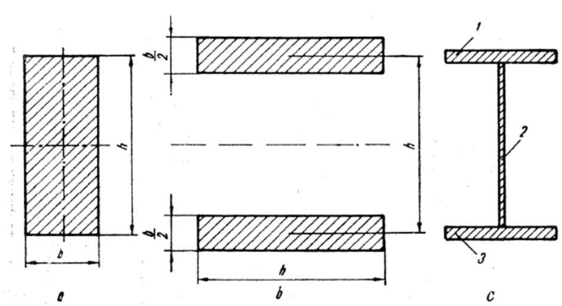
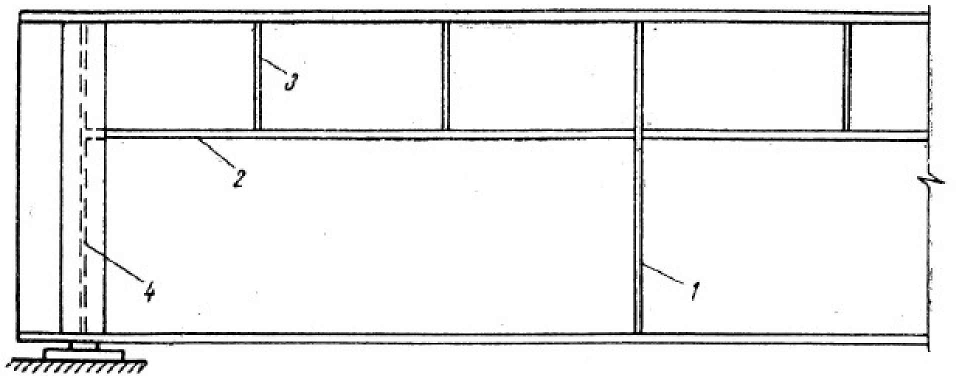
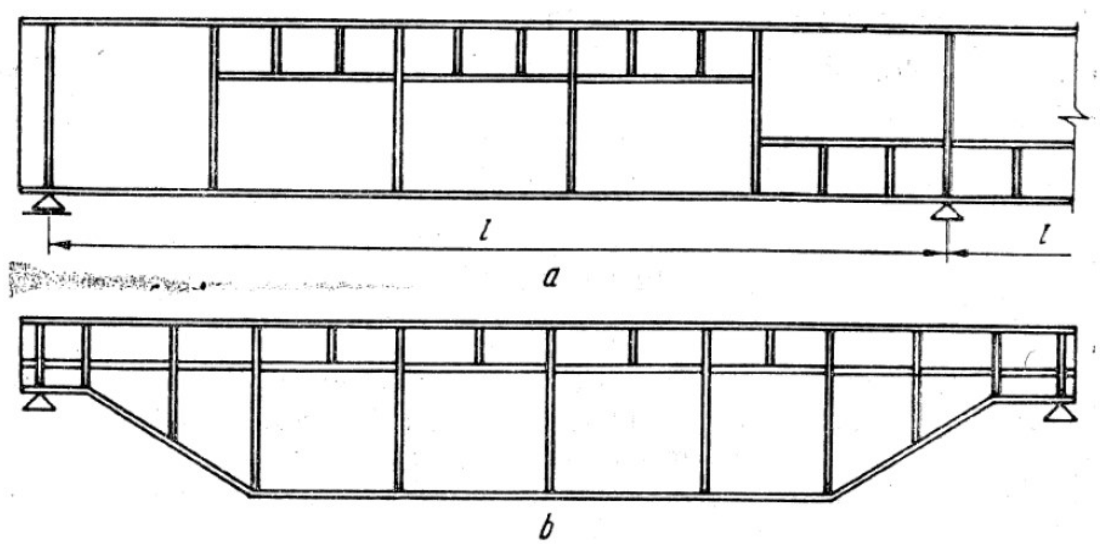


Figure 66. Optimum shape for a beam



Rigidizare:
1 — transversală ; 2 — longitudinală ; 3 — scurtă ; 4 — de reazem.



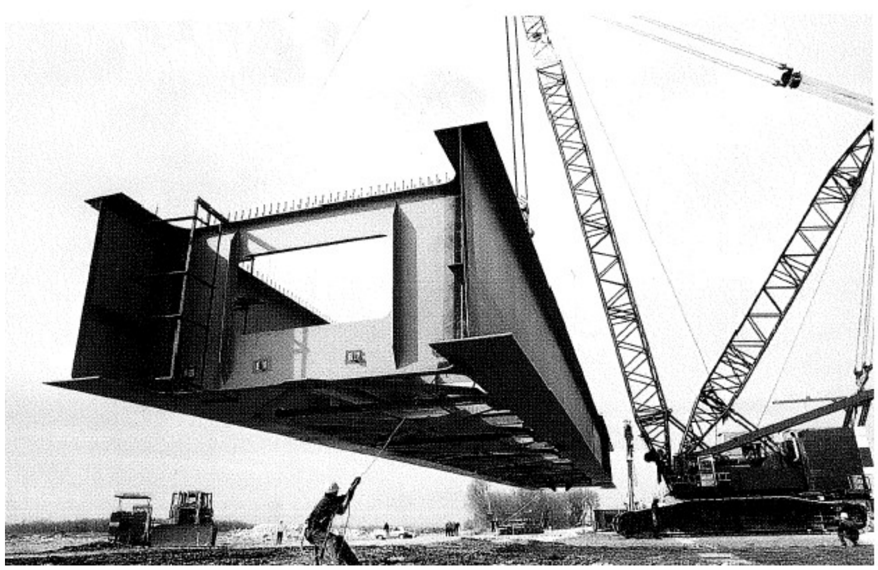


Figure 67: Examples of plated girders

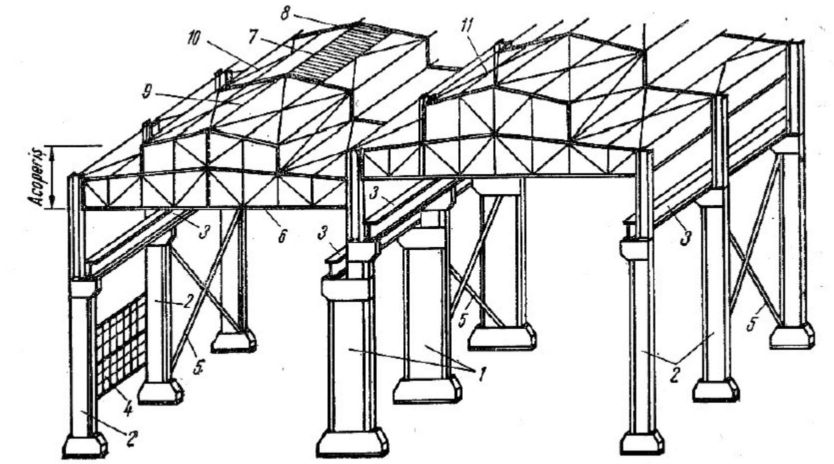


Fig. 9.2. Elementele principale ale halelor industriale metalice:
1 — stîlp central; 2 — stîlp marginal; 3 — grindă de rulare; 4 — perete; 5 — portal de frînare; 6 — fermă; 7 — pane; 8 — învelitoare; 9 — luminator; 10 — contravîntuire longitudinală; 11 — contravîntuire transversală.

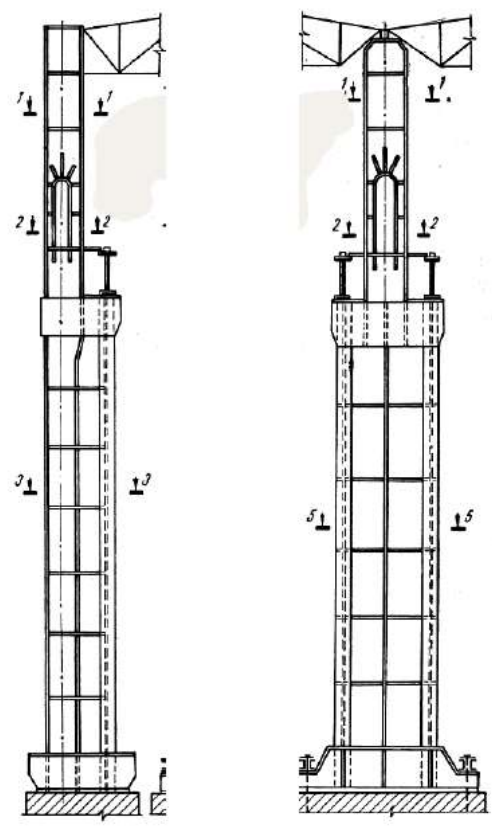


Figure 68: Examples of plated elements (columns) at heavy industrial buildings

11.2. Behavior of plated structural elements with slender web

Plate girders and columns with class 1, 2 and 3 sections are designed according to EN 1993-1-1.

Plate girders and columns with class 4 sections are designed considering the web buckling is possible:

- Effective section (using effective width concept), see EN 1993-1-5.
- Web buckling occurs when:

$$\sigma_{Ed} > \sigma_{cr} \qquad V_{Ed} > V_{cr}$$
$$\tau_{Ed} > \tau_{cr}$$

Plated structural elements are designed using the hypothesis of linear strain distributions for a cross section.

11.3. Deviations from this linear stress distribution

There are three causes for deviations from this linear stress distributions:

a) by exceeding the elastic range, where strain distributions are still linear, but stress responses are not because of exceedance of yield;
b) by local buckling where strain distributions along the original plane elements are considered to be linear but stress responses are not because of the stiffness reduction due to out of plane local buckling;
c) by shear deformations in the plane elements where the strain distributions deviate from linear distributions and cause a nonlinear stress distribution with shear lag.

All these effects may interact and are the more pronounced the more the strain situation approaches the limit states.
By using the concept of effective widths, the nonlinear effects from shear lag, plate buckling and the combination of both may be modelled keeping the hypothesis of linear strain distributions and the easy way to determine cross sectional properties and stresses.
Web buckling may be prevented by using stiffeners. Stiffeners and end posts are used to prevent buckling due to bending and shear as well as local failure under patch loads.

Rigidizări	Transversale		Longitudinale		Longitudinale și scurte
Solicitarea pentru care se recomandă rigidizările	Lunecare Forfecare	Încovoiere	Compresiune	Încovoiere	Acțiuni locale mobile (forțe concentrate mobile)
Dimensiuni aproximative pentru: Construcții metalice hidrotehnice	$a = \lambda$ (distanța între antretoaze) $a \leq 3h_0$ $a \leq 3{,}0\,m$		$h_1 = 0{,}5\,h_0$	Se prevăd dacă: $\frac{h_0}{t_i} > 150$ La distanța $h_1 = \left(\frac{1}{4} \cdots \frac{1}{5}\right)h_0$	$a_1 = \left(\frac{1}{2} \cdots \frac{1}{3}\right)a$
Dimensiuni aproximative pentru: Poduri metalice cu calea sus	$a = 1{,}6 \ldots 2{,}0\,m$ $a \leq 3h_0$		$h_1 = 0{,}5\,h_0$		—
Dimensiuni aproximative pentru: Poduri metalice cu calea jos	$a = \lambda$ (distanța între antretoaze) $a \leq 3h_0$; $a \leq 3{,}0\,m$				
Dimensiuni aproximative pentru: Construcții civile, industriale și agricole	– Dacă $h_0 > 100t_i$ se va lua $a \leq 2h_0$ – Dacă $h_0 \leq 100t_i$ se va lua $a \leq 2{,}5\,h_0$		$h_1 = 0{,}5\,h_0$	$h_1 = \left(\frac{1}{3{,}3} \cdots \frac{1}{5}\right)h_0$	$a_1 = \left(\frac{1}{2} \cdots \frac{1}{3}\right)a$

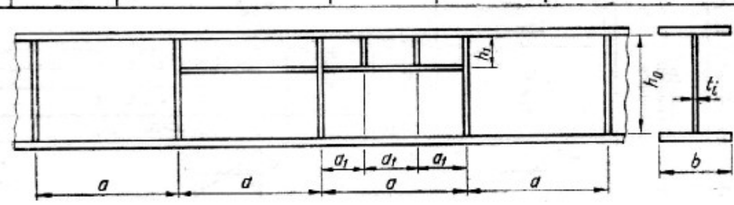


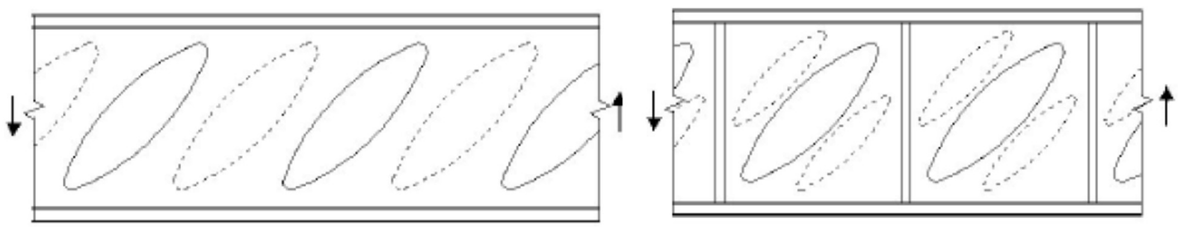
Figure 69. Recommended configuration of stiffeners

11.4. Plated structural element subjected to shear stress

When are subjected to large shear forces, thin and deep webs are vulnerable to shear buckling.
Web buckling could occur before the "full" shear capacity.
Shear buckling strength depends on:

- aspect ratio
- plate thickness
- imperfections
- material properties
- boundary conditions.

Shear buckling could be delayed if appropriate rigid transverse stiffeners are constructed to limit the extent and separate the buckling regions.

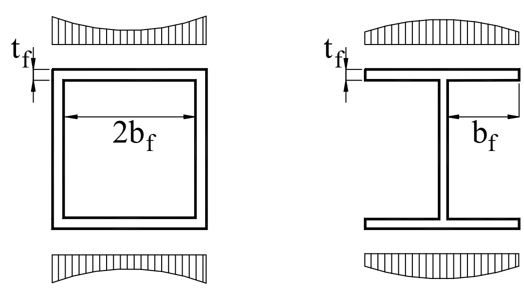


Unstiffened web Stiffened web

Figure 70. Shear buckling of thin web plated girders

11.5. "SHEAR-LAG" effect

Shear lag: Normal stress variation in the beam flanges due to the effect of the shear force



Box-type beam I-section beam

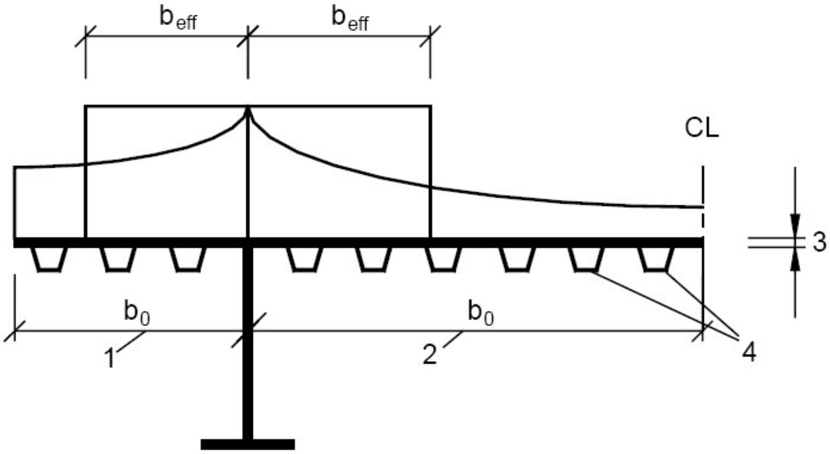
Figure 71. Distribution of stresses in flanges due to shear lag effect

- Shear lag effect is more important in case of wide flange beams.
- Shear lag in flanges may be neglected provided that $b_0 < L_e/50$, where the flange width b_0 is taken as the outstand or half the width of an internal element and L_e is the length between points of zero bending moment
- Where the above limit is exceeded the effect of shear lag in flanges should be considered at serviceability, fatigue and ultimate limit state verifications by the use of an effective width, where the normal stress σ is constant

Definitions and notations for shear lag

The effective width b_{eff} for shear lag under elastic conditions should be determined from:

$$b_{eff} = \beta b_0$$



1 for outstand flange

2 for internal flange

3 plate thickness t

4 stiffeners with $A_{s\ell} = \sum A_{s\ell i}$

Figure 72. Notations for shear lag

The effective width b_{eff} for shear lag varies along the beam span:

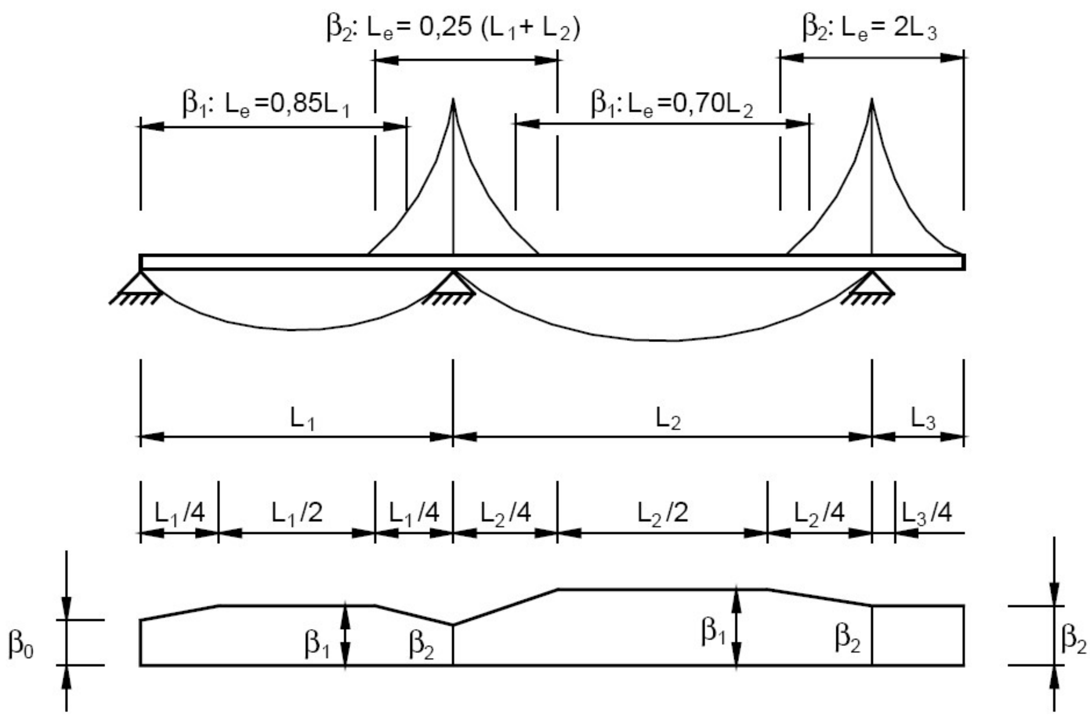


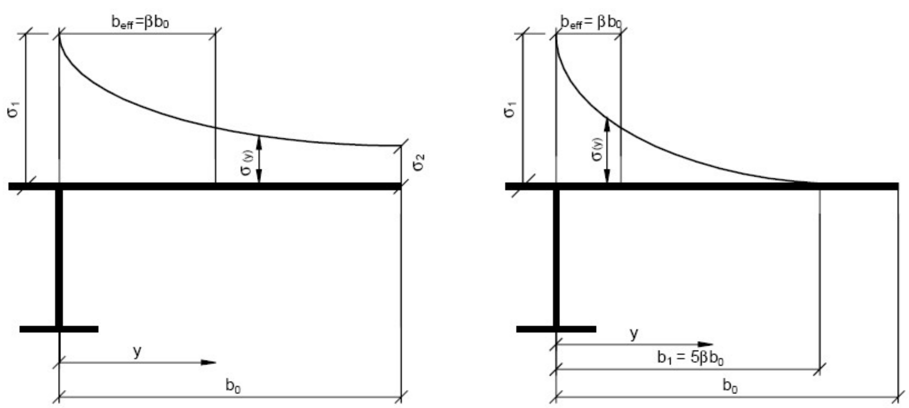
Figure 73. Effective length Le for a continuous beam and the distribution of the effective width

Effective width factor β

κ	location for verification	β – value
$\kappa \le 0{,}02$		$\beta = 1{,}0$
$0{,}02 < \kappa \le 0{,}70$	sagging bending	$\beta = \beta_1 = \frac{1}{1 + 6{,}4\,\kappa^2}$
	hogging bending	$\beta = \beta_2 = \frac{1}{1 + 6{,}0\left(\kappa - \frac{1}{2500\,\kappa}\right) + 1{,}6\,\kappa^2}$
$> 0{,}70$	sagging bending	$\beta = \beta_1 = \frac{1}{5{,}9\,\kappa}$
	hogging bending	$\beta = \beta_2 = \frac{1}{8{,}6\,\kappa}$
all κ	end support	$\beta_0 = (0{,}55 + 0{,}025 / \kappa)\,\beta_1$, but $\beta_0 < \beta_1$
all κ	cantilever	$\beta = \beta_2$ at support and at the end

$\kappa = \alpha_0\, b_0 / L_e$ with $\alpha_0 = \sqrt{1 + \frac{A_{s\ell}}{b_0 t}}$

in which $A_{s\ell}$ is the area of all longitudinal stiffeners within the width b_0 and other symbols are as defined in Figure 3.1 and Figure 3.2.



$\beta > 0{,}20$:

$\sigma_2 = 1{,}25\,(\beta - 0{,}20)\,\sigma_1$

$\sigma(y) = \sigma_2 + (\sigma_1 - \sigma_2)(1 - y / b_0)^4$

$\beta < 0{,}20$:

$\sigma_2 = 0$

$\sigma(y) = \sigma_1 (1 - y / b_1)^4$

σ_1 is calculated with the effective width of the flange b_{eff}

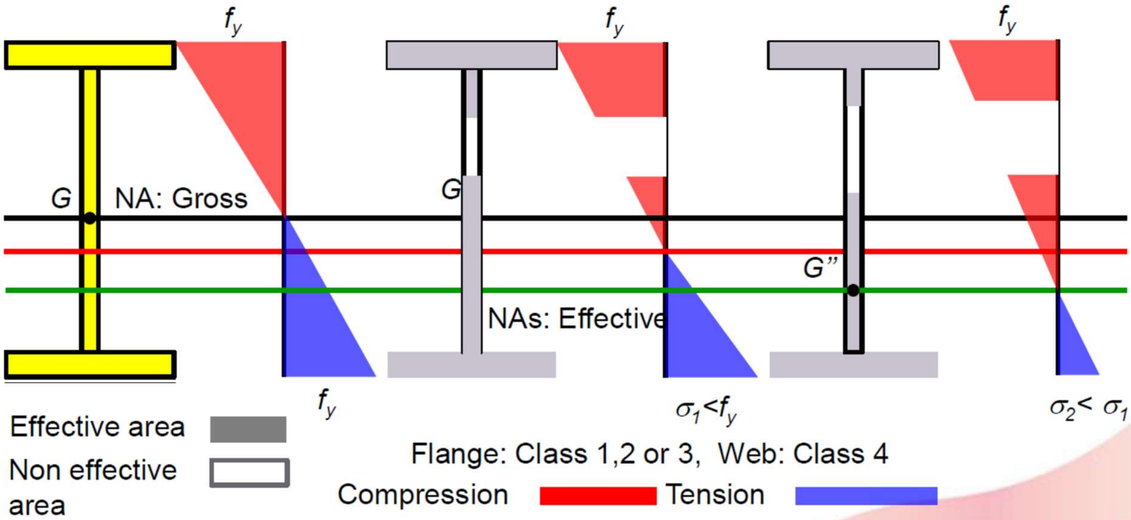
Figure 74. Distribution of stresses across the plate due to shear lag effect

11.6. Design of plate girders

11.6.1 Resistance to bending

- **Flange:** Class 1 or Class 2, **Web:** Almost inevitably Class 4
- EC3 Part 1-5 requires special treatment for Class 4 plate using the **Effective Width Method**

- Part of compressive web become ineffective ➔ relocation of centroid ➔ compressive flange yielded but tension flange remains elastics at ULS
- Bending resistance: $\boldsymbol{M_{y,Rd}=W_{eff}f_y}$



Bending verification:

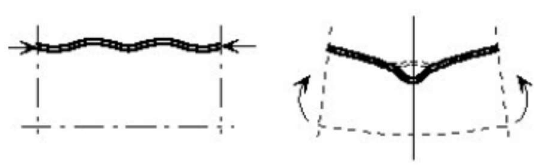
If no axial force is present, verification should be performed as a normal section check using η_1:

$$\eta_1 = \frac{M_{Ed}}{M_{y,Rd}} \leq 1.0$$

where M_{Ed} is the maximum design moment .

To prevent flange induced buckling, we also need to check:

$$\frac{h_w}{t_w} \leq k \frac{E}{f_{yf}} \sqrt{\frac{A_w}{A_{fc}}}$$



where:
A_{fc} is the effective area of the compression flange. The factor *k* is taken as 0.3, 0.4 and 0.55 for Class 1, 2, 3 and 4 flanges, respectively.

11.6.2 Resistance to shear

1. Stocky/Non-stocky check
2. Contribution from the web
3. Contribution from the flanges

Unless the web is Class 1, we need to check whether the web is stocky or not for shear buckling.

- For unstiffened web:

$$\frac{h_w}{t} \leq \frac{72}{\eta}\varepsilon$$

- For transversely stiffened webs

$$\frac{h_w}{t} \leq \frac{31}{\eta}\varepsilon\sqrt{k_\tau}$$

where:
k_τ is the buckling factor
η is the coefficient that includes the increase of shear resistance at smaller web slenderness;
h_w , t are dimensions of web.

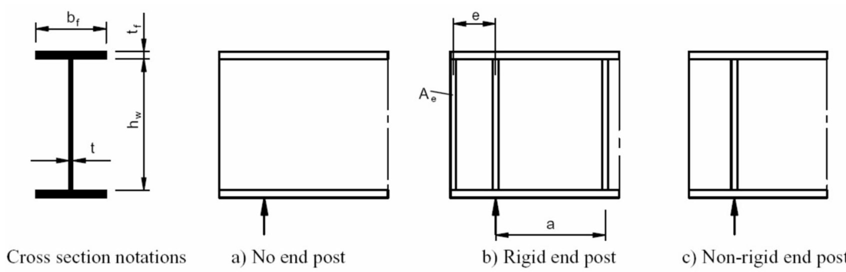


Figure 75. End post

If the web is stocky, no shear buckling of web shall occur and the shear strength of the web is given by EC3 Part 1-1.

$$V_{pl,Rd} = \frac{A_v(f_y/\sqrt{3})}{\gamma_{M0}}$$

If the web is NOT stocky, shear buckling governs the failure:

$$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \leq \frac{\eta f_{yw} h_w t_w}{\sqrt{3}\gamma_{M1}}$$

Web contribution (TFA)   Flange contribution (PH formation)

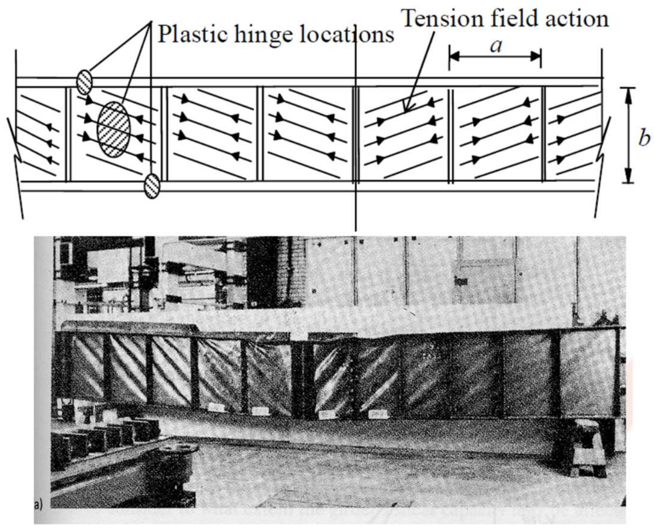


Figure 76. Contribution to shear resistance

<u>Contribution from the web, $V_{bw,Rd}$</u>

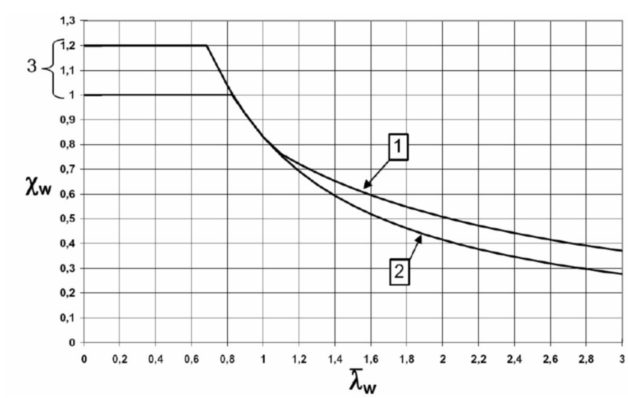
The contribution from the web $V_{bw,Rd}$ is:

$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t}{\sqrt{3}\gamma_{M1}}$$

where:
χ_w is the reduction factor for the shear resistance of the sole web, depending on the web slenderness;

	Rigid end post	Non-rigid end post
$\bar{\lambda}_w < 0{,}83/\eta$	η	η
$0{,}83/\eta \leq \bar{\lambda}_w < 1{,}08$	$0{,}83/\bar{\lambda}_w$	$0{,}83/\bar{\lambda}_w$
$\bar{\lambda}_w \geq 1{,}08$	$1{,}37/\left(0{,}7+\bar{\lambda}_w\right)$	$0{,}83/\bar{\lambda}_w$

a) shear resistance function of the web



b) Reduction factor for shear resistance of the web
Figure 77. Shear resistance of the web

For transverse stiffeners at supports only (e.g. end post only):

$$\bar{\lambda}_w = \frac{h_w}{86,4t\varepsilon}$$

When transverse and intermediate transverse stiffener are present:

$$\bar{\lambda}_w = \frac{h_w}{37,4t\varepsilon\sqrt{k_\tau}}$$

<u>Contribution from the flanges, $V_{bf,Rd}$</u>

- Tests on web panels subjected to shear show that at the ultimate state a kind of plastic mechanism is nearly formed in the flanges (plastic hinges E, H, G and K in Figure 78), caused by the tension field between the flanges.
- Under the assumption that this tension field does not influence the shear resistance of the web obtained on the basis of the rotated stress field theory (Figure 5.10a), the shear resistance coming from the flanges can be added to the contribution of the web.

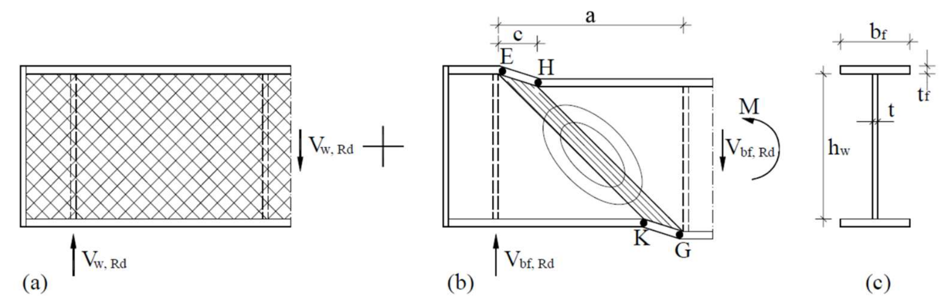


Figure 78. Tension field carried by bending resistance of flanges (Figure 5.10 from EN1993-1-5)

- The shear resistance $V_{bf,Rd}$ provided by the flanges can be calculated based on the plastic mechanism in the flanges:

$$V_{bf,Rd} = \frac{b_f t_f^2 f_{yf}}{c\gamma_{M1}}\left(1 - \left(\frac{M_{Ed}}{M_{f,Rd}}\right)^2\right)$$

$$c = a\left(0,25 + 1,6\frac{M_{pl,f}}{M_{pl,w}}\right) = a\left(0,25 + \frac{1,6 \cdot b_f \cdot t_f^2 \cdot f_{yf}}{t \cdot h_w^2 \cdot f_{yw}}\right)$$

The contribution of flanges can be added to the shear resistance of web panels only when the flanges are not completely utilized in withstanding the bending moments:

$$M_{Ed} \le M_{f,Rd}$$

Where

$$M_{f,Rd} = \frac{M_{f,k}}{\gamma_{M0}}$$

where $M_{f,Rd}$ is the moment of resistance of effective area of the flanges only.

Verification of shear resistance is checked by calculating η_3:

$$\eta_3 = \frac{V_{Ed}}{V_{b,Rd}} \leq 1.0$$

11.6.3 Interaction of bending and shear

If

$$\overline{\eta}_3 = V_{Ed} / V_{bw,Rd} \leq 0.5$$

and

$$M_{Ed} < M_{f,Rd}$$

the design resistance to bending moment need not be reduced to allow for the shear force.
Otherwise, additional verifications are needed (M-V interaction):

$$\overline{\eta}_1 + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)(2\overline{\eta}_3 - 1)^2 \leq 1.0 \quad \overline{\eta}_1 = \frac{M_{Ed}}{M_{pl,Rd}}$$

where $M_{pl,Rd}$ is the design plastic resistance of the cross section consisting of the effective area of the flanges and the fully effective web irrespective of its class.
The above interaction check should be done at all sections other than those located at a distance less than $h_w/2$ from a support with vertical stiffener, as normally the shear force is overestimated there.

11.6.4 Stiffeners and end post design

Intermediate transverse stiffener

- Takes no direct external loading but subjected to internal direct and shear force
- Creates compartments to increase web buckling resistance
- Provides **rigid support** for TFA and flange resistance (PH) for web buckling

Load bearing stiffener

- Prevent yielding, crushing, local and global failures at where heavy patch loads are applied

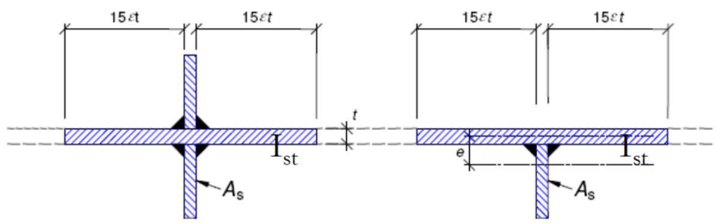
End Post

- A special form of load bearing stiffener
- As “anchor” supports for TFA, flange resistance (PH) and reaction forces

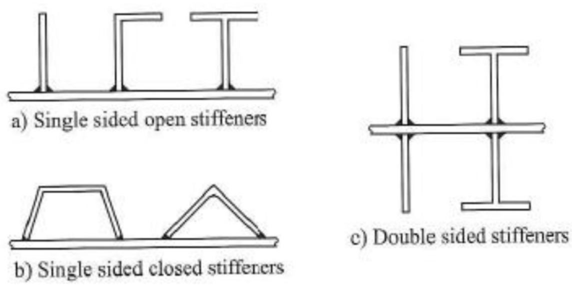
- Rigid and non-rigid end posts are possible

General requirements of all stiffeners

- Independent design check needed
- Cross section of the stiffener consists of
  - (1) gross area of the stiffener itself, and
  - (2) contribution width of $15\varepsilon t_w$ on each side (but no overlapping)
- Normally, thin-walled open section (at least Class 2) are used



Effective cross section of stiffener



Typical cross section of stiffeners

Two general performance requirements for ALL transverse stiffeners (intermediate, load bearing and end post) at section where $M_{Ed} \neq 0$

- Requirement (A) Stress and deflection limits: To verify using a second order elastic analysis that at the ultimate limit state:

$$\sigma_{max} \leq \frac{f_y}{\gamma_{M1}} \quad \text{and} \quad w \leq \frac{b}{300}$$

where σ_{max} is the ultimate stress (elastic) and w is the ultimate lateral deflection while b is the panel height (or web height).

- Requirement (B) Torsional buckling: The stiffener will not fail by torsional buckling

General requirements of all stiffeners

For a stiffener consist of flat plates, the requirement for torsional buckling could be simplified to limit the width (B) to thickness (t) ratio of the plates to those values shown below:

Steel grade	235	275	355	420	460
$B/t \leq$	13.0	12.0	10.6	9.7	9.3

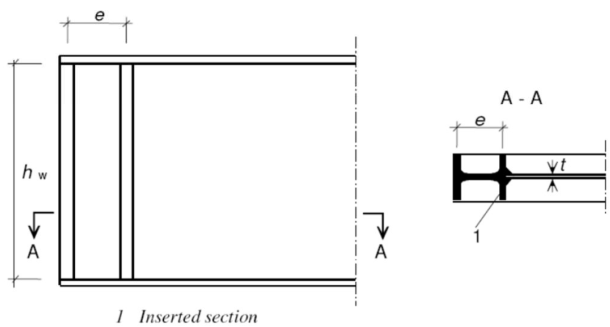
Limiting B/t values to prevent torsional buckling

Design of end post

- Two types of end posts: Rigid and Non-Rigid
- To provide support for TFA and support reaction
- Rigid end post should comprises of two double-sided transverse stiffener which forms the flange of a short column of length h_w and the following two requirements should be met:

$$e \geq 0.1 h_w$$

$$\min(A_e, A_u) \geq \frac{4 h_w t_w^2}{e}$$

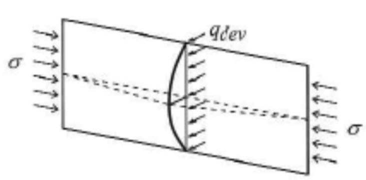


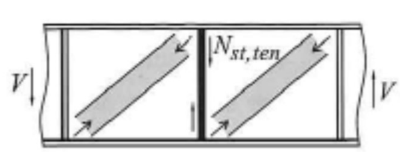
Rolled section forming an end post

- **Requirements (A) and (B)** should still be satisfied for the end post
- As end post often subjected to large reaction force, satisfaction of **Requirement (A)** which limits the maximum stress to f_y may not able to prevent the buckling of the end post as a strut
- EC3 requires separate checking of the end post as a column with buckling length not less than $0.75h_w$.
- **Curve c** from EC3 Part 1-1 should be used for buckling check
- The total force acting on the end post should equal to the sum of reaction, force due to TFA and interaction with direct stress
- If both ends of the end post are fixed laterally, the buckling length could be taken as $0.75h_w$. Otherwise, a larger value should be used.

Design of intermediate transverse stiffener

- The main function of intermediate transverse stiffener is to provide rigid boundary supports for the panel and prevent web buckling due to direct stress (plate-like buckling) and to shear stress (axial force from TFA and PH formations)
- Both **Requirements A and B** should be verified for both direct stress and shear stress





Direct stress — Shear stress

Design of load bearing stiffeners

- Patch loads and concentrated loads are common in plate girder
- Sufficient bearing resistance is needed to resist transverse force acting on the flange plate
- In case that the bearing strength provided by the web plate is insufficient, load bearing stiffener should be constructed.
- In general, **Requirements (A) and (B)** are also needed to be satisfied for loading bearing stiffeners.
- Again, the total force acting on a load bearing stiffener should equal to the sum of reaction, force due to TFA and interaction with direct stress
- Similar to end post, their design is often government by the buckling resistance as a strut (with buckling length ≥0.75h_w) but without the compulsory use of two double-side plates

12. COLD-FORMED THIN GAUGE MEMBERS AND SHEETING

12.1. Introduction

- Wide range of products, with a diversity of shapes, sizes, and applications are produced in steel using the cold forming process.
- In the recent years, cold-formed steel sections were increasingly used as primary framing components
- After their primarily applications as purlins or side rails, the second major one in construction is in the building envelope.
- Options for steel cladding panels range from inexpensive profiled sheeting for industrial applications, through architectural flat panels used to achieve a prestigious look of the building.
- Light steel systems are widely used to support curtain wall panels.
- Cold-formed steel in the form of profiled decking has gained widespread acceptance over the past fifteen years as a basic component, along with concrete, in composite slabs.
- Cold-formed steel members are efficient in terms of both their stiffness and strength.
- In addition, because the steel may be even less than 1 mm thick, the members are lightweight.
- The use of thinner sections and high strength steels leads to design problems for structural engineers which may not normally be encountered in routine structural steel design.
- Structural instability of the sections is more likely to occur as a result of the sections is more likely to occur as a result of the reduced buckling loads (and stresses), and the use of higher strength steel which may make the buckling stress and yield stress of the thin-walled sections approximately equal.
- Further, the shapes which can be cold-formed are often considerably more complex than hot-rolled steel shapes such as I-sections and unlipped channel sections.
- The cold-formed sections commonly have mono-symmetric or point symmetric shapes, and normally have stiffening lips on flanges and intermediate stiffeners in wide flanges and webs.

- Both simple and complex shapes can be formed for structural and non-structural applications.
- Special design standards have been developed for these sections.

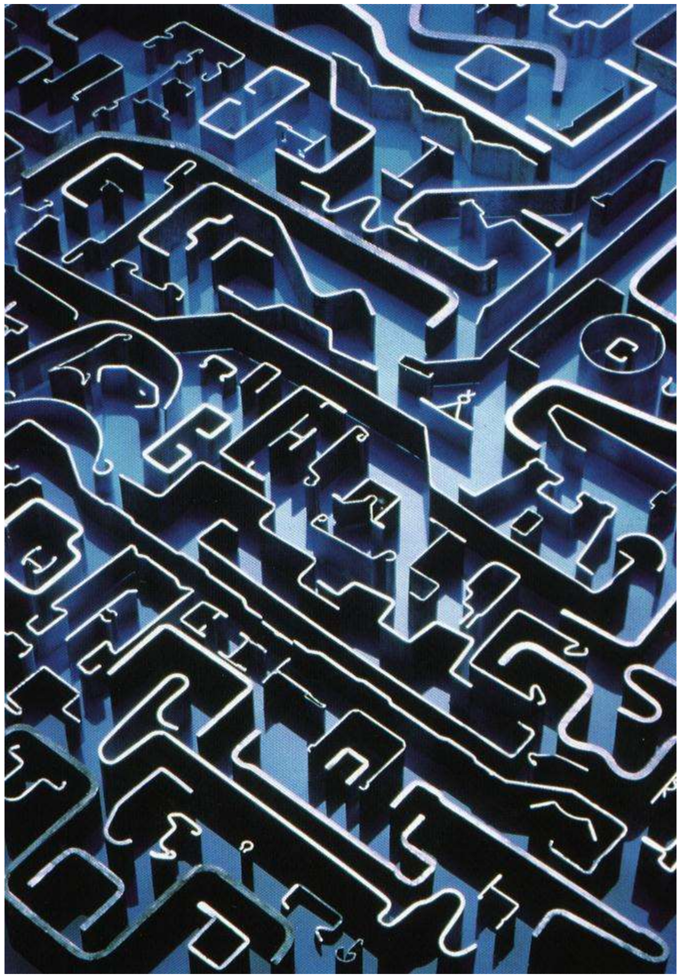


Figure 79. Collection of different cold-formed steel sections shapes (Trebilcock, 1994)

Design of cold formed thin walled members

- In the USA, the Specification for the design of cold-formed steel structural members of the American Iron and Steel Institute was first produced in 1946 and has been regularly updated based on research to the most recent 1996 edition (AISI, 1996, 1999). Recently, the first edition of the unified North American Specification (AISI, 2001) was prepared and issued in 2001. It is applicable to the United States, Canada and Mexico for the design of cold-formed steel structural members.
- In Europe, the ECCS Committee TC7 originally produced the European Recommendations for the design of light gauge steel members in 1987 (ECCS, 1987). This European document has since been further developed and published in 1996 as the European Standard Eurocode 3, Part 1.3 Supplementary rules for cold-formed members and sheeting (EN 1993-1-3, 2006).
- In Australia and New Zeeland, a new limit states design standard AS/NZS 4600 for the design of Cold-formed steel structures was published in December 1996 (AS/NZS, 1996, 1998), followed by the 2003 and 2005 Editions.

Types of Cold-formed Steel Sections

- Cold-formed members and profiled sheets are steel products made from coated or uncoated hot rolled or cold-rolled flat strips or coils. Within the permitted range of tolerances, they have constant or variable cross-section.
- Cold-formed structural members can be classified into two major types:
  - Individual structural framing members;
  - Panels and decks.
- Individual structural members (bar members) obtained from so called "long products" include:
  - single open sections, shown in Figure 80.a;
  - open built-up sections (Figure 80.b);
  - closed built-up sections (Figure 80.c).

Usual, the depth of cold-formed sections for bar members ranges from 50 - 70 mm to 350 - 400 mm, with thickness from 1 to 6 mm about.

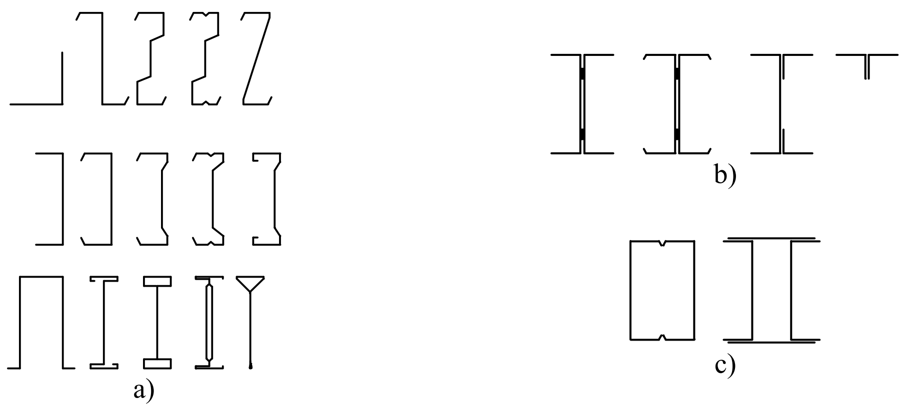


Figure 80. Typical forms of cold formed members

Profiled sheets

- Panels and decks are made from profiled sheets and linear trays (cassettes) shown in figure. The depth of panels usually ranges from 20 to 200 mm, while thickness is from 0.4 to 1.2 (1.5) mm.

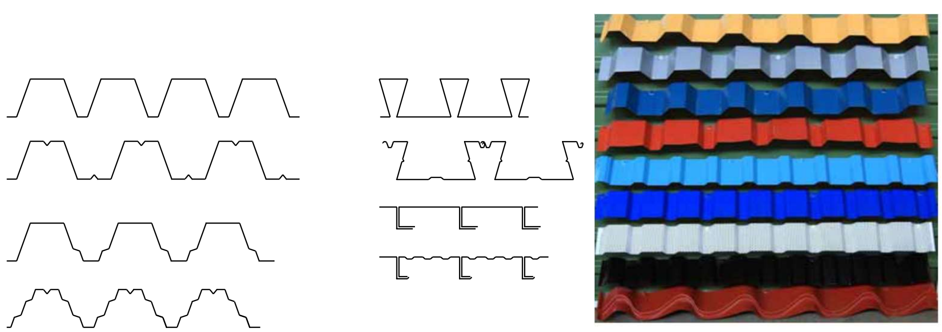


Figure 81. Profiled sheets

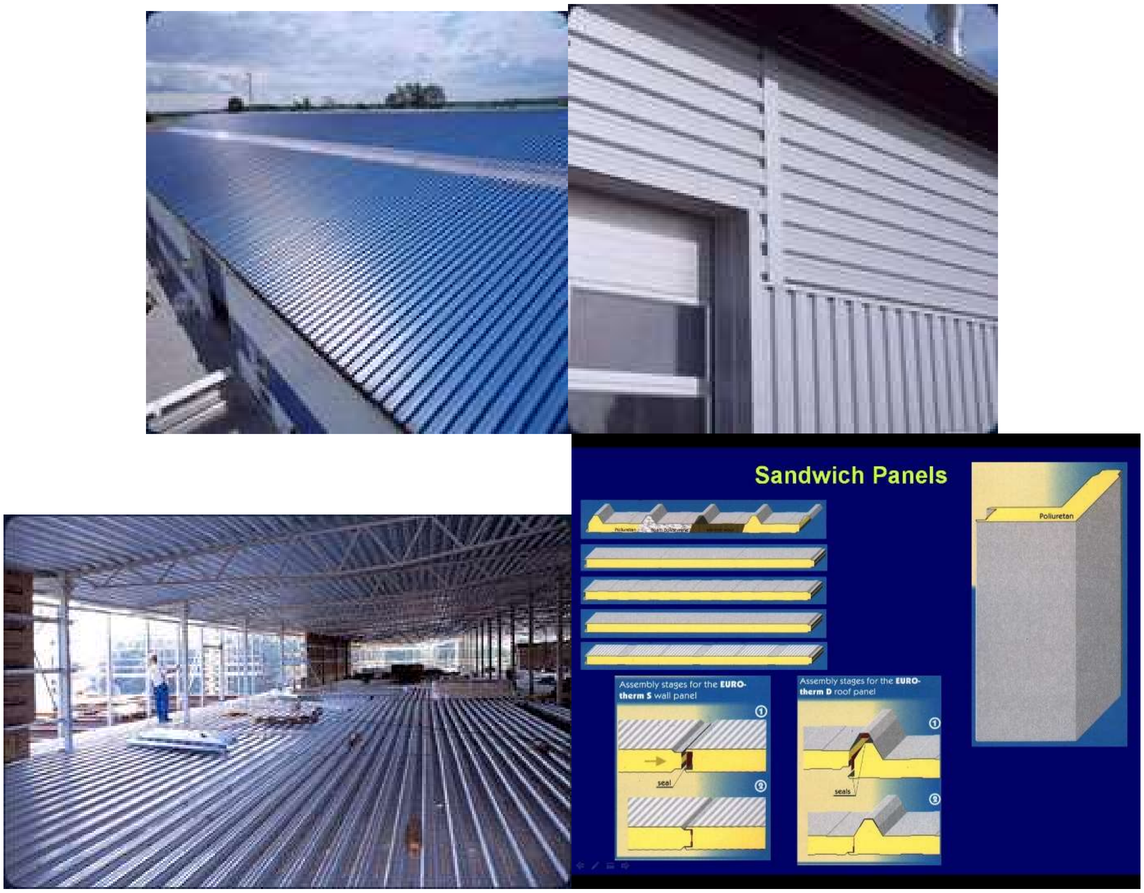


Figure 82. Profiled sheets - examples

Manufacturing

Cold-formed members are normally manufactured by one of two processes. These are:

a) Roll forming;
b) Folding
c) Press braking

a) Roll forming consists of feeding a continuous steel strip through a series of opposing rolls to progressively deform the steel plastically to form the desired shape. Each pair of rolls produces a fixed amount of deformation in a sequence of type shown in Figure **83**. In this example, a Ω section is formed. Each pair of opposing rolls is called a stage. In general, the more complex the cross-sectional shape, the greater the number of stages required. In the case of cold-formed rectangular hollow sections, the rolls initially form the section into a circular section and a weld is applied between the opposing edges of the strip before final rolling (called sizing) into a square or rectangular shape.

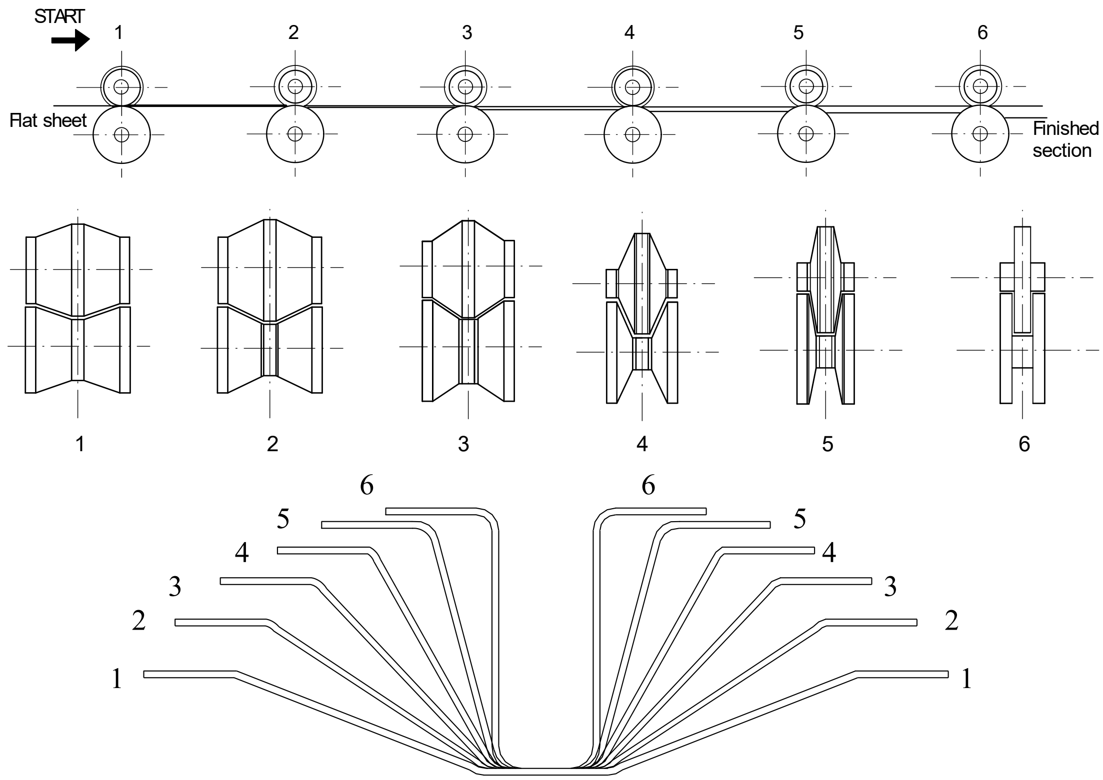


Figure 83. Stages in roll forming a simple section (Rhodes, 1992)

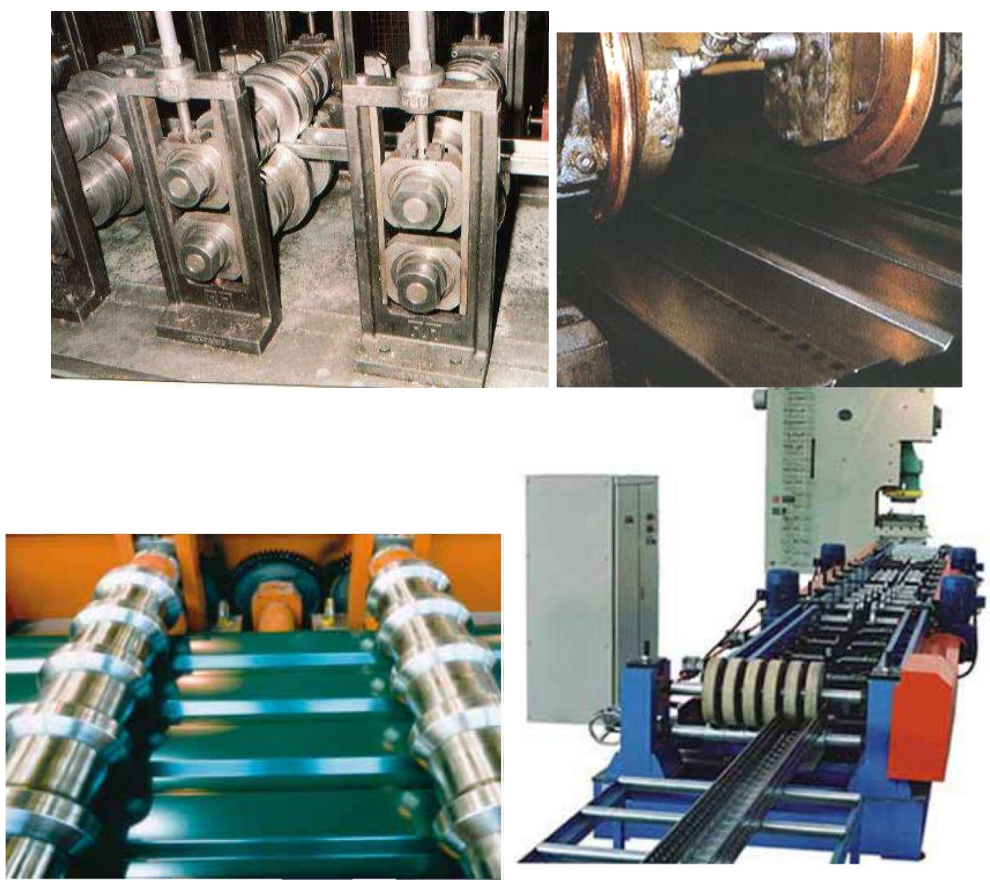
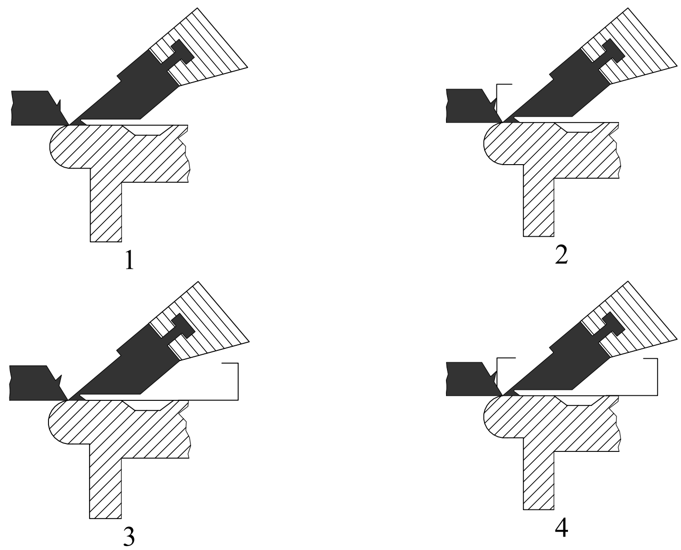


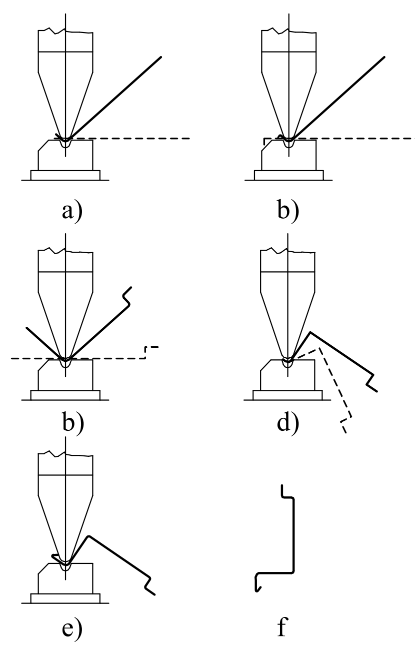
Figure 84. Industrial roll forming lines

b) Folding is the simplest process, in which specimens of short lengths and of simple geometry is produced from a sheet of material by folding a series of bends (see Figure 7). This process has very limited application.



Forming of folding

c) Press-braking is more widely used, and a greater variety of cross-sectional forms can be produced by this process. Here a section is formed from a length of strip by pressing the strip between shaped dies to form the profile shape (see Figure 8). Usually each bend is formed separately. The set up of a typical brake press is illustrated in Figure 8. This process also has limitations on the profiled geometry which can be formed, and, often more importantly, on the lengths of sections which can be produced. Press-braking is normally restricted to sections of length less than 5 m although press brakes capable of production 8 m long members are in use in industry.



Forming steps in press braking process

Some peculiar characteristics of cold-formed sections

- The manufacturing process plays a governing role in some characteristics that have an influence on the buckling behaviour of the profiles.
- It leads to a modification of the stress-strain curve of the steel.
    - With respect to the material, cold-rolling provides an increase of the yield strength and, sometimes, of the ultimate strength that is important in the corners and still appreciable in the flanges, while press braking let these characteristics nearly unchanged in the flanges.
    - Such effects do not appear in case of hot-rolled sections, as shown in Table 1 (Rondal, 1988)
- The increase of the yield strength is due to strain hardening and depends on the type of steel used for cold rolling. On the contrary, the increase of the ultimate strength is related to strain aging that is accompanied by a decrease of the ductility and depends on the metallurgical properties of the material.

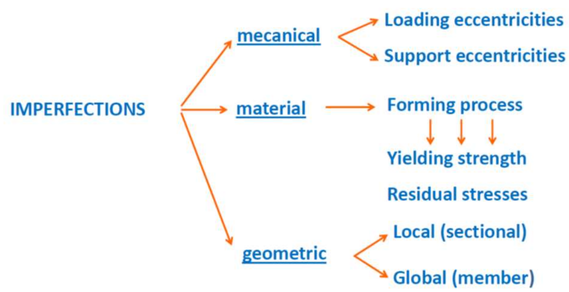


Figure 85. Imperfections in Thin-Walled Cold-Formed Steel Members

Effect of Strain Hardening and Strain Aging on Stress-Strain Characteristics

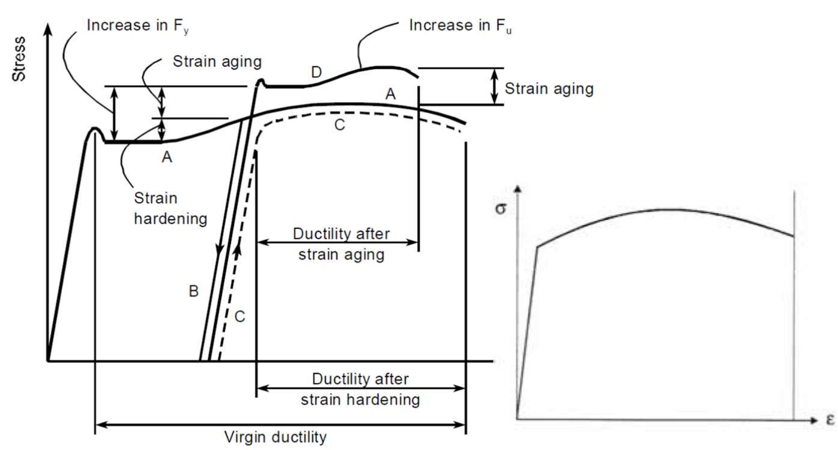


Figure 86. Influence of cold forming on the mechanical characteristic of the steel

Increase of the Yield Strength and Ultimate Strength Due to Cold-Forming

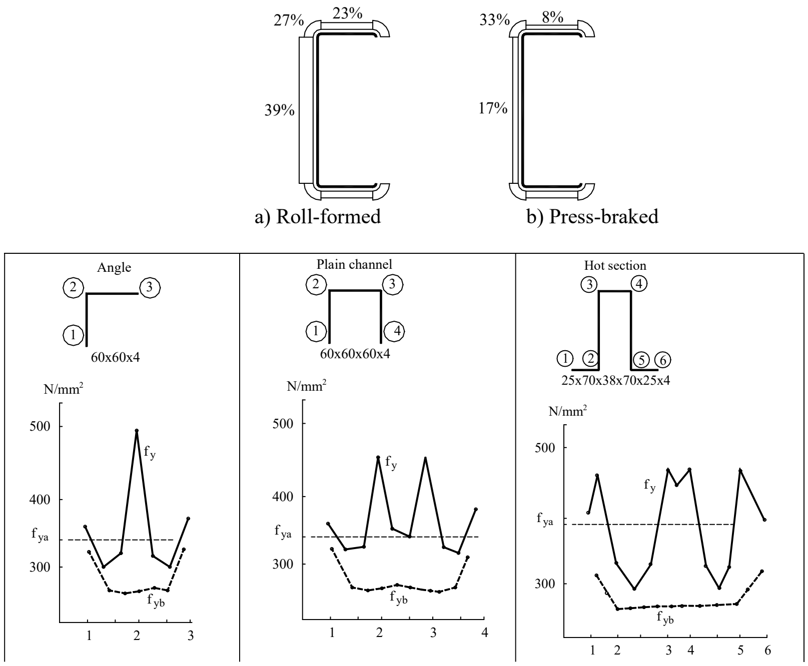


Figure 87. Influence of cold forming

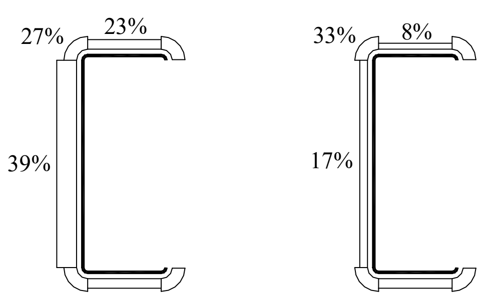
Average value of yield stress increases with the number of bends and may be determined with the eq.:

$$f_{ya} = f_{yb} + (f_u - f_{yb}) \cdot k \cdot n \cdot t^2 / A_g \leq (f_u + f_{yb}) / 2$$

where:

f_{yb} , f_{ub} – yield stress and tensile strength of base material
t – plate thickness
A_g – gross cross section area
k – coefficient depending on the cold forming process (k=7 – rolling, k=5 – other methods);
n – number of bendings with a radius less then *5t* but between 0º - 135º.

Residual Stress In Steel Sections



a) roll-formed b) press-braked
Figure 88. Residual stress in steel sections

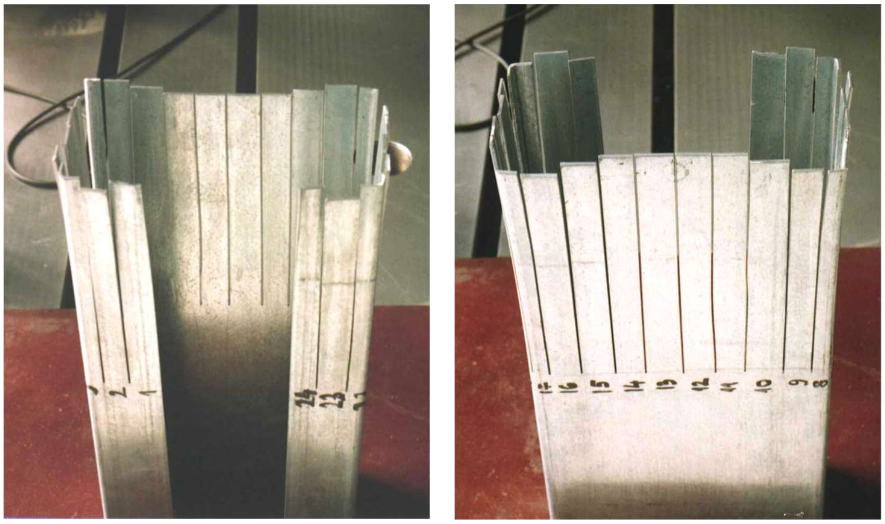


Figure 89. Evidence of flexural residual stresses in a lipped channel cold-formed steel section

12.2. Buckling Strength of Cold-formed Members

- Steel sections may be subject to one of four generic types of buckling, namely **local**, **distortional**, **global**, and **shear**.
- **Local buckling** is particularly prevalent in cold-formed sections and is characterised by the relatively short wavelength buckling of individual plate element.
- **Distortional buckling**, as the term suggests, is buckling which takes place as a consequence of distortion of the cross-section. In cold-formed sections, it is characterised by relative movement of the fold-lines. The wavelength of distortional buckling is generally intermediate between that of local buckling and global buckling. It is a consequence of the increasing complexity of section shapes that local buckling calculation are becoming more complicated and that distortional buckling takes on increasing importance.
- The term "**global buckling**" embraces Euler (flexural) and lateral-torsional buckling of columns and lateral buckling of beams. It is sometimes termed "rigid-body" buckling because any given cross-section moves as a rigid body without any distortion of the cross-section.

- **Local** and **distortional buckling** can be considered as "sectional" modes, and they can interact with each other as well as with global buckling
- Next figure shows single and interactive (coupled) buckling modes for a lipped channel section in compression. The results have been obtained using an elastic eigenbuckling FEM analysis. For given geometrical properties of member cross-section, the different buckling modes depend by the buckling length

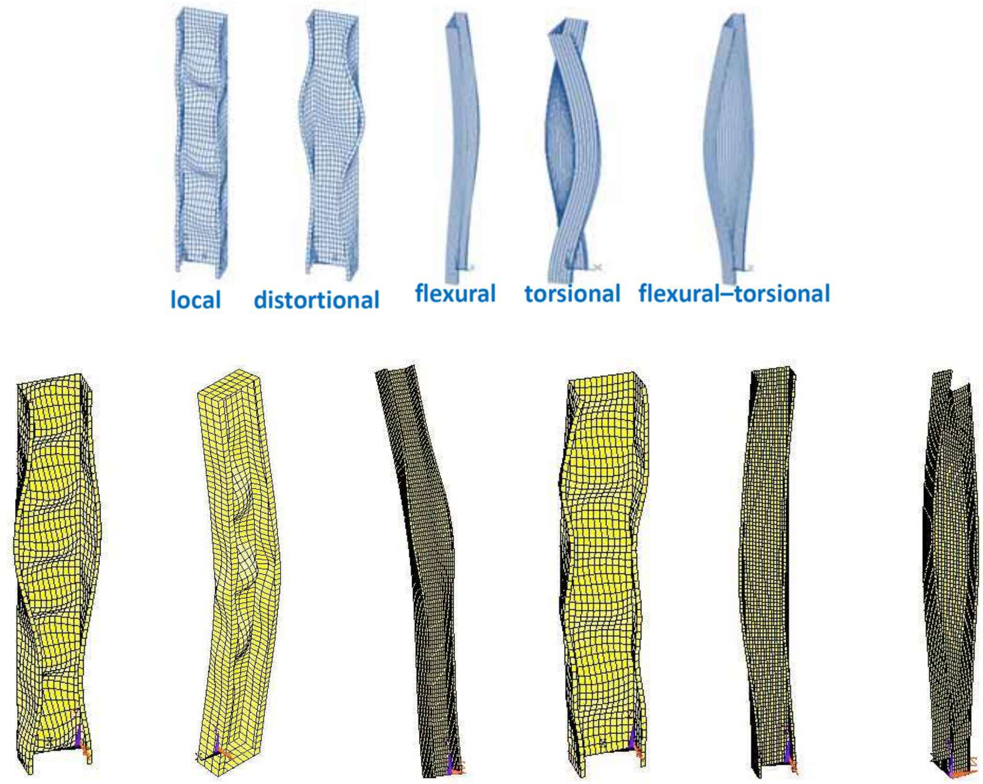


Figure 90. Interactive buckling modes for a lipped channel in compression

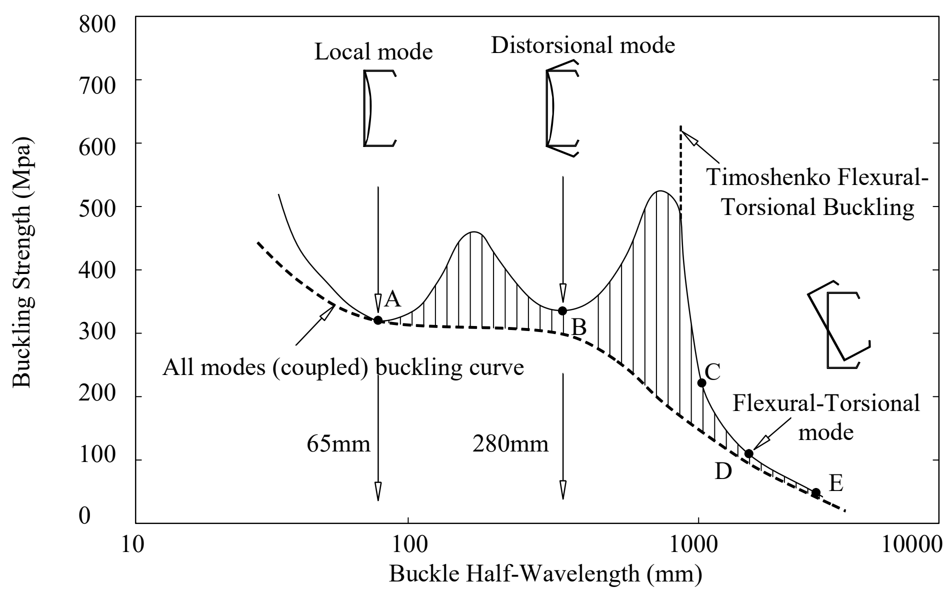


Figure 91. Buckling strength versus half-wavelength for a lipped channel in compression (Hancock)

Effects of buckling modes interaction

- In case of thin-walled bar the sectional buckling, e.g. local or distortional buckling, occurs prior to the initiation of plastification.
- Sectional buckling is characterised by the stable post-critical path and bar does not fail when it occurs, but significantly lose from its stiffness.
- The yielding starts at the corners of cross-section, just before the failure of the bar, when sectional buckling changes into local plastic mechanism quasi-simultaneously with global buckling occurrence.
- In fact, when sectional buckling phenomenon occur prior to global buckling – for very slender members, no local or distortional modes can appear before flexural or flexural-torsional global modes – a “softening” (e.g. weakening of both capacity and stiffness of the bar), the practical design operates with reduced geometrical characteristics of cross-section, i.e. reduced or effective area, moments of inertia, radius of gyration.
- In Figure 16 are shown the comparison between the buckling curves of a lipped channel member in compression, calculated according to EN 1993-1-3, considering the full effective cross-section (e.g. no local buckling effect) and the reduced (effective) cross-section (e.g. when the local buckling occurs and interacts with global buckling).

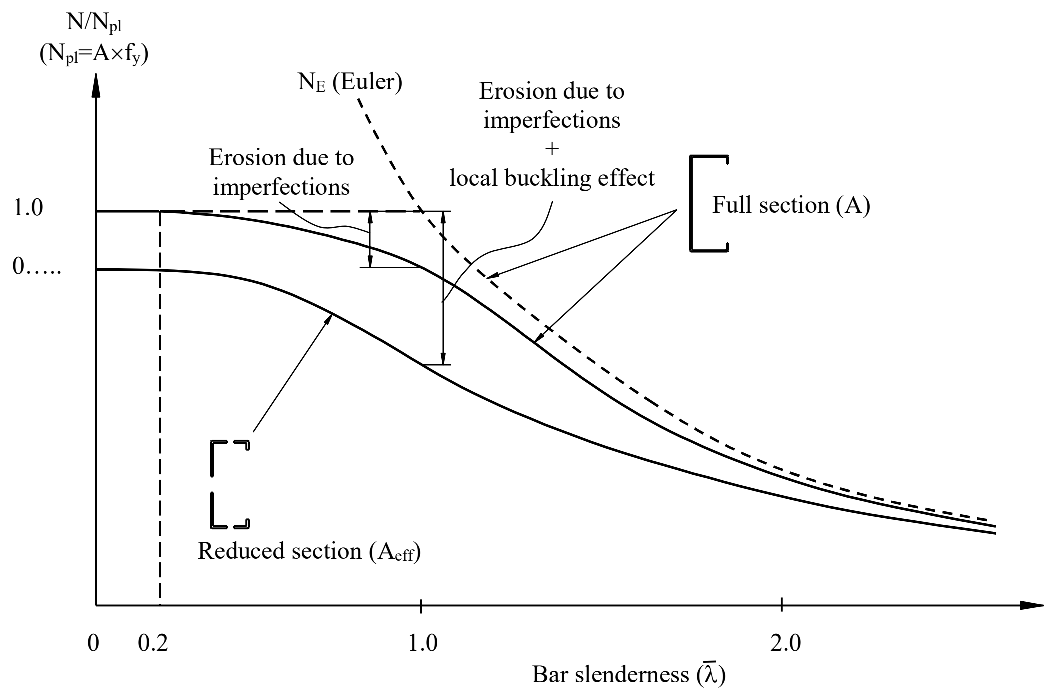


Figure 92. Effect of local buckling on the member capacity

Torsional rigidity

- Cold-formed sections are normally thin and consequently they have a low torsional stiffness.
- Many of the sections produced by cold-forming are mono-symmetric with their shear centre eccentric from their centroid as shown in Figure **93**.a.

- Since the shear centre of a thin-walled beam is the axis through which it must be loaded to produce flexural deformation without twisting, than any eccentricity of the load from this axis will generally produce considerable torsional deformations in a thin-walled beam as shown in Figure **93**.a.
- Consequently beams usually require torsional restraints either at intervals or continuously along them to prevent torsional deformations. Often, this is the case of beams such as Z- and C- purlins which may undergo flexural-torsional buckling because of their low torsional stiffness, if are not properly braced.
- In addition, for columns axially loaded along their centroid axis, the eccentricity of the load from the shear centre axis may cause buckling in the flexural-torsional mode as shown Figure **93**.b at a lower load than the flexural buckling mode also shown in Figure **93**.b. Hence the checking for the flexural-torsional mode of buckling is compulsory in such a case, too.

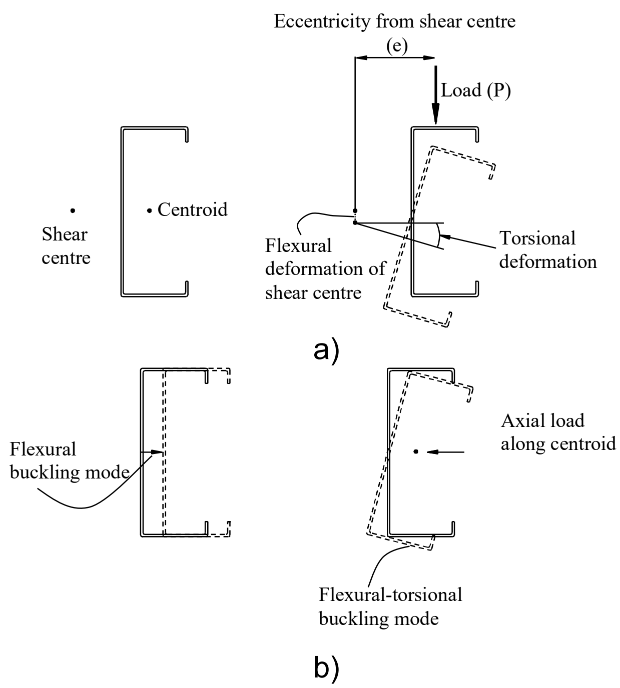


Figure 93. Torsional deformations: a) eccentrically loaded channel beam; b) axially loaded channel column

Web Crippling

- Web crippling at points of concentrated load and supports can be a critical problem in cold-formed steel structural members and sheeting for several reasons. These are:
    1. In cold-formed steel design, it is often not practical to provide load bearing and end bearing stiffeners. This is always the case in continuous sheeting and decking spanning several support points.
    2. The depth-to-thickness ratios of the webs of cold-formed members are usually larger than hot-rolled structural members.
    3. In many cases, the webs are inclined rather than vertical.
    4. The intermediate element between the flange, onto which the load is applied, and the web of a cold-formed member usually consists of a bend of finite radius. Hence the load is applied eccentrically from the web

- Web crippling is really a very peculiar feature of the behaviour of thin-walled cold-formed sections and special design provisions are included in design codes in order to manage this phenomenon

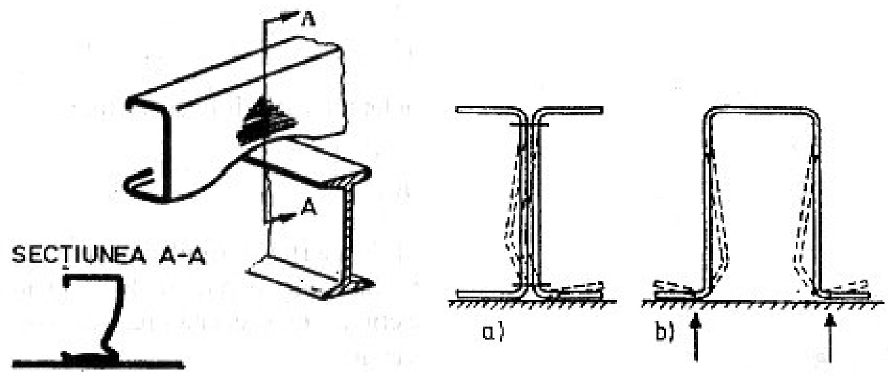


Figure 94. Web crippling at points of concentrated load and supports

Ductility and plastic design

- Due to the sectional buckling phenomena mainly, - cold-formed sections are of class 4 or class 3, at the most, but also due to the effect of cold-forming by stress hardening, the cold-formed steel sections possess a low ductility and are not generally allowed for plastic design.
- Therefore, the previous discussion related to figure 14b revealed the low inelastic capacity reserve for these sections, after the yielding was initiated.
- However, for members in bending, design codes allow to use the inelastic capacity reserve of their cross-section part which is working in tension.
- Moreover, because of their reduced ductility, cold-formed sections cannot dissipate energy in seismic resistant structures.
- Cold-formed sections can be used in seismic resistant structures because there are structural benefits coming from their reduced weight, but only elastic design is allowed and no reduction of shear seismic force is possible.
- Hence, in seismic design, a reduction factor q=1 has to be taken. However, in EUROCODE 8 (EN1998-1), the value of q factor may be taken equal to 1.5 if the structure posses some redundancy (over strength for the elements when some local failures may take place).

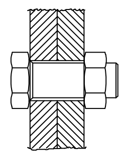
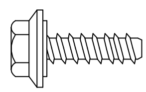
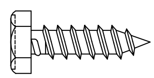
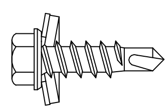
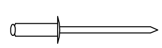
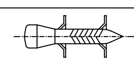
Connections

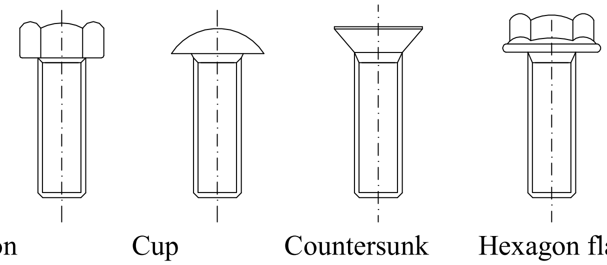
- Because of the reduced wall thickness in cold-formed sections, conventional method for connection used in steel construction, such as bolting and arc-welding are, of course, available but are generally less appropriate and emphasis is on special techniques, more suited to thin materials.
- Long-standing methods for connecting two elements thin material are blind rivets and self drilling, self tapping screws.
- Fired pins are often used to connect thin materials to a ticker supporting member. More recently, press-joining or clinching technology which is very productive requires no additional components and causes no damage to the galvanising or

other metallic coating. This technology has been taken from the automotive industry, but actually it is successfully used in building construction.

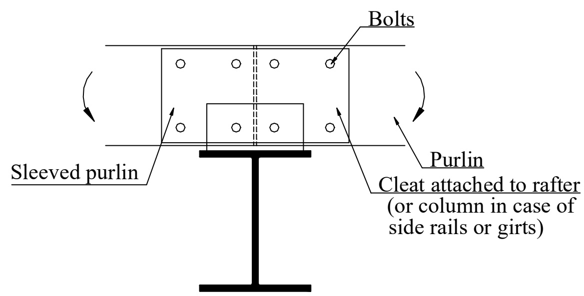
- "Rosette" system is another innovative connecting technology (Makelainen P. and Kesti J., 1999), proper to cold-formed steel structures.
- Therefore, connection technology of cold-formed steel structures is representing one of their particular advantages, both in manufacturing and erection process.

Usual mechanical fasteners for common applications

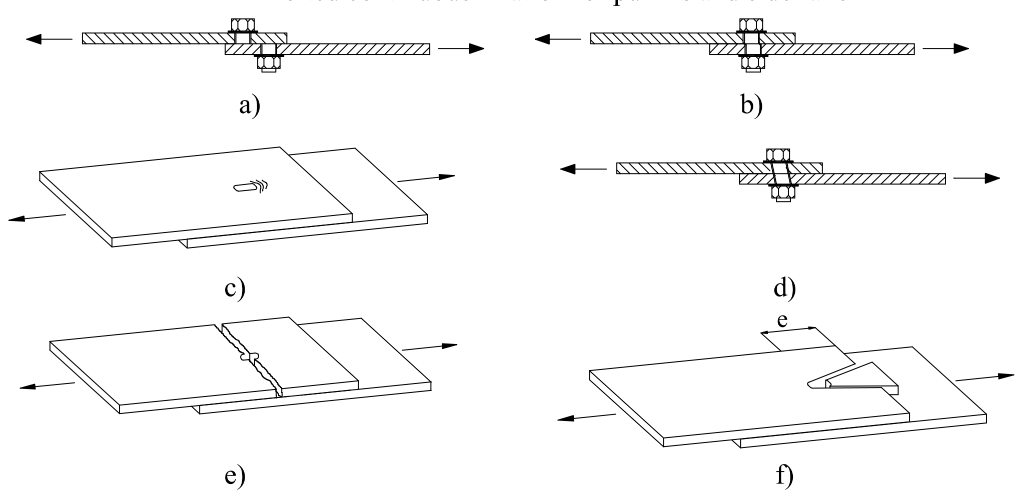
Thin-to-thick	Steel-to-wood	Thin-to-thin	Fasteners	Remark
X		X		Bolts M5-M16
X				Self-tapping screw ϕ6.3 with washer ≥ 16 mm, 1 mm thick with elastomer
	X	X		Hexagon head screw ϕ6.3 or 6.5 with washer ≥ 16 mm, 1 mm thick with elastomer
X		X		Self-drilling screws with diameters: - ϕ4.22 or 4.8 mm - ϕ5.5 mm - ϕ6.3 mm
X				Thread-cutting screw ϕ8 mm with washer ≥ 16 mm, 1 mm thick with or without elastomer
		X		Blind rivets with diameters: - ϕ4.0 mm - ϕ4.8 mm - ϕ6.4 mm
X				Shot (fired) pins
X				Nuts



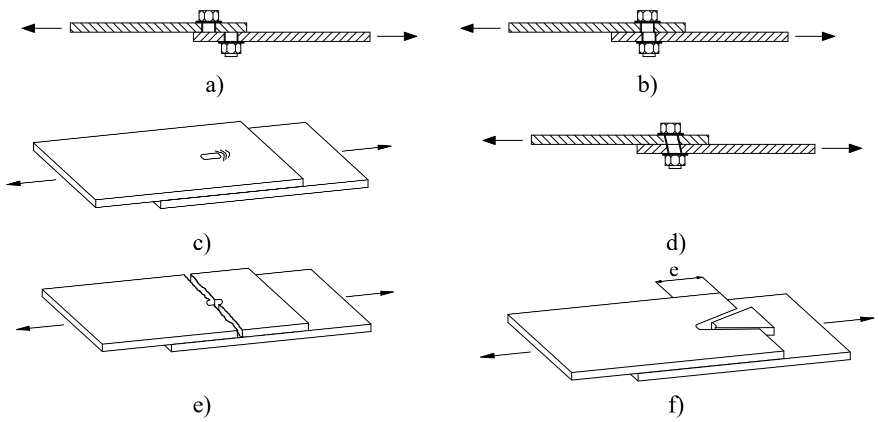
Bolt head shapes



Bolted continuous fixation for purlins and side rails

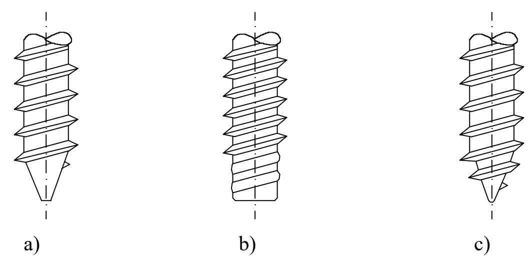


Failure modes for bolted connections in shear

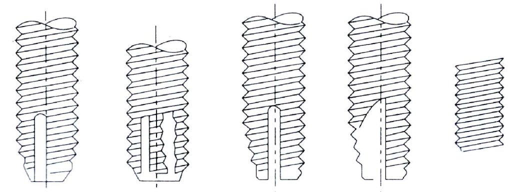


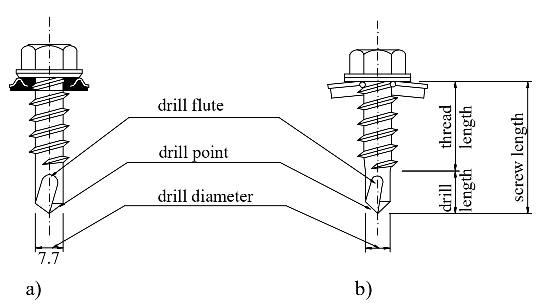
Failure modes for bolted connections in shear

Possible failure modes for bolted connections in tension

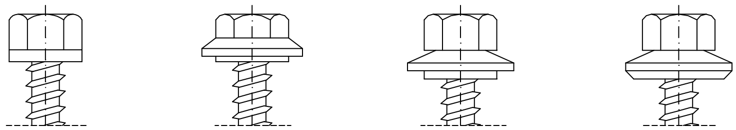


Thread types for thread-forming screws


Thread and points of thread-cutting screws

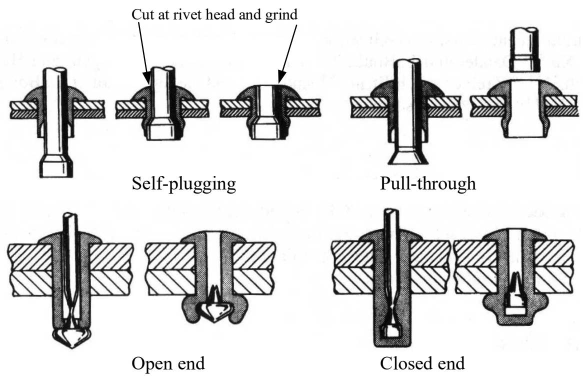


Self-drilling screws: a) drill diameter equal to body diameter for thin-to-thick connections; b) drill diameter smaller than body diameter for thin-to-thin connections

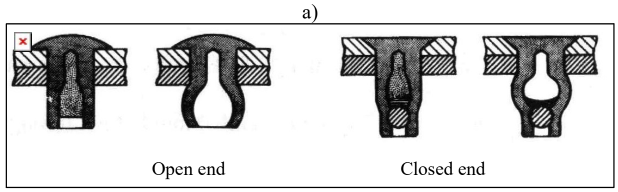


a) b) c) d)

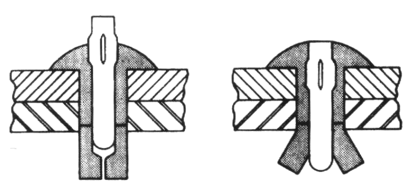
Washers for self-tapping screws: a) metal washer; b) elastomeric washer; c) and d) elastomeric or vulcanized to metal washer



a)

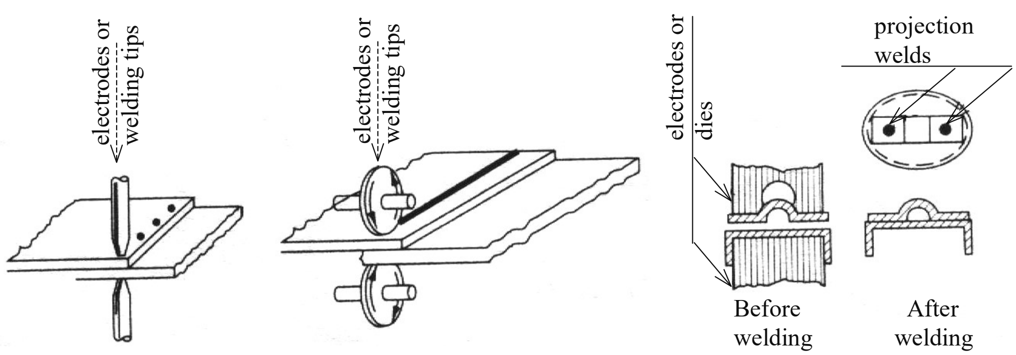


b)



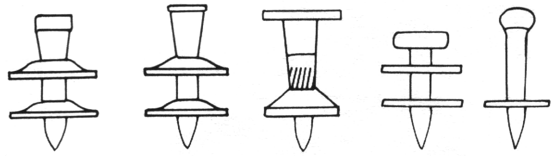
c)

Different types of blind rivets

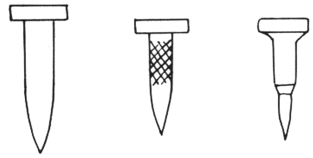


a) b) c)

Resistance welding procedures: a) spot welding; b) seam welding; c) projection welding



Five types of powder actuated fasteners

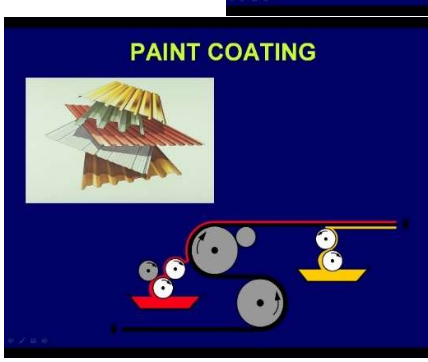


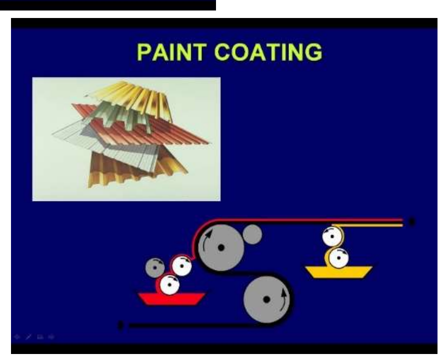
Three types of air driven fasteners

Shot fired pins

Corrosion protection

- The main factor governing the corrosion resistance of cold-formed steel sections is the type and thickness of the protective treatment applied to the steel and not to the base metal thickness.
- Cold-formed steel has the advantage that the protective coatings can be applied to the strip during manufacture and before roll forming.
- Consequently, galvanised strip can be passed through the rolls and requires no further treatment.
- Steel profiles are hot dip galvanised with 275 gram of zinc per square meter (Zn 275), corresponding to a zinc thickness of 20 μm on each side.
- Hot dip galvanised is sufficient to protect the steel profiles against corrosion during the entire life of a building, if it was constructed in the correct manner.
- The most severe effects of corrosion on the steel occur during transport and storage outdoors. When making holes in hot dip galvanised steel framing members, normally no treatment is needed afterwards since the zinc layer a healing effect, i.e. transfers to unprotected surfaces





Applications

- Roof and wall members

Traditionally, a major use of cold-formed steel has been as purlins and side rails to support the cladding in industrial type buildings. These are generally based on the Z section (and its variants) which facilitates incorporation of sleeves and overlaps to improve the efficiency of the members in multi-span applications. Special shapes are made for eaves members etc.

- Steel framing

- Partition walls

- Large panels for housing

Installing of prefabricated units in a single family house

- Floor joists

a)

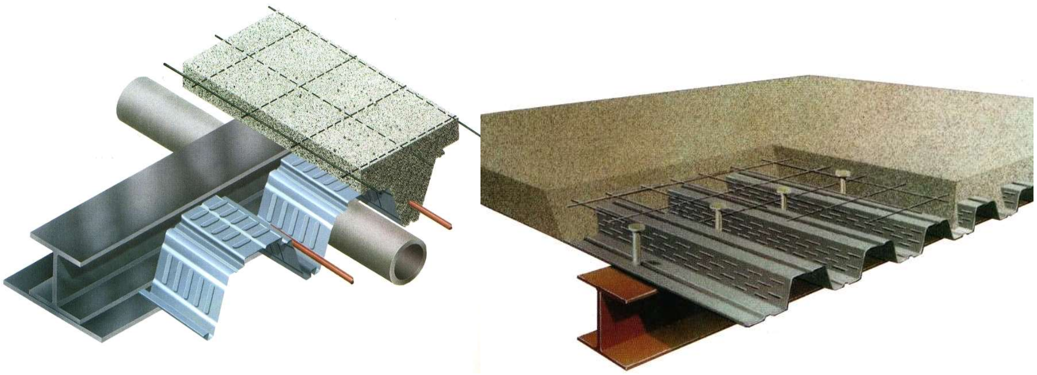
b)

c)

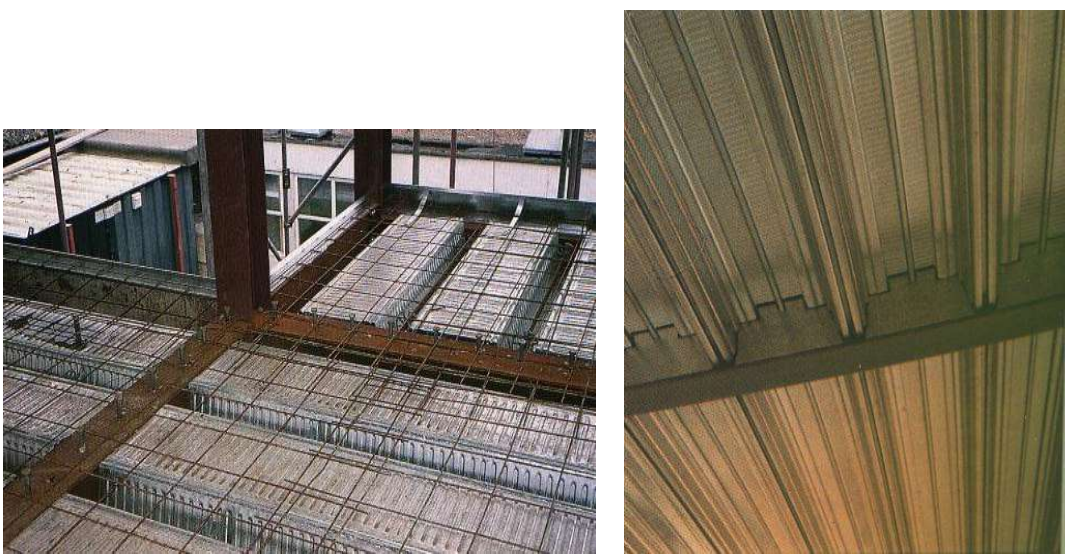
d)

Floor joist:

Floor structure: joist and sheeting on hot rolled steel framing structure



Composite steel concrete floors with sheeting and steel beams



a) b)

Composite steel concrete floor: views of the sheeting positioned on the beams

• Trusses

a)

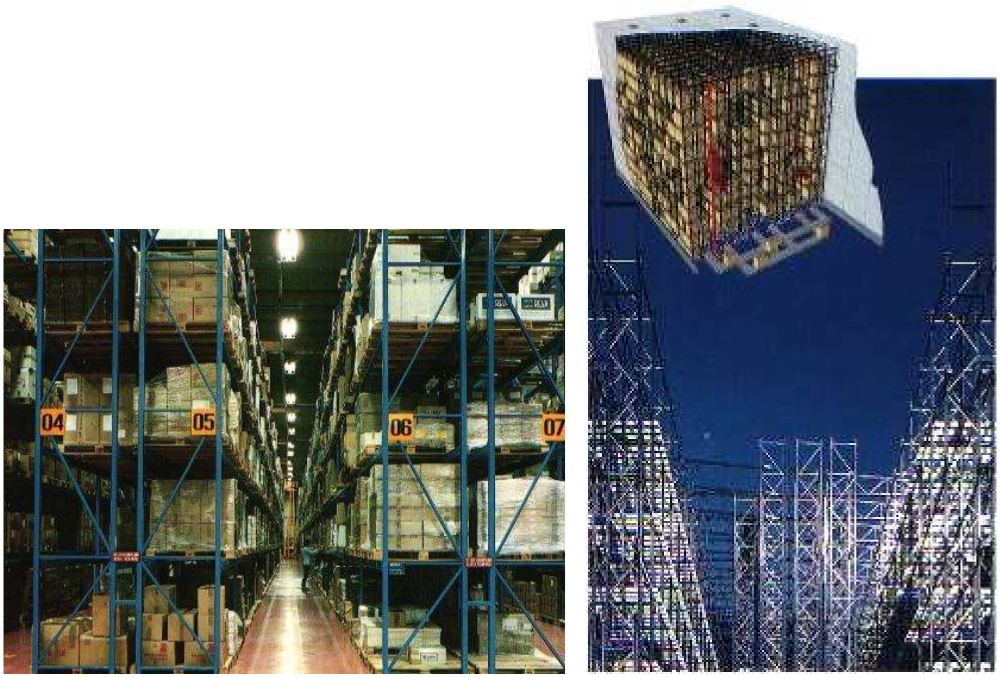
b)

Wall Stud Modular System (WSMS) for small and medium size building facilities using trusses for roof structure and resistance against horizontal actions:

Pitched roof portal frame made by built-up sections (back-to-back C bolted connected by stitches,:

- Storage racking

Storage racking systems for use in warehouses and industrial buildings are made from cold-formed steel perforated sections. Most have special clip attachments, or bolted joints for easy assembly.



Storage rack systems

Member and joint detailing for storage rack systems

- Prefabricated buildings

The transportable prefabricated building unit (such as the ubiquitous site hut) is a common application of the use of cold-formed steel. Other applications are as prefabricated “toilet pod” units in multi-storey buildings.

Prefabricated modular units used in the student residence at the University of Walles, Cardiff

13. PLASTIC ANALYSIS OF STEEL STRUCTURES

13.1. General

- In case of elements subjected to bending in the elastic state, the deformations vary linearly across the cross section height. As the stresses are proportional with deformations, their variation is theoretically linear.
- In the plastic state, the distribution of the stresses across the cross section depends on the σ – ε diagram. For steel with well defined yield plateau, it may be adopted in the practical design the elastic-perfect plastic diagram (Prandtl).

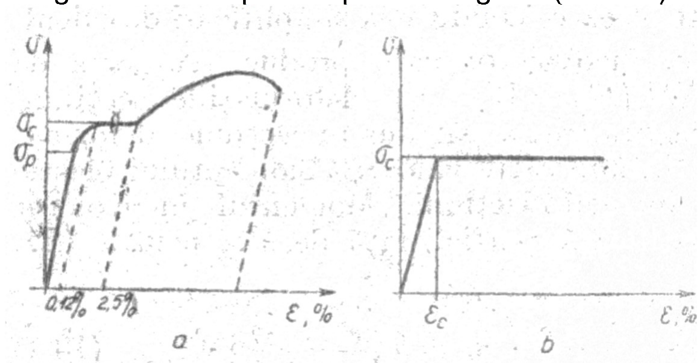


Figure 95. σ – ε diagram for steel with well-defined yield plateau:
a) real; b) conventional (Prandtl)

- For steel with well defined yield plateau, stresses have a linear distribution until they are below the yield stress (at the extreme fiber). At higher loadings the stress remains constant while in the plasticized fibers the deformations are continuously increasing.
- Fibers deformation is lower then those developed in case of pure tension, as the elastic core restrains the deformations of the extreme fibers already plasticized.

Relation between type of analysis and cross-section class:

- The evaluation of internal efforts in the structural elements may be based on an **elastic** or **plastic** global analysis

Ex. - elements in bending

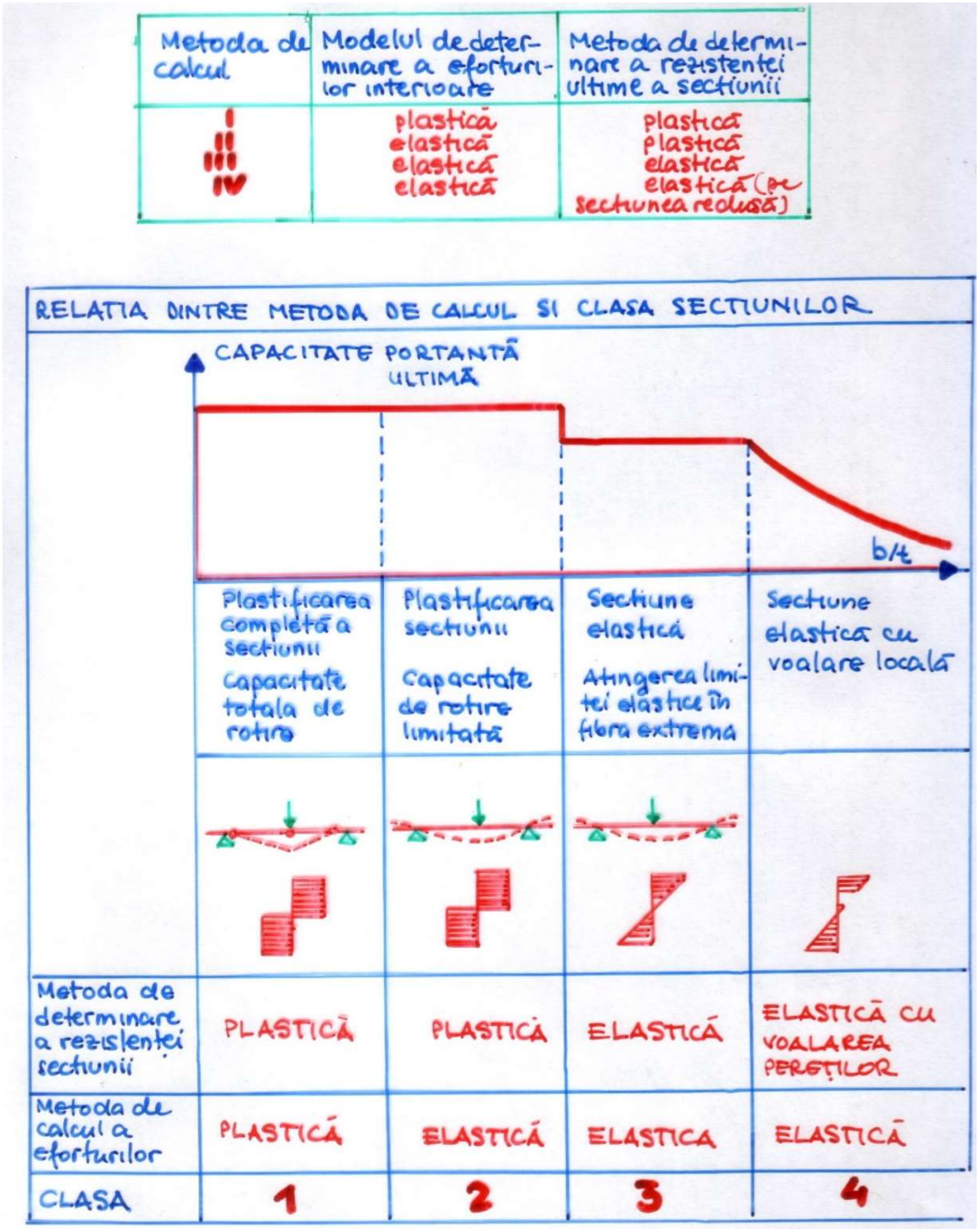
$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1,0 \qquad (6.12)$$

Rezistența de calcul a unei secțiuni transversale supusă la încovoiere în raport cu una din axele principale de inerție se determină astfel:

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl}\, f_y}{\gamma_{M0}} \quad \text{pentru secțiunile Clasa 1 sau 2} \qquad (6.13)$$

$$M_{c,Rd} = M_{el,Rd} = \frac{W_{el,min}\, f_y}{\gamma_{M0}} \quad \text{pentru secțiunile Clasa 3} \qquad (6.14)$$

$$M_{c,Rd} = \frac{W_{eff,min}\, f_y}{\gamma_{M0}} \quad \text{pentru secțiunile Clasa 4} \qquad (.6.15)$$



When **elastic global analysis** is used**,** material behaviour is elastic (Hooke's law is valid) on the entire loading domain. According to EN 1993-1-1, internal efforts must be limited to the plastic resistances of the cross-section (for class 1 and 2) or elastic resistances (for class 3 and 4).

When **plastic global analysis** is used, after the yield stress is attained, the plastic redistribution of the internal efforts on the cross-section and also among different cross-sections is allowed. This leads to the development of plastic hinges until the plastic mechanism is reached (if no other local plastic mechanisms occur – e.g. node mechanism or soft storey mechanism).

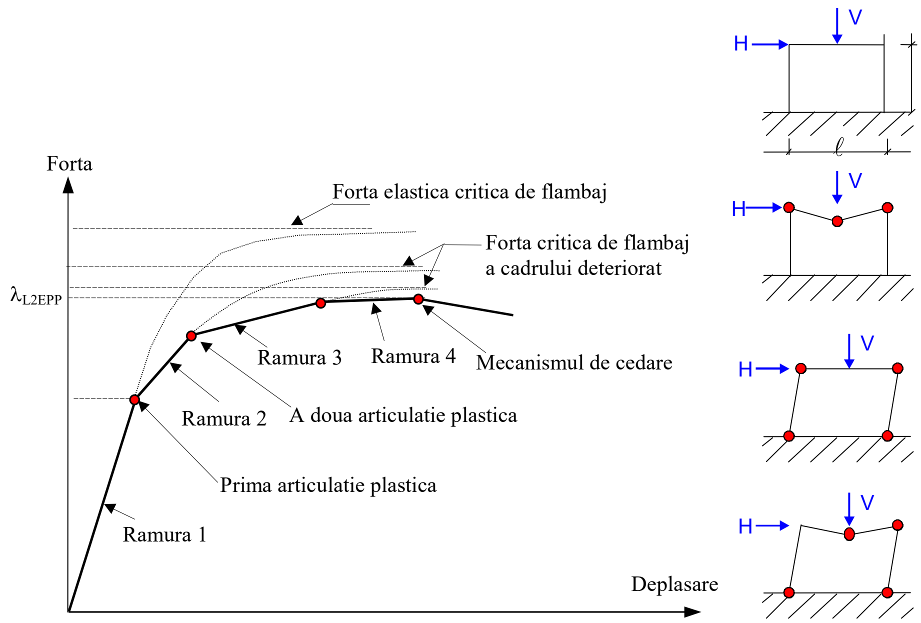


Figure 96. Force – displacement curve in an elastic-perfect plastic analysis

Conditions for the application of plastic analysis:

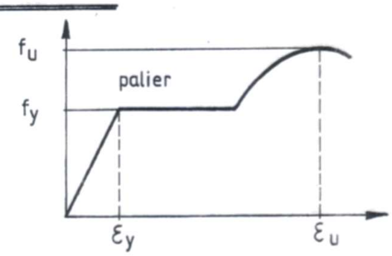
La nivelul secțiunilor susceptibile de plastificare

- Simetria secțiunii în raport cu planul de încărcare
- Capacitate de rotire suficientă
  - ⟹ Secțiuni de clasa 1
  - ⟹ Justificată prin încercări
  - ⟹ Stîlpii satisfac condiția de zveltețe pentru analiză Rigid – Plastică
- Deplasările laterale sunt împiedecate

Materialul din structură este ductil

- elasto - plastic

$f_u / f_y \geq 1,20$



- Alungirea la rupere

$\varepsilon_u \geq 15\%$
$\varepsilon_u \geq 20\,\varepsilon_y$

Solicitare în regim static sau quasi - static

13.2. Elasto-plastic behavior and design of elements

- The extension of member plasticization depends on loading and length of yield plateau
- Until the attainment of the bending moment which induces the yielding in the extreme fiber, denoted as elastic bending moment ($M_{el,Rd} = \frac{W_{el} f_y}{\gamma_{M0}}$), the cross – section and element deformations increase linearly.
- When plasticization develops in the fibers, deformations increases faster and bending moment approaches plastic bending moment $M_{pl,Rd}$.
- Bending moment of partially plasticized rectangular cross-section is given by:

$$M_p = \frac{2bh}{2}\frac{h}{4}f_y - \frac{2bc}{4}\frac{1}{3}\frac{c}{2}f_y = b\left(\frac{h^2}{4} - \frac{c^2}{12}\right)f_y; \tag{133}$$

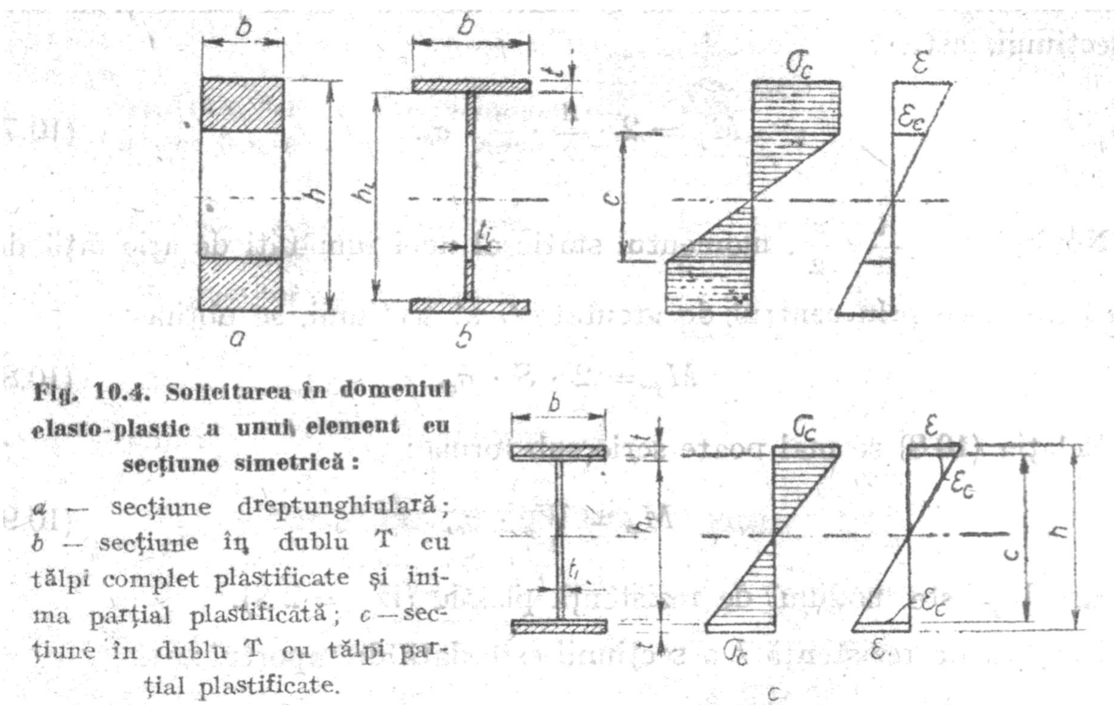


Figure 97. Element with symmetrical cross section in elastic-plastic range

If it is considered c = 0, we obtain the plastic bending moment of the cross-section and plastic section modulus, which, for a rectangular cross-section, have the following relations:

$$M_p = \frac{bh^2}{4} f_y; \; W_p = \frac{bh^2}{4} \tag{134}$$

If we divide the plastic section modulus by the elastic one, we obtain:

$$W_p = \frac{bh^2}{4} = k \times W_e \qquad (135)$$

- In case of a rectangular cross-section, k is equal to 1.5
- In case of an I rectangular cross-section, k is equal to 1.12-1.17
- In case of circular cross-section, k is equal to 1.7
- In case of a tubular cross-section, k is equal to 1.27

When the cross-section is fully plasticized, a plastic hinge develops. This is characterized by large rotations. Unless the mechanical hinge, plastic hinge is loaded with a moment equal to the plastic bending moment M_p

In the next figure the evolution of plasticization of the beam length is presented.

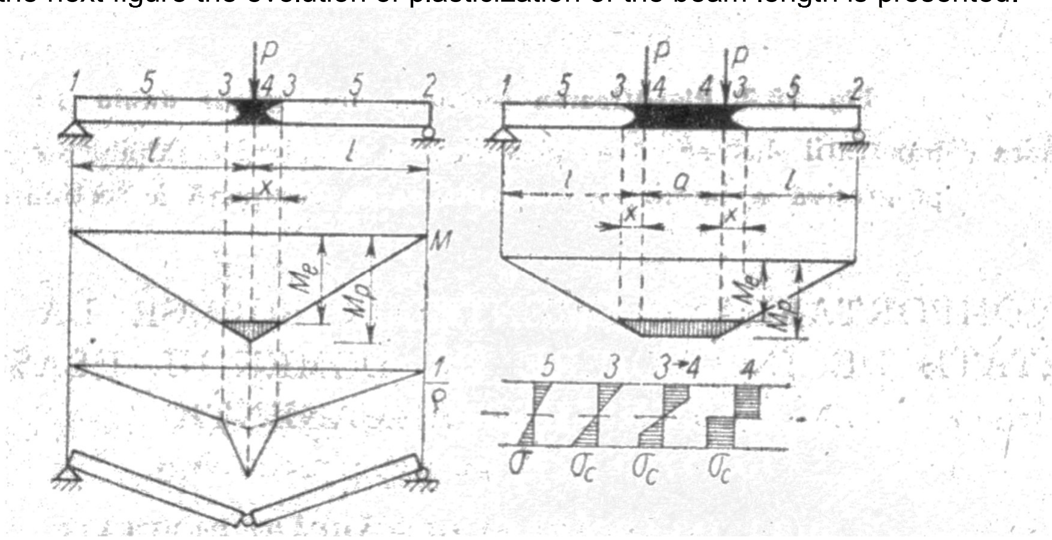


Figure 98. Deformations and stresses after the development of the plastic hinge

13.3. Behavior of elements subjected to plastic cyclic loading

Experimental tests on elements in bending in elastic-plastic range revealed that, when unloading, the element behaves almost entirely elastic, but with a permanent deformation after the load is removed.

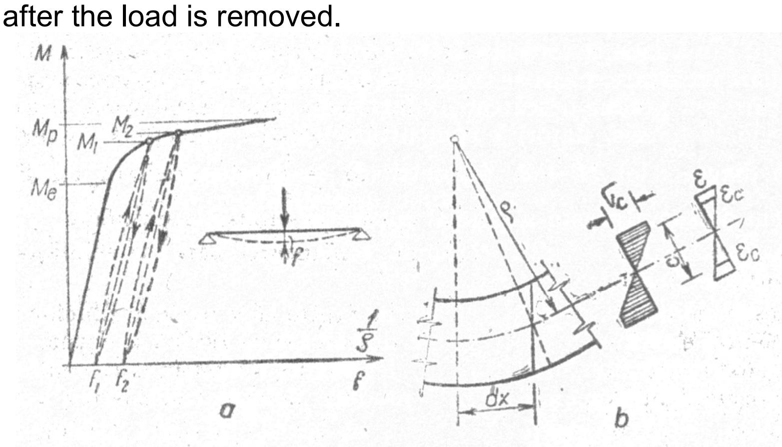


Fig. 10.7. Variația săgeții (*a*) **și a curburii** (*a* și *b*) **în funcție de momentul încovoietor; comportarea la încărcări și descărcări repetate cu formarea buclei de histerezis.**

- When reloading, the element behaves almost elastic until a bending moment equal to the previous moment corresponding to unloading
- Between the two curves takes place the so-called hysteretic loop, due to the imperfect elastic behavior of the steel material
- When unloading, the residual stresses appear on the cross-section. These residual stresses are balanced on the cross section. This phenomenon may be explained by considering a part of the fibers remained in the elastic range and tend, when unloading, to regain the initial shape, while the rest of the fibers suffered permanent deformations.

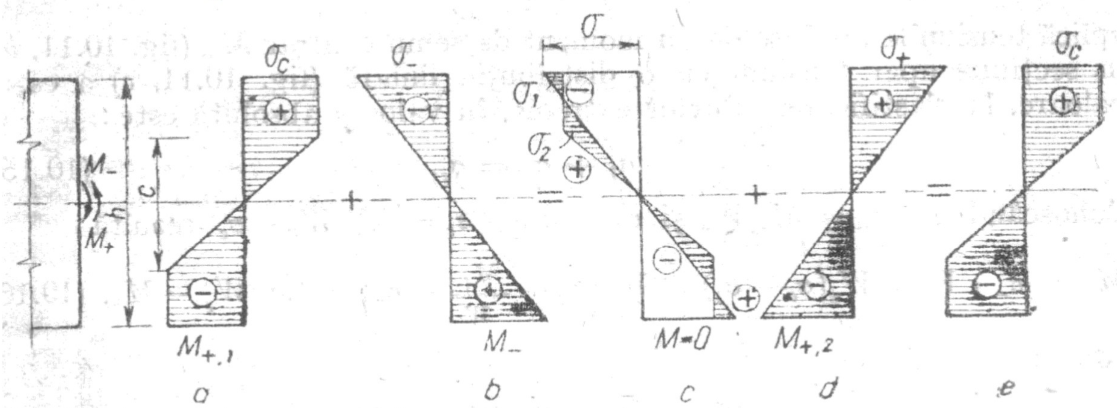


Fig. 10.9. Formarea tensiunilor remanente la acțiunea momentelor de semn contrar:
a, b, c, d, e — etape de analiză.

Behavior of elements that are subjected to opposite bending moments

- Bauschinger effect takes place also in case of elements loaded in plastic range by opposite bending moments.
- They are demonstrated by the reduction of the elastic domain when loaded in the opposite direction, therefore the amplitude of the elastic domain (from M+ to M-) remains constant and is equal to twice the maximum elastic moment ($M_{el} = W_{el}f_y$) of the beam before the first loading.

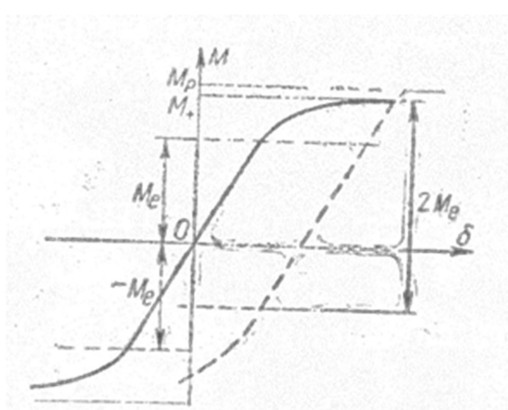


Figure 99. Behavior under reversal cyclic loading – Bauschinger effect

Failure of the elements in bending due to the plastic deformation of the extreme fibers under alternant loading

- In case of alternant loading that produce a large plasticization of the cross-section, the element may fail. This phenomenon is called low cycle fatigue. The experimental tests revealed that, in case of large deformations ($\varepsilon > 0.3\%$), the number of cycles N that leads to failure depends on the amplitude of imposed deformations

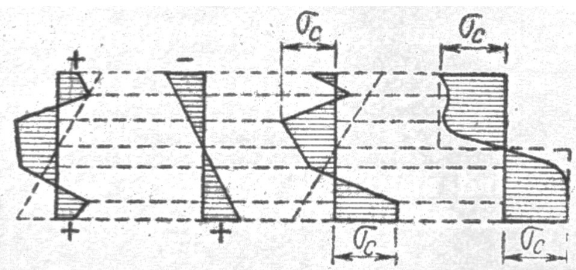
ε[%]	±0,3	±0,5	±1	±2	±5
N [cicluri]	16785	4925	933	177	20

Factors affection the value of plastic bending moment

I. Influence of residual stresses (generated by thermal or mechanical processes)

- In static loading domain, residual stresses do not affect the value of plastic moment, therefore the stress diagram corresponding to plastic moment M_{pl} has the same shape
- However, in the intermediate phases, there may be different stress distributions, which are in balance with the exterior moment.

Fig. 10.12. Efectul tensiunilor remanente asupra formării articulației plastice.



Influence of shear force

- The presence of the shear force in a cross-section leads to a reduction in the plastic resistant moment.
- Condition to achieve the plastification when both σ and τ are present, is given by the following relation (Von Misses criterion), e.g. plane stress state:

$$\sqrt{\sigma^2 + 3\tau^2} = \sigma_c$$

- Simultaneous distribution of σ and τ stresses may consider different shapes – the most closed to real distribution is that corresponding to case a) (both stresses acts on the entire cross section). Due to the difficulties in the design, simplified shapes may be adopted in practice (see figures b) and c).

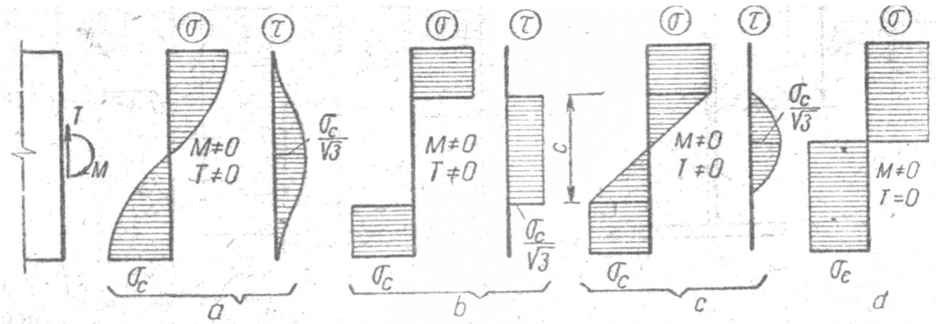


Fig. 10.13. Diferite ipoteze admise privind modul de distribuţie al tensiunilor (a, b, c, d) σ şi τ.

In case described by figure b), M-V ($T \equiv V$ in Eurocode format) interaction formula is given by:

$$\frac{M}{M_p} + \frac{T^2}{T_p^2} = 1$$

In case c), interaction formula M-T is given by:

$$\frac{M}{M_p} + \frac{3T^2}{4T_p^2} = 1$$

In general, the influence of the shear force is reduced. The experimental results confirmed that, as long as the shear force may be taken by the entirely plasticized web, the influence of the value of plastic moment may be neglected.

Influence of axial force

- When axial force is present in a cross-section, plastic deformations will be composed by the deformation ε_N and the rotation φ which correspond to N and M.

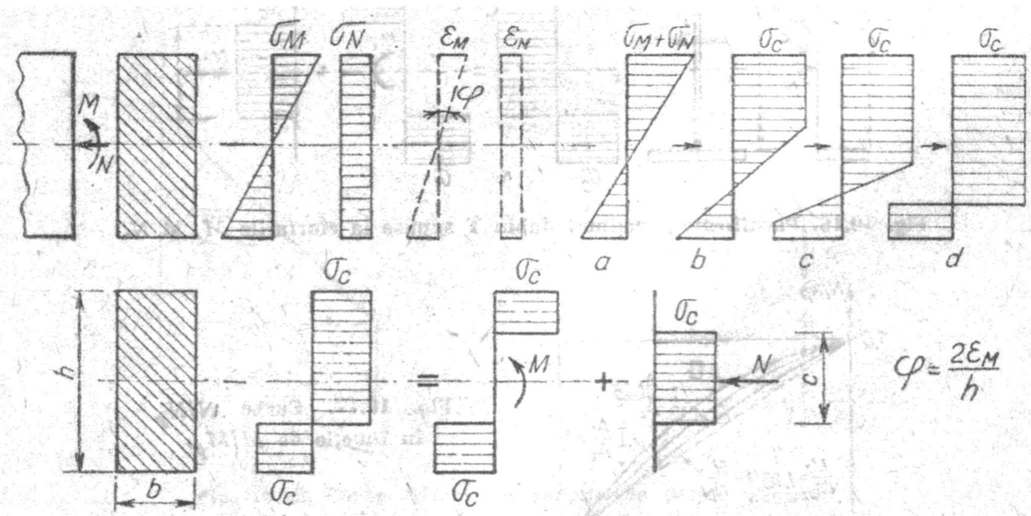


Fig. 10.15. Plastificarea secţiunilor dreptunghiulare supuse la eforturile M şi N:
a, b, c, d — etape succesive de pătrundere a plastificării pe secţiune.

In case of a rectangular cross-section, stress diagram in the plastic stage is composed by a part that forms a couple (produced by M) and another one, produced by axial force N. Finally, it is obtained:

$$N = c \cdot b \cdot \sigma_c; \; M = b\frac{h-c}{2} \cdot \frac{h+c}{2} \cdot \sigma_c = \left(\frac{bh^2}{4} - \frac{bc^2}{4}\right)\sigma_c.$$

$$M = M_p - \frac{N^2}{4b\sigma_c}.$$

If we note $N_p = b \times h \times \sigma_c$, previous equation becomes:

$$\frac{M}{M_p} + \frac{N^2}{N_p^2} = 1.$$

In case of an I cross section, for which c does not exceed the web height (see next figure), we obtain:

$$M_x = M_{p,x} - \frac{t_i c^2}{4}\sigma_c \text{ and } \frac{M_x}{M_{p,x}} = 1 - \frac{t_i c^2}{4W_{p,x}}\sigma_c$$

$$\frac{N}{N_p} = \frac{t_i c \sigma_c}{A\sigma_c} = \frac{t_i c}{A} \text{ and, therefore } c = \frac{N}{N_p}\frac{A}{t_i}$$

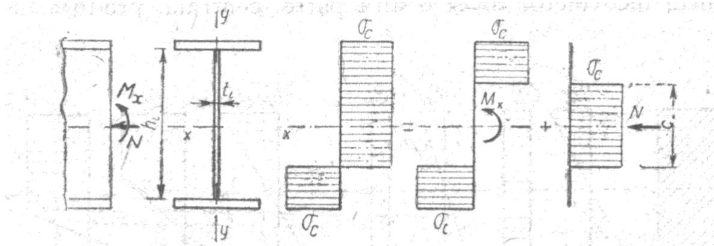


Fig. 10.16. Plastificarea secțiunii dublu T supuse la eforturile M_x și N.

➔

$$\frac{M_x}{M_{p,x}} = 1 - \left(\frac{N}{N_p}\right)^2 \frac{A^2}{4t_i W_{p,x}}.$$

- In the next figure N/N_p – M/M_p curves for two types of cross-sections are plotted:

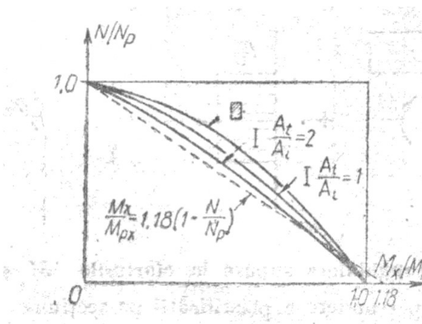


Fig. 10.17. Curbe N/N_p în funcție de M/M_p.

If $N/N_p < 0.15$, the influence of axial force may be neglected, if axial force N is taken by a reduced area of the web *c* and does not significantly affect the bending moment.

If ration $N/N_p > 0.15$, then relation (11) may be replaced by:

$$\frac{M_x}{M_{p,x}} \approx 1{,}18\left(1-\frac{N}{N_p}\right),$$

$$M_x = 1{,}18\, W_{p,x}\left(\sigma_c - \frac{N}{A}\right).$$

Interaction relation for bending along the major y-y axis (13) is presented in the next figure. This relation covers most practical applications.

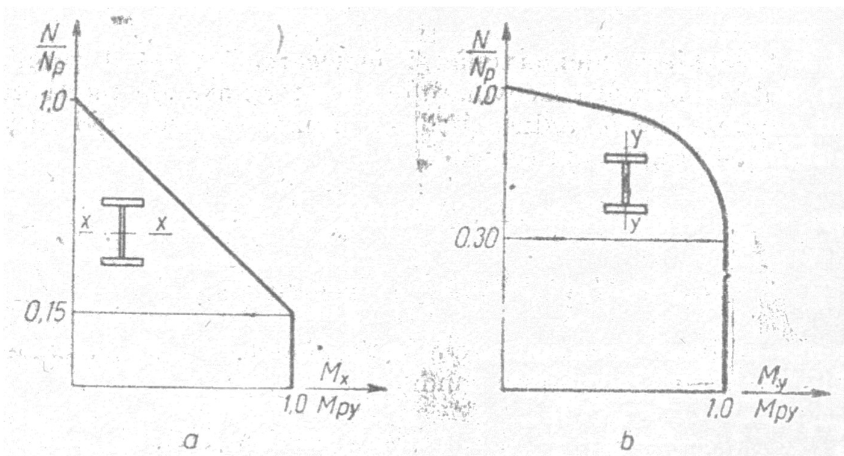


Fig. 10.18. Curbe M/M_p **în funcție de** N/N_p **pentru:**
a — încovoiere față de axa de rigiditate maximă; *b* — încovoiere față de axa de rigiditate minimă.

If bending is along the minor z-z axis, is considered the web is not participating to the bending resistance
Therefore, if:

$$0 \le \frac{N}{N_p} \le \frac{A_i}{A}, \text{ then } \frac{M_y}{M_{p,y}} = 1$$

If $\frac{A_i}{A} \le \frac{N}{N_p} \le 1$, the part from $\frac{N}{N_p}$ which is above $\frac{A_i}{A} \ge 0.30$ reduces the plastic moment of the flanges and instead of linear distribution we will have a parabolic distribution given by the following formula:

$$\frac{M_y}{M_{p,y}} = 1 - \left[\frac{N}{N_p} - \frac{A_i}{A}\right]^2 : \left[1 - \frac{A_i}{A}\right]^2.$$

Plastic deformation capacity under seismic loading

- When steel structures are subjected to strong seismic motions, is some sections the stresses reach the plastic stage and plastic hinges develop. These plastic deformations (plastic rotations) contribute to the dissipation of the seismic energy. By considering this dissipation, it is possible to reduce the design seismic forces. This approach is not possible in case of low dissipative structures, with slender members or sections (class 3 or 4 cross-section).

Ductile behavior

Brittle behavior

- Seismic resistant buildings can be designed according to two concepts:
  a) Dissipative behavior – a part of the seismic energy is taken by plastic deformations in some sections of the structure. Structures designed according to this concept (a) should belong to ductility class medium M or high H.
  b) Low dissipative behavior – stress and deformation distribution is evaluated based on elastic analysis.

Design concept		Behavior factor q	Required ductility class
a	High dissipative structures	$q \geq 4,0$	H (high)
	Medium dissipative structures	$2,0 \leq q < 4,0$	M (medium)
b	Low dissipative structures	$q = 1,0$	L (low)

Dissipative design concept

- In order to design a building according to the dissipative concept, the structure should satisfy the requirements related to the local and global ductility. Thus, local ductility of the members is controlled by limiting the class section and by using steels with good plastic behavior. According to P100/2013 provisions, members should satisfy the following requirements:

- ratio between tensile strength "f_u" and minimum yield strength "f_y" should be at least 1.20 and the elongation at rupture should be at least 20%. Steel used in dissipative structural members should have a distinctive yield plateau and a specific elongation at the end of the plateau of at least 1.5%.
- The correlation between global dissipation capacity (ductility class), expressed by the behavior factor q and local ductility, express by class cross-sections is indicated in the next table.

q factor expresses the capacity of the structure to dissipate energy. This depends on the ratio between horizontal elastic seismic force (which leads to the development of the first plastic hinge) and that corresponding to the development of the collapse mechanism, α_u/α_1

- Moreover, depending on the structural typology, the seismic code requires some other properties for the potential plastic zones. For instance, in case of moment resisting frames, in the potential plastic zones the plastic moment and the plastic deformation capacity shouldn't be reduced in the presence of axial or shear efforts. In order to fulfill this requirement, the following ratio should be verified:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1{,}0$$

$$\frac{N_{Ed}}{N_{pl,Rd}} \leq 0{,}15$$

$$\frac{V_{Ed}}{V_{pl,Rd}} \leq 0{,}5$$

where:

N_{Ed}, M_{Ed}, V_{Ed} are the design internal efforts (axial force, shear force and bending moment) from the combination which includes the seismic action
$N_{pl,Rd}$, $M_{pl,Rd}$, $V_{pl,Rd}$ are the design plastic resistances of the cross sections

14. FATIGUE STRENGTH OF STRUCTURAL STEEL ELEMENTS (EN 1993-1.9)

- Elements of steel structures subjected to many cycles of loading may fail during their service life, at lower stress values then the resistances.
- After a certain number of load fluctuations, the accumulated damage causes the initiation and subsequent propagation of a crack, or cracks.
- This failure mode, which is generated by the development of one or several cracks under many cycles of loading is called fatigue.

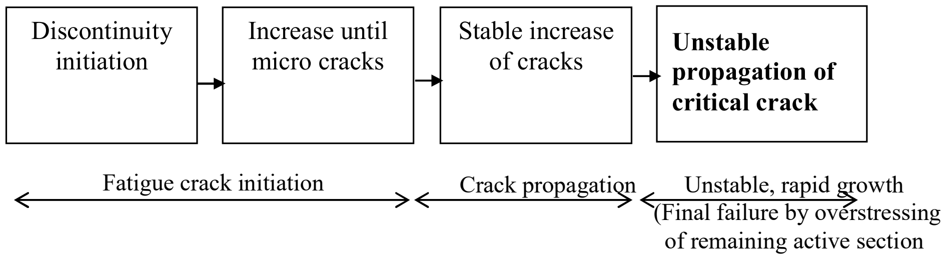


Figure 100. Phases of fatigue failure

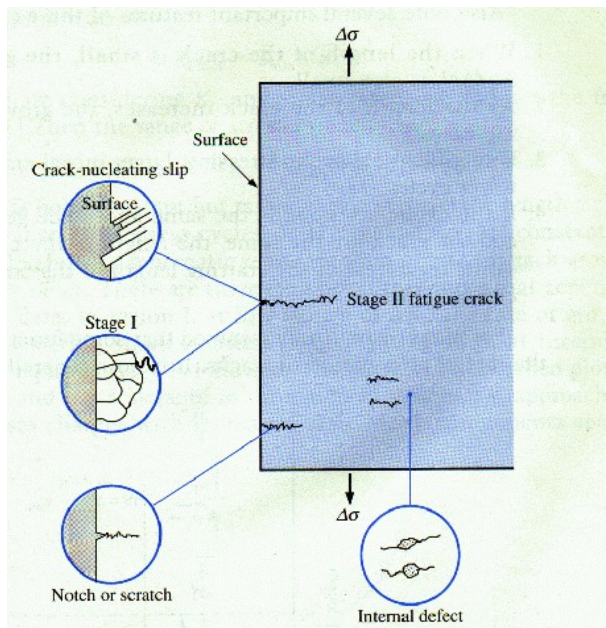


Figure 101. Demonstration of Crack Propagation Due to Fatigue

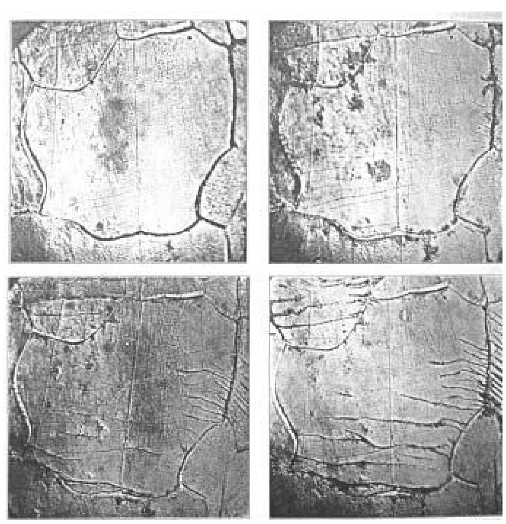


Figure 102. Micrographs showing how surface fatigue cracks grow as material is further loaded

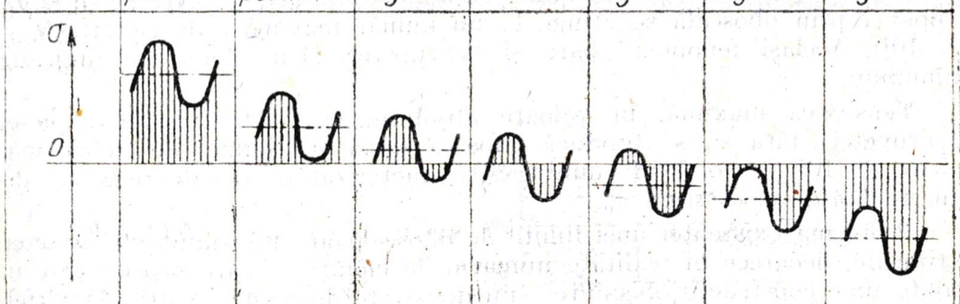
Behavior of material under repeated loads and the fatigue phenomenon may appear in two different modes:

- **high cycle fatigue** is the failure mechanism of many materials whereby a variable stress causes a crack to grow in the material. As the crack grows, the bulk of the material can not carry the imposed load and the crack grows faster until the material suddenly fails. It is considered to develop after $N_f>10_5$ cycles (sometimes $>10^2$ to 10^4 cycles). As the deformations and stresses are in elastic range, the process can be controlled both in stresses and in deformation
- where the stress is high enough for plastic deformation to occur, the account in terms of stress is less useful and the strain in the material offers a simpler description. **Low cycle fatigue** is the general term for fewer than a few thousand stress cycles to failure.

14.1. High cycle fatigue

Depending on the type of variation in time, there are several types of variable loads, represented by load cycle (see table 1).

Table 1. Types of cyclic loads

	Cicluri ondulante pozitive		Cicluri alternante			Cicluri ondulante negative	
	Cicluri asimetrice pozitive			Ciclu simetric	Cicluri asimetrice negative		
	Ciclu ondulant pozitiv	Ciclu pulsant pozitiv	Ciclu alternant negativ		Ciclu alternant negativ	Ciclu pulsant negativ	Ciclu ondulant negativ
σ							
σ_m	>0	$\frac{1}{2}\sigma_{max}>0$	>0	0	<0	$\frac{1}{2}\sigma_{min}<0$	<0
$\lvert\sigma_m\rvert$	$>\lvert\sigma_a\rvert$	$\lvert\sigma_a\rvert$	$<\lvert\sigma_a\rvert$	0	$<\lvert\sigma_a\rvert$	$\lvert\sigma_a\rvert$	$>\lvert\sigma_a\rvert$
σ_{max}	>0	>0	>0	>0	>0	0	<0
σ_{min}	>0	0	<0	<0	<0	<0	<0
σ_a	$\neq 0$	$\lvert\frac{1}{2}\sigma_{max}\rvert$	$\neq 0$	$\sigma_{max}=\sigma_{min}$	$\neq 0$	$\frac{1}{2}\sigma_{min}$	$\neq 0$
ρ	$+1>\rho>0$	0	$0>\rho>-1$	-1	$-1>\rho>\infty$	$\pm\infty$	$+\infty>\rho>+1$

Observatie: Tabelul este valabil și pentru solicitări periodice de forfecare sau răsucire

Beside σ_{max} and σ_{min} stresses, a cycle is also characterized by other parameters, such as:

- Stress amplitude $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$
- Stress range $\Delta\sigma = \sigma_{max} - \sigma_{min} = 2\sigma_a$

- Mean stress $\sigma_{med} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$

- Stress ratio $\rho = \frac{\sigma_{\min}}{\sigma_{\max}}$

Relation between $\sigma_{\max}$ and number of cycles N for fracture may be represented as a $\sigma - N$ curve. This curve is generally called durability curve or Wholer curve.

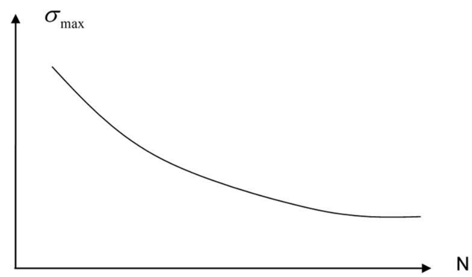


Figure 103. Durability curve for a given ρ

Another possibility to represent the fatigue resistance may use instead of maximum stresses $\sigma_{\max}$ the mean stresses $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$ using a logarithmic scale (**Eroare! Fără sursă de referință.**Figure 104):

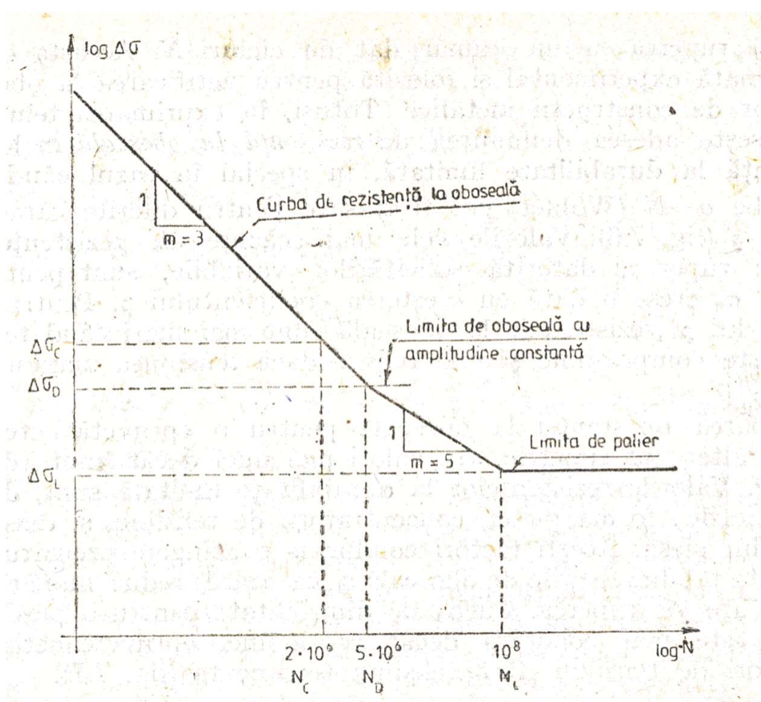


Figure 104**.** Fatigue resistance curve $\Delta\sigma - N$

Factors affecting fatigue resistance (fatigue life):

- Imperfections on the surface of the steel material
- Screw or river holes
- Welding process:
    - o Notch effect, residual stresses
- Quality of steel material

14.2. Low cycle fatigue

In case of low cycle fatigue, the stresses associated to the reduced number of cycles can be large enough to produce plastic deformations and therefore the $\sigma - \varepsilon$ relation is no longer linear but shows hysteretic loops.

Application: Behavior of steel structures under seismic action.

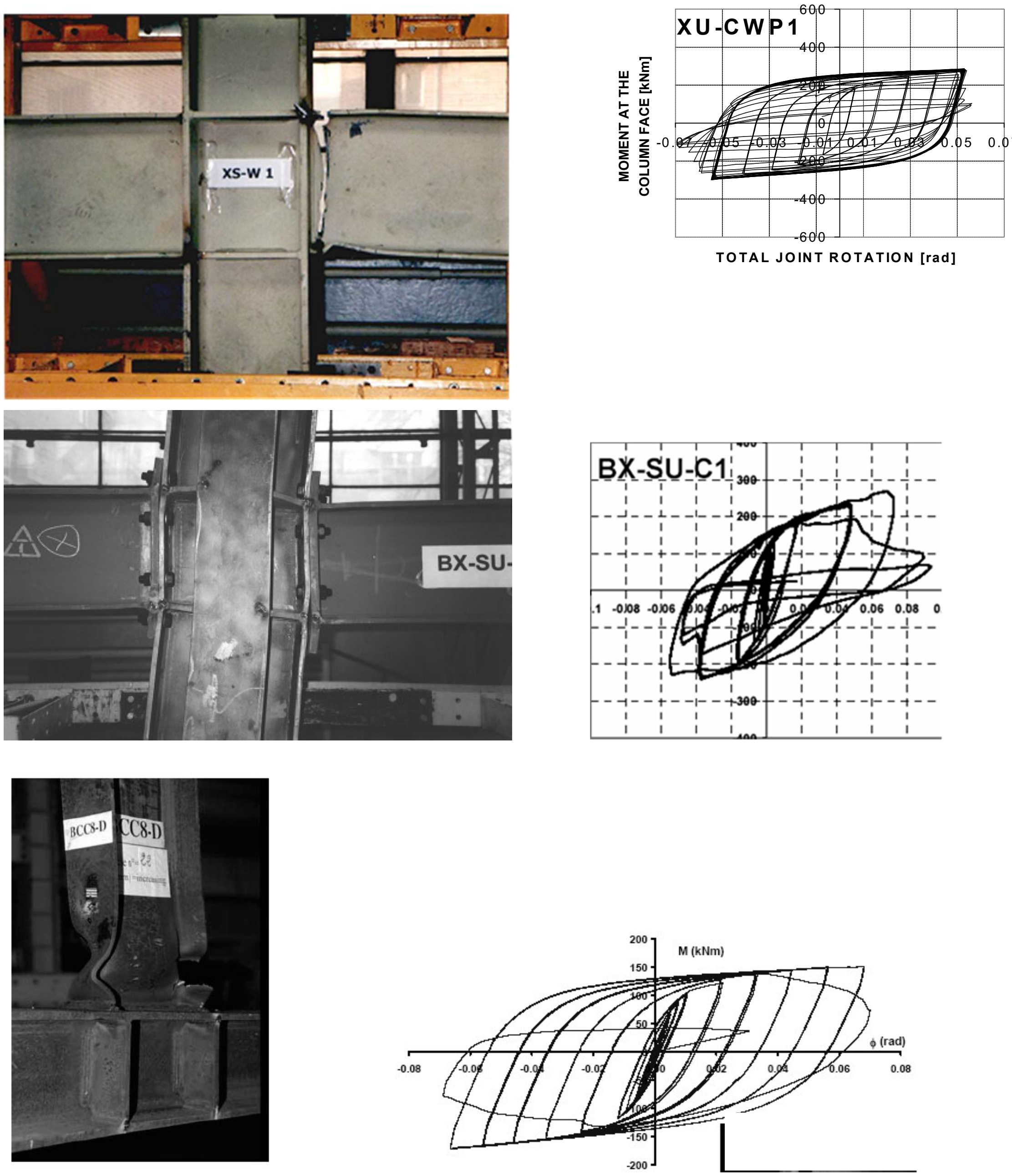


Figure 105. Hysteretic loops for elements and connections (welded and bolted)

7 Fatigue strength

7.1 General

(1) The fatigue strength for nominal stress ranges is represented by a series of (log $\Delta\sigma_R$) – (log N) curves and (log $\Delta\tau_R$) – (log N) curves (S-N-curves), which correspond to typical detail categories. Each detail category is designated by a number which represents, in N/mm², the reference value $\Delta\sigma_C$ and $\Delta\tau_C$ for the fatigue strength at 2 million cycles.

(2) For constant amplitude nominal stresses fatigue strengths can be obtained as follows:

$$\Delta\sigma_R^m\ N_R = \Delta\sigma_C^m\ 2\times10^6 \quad \text{with } m = 3 \ \text{ for } N \leq 5\times10^6$$ **, see**

Figure 7.1

$$\Delta\tau_R^m\ N_R = \Delta\tau_C^m\ 2\times10^6 \quad \text{with } m = 5 \ \text{ for } N \leq 10^8$$, see Figure 7.2

$$\Delta\sigma_D = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_C = 0{,}737\Delta\sigma_C$$ **is the constant amplitude fatigue limit, see**

Figure 7.1, and

$$\Delta\tau_L = \left(\frac{2}{100}\right)^{1/5} \Delta\tau_C = 0{,}457\Delta\tau_C$$ is the cut off limit, see Figure 7.2.

(3) For nominal stress spectra with stress ranges above and below the constant amplitude fatigue limit $\Delta\sigma_D$ the fatigue strength should be based on the extended fatigue strength curves as follows:

$$\Delta\sigma_R^m\ N_R = \Delta\sigma_C^m\ 2\times10^6 \quad \text{with } m = 3 \ \text{ for } N \leq 5\times10^6$$

$$\Delta\sigma_R^m\ N_R = \Delta\sigma_D^m\ 5\times10^6 \quad \text{with } m = 5 \ \text{ for } 5\times10^6 \leq N \leq 10^8$$

$$\Delta\sigma_L = \left(\frac{5}{100}\right)^{1/5} \Delta\sigma_D = 0{,}549\Delta\sigma_D$$ **is the cut off limit, see**

Figure 7.1.

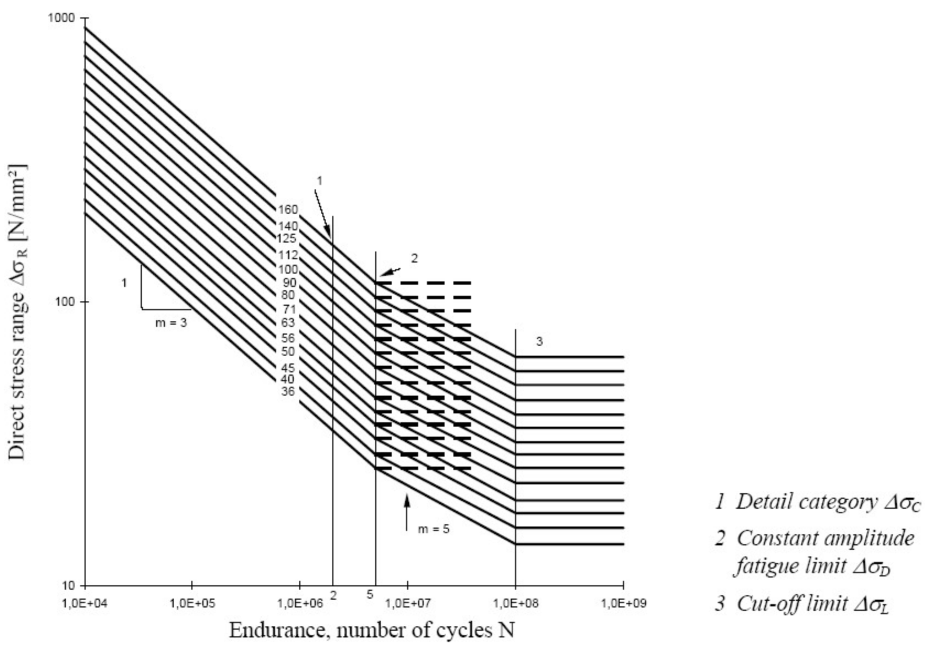


Figure 7.1: Fatigue strength curves for direct stress ranges

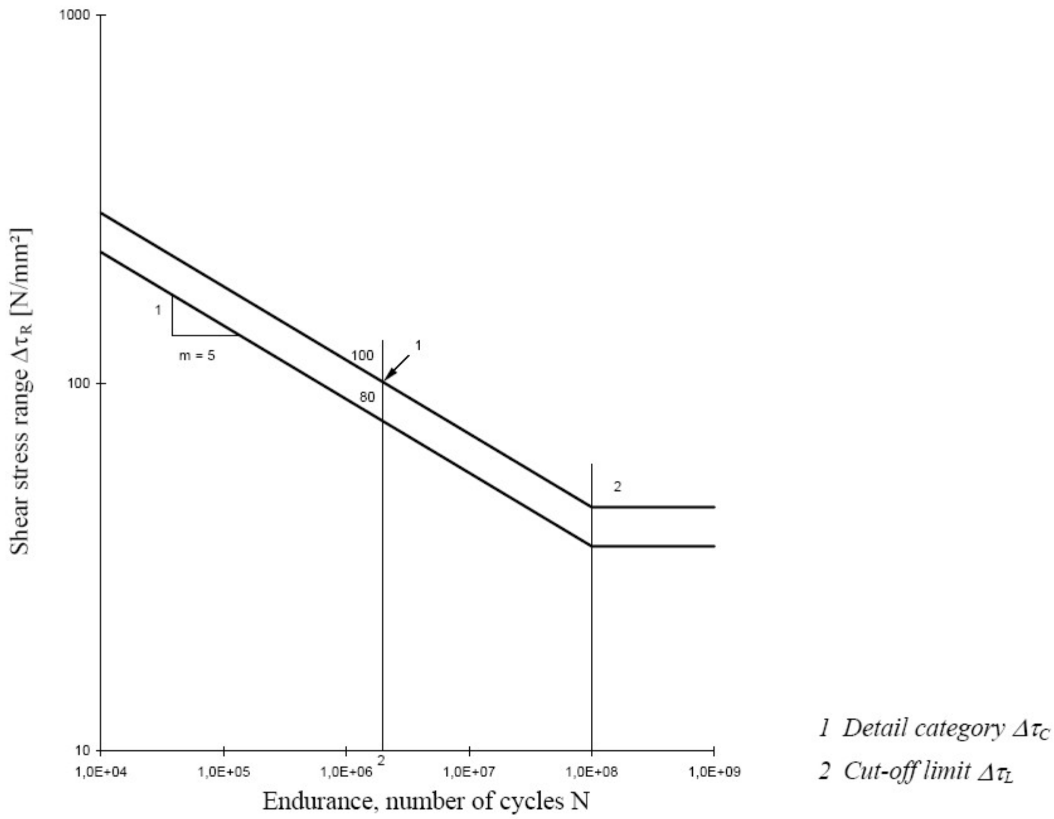


Figure 7.2: Fatigue strength curves for shear stress ranges

NOTE 1 When test data were used to determine the appropriate detail category for a particular constructional detail, the value of the stress range $\Delta\sigma_C$ corresponding to a value of N_C = 2 million cycles were calculated for a 75% confidence level of 95% probability of survival for log N, taking into account the standard deviation and the sample size and residual stress effects. The number of data points (not lower than 10) was considered in the statistical analysis, see annex D of EN 1990.

NOTE 2 The National Annex may permit the verification of a fatigue strength category for a particular application provided that it is evaluated in accordance with NOTE 1.

NOTE 3 **Test data for some details do not exactly fit the fatigue strength curves in**

Figure 7.1. In order to ensure that non conservative conditions are avoided, such details, marked with an asterisk, are located one detail category lower than their fatigue strength at 2×10^6 cycles would require. An alternative assessment may increase the classification of such details by one detail category provided that the constant amplitude fatigue limit $\Delta\sigma_D$ is defined as the fatigue strength at 10^7 cycles for m=3 (see Figure 7.3).

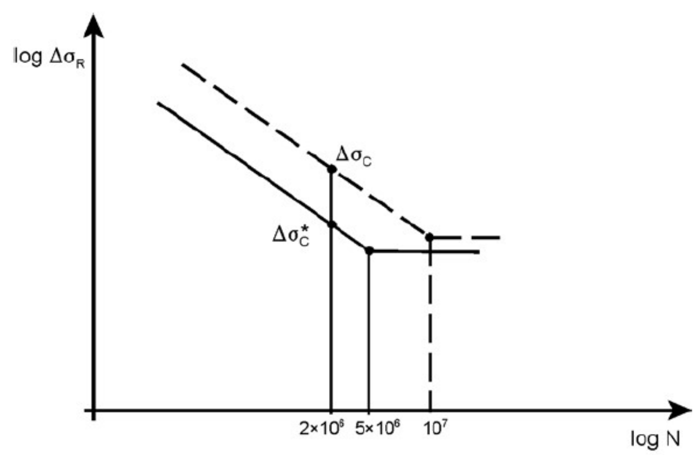


Figure 7.3: Alternative strength $\Delta\sigma_C$ for details classified as $\Delta\sigma_C^*$

(4) Detail categories $\Delta\sigma_C$ and $\Delta\tau_C$ for nominal stresses are given in

Table 8.1 for plain members and mechanically fastened joints

Table 8.2 for welded built-up sections

Table 8.3 for transverse butt welds

Table 8.4 for weld attachments and stiffeners

Table 8.5 for load carrying welded joints

Table 8.6 for hollow sections

Table 8.7 for lattice girder node joints

Table 8.8 for orthotropic decks – closed stringers

Table 8.9 for orthotropic decks – open stringers

Table 8.10 for top flange to web junctions of runway beams

(5) The fatigue strength categories $\Delta\sigma_C$ for geometric stress ranges are given in Annex B.

NOTE The National Annex may give fatigue strength categories $\Delta\sigma_C$ and $\Delta\tau_C$ for details not covered by Table 8.1 to Table 8.10 and by Annex B.

7.2 Fatigue strength modifications

7.2.1 Non-welded or stress-relieved welded details in compression

(1) In non-welded details or stress-relieved welded details, the mean stress influence on the fatigue strength may be taken into account by determining a reduced effective stress range $\Delta\sigma_{E,2}$ in the fatigue assessment when part or all of the stress cycle is compressive.

(2) The effective stress range may be calculated by adding the tensile portion of the stress range and 60% of the magnitude of the compressive portion of the stress range, see Figure 7.4.

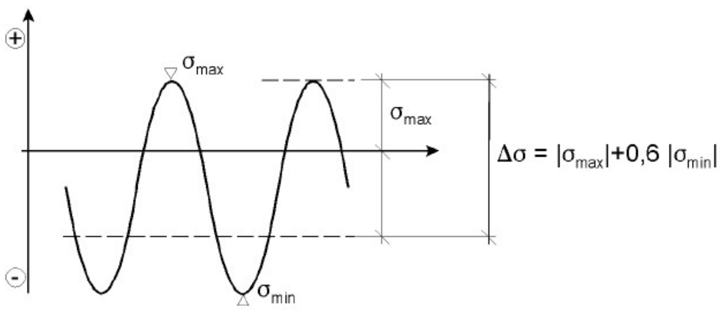


Figure 7.4: Modified stress range for non-welded or stress relieved details

7.2.2 Size effect

(1) The size effect due to thickness or other dimensional effects should be taken into account as given in Table 8.1 to Table 8.10. The fatigue strength then is given by:

$$\Delta\sigma_{C,red} = k_s \Delta\sigma_C \tag{7.1}$$

8 Fatigue verification

(1) Nominal, modified nominal or geometric stress ranges due to frequent loads $\psi_1\ Q_k$ (see EN 1990) should not exceed

$$\begin{aligned} &\Delta\sigma \le 1{,}5\ f_y && \text{for direct stress ranges} \\ &\Delta\tau \le 1{,}5\ f_y / \sqrt{3} && \text{for shear stress ranges} \end{aligned} \tag{8.1}$$

(2) It should be verified that under fatigue loading

$$\frac{\gamma_{Ff}\ \Delta\sigma_{E,2}}{\Delta\sigma_C / \gamma_{Mf}} \le 1{,}0$$

and (8.2)

$$\frac{\gamma_{Ff}\ \Delta\tau_{E,2}}{\Delta\tau_C / \gamma_{Mf}} \le 1{,}0$$

NOTE Table 8.1 to Table 8.9 require stress ranges to be based on principal stresses for some details.

(3) Unless otherwise stated in the fatigue strength categories in Table 8.8 and Table 8.9, in the case of combined stress ranges $\Delta\sigma_{E,2}$ and $\Delta\tau_{E,2}$ it should be verified that:

$$\left(\frac{\gamma_{Ff}\ \Delta\sigma_{E,2}}{\Delta\sigma_C / \gamma_{Mf}}\right)^3 + \left(\frac{\gamma_{Ff}\ \Delta\tau_{E,2}}{\Delta\tau_C / \gamma_{Mf}}\right)^5 \le 1{,}0 \tag{8.3}$$

(4) When no data for $\Delta\sigma_{E,2}$ or $\Delta\tau_{E,2}$ are available the verification format in Annex A may be used.

NOTE 1 Annex A is presented for stress ranges in longitudinal direction. This presentation may be adapted for shear stress ranges.

NOTE 2 The National Annex may give information on the use of Annex A.

Table 8.1: Plain members and mechanically fastened joints

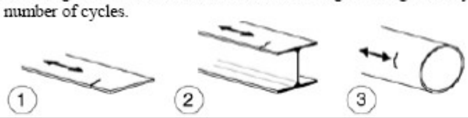
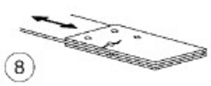
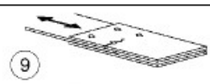
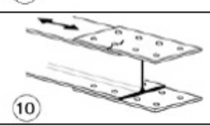
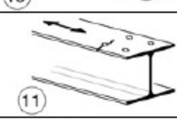
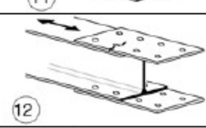
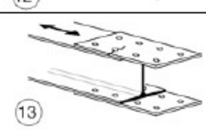
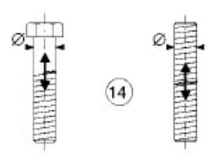
Detail category	Constructional detail	Description	Requirements
160	**NOTE** The fatigue strength curve associated with category 160 is the highest. No detail can reach a better fatigue strength at any number of cycles. 	Rolled and extruded products: 1) Plates and flats; 2) Rolled sections; 3) Seamless hollow sections, either rectangular or circular.	Details 1) to 3): Sharp edges, surface and rolling flaws to be improved by grinding until removed and smooth transition achieved.
140		Sheared or gas cut plates: 4) Machine gas cut or sheared material with subsequent dressing. 5) Material with machine gas cut edges having shallow and regular drag lines or manual gas cut material, subsequently dressed to remove all edge discontinuities. Machine gas cut with cut quality according to EN 1090.	4) All visible signs of edge discontinuities to be removed. The cut areas are to be machined or ground and all burrs to be removed. Any machinery scratches for example from grinding operations, can only be parallel to the stresses. Details 4) and 5): - Re-entrant corners to be improved by grinding (slope ≤ ¼) or evaluated using the appropriate stress concentration factors. - No repair by weld refill.
125			
100 m = 5		6) and 7) Rolled and extruded products as in details 1), 2), 3)	Details 6) and 7): $\Delta\tau$ calculated from: $\tau = \frac{V\,S(t)}{I\,t}$
For detail 1 – 5 made of weathering steel use the next lower category.			
112		8) Double covered symmetrical joint with preloaded high strength bolts. 8) Double covered symmetrical joint with preloaded injection bolts.	8) $\Delta\sigma$ to be calculated on the gross cross-section. 8) ... gross cross-section. For bolted connections (Details 8) to 13)) in general: End distance: $e_1 \geq 1{,}5\ d$ Edge distance: $e_2 \geq 1{,}5\ d$ Spacing: $p_1 \geq 2{,}5\ d$ Spacing: $p_2 \geq 2{,}5\ d$ Detailing to EN 1993-1-8, Figure 3.1
90		9) Double covered joint with fitted bolts. 9) Double covered joint with non preloaded injection bolts.	9) ... net cross-section. 9) ... net cross-section.
		10) One sided connection with preloaded high strength bolts. 10) One sided connection with preloaded injection bolts.	10) ... gross cross-section. 10) ... gross cross-section.
		11) Structural element with holes subject to bending and axial forces	11) ... net cross-section.
80		12) One sided connection with fitted bolts. 12) One sided connection with non-preloaded injection bolts.	12) ... net cross-section. 12) ... net cross-section.
50		13) One sided or double covered symmetrical connection with non-preloaded bolts in normal clearance holes. No load reversals.	13) ... net cross-section.
50	size effect for $t > 30$mm: $k_s = (30/t)^{0,25}$ 	14) Bolts and rods with rolled or cut threads in tension. For large diameters (anchor bolts) the size effect has to be taken into account with k_s.	14) $\Delta\sigma$ to be calculated using the tensile stress area of the bolt. Bending and tension resulting from prying effects and bending stresses from other sources must be taken into account. For preloaded bolts, the reduction of the stress range may be taken into account.

Table 8.1 (continued): Plain members and mechanically fastened joints

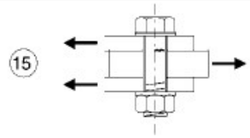
Detail category	Constructional detail	Description	Requirements
100 m=5		Bolts in single or double shear Thread not in the shear plane 15) - Fitted bolts - normal bolts without load reversal (bolts of grade 5.6, 8.8 or 10.9)	15) $\Delta\tau$ calculated on the shank area of the bolt.

Table 8.2: Welded built-up sections

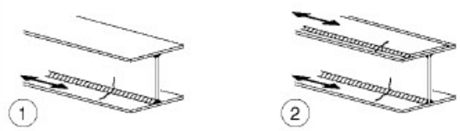
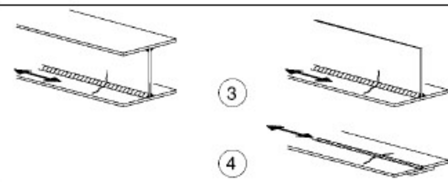
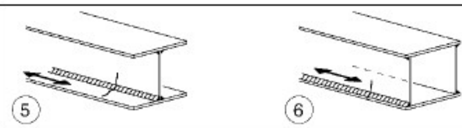
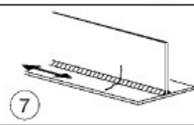
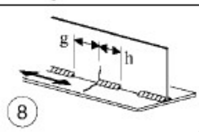
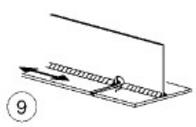
Detail category	Constructional detail	Description	Requirements
125		Continuous longitudinal welds: 1) Automatic butt welds carried out from both sides. 2) Automatic fillet welds. Cover plate ends to be checked using detail 6) or 7) in Table 8.5.	Details 1) and 2): No stop/start position is permitted except when the repair is performed by a specialist and inspection is carried out to verify the proper execution of the repair.
112		3) Automatic fillet or butt weld carried out from both sides but containing stop/start positions. 4) Automatic butt welds made from one side only, with a continuous backing bar, but without stop/start positions.	4) When this detail contains stop/start positions category 100 to be used.
100		5) Manual fillet or butt weld. 6) Manual or automatic butt welds carried out from one side only, particularly for box girders	5), 6) A very good fit between the flange and web plates is essential. The web edge to be prepared such that the root face is adequate for the achievement of regular root penetration without break-out.
100		7) Repaired automatic or manual fillet or butt welds for categories 1) to 6).	7) Improvement by grinding performed by specialist to remove all visible signs and adequate verification can restore the original category.
80	 $g/h \leq 2,5$	8) Intermittent longitudinal fillet welds.	8) $\Delta\sigma$ based on direct stress in flange.
71		9) Longitudinal butt weld, fillet weld or intermittent weld with a cope hole height not greater than 60 mm. For cope holes with a height > 60 mm see detail 1) in Table 8.4	9) $\Delta\sigma$ based on direct stress in flange.
125		10) Longitudinal butt weld, both sides ground flush parallel to load direction, 100% NDT	
112		10) No grinding and no start/stop	
90		10) with start/stop positions	
140		11) Automatic longitudinal seam weld without stop/start positions in hollow sections	11) Free from defects outside the tolerances of EN 1090. Wall thickness $t \leq 12,5$ mm.
125		11) Automatic longitudinal seam weld without stop/start positions in hollow sections	11) Wall thickness $t > 12,5$ mm.
90		11) with stop/start positions	
For details 1 to 11 made with fully mechanized welding the categories for automatic welding apply.			

Table 8.3: Transverse butt welds

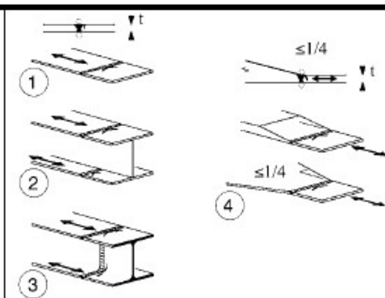
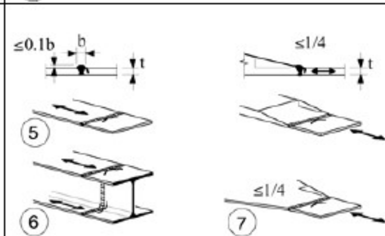
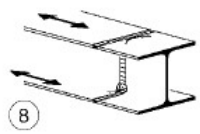
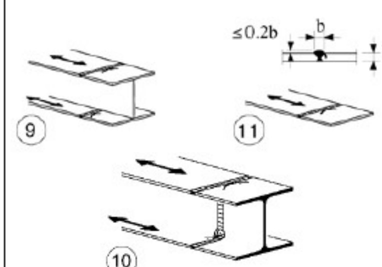
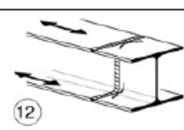
Detail category	Constructional detail	Description	Requirements
112	size effect for t>25mm: $k_s=(25/t)^{0.2}$ 	Without backing bar: 1) Transverse splices in plates and flats. 2) Flange and web splices in plate girders before assembly. 3) Full cross-section butt welds of rolled sections without cope holes. 4) Transverse splices in plates or flats tapered in width or in thickness, with a slope ≤ ¼.	- All welds ground flush to plate surface parallel to direction of the arrow. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Detail 3): Applies only to joints of rolled sections, cut and rewelded.
90	size effect for t>25mm: $k_s=(25/t)^{0.2}$ 	5) Transverse splices in plates or flats. 6) Full cross-section butt welds of rolled sections without cope holes. 7) Transverse splices in plates or flats tapered in width or in thickness with a slope ≤ ¼. Translation of welds to be machined notch free.	- The height of the weld convexity to be not greater than 10% of the weld width, with smooth transition to the plate surface. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Details 5 and 7: Welds made in flat position.
90	size effect for t>25mm: $k_s=(25/t)^{0.2}$ 	8) As detail 3) but with cope holes.	- All welds ground flush to plate surface parallel to direction of the arrow. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. - Rolled sections with the same dimensions without tolerance differences
80	size effect for t>25mm: $k_s=(25/t)^{0.2}$ 	9) Transverse splices in welded plate girders without cope hole. 10) Full cross-section butt welds of rolled sections with cope holes. 11) Transverse splices in plates, flats, rolled sections or plate girders.	- The height of the weld convexity to be not greater than 20% of the weld width, with smooth transition to the plate surface. - Weld not ground flush - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Detail 10: The height of the weld convexity to be not greater than 10% of the weld width, with smooth transition to the plate surface.
63		12) Full cross-section butt welds of rolled sections without cope hole.	- Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides.

Table 8.4: Weld attachments and stiffeners

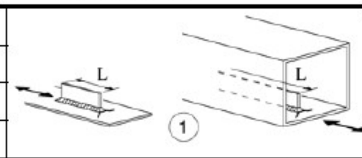
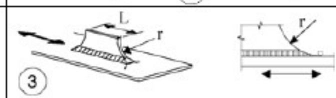
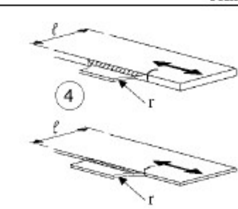
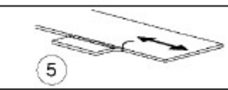
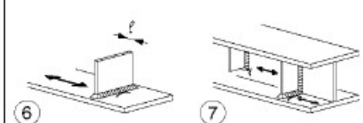
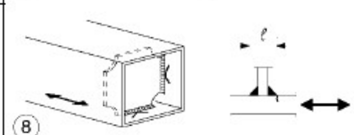
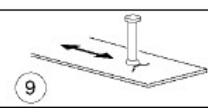
Detail category	Constructional detail		Description	Requirements
80	$L \leq 50mm$		Longitudinal attachments: 1) The detail category varies according to the length of the attachment L.	The thickness of the attachment must be less than its height. If not see Table 8.5, details 5 or 6.
71	$50 < L \leq 80mm$			
63	$80 < L \leq 100mm$			
56	$L > 100mm$			
71	$L > 100mm$ $\alpha < 45°$		2) Longitudinal attachments to plate or tube.	
80	$r > 150mm$	 reinforced	3) Longitudinal fillet welded gusset with radius transition to plate or tube; end of fillet weld reinforced (full penetration); length of reinforced weld > r.	Details 3) and 4): Smooth transition radius r formed by initially machining or gas cutting the gusset plate before welding, then subsequently grinding the weld area parallel to the direction of the arrow so that the transverse weld toe is fully removed.
90	$\frac{r}{L} \geq \frac{1}{3}$ or $r > 150mm$		4) Gusset plate, welded to the edge of a plate or beam flange.	
71	$\frac{1}{6} \leq \frac{r}{L} \leq \frac{1}{3}$			
50	$\frac{r}{L} < \frac{1}{6}$	L: attachment length as in detail 1, 2 or 3		
40			5) As welded, no radius transition.	
80	$\ell \leq 50mm$		Transverse attachments: 6) Welded to plate. 7) Vertical stiffeners welded to a beam or plate girder. 8) Diaphragm of box girders welded to the flange or the web. May not be possible for small hollow sections. The values are also valid for ring stiffeners.	Details 6) and 7): Ends of welds to be carefully ground to remove any undercut that may be present. 7) $\Delta\sigma$ to be calculated using principal stresses if the stiffener terminates in the web, see left side.
71	$50 < \ell \leq 80mm$			
80			9) The effect of welded shear studs on base material.	

Table 8.5: Load carrying welded joints

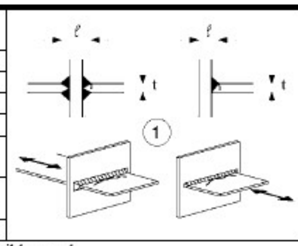
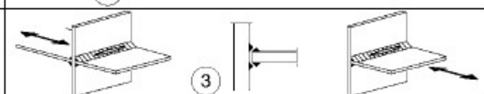
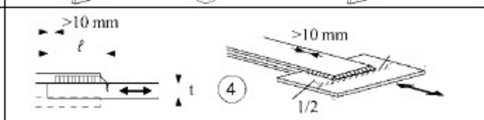
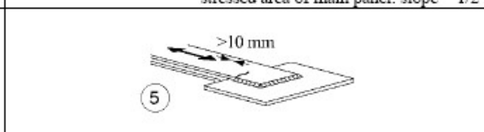
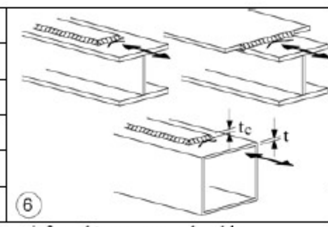
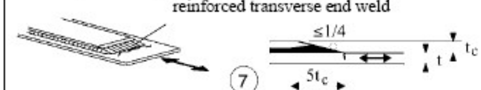
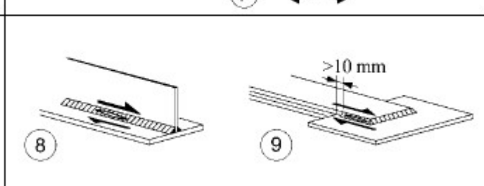
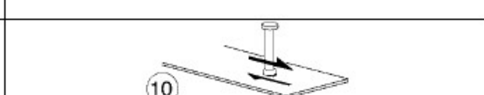
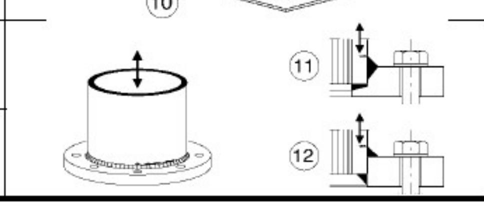
Detail category			Constructional detail	Description	Requirements
80	ℓ<50 mm	all t [mm]		Cruciform and Tee joints: 1) Toe failure in full penetration butt welds and all partial penetration joints.	1) Inspected and found free from discontinuities and misalignments outside the tolerances of EN 1090. 2) For computing $\Delta\sigma$, use modified nominal stress. 3) In partial penetration joints two fatigue assessments are required. Firstly, root cracking evaluated according to stresses defined in section 5, using category 36* for $\Delta\sigma_w$ and category 80 for $\Delta\tau_w$. Secondly, toe cracking is evaluated by determining $\Delta\sigma$ in the load-carrying plate. Details 1) to 3): The misalignment of the load-carrying plates should not exceed 15 % of the thickness of the intermediate plate.
71	50<ℓ≤80	all t			
63	80<ℓ≤100	all t			
56	100<ℓ≤120	all t			
56	ℓ>120	t≤20			
50	120<ℓ≤200 ℓ>200	t>20 20<t≤30			
45	200<ℓ≤300 ℓ>300	t>30 30<t≤50			
40	ℓ>300	t>50			
As detail 1 in Table 8.5			flexible panel 	2) Toe failure from edge of attachment to plate, with stress peaks at weld ends due to local plate deformations.	
36*				3) Root failure in partial penetration Tee-butt joints or fillet welded joint and effective full penetration in Tee-butt joint.	
As detail 1 in Table 8.5			 stressed area of main panel: slope = 1/2	Overlapped welded joints: 4) Fillet welded lap joint.	4) $\Delta\sigma$ in the main plate to be calculated on the basis of area shown in the sketch. 5) $\Delta\sigma$ to be calculated in the overlapping plates. Details 4) and 5): - Weld terminations more than 10 mm from plate edge. - Shear cracking in the weld should be checked using detail 8).
45*				Overlapped: 5) Fillet welded lap joint.	
	t_c<t	t_c≥t		Cover plates in beams and plate girders: 6) End zones of single or multiple welded cover plates, with or without transverse end weld.	6) If the cover plate is wider than the flange, a transverse end weld is needed. This weld should be carefully ground to remove undercut. The minimum length of the cover plate is 300 mm. For shorter attachments size effect see detail 1).
56*	t≤20	-			
50	20<t≤30	t≤20			
45	30<t≤50	20<t≤30			
40	t>50	30<t≤50			
36	-	t>50			
56			reinforced transverse end weld 	7) Cover plates in beams and plate girders. $5t_c$ is the minimum length of the reinforcement weld.	7) Transverse end weld ground flush. In addition, if t_c>20mm, front of plate at the end ground with a slope < 1 in 4.
80 m=5				8) Continuous fillet welds transmitting a shear flow, such as web to flange welds in plate girders. 9) Fillet welded lap joint.	8) $\Delta\tau$ to be calculated from the weld throat area. 9) $\Delta\tau$ to be calculated from the weld throat area considering the total length of the weld. Weld terminations more than 10 mm from the plate edge, see also 4) and 5) above.
see EN 1994-2 (90 m=8)				Welded stud shear connectors: 10) For composite application	10) $\Delta\tau$ to be calculated from the nominal cross section of the stud.
71				11) Tube socket joint with 80% full penetration butt welds.	11) Weld toe ground. $\Delta\sigma$ computed in tube.
40				12) Tube socket joint with fillet welds.	12) $\Delta\sigma$ computed in tube.

Table 8.6: Hollow sections ($t \leq 12{,}5$ mm)

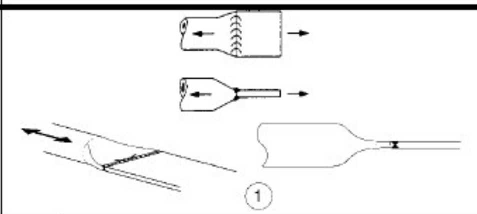
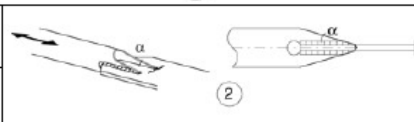
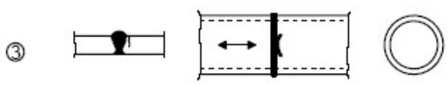
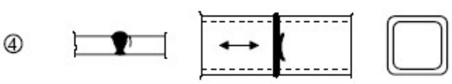
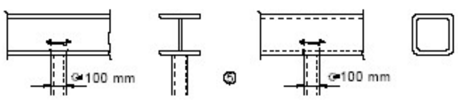
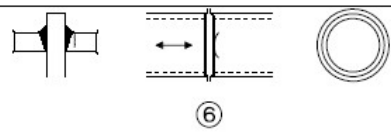
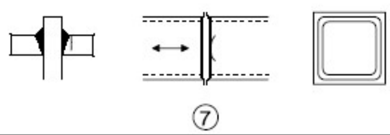
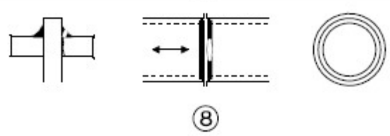
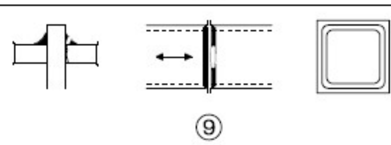
Detail category	Constructional detail	Description	Requirements
71		1) Tube-plate joint, tubes flatted, butt weld (X-groove)	1) $\Delta\sigma$ computed in tube. Only valid for tube diameter less than 200 mm.
71	$\alpha \leq 45°$ 	2) Tube-plate joint, tube slitted and welded to plate. Holes at end of slit.	2) $\Delta\sigma$ computed in tube. Shear cracking in the weld should be verified using Table 8.5, detail 8).
63	$\alpha > 45°$		
71		Transverse butt welds: 3) Butt-welded end-to-end connections between circular structural hollow sections.	Details 3) and 4): - Weld convexity $\leq$ 10% of weld width, with smooth transitions. - Welded in flat position, inspected and found free from defects outside the tolerances EN 1090. - Classify 2 detail categories higher if $t > 8$ mm.
56		4) Butt-welded end-to-end connections between rectangular structural hollow sections.	
71		Welded attachments: 5) Circular or rectangular structural hollow section, fillet-welded to another section.	5) - Non load-carrying welds. - Width parallel to stress direction $\ell \leq 100$ mm. - Other cases see Table 8.4.
50		Welded splices: 6) Circular structural hollow sections, butt-welded end-to-end with an intermediate plate.	Details 6) and 7): - Load-carrying welds. - Welds inspected and found free from defects outside the tolerances of EN 1090. - Classify 1 detail category higher if $t > 8$ mm.
45		7) Rectangular structural hollow sections, butt welded end-to-end with an intermediate plate.	
40		8) Circular structural hollow sections, fillet-welded end-to-end with an intermediate plate.	Details 8) and 9): - Load-carrying welds. - Wall thickness $t \leq 8$ mm.
36		9) Rectangular structural hollow sections, fillet-welded end-to-end with an intermediate plate.	

Table 8.7: Lattice girder node joints

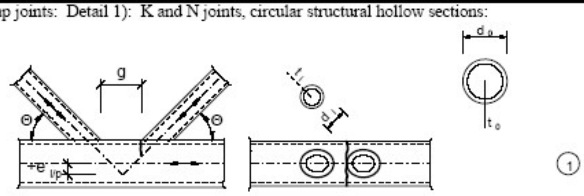
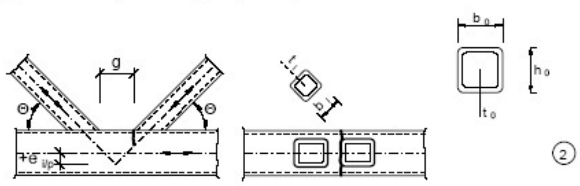
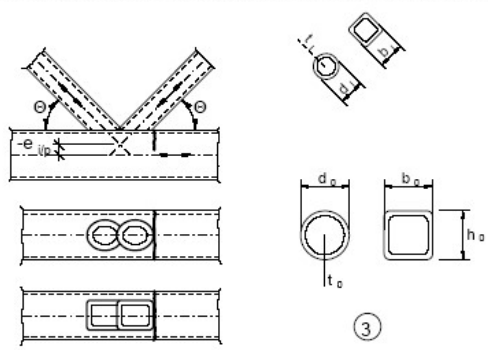
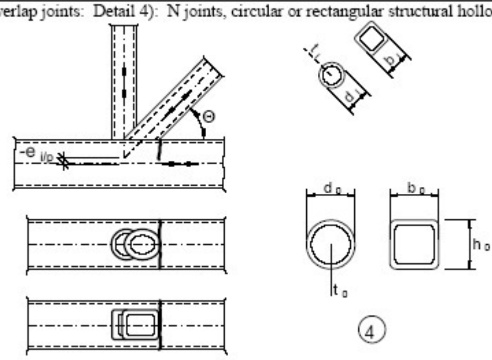
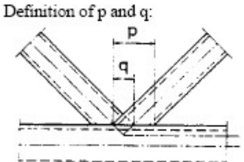
Detail category		Constructional detail	Requirements
90 m=5	$\frac{t_o}{t_i} \geq 2{,}0$	Gap joints: Detail 1): K and N joints, circular structural hollow sections: 	Details 1) and 2): - Separate assessments needed for the chords and the braces. - For intermediate values of the ratio t_o/t_i interpolate linearly between detail categories. - Fillet welds permitted for braces with wall thickness $t \leq$ 8 mm. - t_0 and $t_i \leq 8$mm - $35° \leq \theta \leq 50°$ - $b_0/t_0 \times t_0/t_i \leq 25$ - $d_0/t_0 \times t_0/t_i \leq 25$ - $0{,}4 \leq b_i/b_0 \leq 1{,}0$ - $0{,}25 \leq d_i/d_0 \leq 1{,}0$ - $b_0 \leq 200$ mm - $d_0 \leq 300$ mm - $-0{,}5h_0 \leq e_{i/p} \leq 0{,}25h_0$ - $-0{,}5d_0 \leq e_{i/p} \leq 0{,}25d_0$ - $e_{o/p} \leq 0{,}02b_0$ or $\leq 0{,}02d_0$
45 m=5	$\frac{t_o}{t_i} = 1{,}0$		
71 m=5	$\frac{t_o}{t_i} \geq 2{,}0$	Gap joints: Detail 2): K and N joints, rectangular structural hollow sections: 	
36 m=5	$\frac{t_o}{t_i} = 1{,}0$		[$e_{o/p}$ is out-of-plane eccentricity] Detail 2): $0{,}5(b_o - b_i) \leq g \leq 1{,}1(b_o - b_i)$ and $g \geq 2t_o$
71 m=5	$\frac{t_o}{t_i} \geq 1{,}4$	Overlap joints: Detail 3): K joints, circular or rectangular structural hollow sections: 	Details 3) and 4): - $30\ \% \leq$ overlap $\leq 100\ \%$ - overlap = $(q/p) \times 100\ \%$ - Separate assessments needed for the chords and the braces. - For intermediate values of the ratio t_o/t_i interpolate linearly between detail categories. - Fillet welds permitted for braces with wall thickness $t \leq$ 8 mm. - t_0 and $t_i \leq 8$mm - $35° \leq \theta \leq 50°$ - $b_0/t_0 \times t_0/t_i \leq 25$ - $d_0/t_0 \times t_0/t_i \leq 25$ - $0{,}4 \leq b_i/b_0 \leq 1{,}0$ - $0{,}25 \leq d_i/d_0 \leq 1{,}0$ - $b_0 \leq 200$ mm - $d_0 \leq 300$ mm - $-0{,}5h_0 \leq e_{i/p} \leq 0{,}25h_0$ - $-0{,}5d_0 \leq e_{i/p} \leq 0{,}25d_0$ - $e_{o/p} \leq 0{,}02b_0$ or $\leq 0{,}02d_0$
56 m=5	$\frac{t_o}{t_i} = 1{,}0$		
71 m=5	$\frac{t_o}{t_i} \geq 1{,}4$	Overlap joints: Detail 4): N joints, circular or rectangular structural hollow sections: 	[$e_{o/p}$ is out-of-plane eccentricity]
50 m=5	$\frac{t_o}{t_i} = 1{,}0$		Definition of p and q: 

Table 8.8: Orthotropic decks – closed stringers

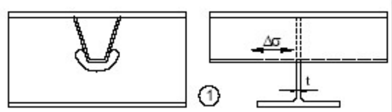
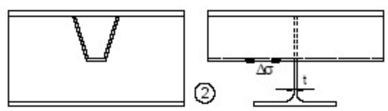
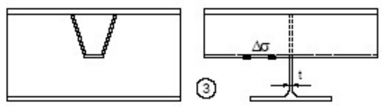
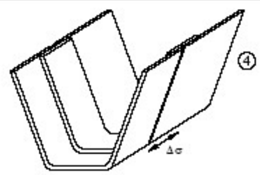
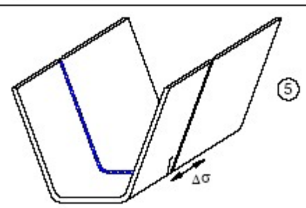
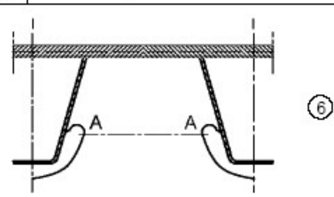
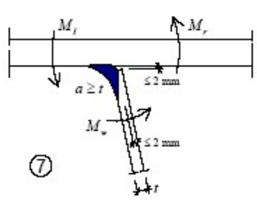
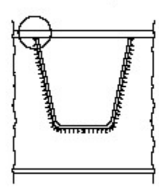
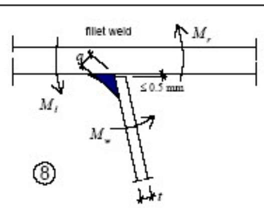
Detail category	Constructional detail		Description	Requirements
80	t≤12mm		1) Continuous longitudinal stringer, with additional cutout in cross girder.	1) Assessment based on the direct stress range Δσ in the longitudinal stringer.
71	t>12mm			
80	t≤12mm		2) Continuous longitudinal stringer, no additional cutout in cross girder.	2) Assessment based on the direct stress range Δσ in the stringer.
71	t>12mm			
36			3) Separate longitudinal stringer each side of the cross girder.	3) Assessment based on the direct stress range Δσ in the stringer.
71			4) Joint in rib, full penetration butt weld with steel backing plate.	4) Assessment based on the direct stress range Δσ in the stringer.
112	As detail 1, 2, 4 in Table 8.3		5) Full penetration butt weld in rib, welded from both sides, without backing plate.	5) Assessment based on the direct stress range Δσ in the stringer. Tack welds inside the shape of butt welds.
90	As detail 5, 7 in Table 8.3			
80	As detail 9, 11 in Table 8.3			
71			6) Critical section in web of cross girder due to cut outs.	6) Assessment based on stress range in critical section taking account of Vierendeel effects. **NOTE** In case the stress range is determined according to EN 1993-2, 9.4.2.2(3), detail category 112 may be used.
71		$\Delta\sigma = \frac{\Delta M_w}{W_w}$ 	Weld connecting deck plate to trapezoidal or V-section rib 7) Partial penetration weld with a ≥ t	7) Assessment based on direct stress range from bending in the plate.
50			8) Fillet weld or partial penetration welds out of the range of detail 7)	8) Assessment based on direct stress range from bending in the plate.

Table 8.9: Orthotropic decks – open stringers

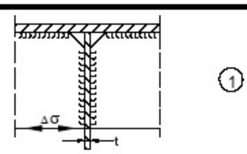
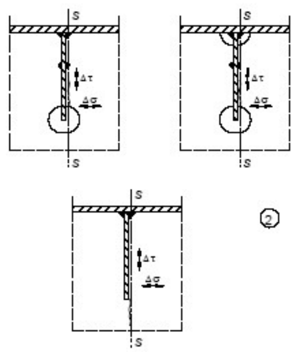
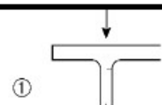
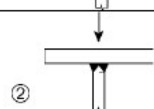
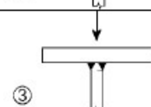
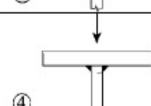
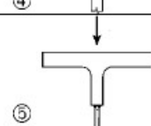
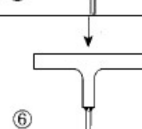
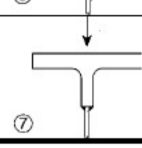
Detail category	Constructional detail	Description	Requirements
80	t≤12mm 	1) Connection of longitudinal stringer to cross girder.	1) Assessment based on the direct stress range $\Delta\sigma$ in the stringer.
71	t>12mm		
56		2) Connection of continuous longitudinal stringer to cross girder. $\Delta\sigma = \frac{\Delta M_s}{W_{net,s}}$ $\Delta\tau = \frac{\Delta V_s}{A_{w,net,s}}$ Check also stress range between stringers as defined in EN 1993-2.	2) Assessment based on combining the shear stress range $\Delta\tau$ and direct stress range $\Delta\sigma$ in the web of the cross girder, as an equivalent stress range: $\Delta\sigma_{eq} = \frac{1}{2}\left(\Delta\sigma + \sqrt{\Delta\sigma^2 + 4\Delta\tau^2}\right)$

Table 8.10: Top flange to web junction of runway beams

Detail category	Constructional detail	Description	Requirements
160		1) Rolled I- or H-sections	1) Vertical compressive stress range $\Delta\sigma_{vert.}$ in web due to wheel loads
71		2) Full penetration tee-butt weld	2) Vertical compressive stress range $\Delta\sigma_{vert.}$ in web due to wheel loads
36*		3) Partial penetration tee-butt welds, or effective full penetration tee-butt weld conforming with EN 1993-1-8	3) Stress range $\Delta\sigma_{vert.}$ in weld throat due to vertical compression from wheel loads
36*		4) Fillet welds	4) Stress range $\Delta\sigma_{vert.}$ in weld throat due to vertical compression from wheel loads
71		5) T-section flange with full penetration tee-butt weld	5) Vertical compressive stress range $\Delta\sigma_{vert.}$ in web due to wheel loads
36*		6) T-section flange with partial penetration tee-butt weld, or effective full penetration tee-butt weld conforming with EN 1993-1-8	6) Stress range $\Delta\sigma_{vert.}$ in weld throat due to vertical compression from wheel loads
36*		7) T-section flange with fillet welds	7) Stress range $\Delta\sigma_{vert.}$ in weld throat due to vertical compression from wheel loads