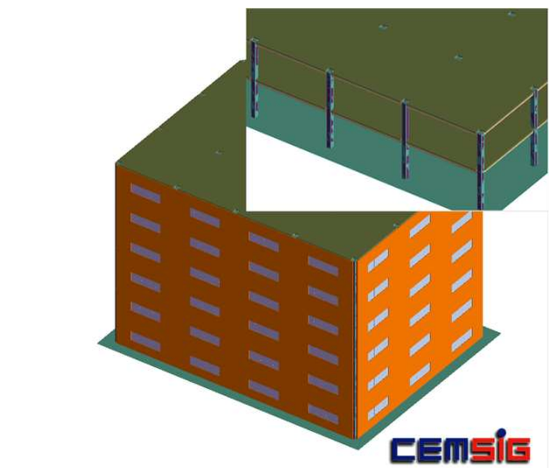
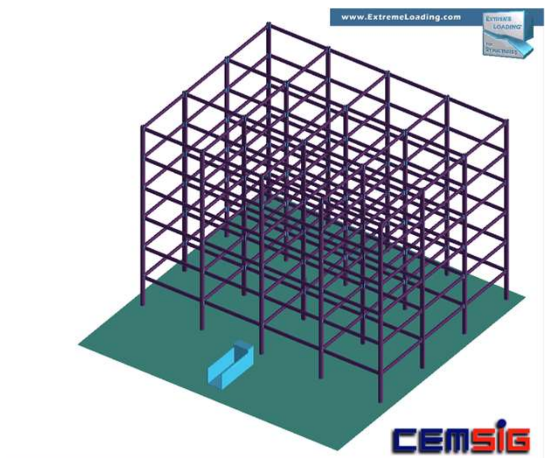
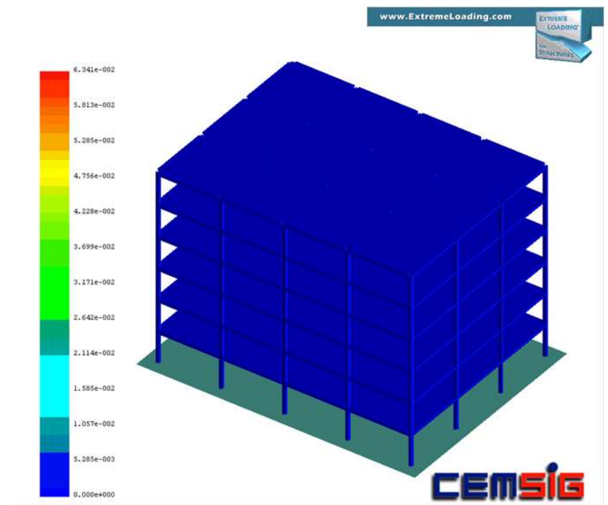
Universitatea Politehnica Timişoara

# Robustness of structures

## Structural dynamics




**2018-2019**

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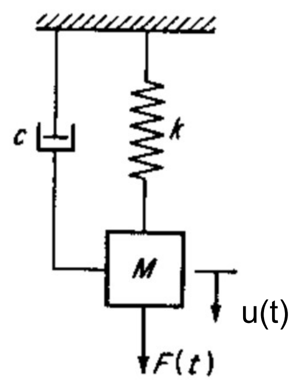
## Structural response to blast loading

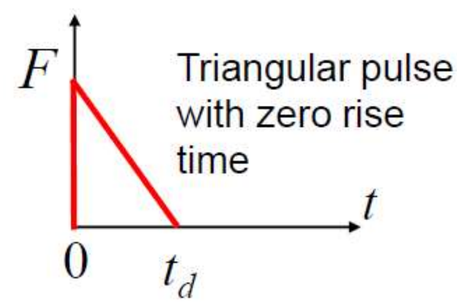
- Analysis and design of structures subjected to blast loads require:
  - understanding of blast phenomena
  - understanding of the dynamic response of various structural elements.
- Complexity in analyzing the response:
  - uncertainties of blast load calculations
  - time-dependent deformations
  - effect of high strain rates
  - non-linear inelastic material behavior
- To simplify the analysis, a number of assumptions related to the response of structures and the loads has been proposed and widely accepted:
  - Elastic SDOF Systems
  - Elasto-Plastic SDOF Systems
- Blast loading effects:
  - Global structural behavior
  - Localised structural behavior
  - Pressure-Impulse (P-I) Diagrams

## Elastic SDOF systems

- The simplest discretization of transient problems is by means of the SDOF approach
- A SDOF is a system with only one type of motion (position at any instant can be defined using a single coordinate)
- The actual structure can be replaced by an equivalent system of one concentrated mass and one weightless spring representing the resistance of the structure against deformation.
- The blast load can be idealized as a triangular pulse



SDOF system



Blast loading

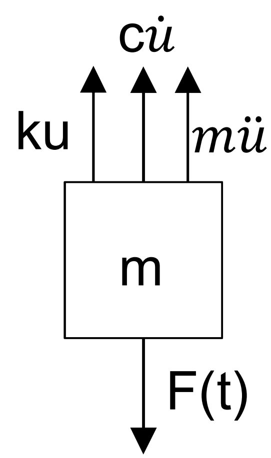
## Formulation of the problem

:

- Consider a SDOF and the state of equilibrium shown in figure
- Equation of motion as follows:

$$m\ddot{u}(t) + ku(t) + c\dot{u}(t) = F(t)$$

- The effects of damping are generally small and are often neglected



## - Free Vibration

- The force equals zero and there is no damping, c=0
- Motion will occur if the system is given an initial disturbance - initial displacement $u_0$ or initial velocity $\dot{u}_0$ (e.g. produced by an impulse) - or a combination of the two
- Equation of motion as follows:

$$m\ddot{u}(t) + ku(t) \ = 0 \ \rightarrow \ \ddot{u}(t) + \frac{k}{m}u(t) \ = 0$$

- Solution to equation of motion is:

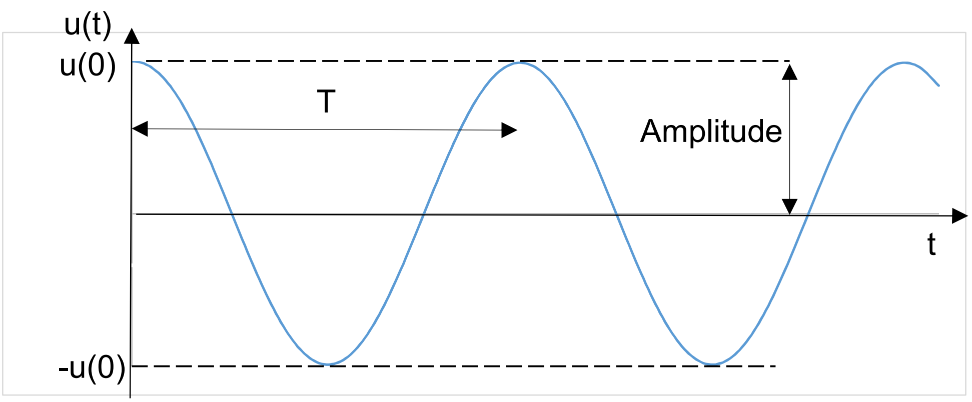
$$u(t) = C_1 sin\sqrt{\frac{k}{m}}t \ + C_2\sqrt{\frac{k}{m}}t \qquad ; \sqrt{\frac{k}{m}} = \omega$$

$$\Rightarrow u(t) = C_1 sin\omega t \ + C_2 cos\omega t$$

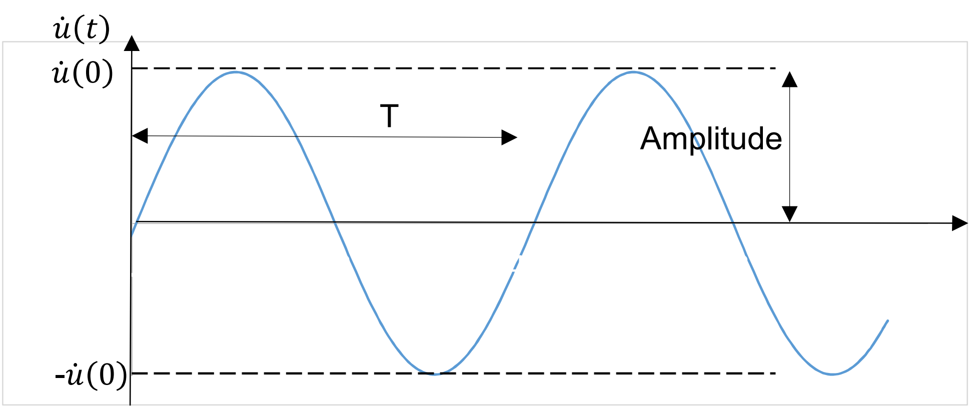
ω = natural circular frequency

- Constants $C_1$, $C_2$ depend upon initial conditions, $u(0), \dot{u}(0)$
- This results in: $C_2$=u(0); $C1=\frac{\dot{u}(0)}{\omega}$
- The solution for zero external load is therefore:

$$u(t) = \frac{\dot{u}(0)}{\omega} sin\omega t \ + u(0) cos\omega t$$




Free vibration of SDOF, undamped, initial displacement




Free vibration of SDOF, undamped, initial velocity

$\omega =$ *Natural circular frequency*
$T$ = *Natural period*
$f$ = *Natural frequency*

Since one complete cycle occurs for each angular increment $\omega t = 2\pi$ , T and f are given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

## - Forced Vibration

- The motion is the result of the force F(t), and there is no damping, c=0
- The system may be at rest (displacement and velocity are zero at t=0), or with an initial disturbance (initial displacement $u_0$ or initial velocity $\dot{u}_0$, or a combination of the two).
- The simplest case corresponds to a force F(t) with a constant magnitude (suddenly applied and remains constant indefinitely)
- Equation of motion as follows:

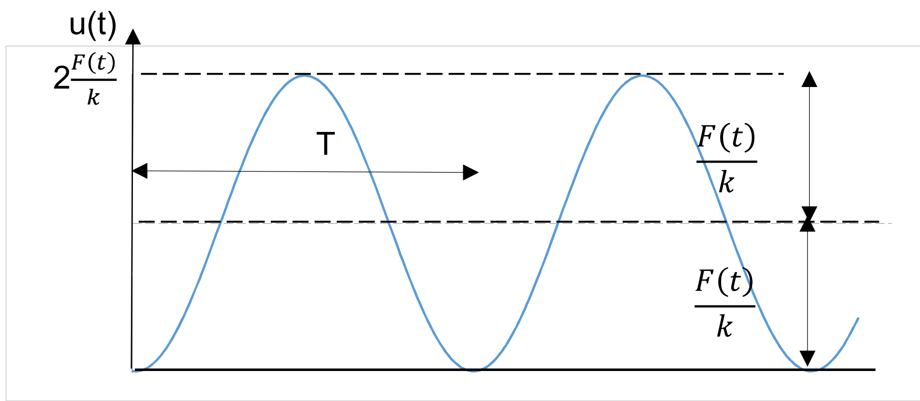
$$m\ddot{u}(t) + ku(t) = F(t) \Rightarrow \ddot{u}(t) + \frac{k}{m}u(t) = \frac{F(t)}{m}$$

- Solution to equation of motion is:

$$u(t) = C_1 sin\omega t + C_2 cos\omega t + \frac{F(t)}{k}$$

- Constants $C_1$, $C_2$ depend upon initial conditions, $u(0) = 0, \dot{u}(0) = 0$
- This results in: $C_2 = -\frac{F(t)}{k}$; $C_1 = 0$
- The solution is:

$$u(t) = \frac{F(t)}{k}\mathbf{(1 - cos\omega t)}$$




Undamped SDOF, suddenly applied constant force

- The solution is similar to that of an undamped SDOF, free vibration. The only difference is that the axis is shifted with $\frac{F(t)}{k}$
- The maximum displacement $2\frac{F(t)}{k}$ is twice the displacement for a statically applied force $F(t)$.
- If a constant force is suddenly applied to a linear elastic system, the resulting displacement is twice that for the same force applied statically.
- Dynamic load factor DLF = ratio of the dynamic deflection at any time to the deflection that would result from the static application of the load F.

$$DLF = \frac{u}{u_{st}} = \frac{u}{F/k} = \frac{ku}{F} \quad \Rightarrow \quad DLF = (1 - cos\omega t)$$

## - Various load-time functions

- Let us consider a system at rest, subjected to a constant force F, with the duration $t_d$.
- The mass m begins to move with acceleration $\ddot{u} = \frac{F}{m}$
- If $t_d$ is short enough, acceleration is constant, and the velocity at time $t_d$ is:

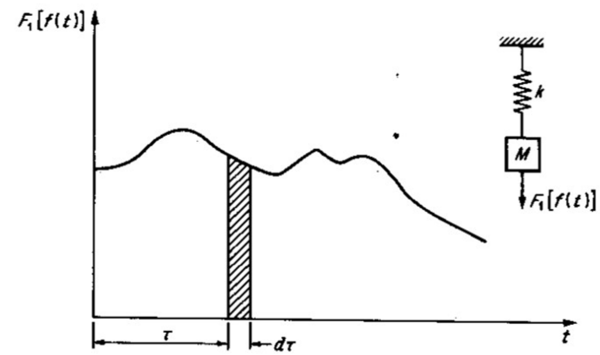
$$\dot{u} = \ddot{u}t_d = \frac{F}{m}t_d = \frac{i}{m}$$

i is the applied impulse (equal to the area below the load-time curve)

- Let us consider a general load function. For a time $\mathrm{d}\tau$ , the area below the curve is the pure impulse. This impulse causes an increment of the velocity at time $\tau$ equal to $\frac{Fd\tau}{m}$ - this velocity can be considered as an initial velocity imparted to the system at rest. The displacement at time t due to this impulse (load applied during $d\tau$) is (the initial displacement u(0)=0):

$$u(t) = \frac{\dot{u}(0)}{\omega}sin\omega t + u(0)cos\omega t \quad \Rightarrow \quad u(t) = \frac{\dot{u}(0)}{\omega}sin\omega t = \frac{Fd\tau}{m\omega}sin\omega(t-\tau)$$

- The total displacement is therefore the sum of the effects of all elements of impulse between zero and t. Thus:




$$u = \int_0^t \frac{Fd\tau}{m\omega} sin\omega(t-\tau)\, d\tau)$$

- The static deflection $u_{st}$ due to F may be expressed as:

$$u_{st} = \frac{F}{k} = \frac{F}{\omega^2 m}$$

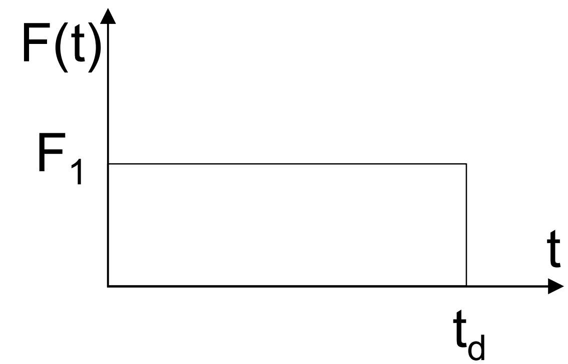
$$\Rightarrow \quad u = ust\omega \int_0^t f(\tau) sin\omega(t-\tau)\, d\tau$$

- If the effects of initial displacement and velocity are also included, the general expression for the response of undamped, liner elastic SDOF system subjected to any load function and/or initial condition is:

$$u = u_0 cos\omega t + \frac{\dot{u}_0}{\omega} sin\omega t + u_{st}\omega \int_0^t f(\tau) sin\omega(t-\tau)\, d\tau$$

## - Rectangular–pulse load

- In this case, a constant load is sudenly applied within with a limited duration, $t_d$
- Up to time $t_d$, the following equations apply:




$$u_{td} = \frac{F_1}{k}(1 - cos\omega t_d)$$

$$\dot{u}_{td} = \frac{F_1}{k}\omega sin\omega t_d$$

- For the response after $t_d$, we consider as initial condition the velocity at $t_d$. Replacing t by t-$t_d$ , displacement and velocity $u_0$ and $\dot{u}_0$ by $u_{td}$ and $\dot{u}_{td}$ and considering $f(\tau)$ =0, we obtain:

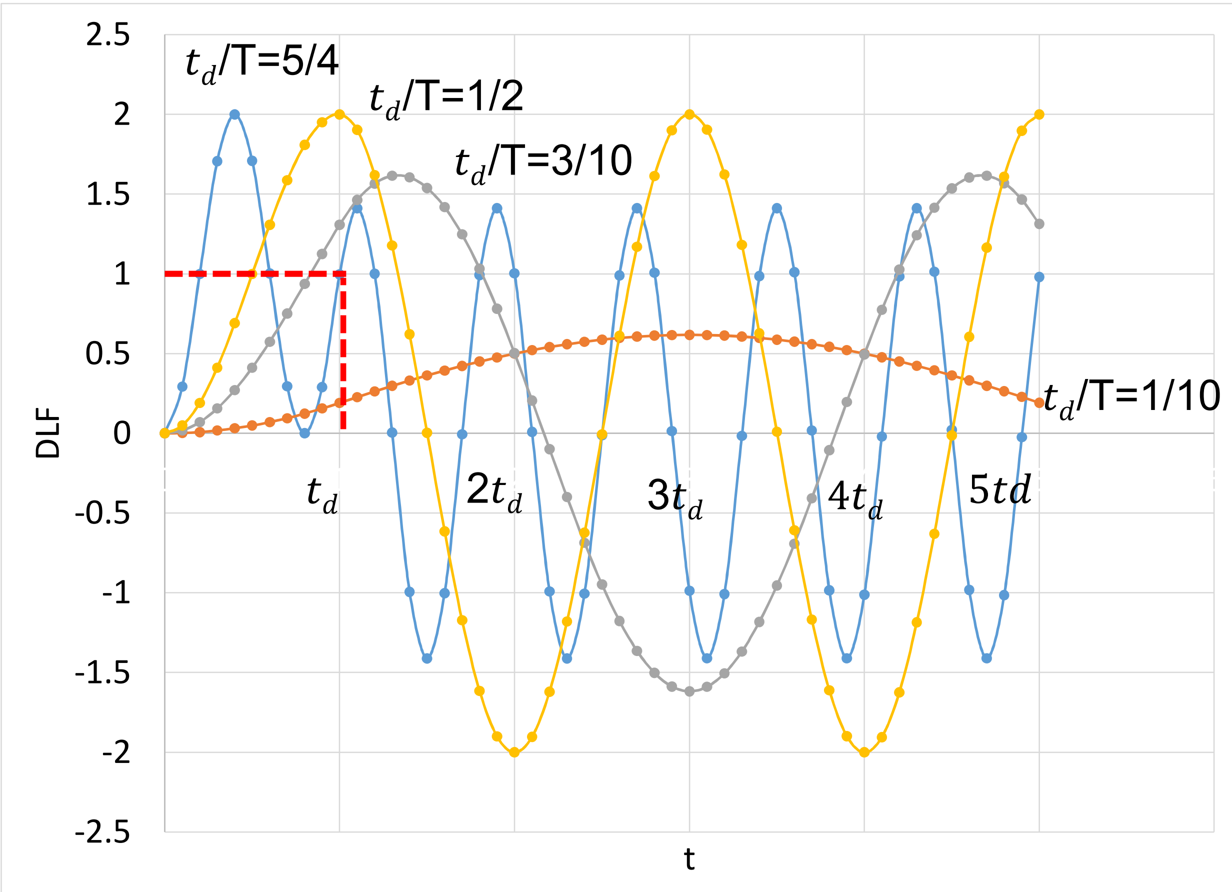
$$u = \frac{F_1}{k}(1 - cos\omega t_d)cos\omega(t - td) + \frac{F_1}{k}sin\omega t_d sin\omega(t - td) =$$

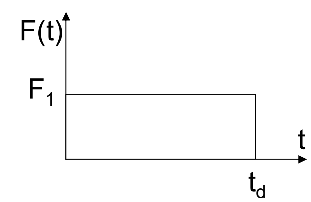
$$= \frac{F1}{k}[cos\omega(t - t_d) - cos\omega t]$$

$$u_{st} = \frac{F_1}{k} \Rightarrow DLF = 1 - cos\omega t = 1 - cos2\pi\frac{t}{T} \quad \textbf{for } t<=t_d$$

$$DLF = \frac{u}{u_{st}} \Rightarrow DLF = cos\omega(t - t_d) - cos\omega t = cos2\pi\left(\frac{t}{T} - \frac{t_d}{T}\right) - cos2\pi\frac{t}{T} \quad \textbf{for } t>=t_d$$

- It is more convenient to normalize the time parameter
  - If $t_d$ is small (approaches zero), the maximum deflection (and stress) tends to zero
  - For $t_d/T>0.5$, the maximum response is the same as for infinite duration load



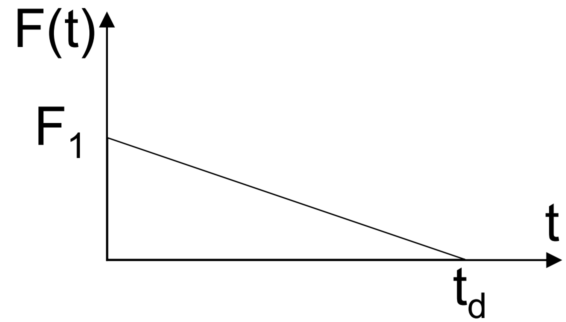
## - Triangular–pulse load

- In this case, the system is at rest and is subjected to a force F which has the value $F_1$ - suddenly applied and then decreasing linearly to zero at $t_d$
- For t<=$t_d$, $u_0$=0, $\dot{u}_0$ =0, $F(t) = F_1(1-\frac{t}{t_d})$ and the following equations apply:

$$u = \frac{F_1}{k}(1-cos\omega t) + \frac{F_1}{kt_d}(\frac{sin\omega t}{\omega} - t)$$

or

$$DLF = 1 - cos\omega t + \frac{sin\omega t}{\omega t_d} - \frac{t}{t_d}$$




- For the response after $t_d$, we obtain:

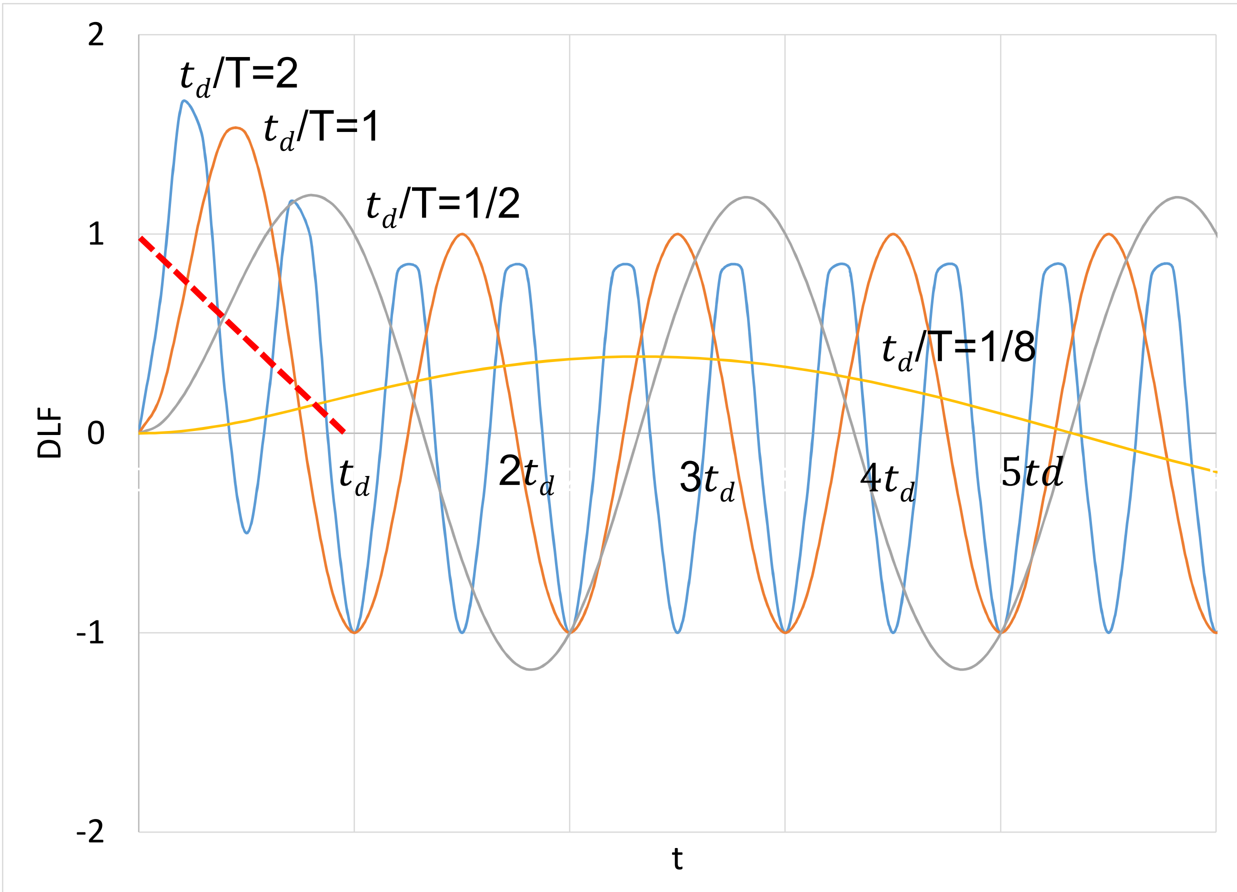
$$u_0 = \frac{F_1}{k}(\frac{sin\omega td}{\omega t_d} - cos\omega t_d) \qquad \dot{u}_0 = \frac{F_1}{k}(\omega sin\omega t_d + \frac{cos\omega td}{t_d} - \frac{1}{t_d}) \qquad f(\tau)=0$$

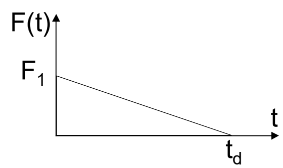
- Substituting in general equation and replacing t by t-$t_d$, we obtain:

$$u = \frac{F_1}{k\omega t_d}[sin\omega t - sin\omega(t-td)] - \frac{F_1}{k}cos\omega t$$

$$DLF = \frac{1}{\omega td}[sin\omega t - sin\omega(t-td)] - cos\omega t$$

- It is more convenient to normalize the time parameter
  - If the ratio $t_d/T$ becomes greater, more oscillations occur during the presence of the forcing function.
  - $t_d/T \to \infty$ (step force)



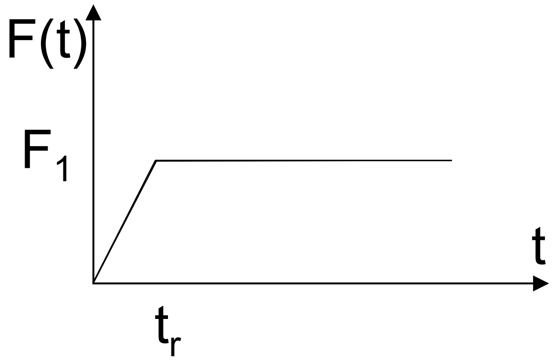



- Constant force with finite rise time

- As a force is never applied instantaneously, it is useful to see the influence of the response for intermediate cases. In this case, the load-time functions are as follows:

$$f(t) = \frac{t}{t_r} \qquad \text{for } t<t_r$$
$$f(t) = 1 \qquad \text{for } t>=t_r$$

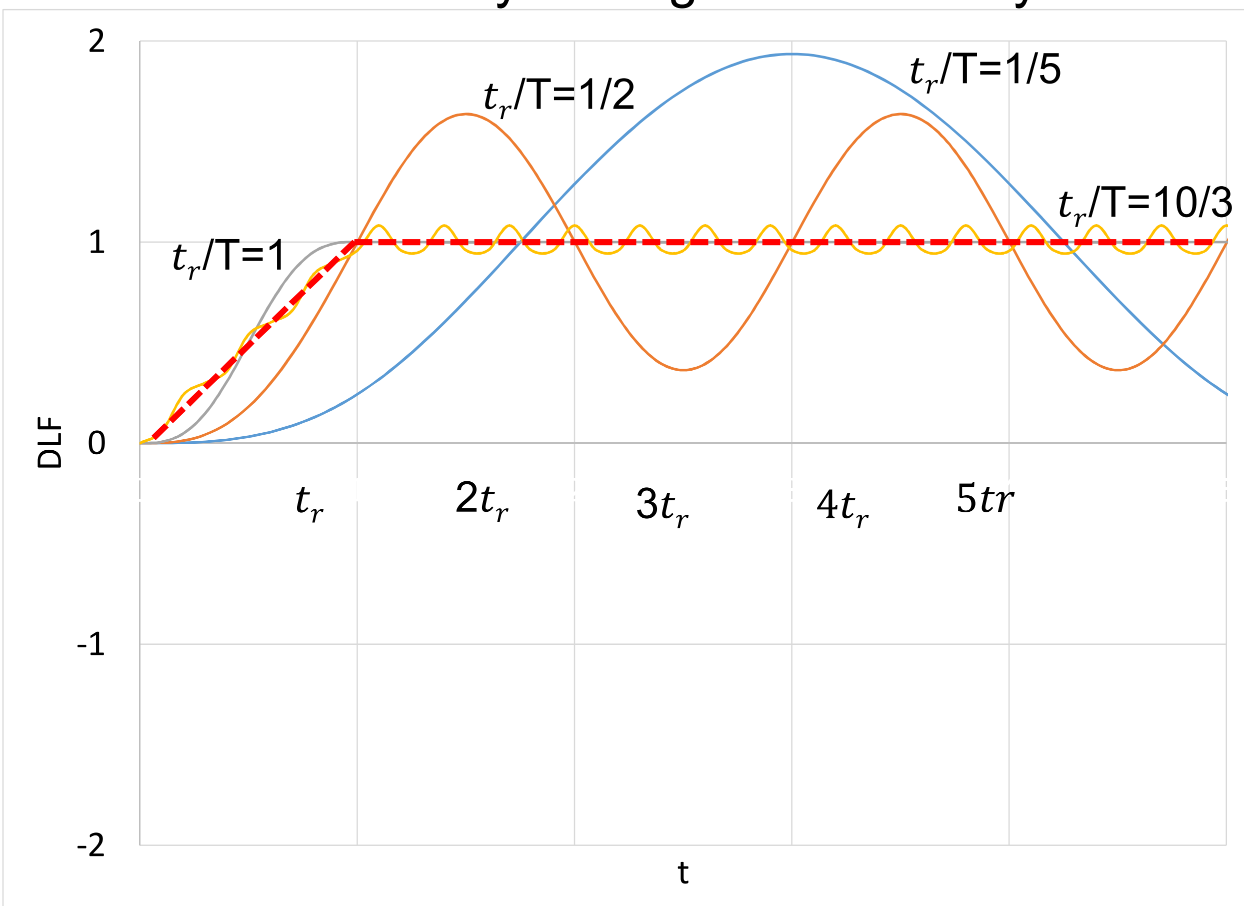
where $t_r$ is the rise time

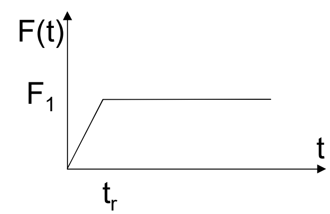


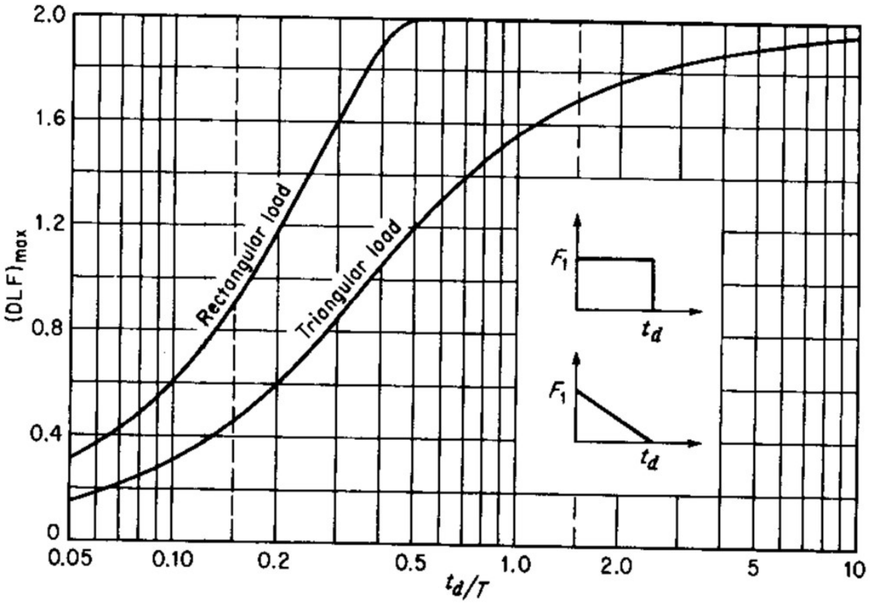

- Up to time $t_r$, the following equations apply:

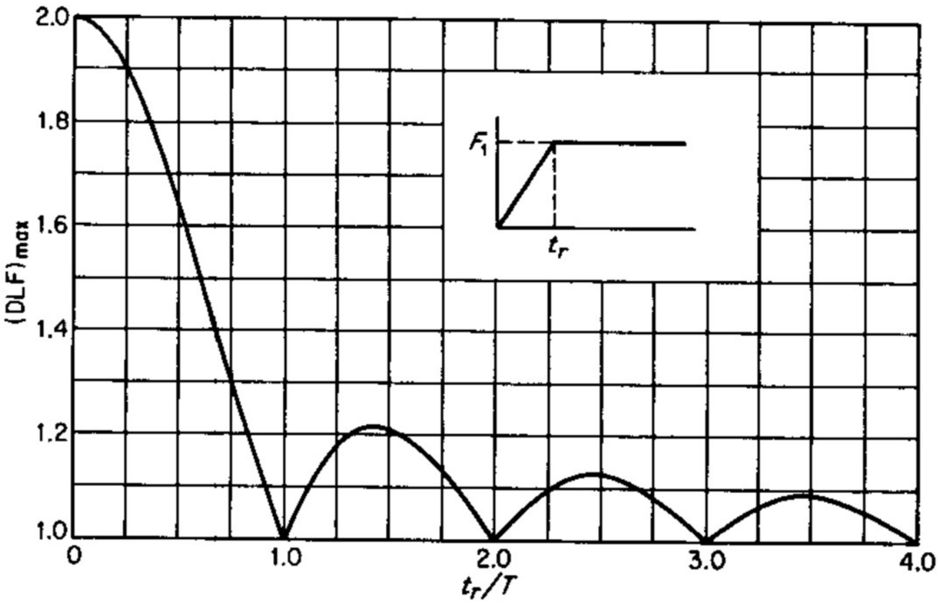
$$\Rightarrow DLF = \frac{1}{t_r}\left(t - \frac{sin\omega t}{\omega}\right) \qquad \text{for } t<t_r$$
$$\Rightarrow DLF = 1 + \frac{1}{\omega t_r}[sin\omega(t-tr) - sin\omega t] \qquad \text{for } t>=t_r$$

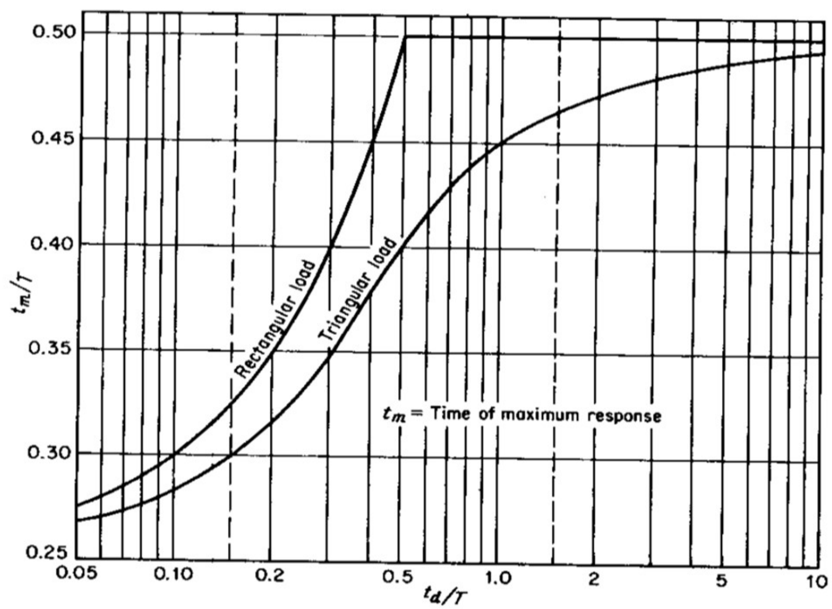
- It is more convenient to normalize the time parameter
  - If $t_r$ increases – relative to T, the response simply follows the applied load and dynamic effect is negligible
  - If $t_r$ <=1/4T, the effect is similar to that of a suddenly applied load ➔ small rise times may be neglected in analysis

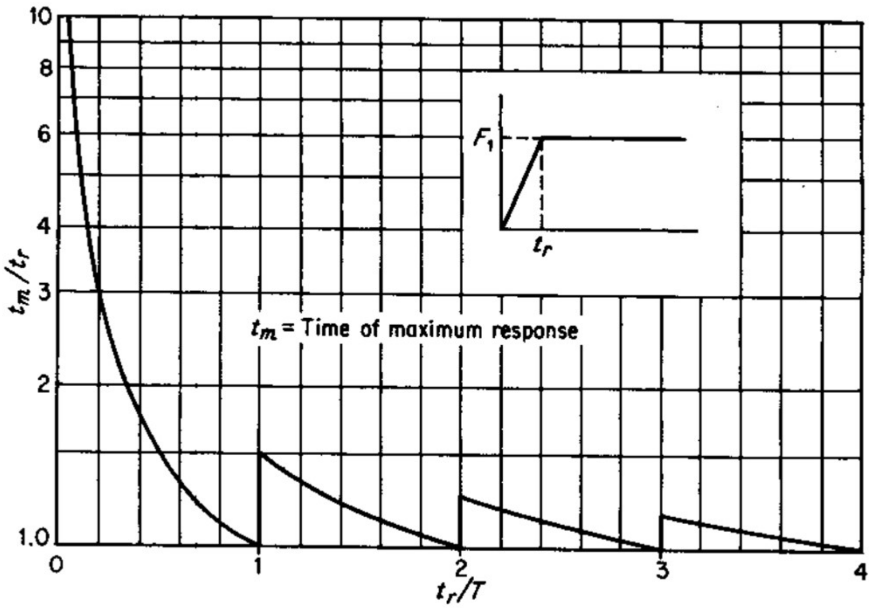


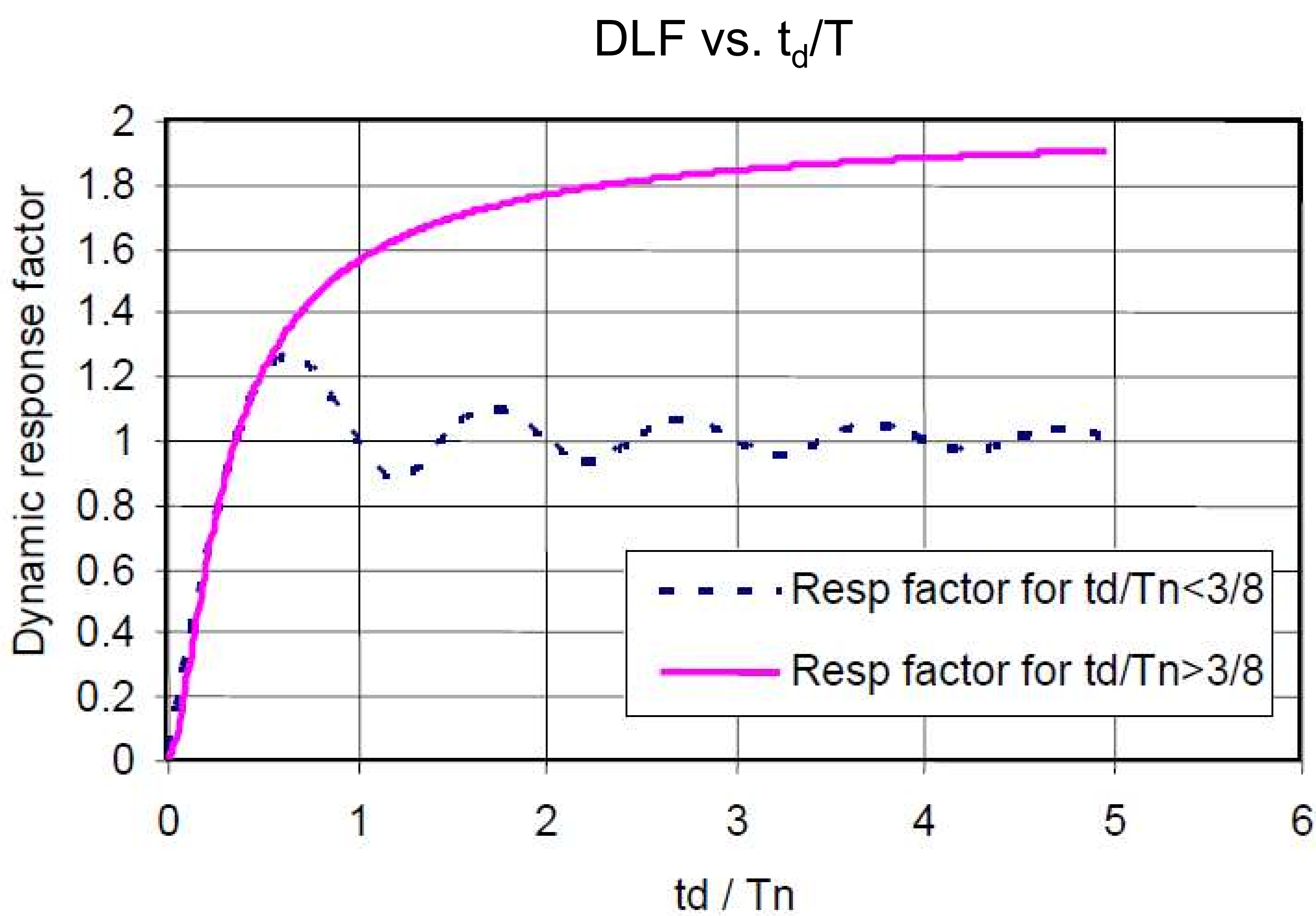

$(DLF)_{max}$
2.0
1.6
1.2
0.8
0.4
0
0.05
0.10
0.2
0.5
1.0
2.0
5
10
$t_d/T$
Rectangular load
Triangular load
$F_1$
$t_d$
$F_1$
$t_d$


$(DLF)_{max}$
2.0
1.8
1.6
1.4
1.2
1.0
0
1.0
2.0
3.0
4.0
$t_r/T$
$F_1$
$t_r$


$t_m/T$
0.50
0.45
0.40
0.35
0.30
0.25
0.05
0.10
0.2
0.5
1.0
2
5
10
$t_d/T$
Rectangular load
Triangular load
$t_m$ = Time of maximum response

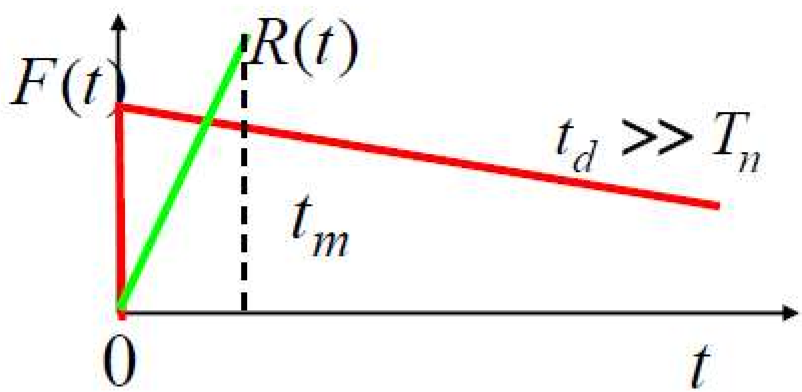

$t_m/t_r$
10
8
6
4
3
2
1.0
0
1
2
3
4
$t_r/T$
$F_1$
$t_r$
$t_m$ = Time of maximum response

- Triangular–pulse load

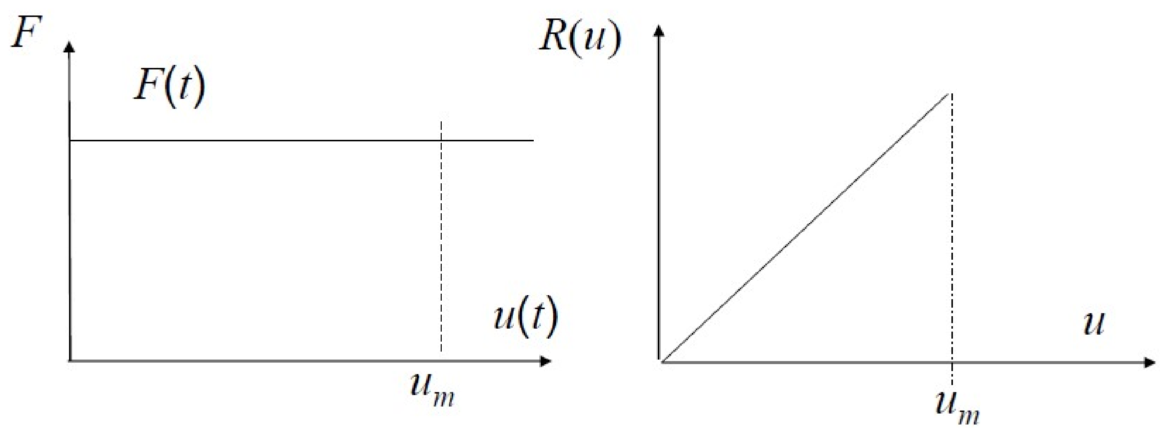


- The spectrum curve can be constructed in a simpler way by looking at two extreme situations:

1) Quasi-static or pressure loading: long $t_d$, short $T_n$




SDOF reaches $u_m$ before load has any significant decay, $F(t) \approx F$



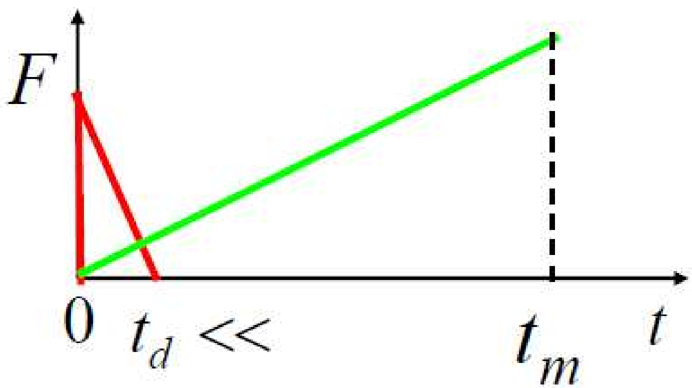

Here consider system energy:

$$Fu_m = \frac{1}{2}ku_m^2$$

$$\frac{u_m}{F/k} = 2$$ or $$\boxed{DLF = \frac{u_m}{u_{st}} = 2}$$

**Quasi-static asymptote**

## 2) Impulsive loading: very short $t_d$, long $T_n$




The load is applied so quickly even before the SDOF system has any movement. Response treated as free vibration with initial velocity due to impulse. Strain energy stored is the same as previous case.

$$I = \frac{1}{2} F t_d = m\dot{u}_0 \qquad \dot{u}_0 = \frac{I}{m}$$

Kinetic energy: $$KE = \frac{1}{2} m\dot{u}_0^2 = \frac{I^2}{2m}$$

Equating kinetic energy with stored strain energy:

$$\frac{I^2}{2m} = \frac{1}{2} k u_m^2 \Rightarrow u_m = \frac{I}{\sqrt{km}}$$

$$DLF = \frac{u_m}{F/k} = \frac{I}{\sqrt{km}(F/k)}$$ but $$I = \frac{1}{2} F t_d = m\dot{u}_0$$

$$DLF = \frac{1/2 F t_d}{\sqrt{km}(F/k)} = \frac{1}{2} \omega_n t_d$$

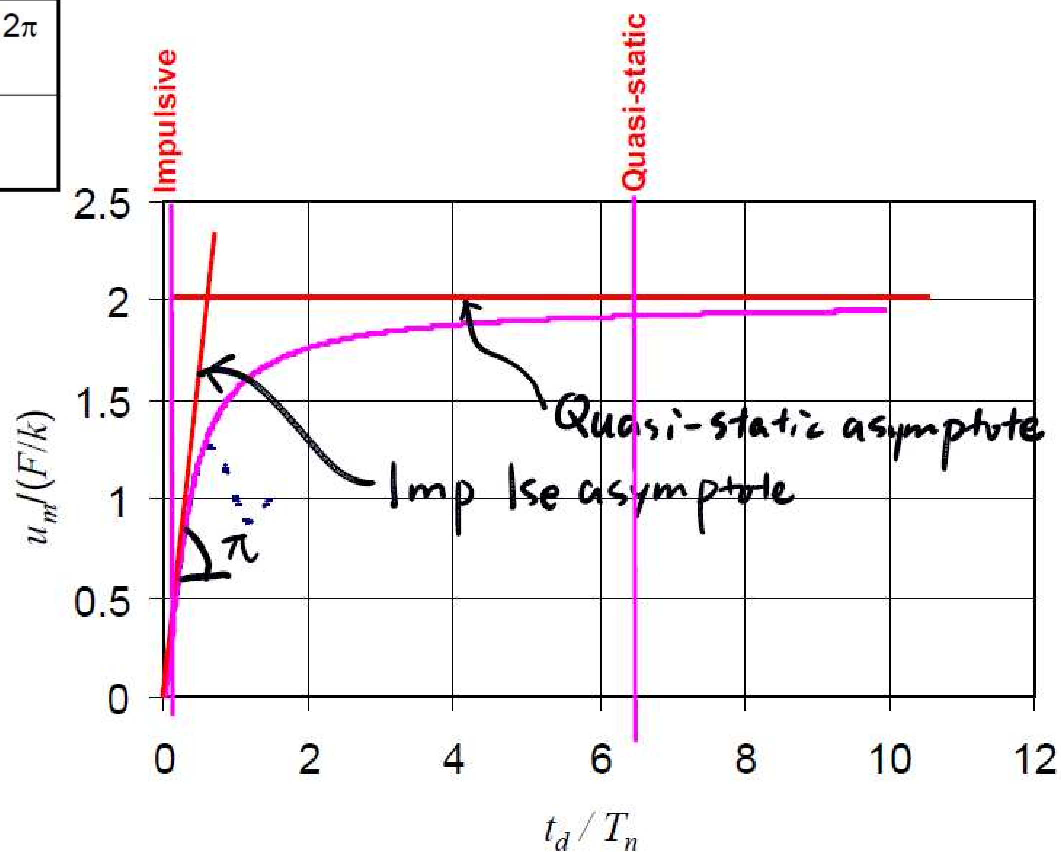
$$\boxed{DLF = \pi \frac{t_d}{T_n}}$$

**Impulsive asymptote**

## Summary of three regimes

Boundaries of three regimes can be specified in terms of the product $\omega t_d$ or $t_d/T_n$ as below:

| | | |
|---|---|---|
| Impulsive loading | $0.4 > \omega t_d$ | $0.02\pi > t_d/T_n$ |
| Dynamic loading | $0.4 < \omega t_d < 40$ | $0.02\pi < t_d/T_n < 2\pi$ |
| Quasi-static loading | $40 < \omega t_d$ | $2\pi < t_d/T_n$ |



## Elasto-plastic SDOF systems

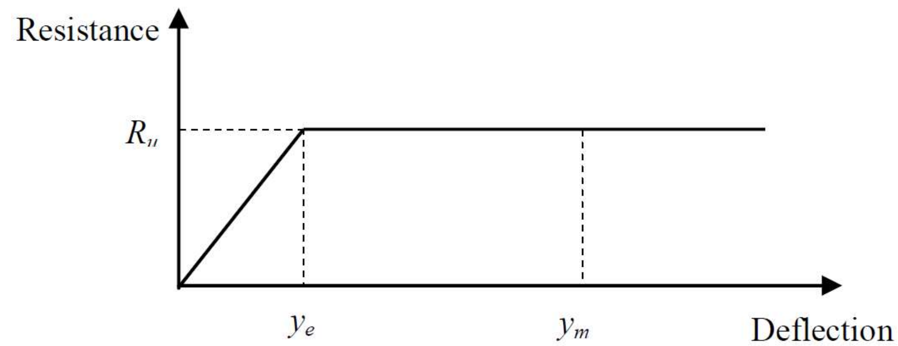
- Structural elements are expected to undergo large inelastic deformation under blast load or high velocity impact.
- Exact analysis of dynamic response is then only possible by step-by-step numerical solution requiring nonlinear dynamic finite-element software.
- However, the degree of uncertainty in both the determination of the loading and the interpretation of acceptability of the resulting deformation is such that solution of a postulated equivalent ideal elasto-plastic SDOF system is commonly used (Biggs, 1964).
- Interpretation is based on the required ductility factor $\mu = y_m/y_e$.

- For example, uniform simply supported beam has first mode shape and the equivalent mass:

$$\varphi(x) = \sin \pi x/L$$

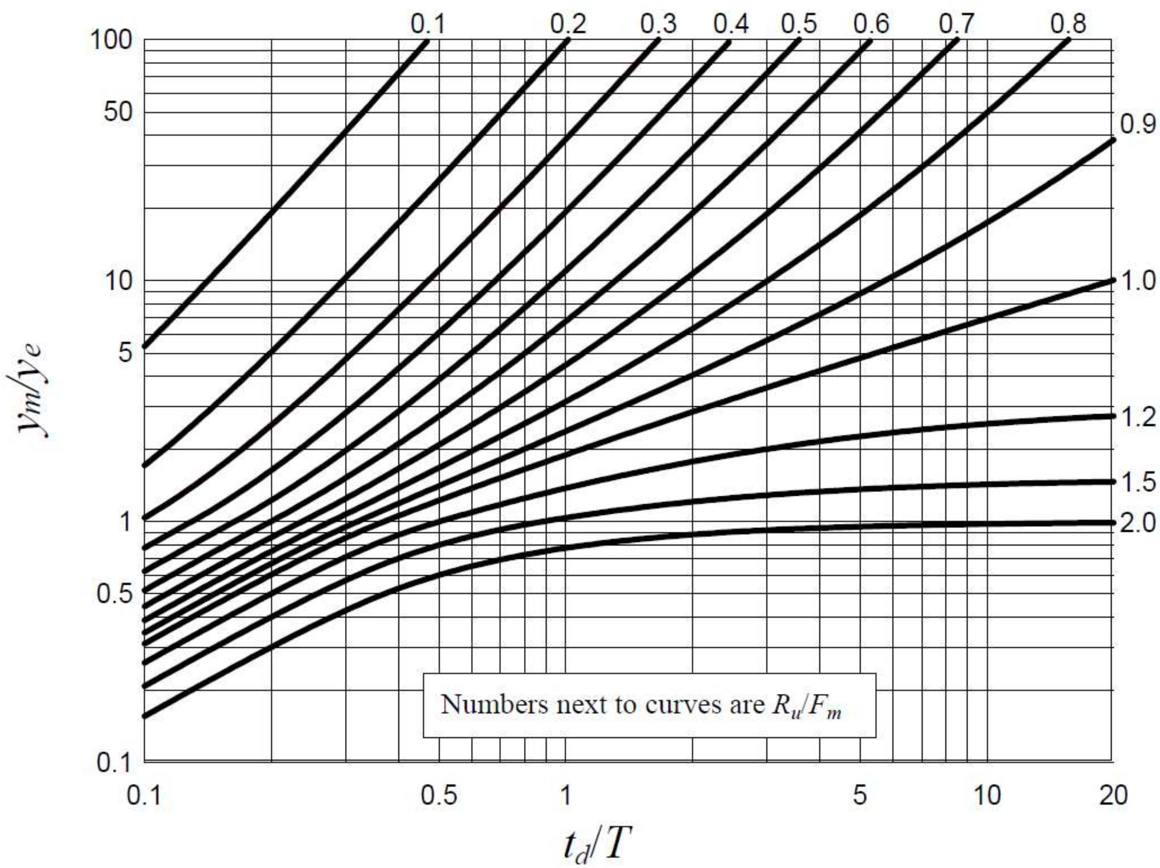
$$M = (1/2)mL$$

where $L$ is the span of the beam and $m$ is mass per unit length.




Simplified resistance function of an elasto-plastic SDOF system

- The equivalent force corresponding to a uniformly distributed load of intensity $p$ is $F = (2/\pi)pL$.
- The response of the ideal bilinear elasto-plastic system can be evaluated in closed form for the triangular load pulse comprising rapid rise and linear decay, with maximum value $F_m$ and duration $t_d$.
- The result for the maximum displacement is generally presented in chart form as a family of curves for selected values of $R_u/F_m$ showing the required ductility μ as a function of $t_d/T$, in which $R_u$ is the structural resistance of the beam and $T$ is the natural period




Maximum response of elasto-plastic SDF system to a triangular load

## Blast loading effects

- Blast loading effects on structural members may produce both local and global responses associated with different failure modes
- The type of structural response depends mainly on:
  - the loading rate
  - the orientation of the target with respect to the direction of the blast wave propagation
  - boundary conditions
- Failure modes associated with global response: flexure, direct shear or punching shear
- Failure modes associated with local response (close-in effects): localized breaching and spalling

Global structural behavior

- The global response of structural elements is generally a consequence of transverse (out-of-plane) loads with long exposure time (quasi-static loading):
  - global membrane (bending)
  - shear responses:
    - diagonal tension,
    - diagonal compression
    - punching shear
    - direct (dynamic) shear

Have relatively minor structural effect in case of blast loading and can be neglected

The high shear stresses may lead to direct global shear failure and may occur prior to any occurrence of significant bending deformations.

## Local structural behavior

- The close-in effect of explosion may cause localized shear (localized punching - or breaching and spalling) or flexural failure in the closest structural elements.
- Breaching failures are typically accompanied by spalling and scabbing of concrete covers as well as fragments and debris

(a) RC Column: Breach (b) RC Column: Partial Breach (c) Steel Column: k-line fracture (d) Steel Column: Brittle fracture

Column responses subject to near-contact blast charges (T. Brewer et al., 2016)

Pressure-Impulse (P-I) Diagrams (Iso-damage curves)

- The pressure-impulse (*P-I*) diagram is an easy way to mathematically relate a specific damage level to a combination of blast pressures and impulses imposes on a particular structural element
- There are *P-I* diagrams that concern with human response to blast as well. In this case, there are three categories of blast-induced injury, namely: primary, secondary, and tertiary injury

## From SDOF to P-I diagram

- Modify the axis of diagram $u_m/(F/k)$ vs. $t_d/T_n$ to become normalized force (pressure) vs. normalized impulse (force x duration) with respect to displacement

**Step 1:** inverting vertical axis and scale to

$$\boxed{\frac{u_m}{F/k} = 2} \qquad y = \frac{2F}{ku_m} \qquad \frac{\text{load (pressure)}}{\text{max. resistance}}$$

Hence quasi-static asymptote becomes:

$$y = \frac{2F}{ku_m} = 1$$

**Step 2:** multiply abscissa (duration) by the new ordinate (already force measure) and scaling:

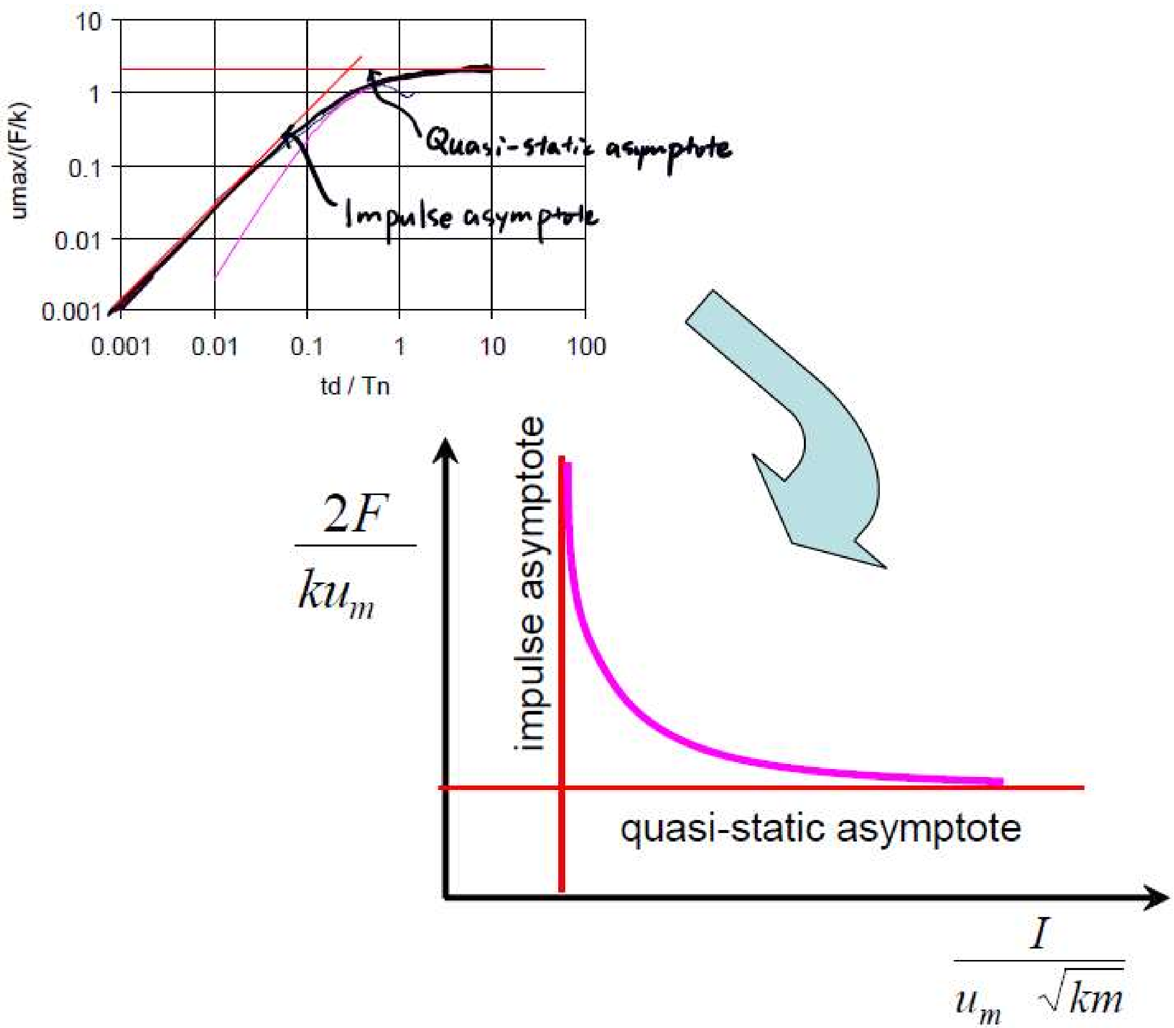
$$x = \pi \frac{t_d}{T_n}\left(\frac{F}{ku_m}\right) = \frac{1}{2}\omega_n t_d \left(\frac{F/k}{u_m}\right) = \frac{1/2Ft_d}{u_m\sqrt{km}}$$

$$x = \frac{I}{u_m\sqrt{km}}$$

$\boxed{\text{non-dimensional impulse}}$

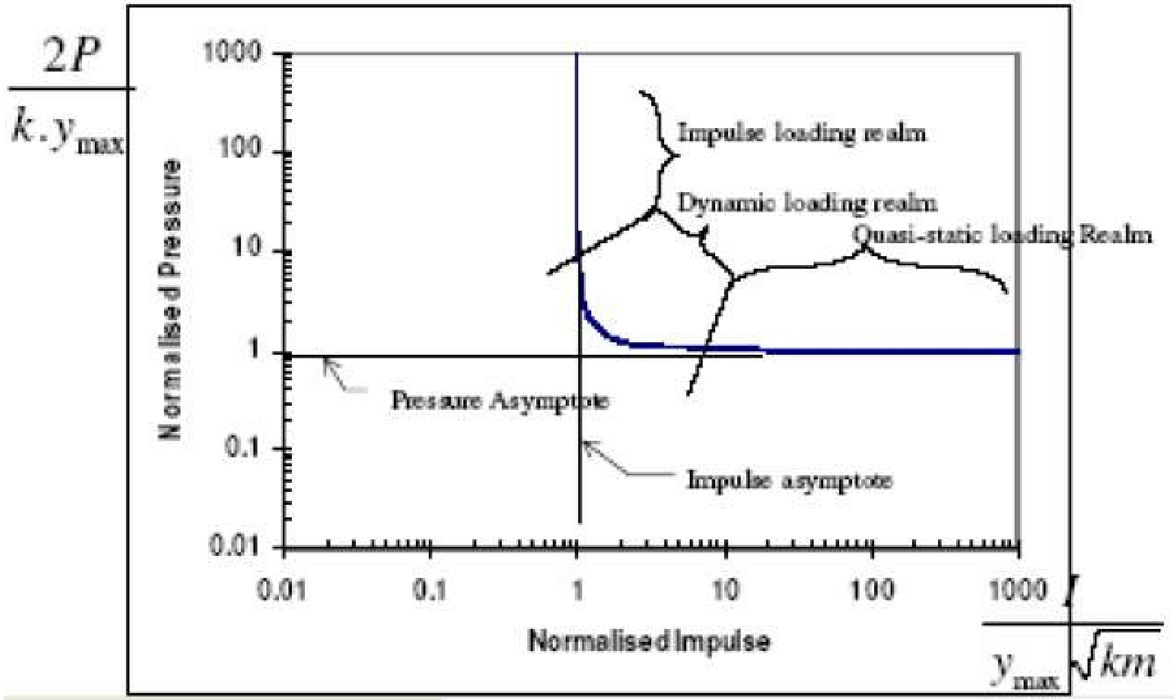
Hence impulse asymptote $\dfrac{u_m}{\cancel{F/k}} = \dfrac{I}{\sqrt{km}\cancel{(F/k)}}$ becomes:

$$\frac{I}{u_m\sqrt{km}} = 1$$

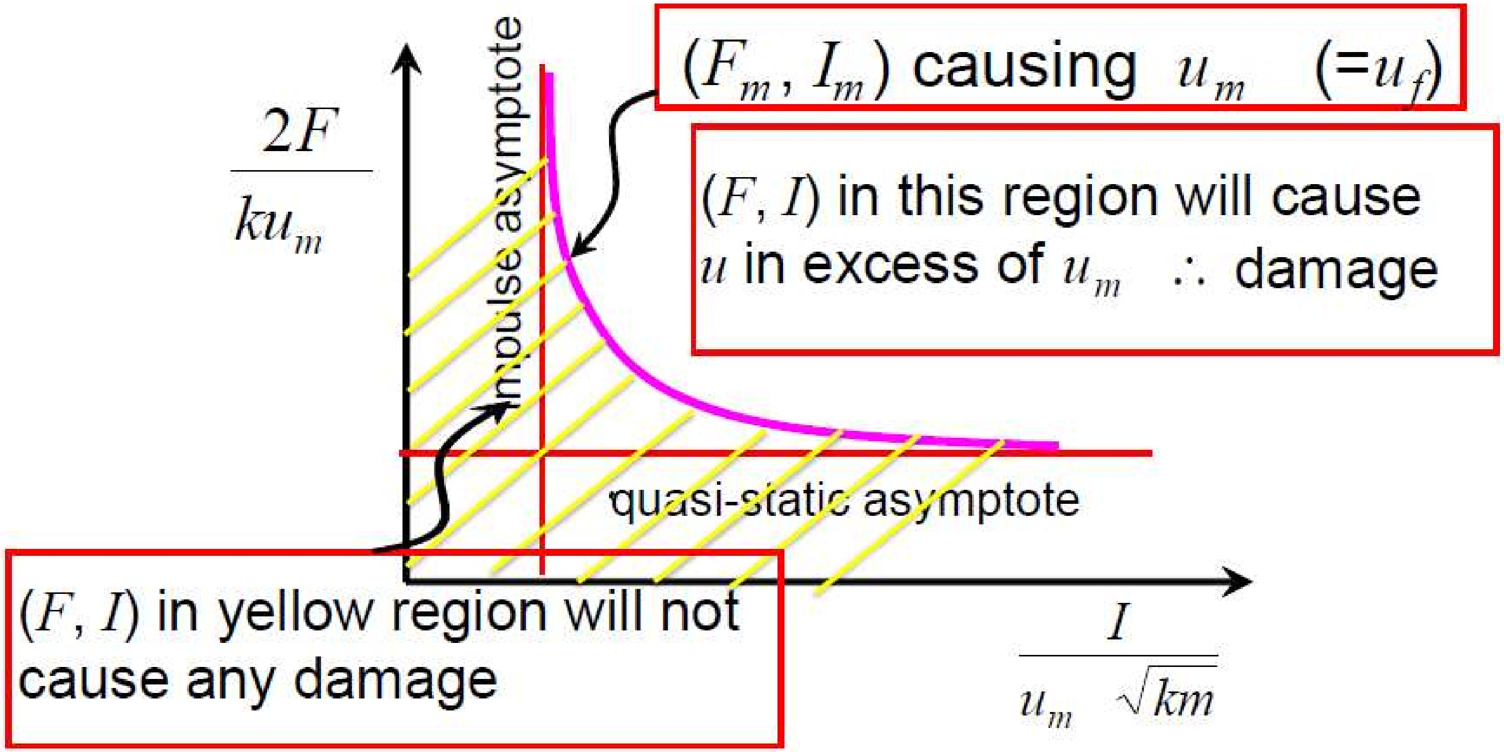


10
1
0.1
0.01
0.001
umax/(F/k)
0.001
0.01
0.1
1
10
100
td / Tn
Quasi-static asymptote
Impulse asymptote
impulse asymptote
quasi-static asymptote
$\frac{2F}{ku_m}$
$\frac{I}{u_m \sqrt{km}}$

# Pressure – impulse diagrams

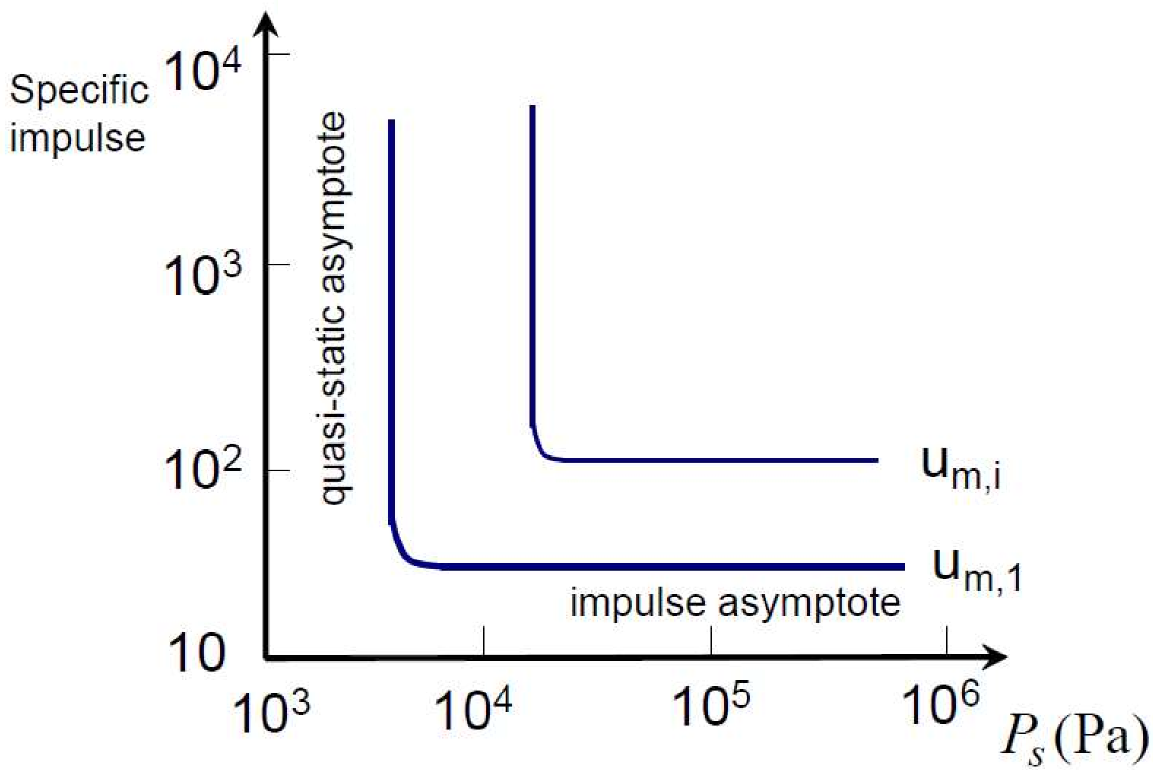



| Load Regime | Limits | |
|---|---|---|
| Impulsive | $\omega t_d < 0.4$ | $t_d/T < 6.37 \times 10^{-2}$ |
| Dynamic | $0.4 < \omega t_d < 40$ | $6.37 \times 10^{-2} < t_d/T < 6.37$ |
| Quasi-static | $\omega t_d > 40$ | $t_d/T > 6.37$ |



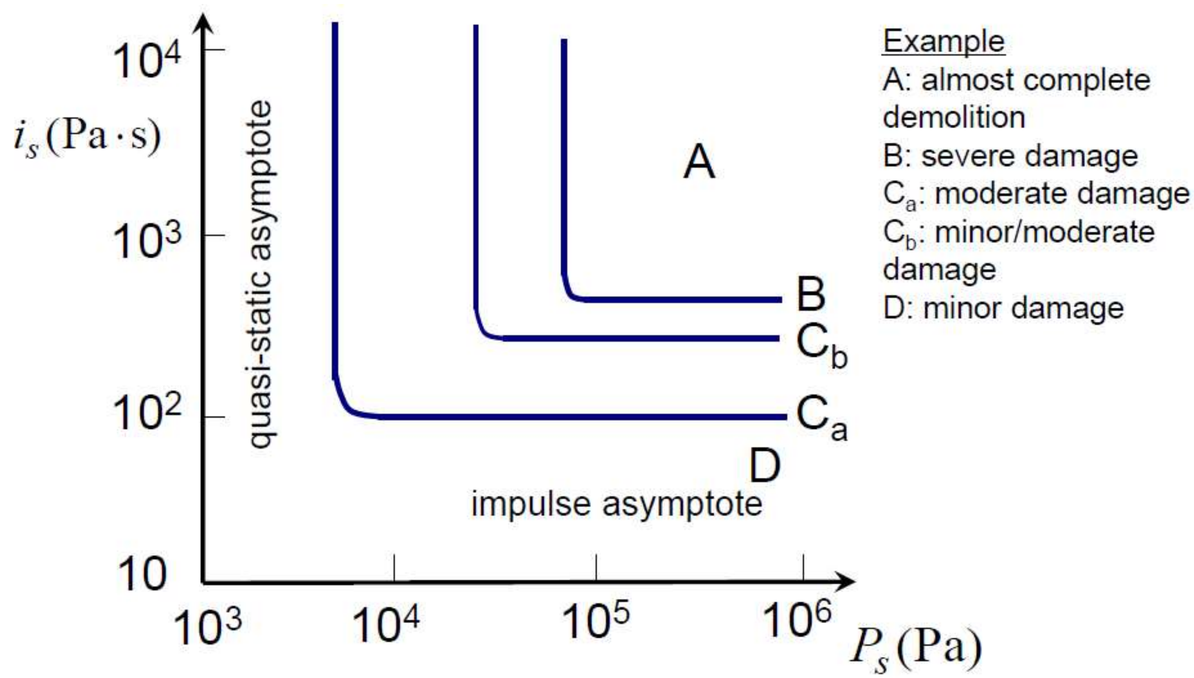
$(F_m, I_m)$ causing $u_m$ $(=u_f)$
$(F, I)$ in this region will cause $u$ in excess of $u_m$ $\therefore$ damage
$\frac{2F}{ku_m}$
impulse asymptote
quasi-static asymptote
$(F, I)$ in yellow region will not cause any damage
$\frac{I}{u_m \sqrt{km}}$

For a particular type of structure, diagram are presented in absolute impulse (specific) vs. overpressure terms, for different damage ($u_m$) levels



Example (for illustration only)

In practice, such diagrams are often constructed on empirical basis, not necessarily with explicit SDOF/limit displacement values




P-I diagram for damage to some small buildings

# Typical Blast Damage to Structures

| Pressure | | Damage |
|---|---|---|
| (psi) | (kPa) | |
| 0.02 | 0.14 | Annoying Noise (137 dB), if of low frequency |
| 0.03 | 0.21 | Occasional Breakage of large glass windows already under strain |
| 0.04 | 0.28 | Loud Noise (143 dB). Sonic boom glass failure |
| 0.10 | 0.70 | Breakage of small windows under strain |
| 0.15 | 1.0 | Typical pressure for glass failure |
| 0.30 | 2.1 | Some damage to ceilings, limit of missiles |
| 0.40 | 2.8 | Limited minor structural damage |
| 0.50-1.0 | 3.5-7.0 | Large and small windows shattered, occasional damage to window frames |
| 0.75 | 5.2 | Minor damage to house structures; 20-50% roof tiles displaced |
| 0.90 | 6.3 | Roof damage to oil storage tanks |
| 1.0 | 7.0 | Partial demolition of houses, made uninhabitable |

Source: Clancy, 1972, from WWII Data

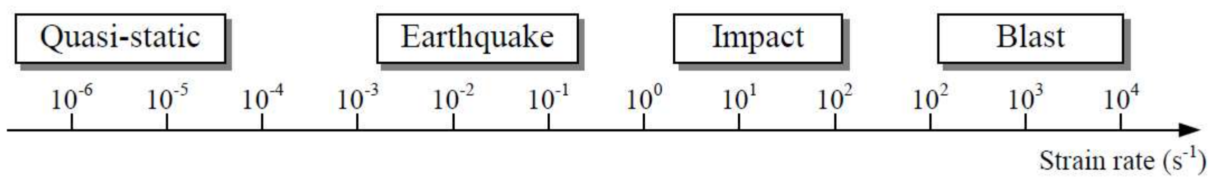
# Typical Blast Damage to Structures

| Pressure | | Damage |
|---|---|---|
| (psi) | (kPa) | |
| 5.0 | 35 | Wooden utility poles damaged |
| 7.0 | 49 | Rail cars overturned |
| 7-8.0 | 49-56 | Brick panels (8-12"), not reinforced, fail by flexure |
| 7-9 | 49-63 | Collapse of steel girder framed building |
| 7-10 | 49-70 | Cars severely crushed |
| 8-10 | 56-70 | Brick walls completely demolished |
| 9 | 63 | Collapse of steel truss type bridges; loaded train wagons demolished |
| >10 | >70 | Complete destruction of all un-reinforced buildings |
| 13 | 91 | 18" brick walls completely destroyed |
| 70 | 490 | Collapse of heavy masonry or concrete bridges |
| | | |

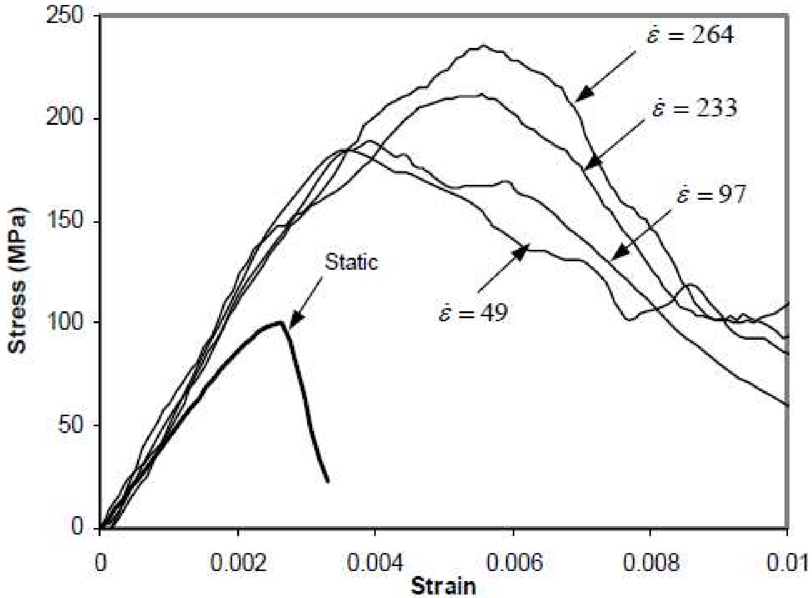
Source: Clancy, 1972, from WWII Data

## Material behaviors at high strain rate

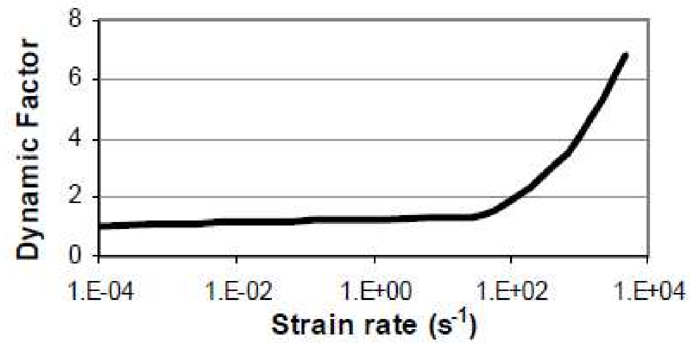
- Blast loads typically produce very high strain rates in the range of $10^2$ - $10^4$ $s^{-1}$.
- This high straining (loading) rate would alter the dynamic mechanical properties of target structures and, accordingly, the expected damage mechanisms for various structural elements.
- It can be seen that ordinary static strain rate is located in the range: $10^{-6}$-$10^{-5}$ $s^{-1}$, while blast pressures normally yield loads associated with strain rates in the range: $10^2$-$10^4$ $s^{-1}$.
- For reinforced concrete structures subjected to blast effects the strength of concrete and steel reinforcing bars can increase significantly due to strain rate effects.
- The typical effects of increased strain rate on the response of structural steels are an increase in yield stress; an increase in ultimate strength, even smaller than for yield stress; and a reduction in the elongation at rupture



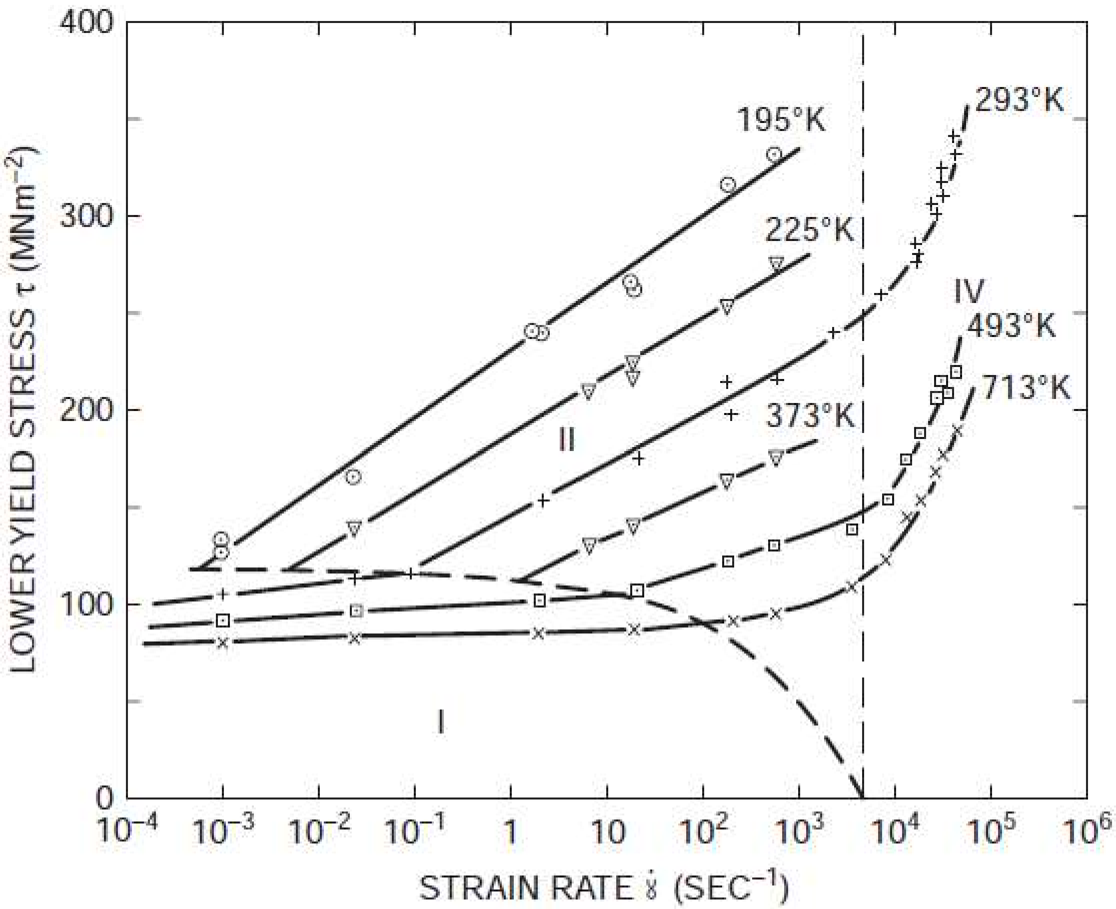

Strain rates associated with different types of loading




Stress-strain curves of concrete at different strain rates




Dynamic increase factor for peak stress of concrete



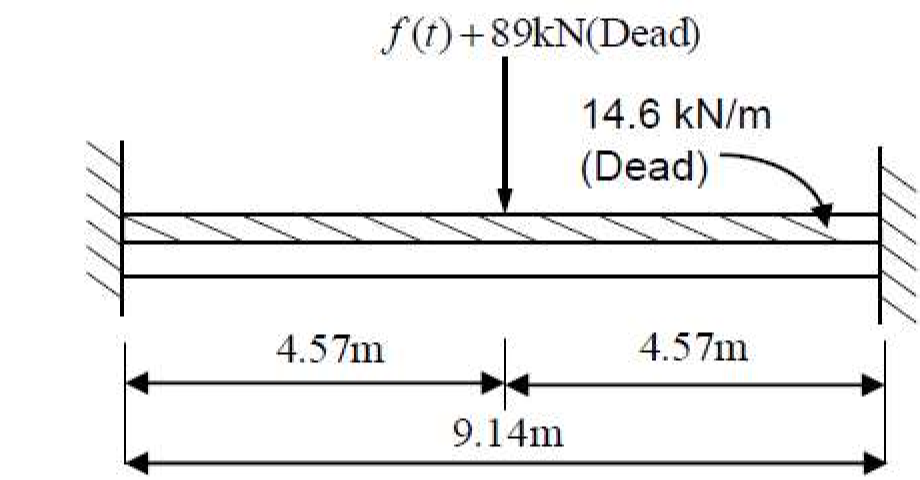

Effect of strain rate on mild steel

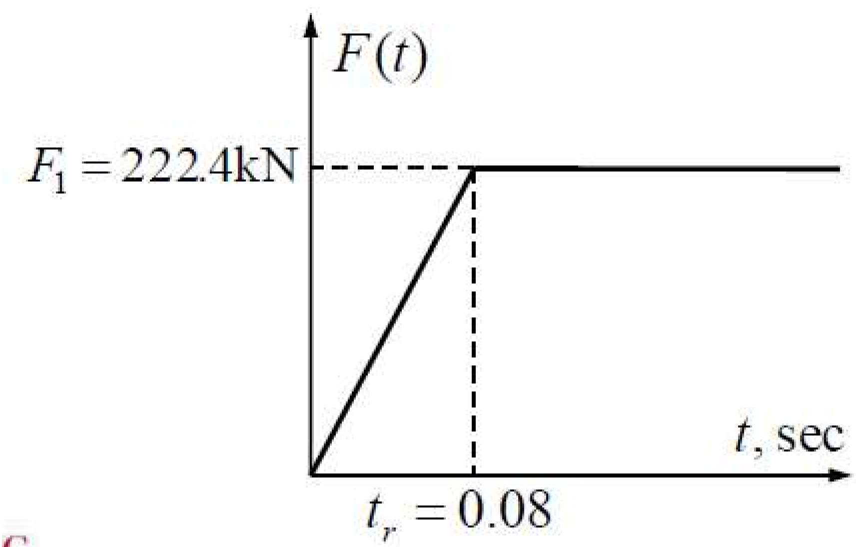
## Design Example

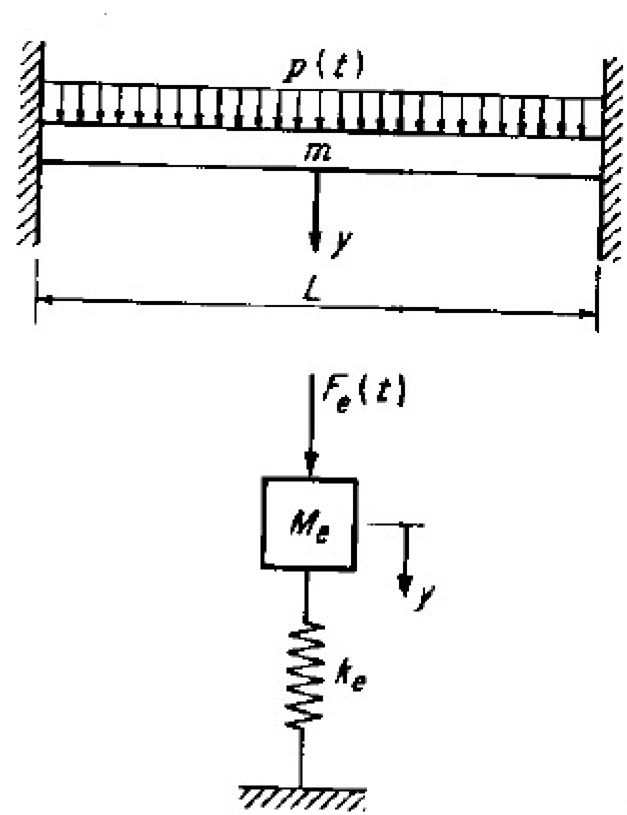
Design a steel beam for elastic design response:

A steel beam of 9.14 m length is clamped at both ends. For elastic design, the bending stress should be less than 207 MPa.

The beam is subjected to dead loads of 14.6 kN/m and 89 kN at mid span.
The beam is braced at load point so that there is no LTB. It is subjected to a dynamic point load F(t) at mid-span as shown in the figures.



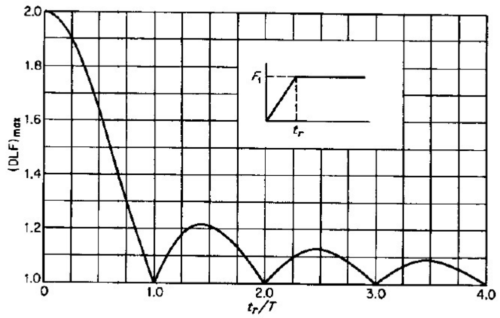




$F_e$, $K_e$, $M_e$ = characteristics of the equivalent SDOF system

$K_M$ =mass factor = $M_e/M$

$K_L$=load factor = $F_e/F$




## Elastic Design

Assume $t_r/T \approx 2/3$; from Fig. 2.9, DLF $\approx$ 1.4. This only affects the dynamic point load $P(t)$ but not the dead point

$$M_{\max} = \frac{w_D L^2}{12} + \frac{F_D L}{8} + \frac{F_1 L}{8}(\mathrm{DFL})_{\max}$$

$$M_{\max} = \frac{14.6 \times 9.14^2}{12} + \frac{89 \times 9.14}{8} + \frac{222.4 \times 9.14}{8} \times 1.4 = 559.1\mathrm{kNm}$$

Required Elastic Section Modulus:

$$W_{el-req} = \frac{M_{\max}}{\sigma} = \frac{559.1}{207 \times 10^3} = 2.7 \times 10^{-3}\,\mathrm{m}^3$$

Most economical section is 24WF76 (UB 610x229x113)

having $I$=8.72x10$^{-4}$ m$^4$ and $W_{el}$=2.87x10$^{-3}$ m$^3$

$$k = \frac{192EI}{L^3} = \frac{192 \times (210 \times 10^6) \times 8.72 \times 10^{-4}}{9.14^3} = 46{,}047\mathrm{kN/m}$$

k=spring constant

From Table 5.2, for elastic clamped beams subjected to mid-span point load,

$$K_L = 1.0 \quad K_M = 1.0 \qquad \textit{concentrated mass (for point load)}$$

$$K_M = 0.37 \qquad \textit{distributed mass (for dead load u.d.l.)}$$

$$M_e = \sum K_M M = \frac{89\times1.0+14.6\times9.14\times0.37}{9.81} = 14.11\,\mathrm{kNsec^2/m.}$$

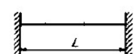
$$K_e = K_L k = 46{,}047\times1.0 = 46{,}047\,\mathrm{kN/m}$$

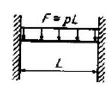
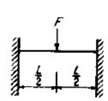
$$T = 2\pi\sqrt{\frac{M_e}{K_e}} = 2\pi\sqrt{\frac{14.11}{46{,}047}} = 0.111\,\mathrm{sec}$$

$$\frac{t_d}{T} = 0.08/0.111 = 0.72$$

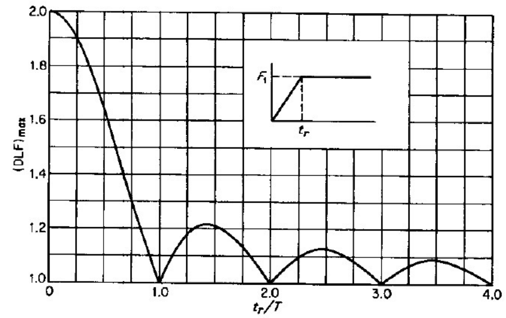
***Table 5.2 Transformation Factors for Beams and One-way Slabs***

$\mathfrak{M}_{Ps}$ = ultimate moment capacity at support
$\mathfrak{M}_{Pm}$ = ultimate moment capacity at midspan




| Loading diagram | Strain range | Load factor $K_L$ | Mass factor $K_M$: Concentrated mass* | Mass factor $K_M$: Uniform mass | Load-mass factor $K_{LM}$: Concentrated mass* | Load-mass factor $K_{LM}$: Uniform mass | Maximum resistance $R_m$ | Spring constant $k$ | Effective spring constant $k_E$† | Dynamic reaction $V$ |
|---|---|---|---|---|---|---|---|---|---|---|
|   | Elastic | 0.53 | ... | 0.41 | ... | 0.77 | $\frac{12\mathfrak{M}_{Ps}}{L}$ | $\frac{384EI}{L^3}$ | ...... | $0.36R + 0.14F$ |
| | Elastic-plastic | 0.64 | ... | 0.50 | ... | 0.78 | $\frac{8}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | $\frac{384EI}{5L^3}$ | $\frac{307EI}{L^3}$ | $0.39R + 0.11F$ |
| | Plastic | 0.50 | ... | 0.33 | ... | 0.66 | $\frac{8}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | 0 | ...... | $0.38R_m + 0.12F$ |
|   | Elastic | 1.0 | 1.0 | 0.37 | 1.0 | 0.37 | $\frac{4}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | $\frac{192EI}{L^3}$ | ...... | $0.71R - 0.21F$ |
| | Plastic | 1.0 | 1.0 | 0.33 | 1.0 | 0.33 | $\frac{4}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | 0 | ...... | $0.75R_m - 0.25F$ |

* Concentrated mass is lumped at the concentrated load.




2nd round iteration from Fig. 2.9,

*$(DLF)_{max}$=1.35*

$$M_{max} = \frac{14.6 \times 9.14^2}{12} + \frac{89 \times 9.14}{8} + \frac{222.4 \times 9.14}{8} \times 1.35 = 546.3\text{kNm}$$

Required Elastic Section Modulus:

$$\sigma_{max} = \frac{M_{max}}{W_{el}} = \frac{546.3}{2.87 \times 10^{-3}} = 190.3 \times 10^3 \text{kN/m}^2 = 190.3\text{MPa}$$

From Fig. 2.9(a), $t_r/T=0.72$, DLF = 1.35

$\Rightarrow R_m$ =1.35 x 222.4 = 300.2 kN

$$V_{max} = 0.71 \times 300.2 - 0.21 \times 222.4 + \text{dead}$$
$$= 213.1 - 46.7 + 89 \times 0.5 + 14.6 \times 9.14 \times 0.5 = 277.6\text{kN}$$

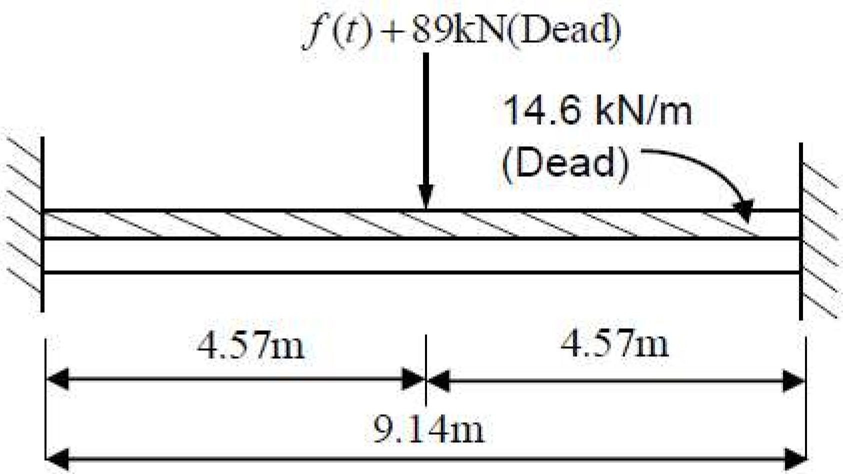
## Elasto-plastic design

Design a steel beam for plastic design response:

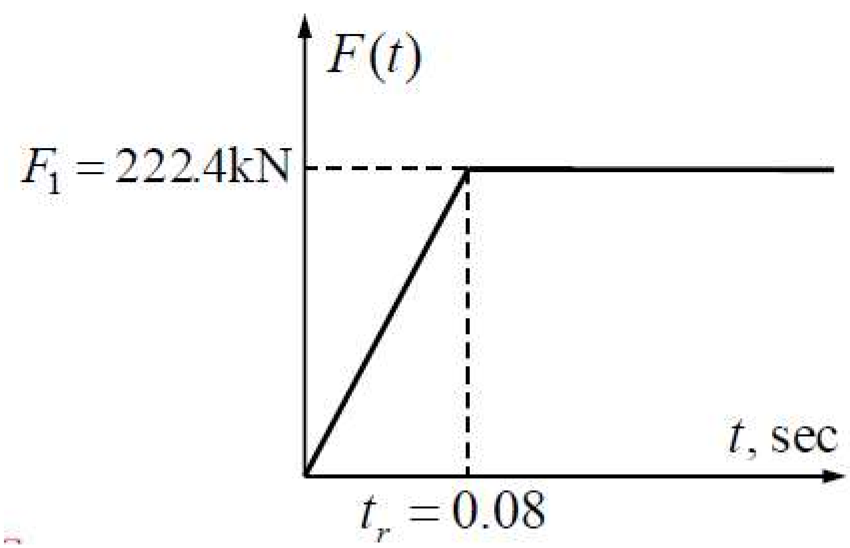
A steel beam of 9.14 m length is clamped at both ends. For plastic design, the bending stress should be less than its yield strength of 344 MPa

The beam is subjected to dead loads of 14.6 kN/m and 89 kN at mid span.

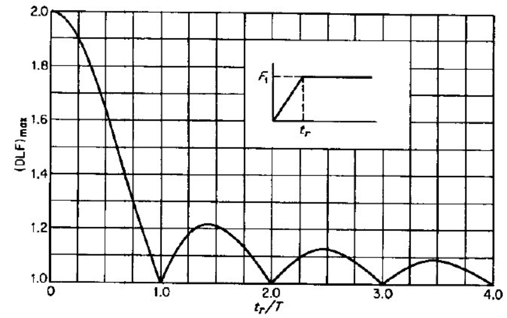
The beam is braced at load point so that there is no LTB. It is subjected to a dynamic point load F(t) at mid-span as shown in the figures.



f(t) + 89kN(Dead)
14.6 kN/m
(Dead)
4.57m
4.57m
9.14m

F(t)
F1 = 222.4kN
t, sec
tr = 0.08




## Plastic Design

Assume $t_r/T \approx 2/3$ and from Fig. 2.9, DLF $\approx$ 1.4. This only affects the dynamic point load $P(t)$ but not the dead point

$$M_{\max} = \frac{w_D L^2}{12} + \frac{F_D L}{8} + \frac{F_1 L}{8}(\mathrm{DFL})_{\max}$$

$$M_{\max} = \frac{14.6 \times 9.14^2}{12} + \frac{89 \times 9.14}{8} + \frac{222.4 \times 9.14}{8} \times 1.4 = 559.1\mathrm{kNm}$$

Required Plastic Section Modulus:

$$W_{pl-req} = \frac{M_{\max}}{\sigma} = \frac{559.1}{344 \times 10^3} = 1.63 \times 10^{-3}\,\mathrm{m}^3$$

Most economical section is UB 533x210x82 having $I$=4.75x10$^{-4}$ m$^4$ and $W_{pl}$=2.06x10$^{-3}$ m$^3$ and Moment capacity:

$$M_s = M_m = W_{pl} \times \sigma = 2.06 \times 344 = 708.6\mathrm{kNm}$$

From Table 5.2, for plastic clamped beams subjected to mid-span point load, maximum resistance is:

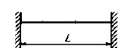
$$R_m = 4(M_s + M_m)/L = 4 \times 2 \times 708.6/9.14 = 620\text{kN}$$

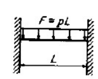
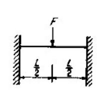
Dynamic reaction is:

$$V_{\max} = 0.75 \times 620 - 0.25 \times 222.4 + \text{dead}$$
$$= 465 - 55.6 + 0.5 \times 89 + 0.5 \times 14.6 \times 9.14 = 520.6\,\text{kN}$$

*Table 5.2 Transformation Factors for Beams and One-way Slabs*

$\mathfrak{M}_{Ps}$ = ultimate moment capacity at support
$\mathfrak{M}_{Pm}$ = ultimate moment capacity at midspan




| Loading diagram | Strain range | Load factor $K_L$ | Mass factor $K_M$: Concentrated mass* | Mass factor $K_M$: Uniform mass | Load-mass factor $K_{LM}$: Concentrated mass* | Load-mass factor $K_{LM}$: Uniform mass | Maximum resistance $R_m$ | Spring constant $k$ | Effective spring constant $k_E$† | Dynamic reaction $V$ |
|---|---|---|---|---|---|---|---|---|---|---|
|   | Elastic | 0.53 | ... | 0.41 | ... | 0.77 | $\frac{12\mathfrak{M}_{Ps}}{L}$ | $\frac{384EI}{L^3}$ | ...... | $0.36R + 0.14F$ |
| | Elastic-plastic | 0.64 | ... | 0.50 | ... | 0.78 | $\frac{8}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | $\frac{384EI}{5L^3}$ | $\frac{307EI}{L^3}$ | $0.39R + 0.11F$ |
| | Plastic | 0.50 | ... | 0.33 | ... | 0.66 | $\frac{8}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | 0 | ...... | $0.38R_m + 0.12F$ |
|   | Elastic | 1.0 | 1.0 | 0.37 | 1.0 | 0.37 | $\frac{4}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | $\frac{192EI}{L^3}$ | ...... | $0.71R - 0.21F$ |
| | Plastic | 1.0 | 1.0 | 0.33 | 1.0 | 0.33 | $\frac{4}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$ | 0 | ...... | $0.75R_m - 0.25F$ |

* Concentrated mass is lumped at the concentrated load.

# Comparison of Design Example 2

| | Elastic design | Plastic design |
|---|---|---|
| Beam chosen | UB 610x229x113 | UB 533x210x82 |
| Maximum resistance | 300.2 kN | 620 kN |
| Dynamic reaction | 277.6 kN | 520.6 kN |