Universitatea Politehnica Timişoara
Facultatea de Construcţii
Departamentul de Construcţii Metalice şi Mecanica Construcţiilor

# COMPOSITE STEEL-CONCRETE STRUCTURES

- CURS 5-a -

Composite Columns (2)

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**Notele de curs pot fi descărcate de pe pagina de web**
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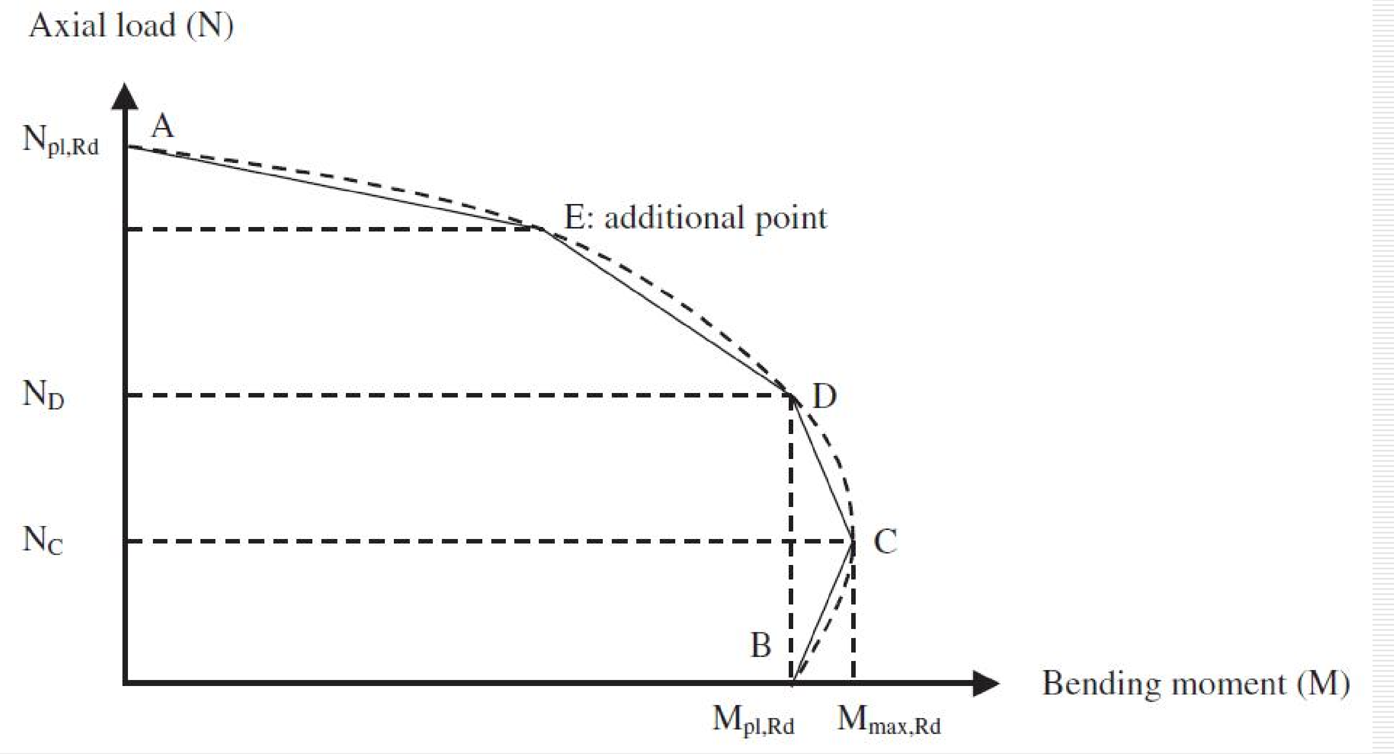
# § 3.3 Combined Compression and Bending
## N –M INTERACTION DIAGRAM

- Due to the assumption that concrete has no tensile strength, the *N*–*M* interaction curve is convex as shown in figure below:

Axial force – bending moment diagram of a composite cross-section




Obs: The above described general approach is time consuming and is best carried out by computers. To simplify calculations, the concrete stress distribution may be approximated by a uniform stress block with a reduced depth of compression.

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM

- For a composite cross-section symmetrical about the axis of bending, Eurocode 4 presents a simple method to evaluate its *N–M* interaction diagram.
- As shown in the figure above, in this method, instead of determining the continuous *N–M* interaction curve, only a few key points in the curve are determined. The *N–M* curve is then constructed by joining these key points by straight lines.
- When evaluating these key points, rigid-plastic material behaviour is assumed.
  - Thus, steel is assumed to have reached yield in either tension or compression.
  - Concrete is assumed to have reached its peak stress in compression and its tensile strength is zero.

Obs: However, results of a recent studies indicate that for high strength concrete ($f_{ck} \geq 100$ N/mm$^2$) filled columns, the assumption of rigid-plastic behaviour in the simplified approach is unconservative and the general procedure should be used. Similarly, if steel of very high strength is used, steel may not fully yield at concrete crushing and the general approach should be used.

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM

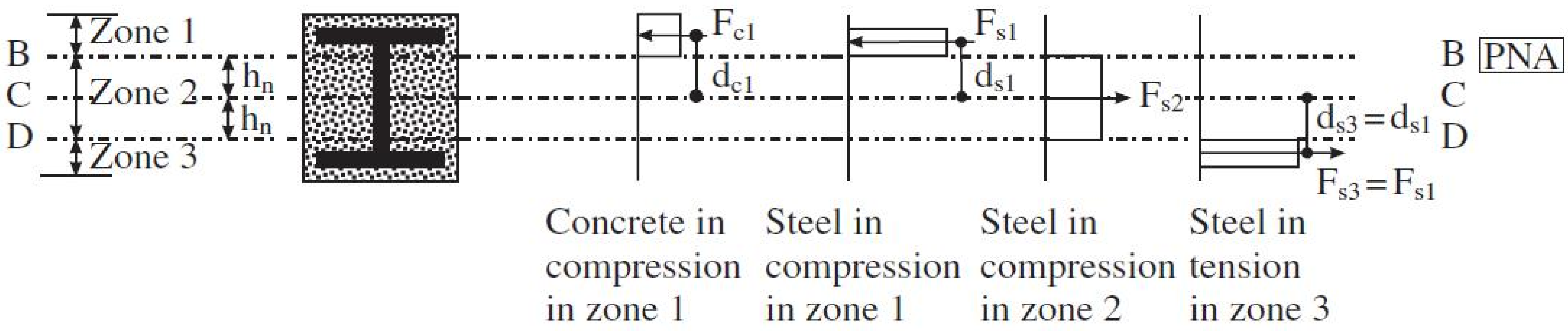
- The key points in the *N-M* interaction diagram to be evaluated are:
  - point A: Squash load point ($N_{pl,Rd}$, $0$),
  - point B: Pure flexural bending point ($0$, $M_{pl,Rd}$),
  - point C: The maximum bending moment point ($N_C$, $M_{max,Rd}$),
  - point D: Point ($N_D$, $M_{pl,Rd}$) with bending moment equal to the pure bending moment capacity,
  - point E: A point between A and D to refine the *N–M* interaction diagram.

- The squash load $\boldsymbol{N_{pl,Rd}}$ has been determined in paragraph 3.2.
- In the following are determined the values of $N_D$, $N_C$, $M_{pl,Rd}$ and $M_{max,Rd}$ that defines the characteristic points of the *M-N* interaction diagram.

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM

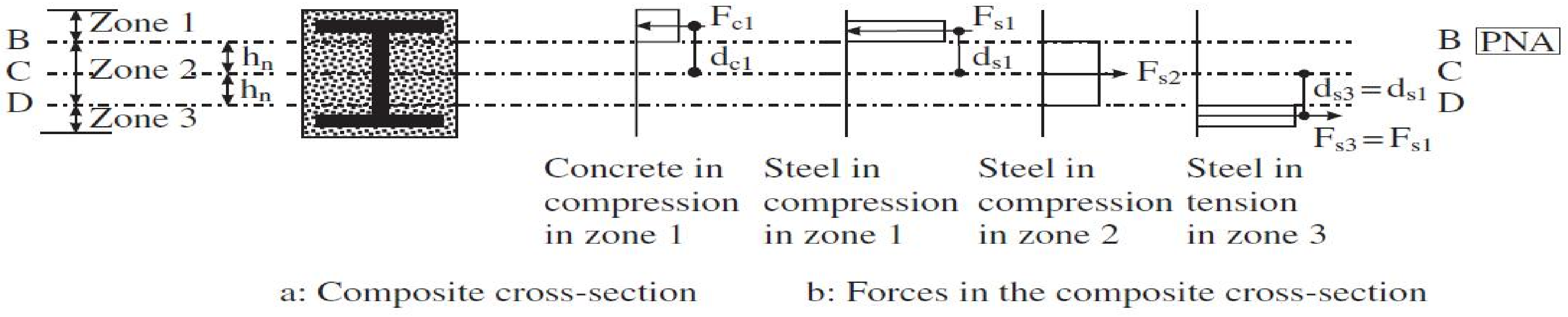
- ***$N_D$*** represents the axial compression force at which the composite cross-sectional bending moment resistance is the same as that under pure bending.
- Considering the figure below with the position of the plastic neutral axis (PNA) of the composite cross-section under pure bending. Assume that the part below the PNA is in tension and the part above in compression.




a: Composite cross-section b: Forces in the composite cross-section

# § 3.3 Combined Compression and Bending

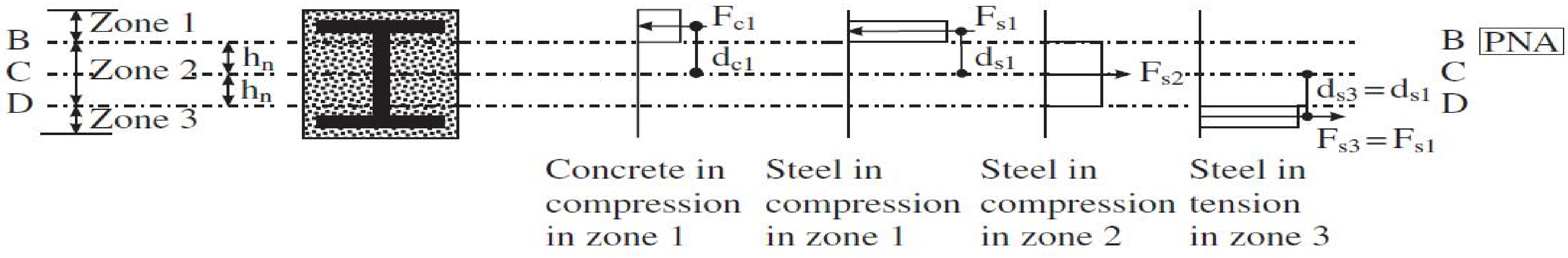
## N –M INTERACTION DIAGRAM




- Since concrete is assumed to have no tensile resistance, the plastic neutral axis for pure bending must be above the axis of symmetry C–C. Assume it is at a distance $h_n$ from the axis of symmetry at B–B.
- If the composite cross-section is divided into three zones as shown, the forces in the composite cross-section may be divided into four parts:
  - concrete in compression in zone 1 above the plastic neutral axis ($F_{c1}$),
  - steel in compression in zone 1 above the plastic neutral axis ($F_{s1}$),
  - steel in tension in zone 2 within a distance $h_n$ on either side of the axis of symmetry ($F_{s2}$) and
  - steel in tension in zone 3 ($F_{s3}$).

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM

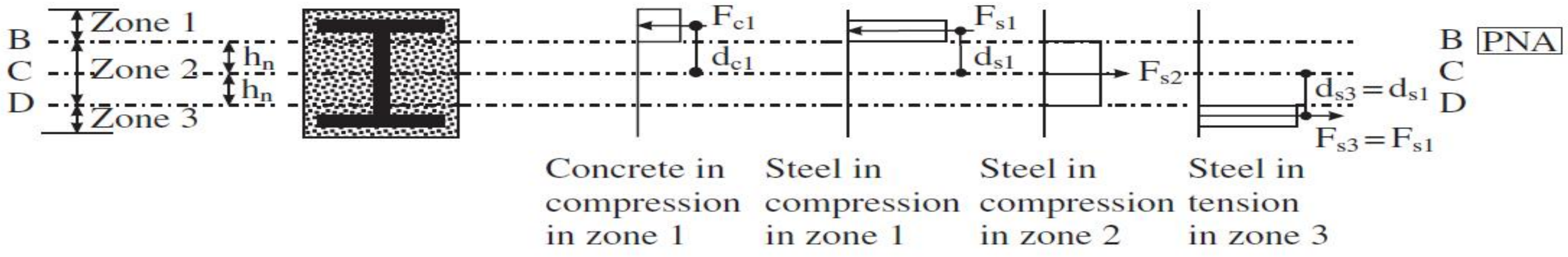



a: Composite cross-section b: Forces in the composite cross-section

- From symmetry: $F_{s1} = F_{s3}$
- Since the resultant axial force in the composite cross-section is zero under pure bending, it follows: $F_{c1} + F_{s1} = F_{s2} + F_{s3}$
- By using the first equation it results: $F_{c1} = F_{s2}$
- The plastic bending moment capacity of the composite cross-section under pure bending is that about the axis of symmetry (C–C) and is then given by: $M_{pl,Rd} = F_{c1} * d_{c1} + 2 * F_{s1} * d_{s1}$
- Now consider the case where the plastic neutral axis is moved from B–B to D–D, the forces in both concrete and steel in zone 2 changing from tension to compression.

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM




a: Composite cross-section b: Forces in the composite cross-section

- But since the centroid of these forces is at the axis of symmetry, their bending moment contribution is zero. Thus the bending moment in the composite cross-section is unchanged by moving the plastic neutral axis from B–B to D–D, but there is now a net compressive force in the composite cross-section. This force $N_D$ is given by:

$$N_D = F_{c1} + F_{c2} + F_{s1} + F_{s2} - F_{s3} = F_{c1} + F_{c2} + F_{s2}$$

- Recognizing $F_{c1}$=$F_{c3}$ it follows $F_{s2}$ =$F_{c3}$ . Hence:

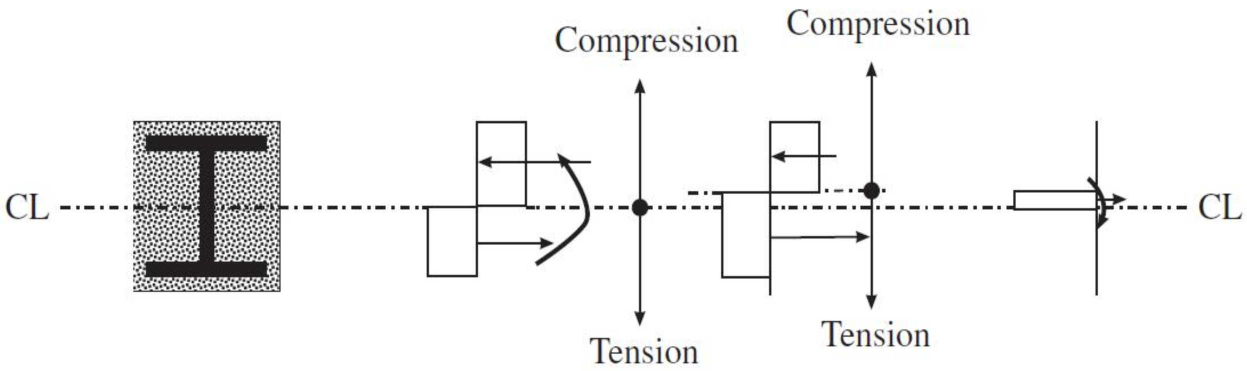
$$N_D = F_{c1} + F_{c2} + F_{c3} = N_{c,Rd}$$

where $N_{c,Rd}$ is the compressive resistance of the entire concrete part of the composite cross-section.

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM

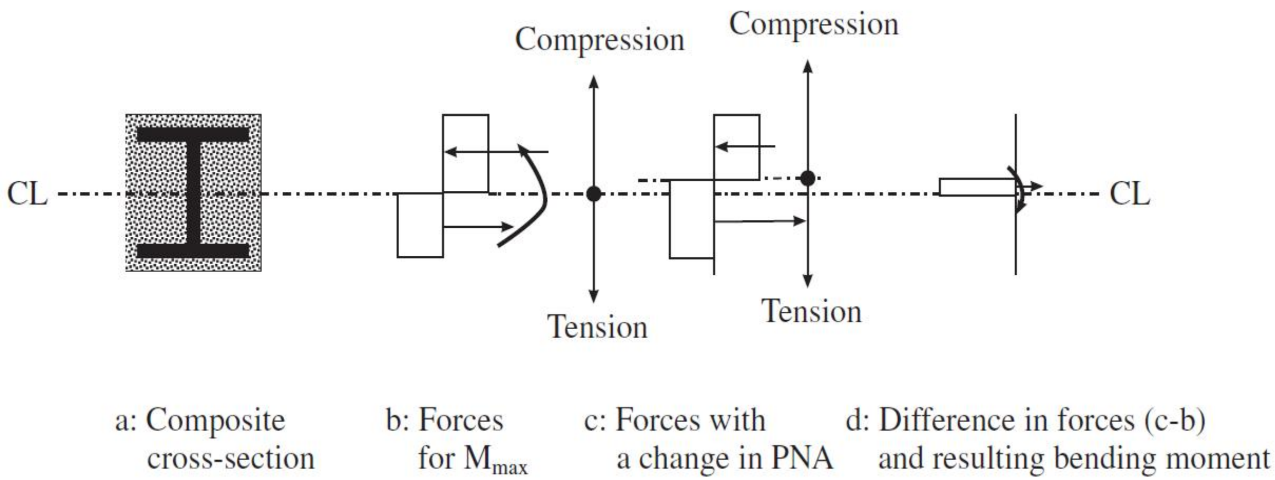
- ***$M_{max,Rd}$***. The bending moment of a composite cross-section is taken about the axis of symmetry. Therefore, the maximum bending moment is obtained by placing the plastic neutral axis at the axis of symmetry of the composite cross-section.
- This conclusion can be obtained by examining the change in the bending moment of the composite cross-section by making a small change in the position of the plastic neutral axis.
- Referring to figure right, it is assumed the positive bending moment anti-clockwise and causes tension below and compression above the plastic neutral axis.




a: Composite cross-section
b: Forces for $M_{max}$
c: Forces with a change in PNA
d: Difference in forces (c-b) and resulting bending moment

# § 3.3 Combined Compression and Bending

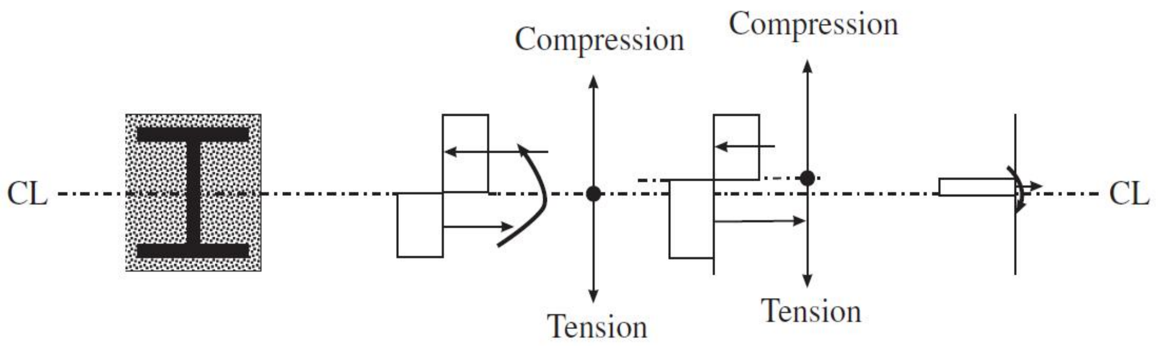
## N –M INTERACTION DIAGRAM




- Consider the plastic neutral axis is moved up from the axis of symmetry. Compared to the case where the plastic neutral axis is at the axis of symmetry, this movement in the plastic neutral axis gives a net increase in the tension force.
- This net tension force acts at an eccentricity above the axis of symmetry, causing a clockwise bending moment about this axis, which reduces the bending moment of the composite cross-section.
- Similarly, if the plastic neutral axis is moved down from the axis of symmetry, a net compressive force is obtained acting below this axis, again causing a clockwise bending moment about the axis of symmetry and a reduction in the bending moment of the composite cross-section.

# § 3.3 Combined Compression and Bending
## N –M INTERACTION DIAGRAM

- Thus, moving the plastic neutral axis of the composite cross-section either above or below the axis of symmetry, the bending moment of the




a: Composite cross-section b: Forces for $M_{max}$ c: Forces with a change in PNA d: Difference in forces (c-b) and resulting bending moment

composite cross-section is reduced, implying that the bending moment is maximum if the plastic neutral axis is placed at the axis of symmetry.

- The maximum bending moment of the composite cross-section is then given by:

$$M_{max,Rd} = \left( W_{pa} f_y + \frac{1}{2} W_{pc} f_{ck} + W_{ps} f_{sk} \right)$$

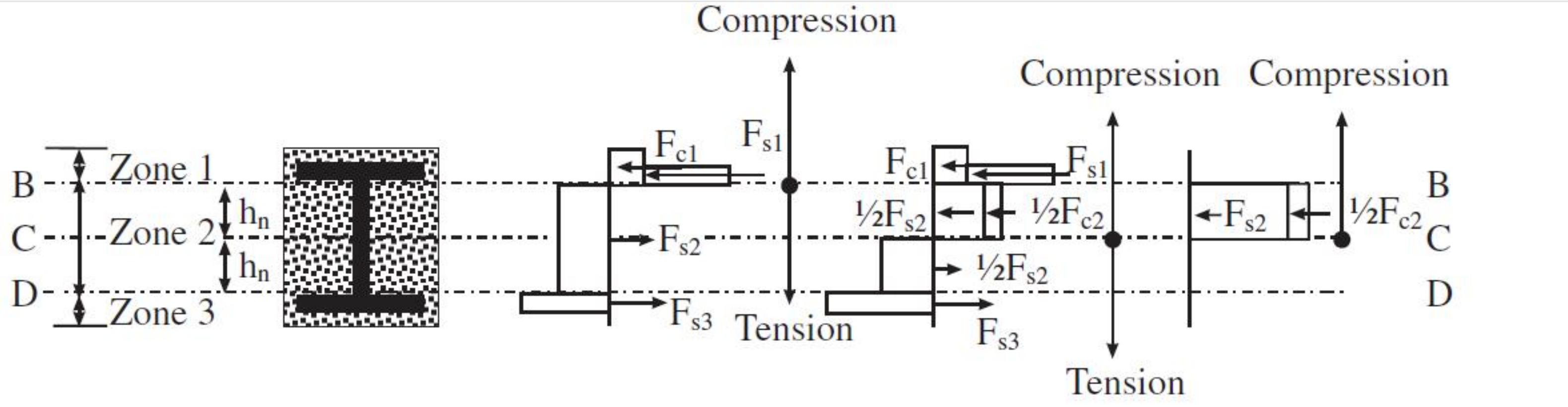
where $W_{pa}$, $W_{pc}$ and $W_{ps}$ are plastic modulus of steel, overall concrete and reinforcement about the axis of symmetry of the composite cross-section.

# § 3.3 Combined Compression and Bending

## N–M INTERACTION DIAGRAM

- The coefficient "1/2" in above formula is a result of the assumption that concrete has no tensile strength and only the compressive strength contributes to the bending moment capacity.

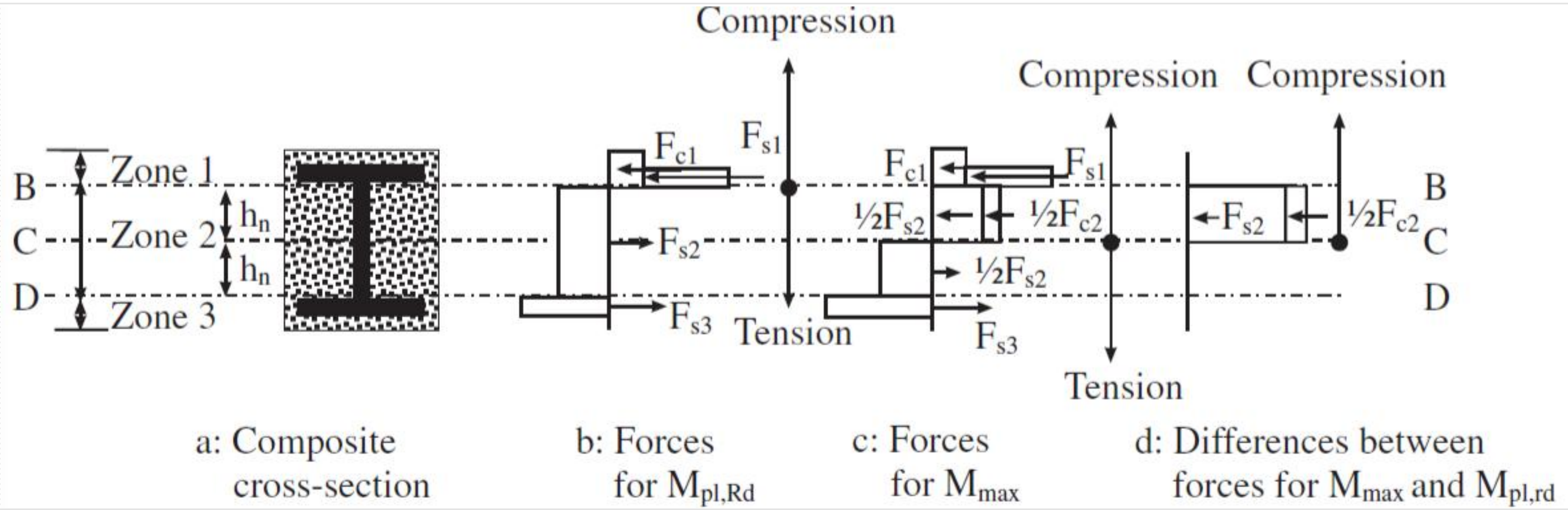
- $M_{pl,Rd}$. Comparing the stress diagram of the composite cross-section for the maximum bending moment (figure c below) with the stress diagram for pure bending (zero axial force) in figure b, the differential stress diagram (figure d) has stress resultants $1/2F_{c2}$ for concrete and $F_{s2}$ for steel within a distance $h_n$ above the axis of symmetry.




a: Composite cross-section  b: Forces for $M_{pl,Rd}$  c: Forces for $M_{max}$  d: Differences between forces for $M_{max}$ and $M_{pl,rd}$

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM




- Thus, the plastic bending moment capacity of the composite cross-section under pure bending may be evaluated by the relation:

$$M_{pl,Rd} = M_{max,Rd} - \left(W_{pan} f_y + \frac{1}{2} W_{pcn} f_{ck} + W_{psn} f_{sk}\right)$$

where $W_{pan}$, $W_{pcn}$ and $W_{psn}$ are plastic modulus of steel, overall concrete and reinforcement within zone 2. To obtain their values, the value of $h_n$ should be found.

Obs: In Eurocode 4, detailed analytical equations have been provided for calculating $h_n$ for various types of composite cross-section.

# § 3.3 Combined Compression and Bending

## N –M INTERACTION DIAGRAM

- ***$N_c$***. The corresponding axial compression to the maximum bending moment is:

$$N_C = F_{s2} + \frac{1}{2}F_{c2} = F_{c1} + \frac{1}{2}F_{c2} = \frac{1}{2}(F_{c1} + F_{c2} + F_{c3}) = \frac{1}{2}N_{c,Rd}$$

where all the forces are described before.

- ***Additional Point E***.
- Due to the way in which the *N–M* interaction curve is used to calculate the column strength, using a polygonal curve will result in overestimation of the column strength compared with using the exact *N–M* curve. In order to reduce this overestimation, sometimes an additional point (point E) in the *N–M* curve is evaluated. According to Eurocode 4, this point corresponds to the case in which the PNA is at half way between the compressed edge of the composite cross-section and (D–D) line (see the figure with the forces in a composite cross-section). Detailed analytical equations are provided in Eurocode 4 for the determination of point E in the *N–M* curve.

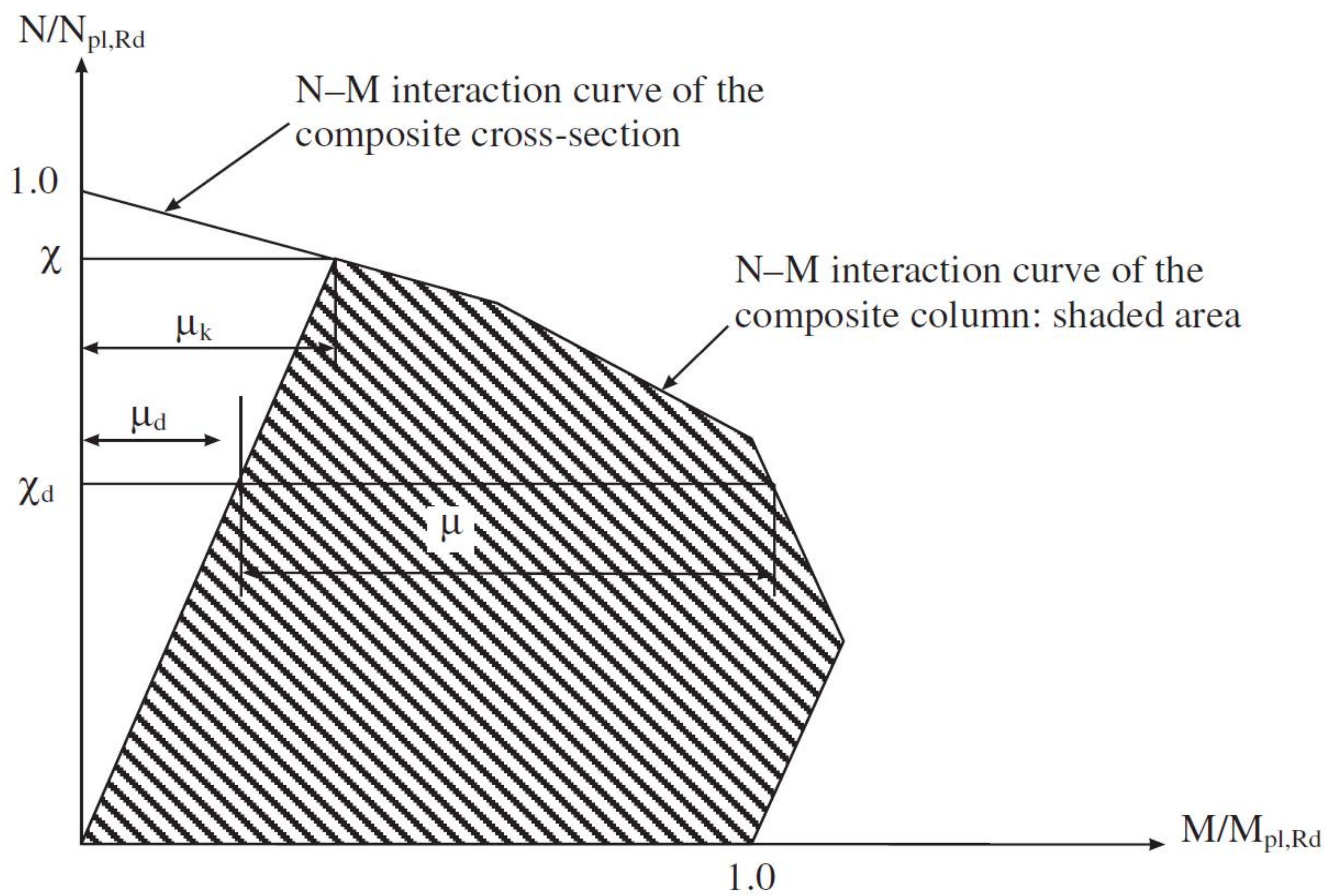
# § 3.3 Combined Compression and Bending

## STRENGTH OF COMPOSITE COLUMN WITH AXIAL FORCE AND BENDING MOMENT ABOUT ONE AXIS

- **The *N–M* interaction diagram represents the failure surface of a composite cross-section** under compression and bending about one axis. The failure load of a composite column is always less than the capacity of the composite cross-section and this is because of the secondary moment associated with the column's imperfections.
- Refer to figure below, which shows the non-dimensional *N–M* interaction curve of a composite cross-section, normalized with respect to $N_{pl,Rd}$ and $M_{pl,Rd}$. For a composite column under pure compression, its failure load is indicated by $\chi$.
- This implies that at the instant of column failure, the column straightness imperfections cause an equivalent bending moment $\mu_k$ to cause failure at the most highly stressed cross-section of the composite column.

# § 3.3 Combined Compression and Bending

## STRENGTH OF COMPOSITE COLUMN ABOUT 1 AXIS




Design *N–M* diagram for a composite column under compression and uniaxial bending

# § 3.3 Combined Compression and Bending

## STRENGTH OF COMPOSITE COLUMN ABOUT 1 AXIS

- If an additional external column bending moment is applied, the column compressive resistance will be even smaller than the column resistance under pure compression, thus the second order bending moment arising from imperfection is also reduced.
- Assume that the imperfection-induced second order bending moment varies linearly with column axial load. Under the applied load $\chi_d$, the second order bending moment is $\mu_d$ and the usable bending moment resistance is $\mu$.
- Thus, the **interaction curve for the composite column** (as opposed to the composite cross-section) is the shaded part in the above figure.
- For design checks, the following equation should be satisfied:

$$M_{Sd} \leq 0.9\,\mu M_{pl,Rd}$$

- The constant *0.9* used in above equation is used to account for approximations in the determination of the *N–M* interaction curve of the composite cross-section.

# § 3.3 Combined Compression and Bending

## STRENGTH OF COMPOSITE COLUMN ABOUT 1 AXIS

- In this calculation, it is assumed that there is no reduction in the column bending moment capacity due to lateral torsional buckling when it is subjected to pure bending. Since the lateral torsional stiffness of a composite column is high, this assumption is reasonable.

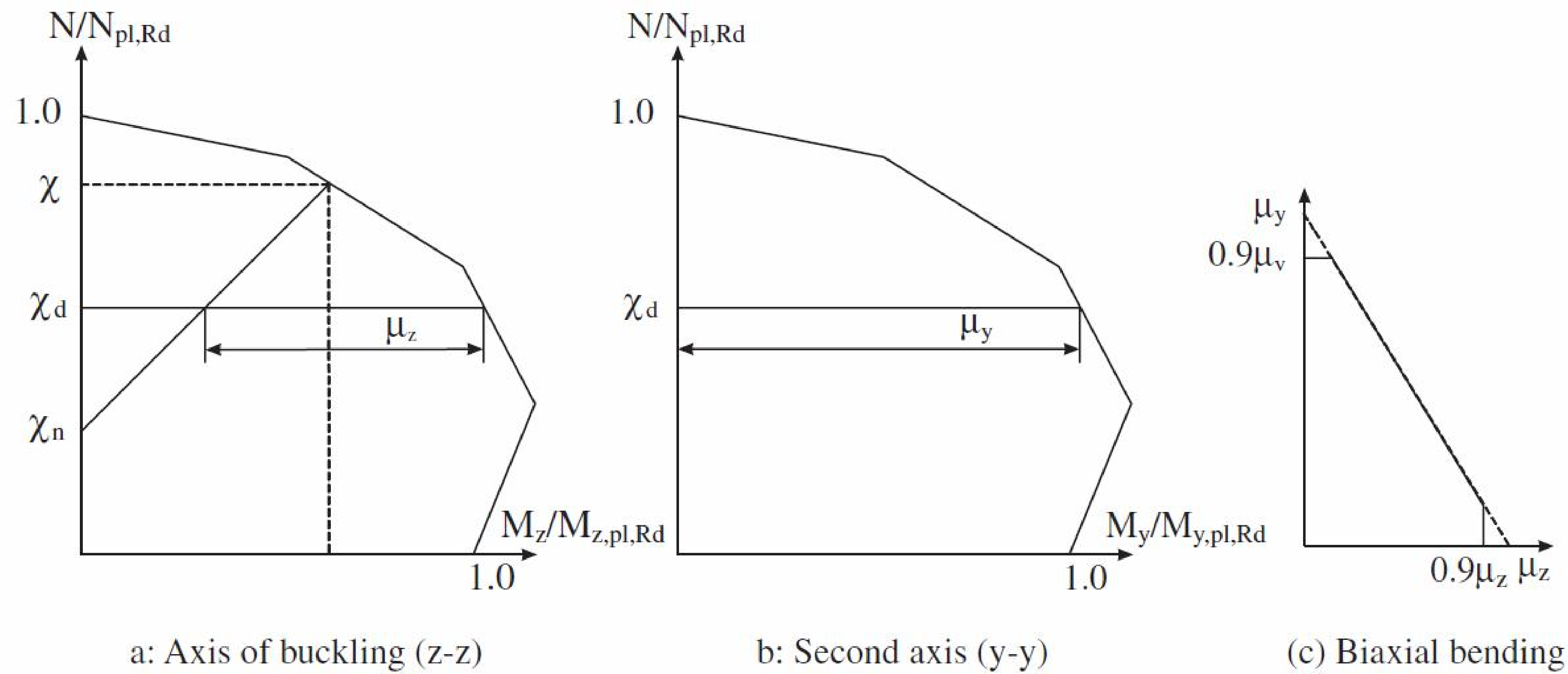
## COLUMN STRENGTH UNDER BIAXIAL BENDING

- To check the composite column capacity against combined compression and biaxial bending, the *N–M* interaction curve of the composite cross-section should be evaluated about both principal axes.
- From these two *N–M* interaction curves, the bending moment capacities of the composite column under axial compression are obtained separately. When calculating these values, it is assumed that the second order bending moment arising from axial compression acting on imperfection is only effective in the expected plane of column buckling. In the other plane, column deflection and second order bending moment are assumed small.

# § 3.3 Combined Compression and Bending

## COLUMN STRENGTH UNDER BIAXIAL BENDING

- Thus, if the composite column is expected to buckle about the z–z plane, the column bending moment capacities $\mu_z$ and $\mu_y$ are obtained as shown in figure below.




Design *N–M* interaction diagrams for biaxial buckling

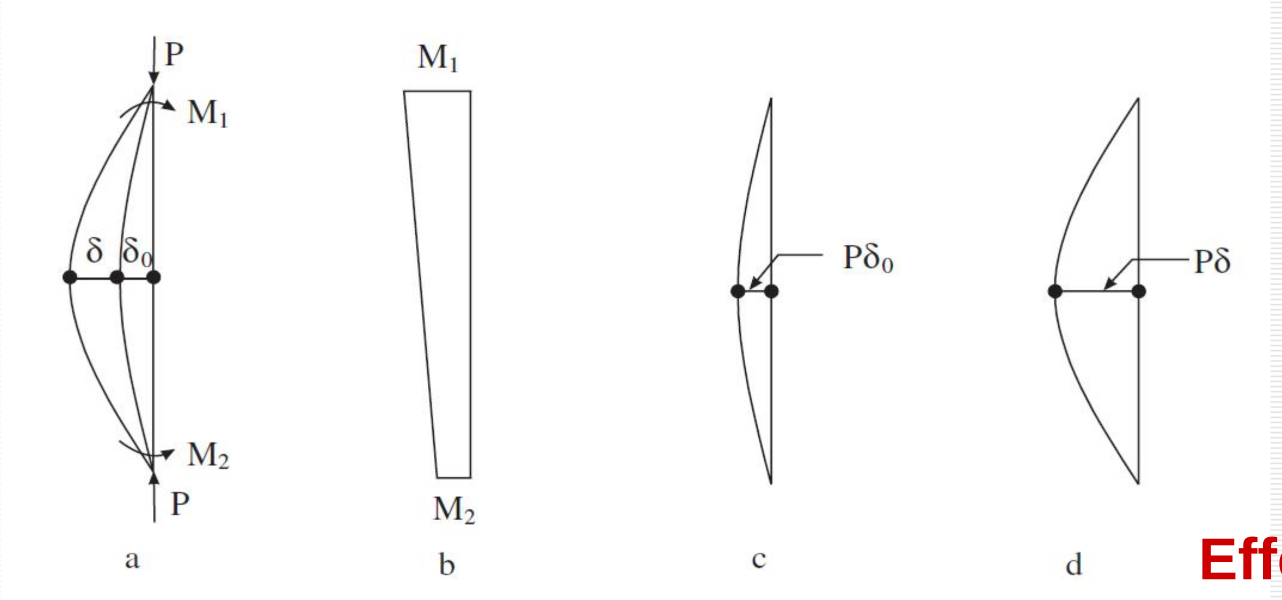
# § 3.3 Combined Compression and Bending
## COLUMN BENDING MOMENTS

- When a column is under combined compression and bending, the bending moment may be regarded as comprising of three parts as shown in figure below:
  - the primary bending moment (figure b),
  - the secondary bending moment arising from initial imperfections (fig. c)
  - and the secondary bending moment from $P-\delta$ effect (figure d).
- All these bending moments should be considered in design calculations to give the maximum overall bending moment in the column.

- **Effect of Imperfections**
- When calculating the second order bending moments, a column is often assumed to have an equivalent half-sine form of initial imperfection. When acting with equal end bending moments, the maximum second order bending moment is directly added to the primary bending moment.

# § 3.3 Combined Compression and Bending

## COLUMN BENDING MOMENTS




a) Column loads and deflections.

b) Primary bending moment distribution

c) Bending moment distribution from initial deflections

d) Bending moment distribution from displacements

**Effect of Imperfections**

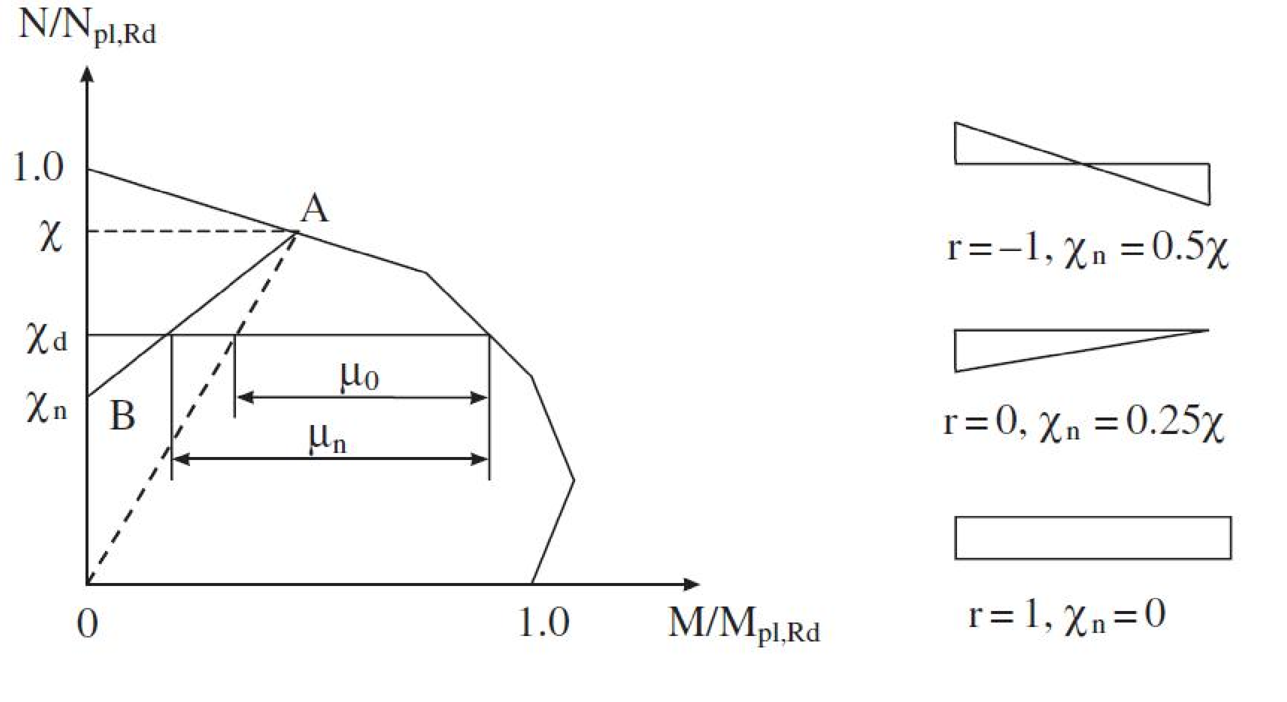
- With other applied end bending moments, because the maximum second order bending moment is not at the same position as the maximum applied bending moment, the effect of the second order bending moment is less severe. However, it is difficult to evaluate precisely the secondary bending moment induced by initial imperfections. In Eurocode 4, this is considered by increasing the axial load at which the column *N–M* interaction diagram is affected.

# § 3.3 Combined Compression and Bending

## COLUMN BENDING MOMENTS

**Effect of Imperfections**

Figure: effect of bending moment distribution on *N–M* interaction diagram




a: Interaction diagram

b: Value of $\chi_n$

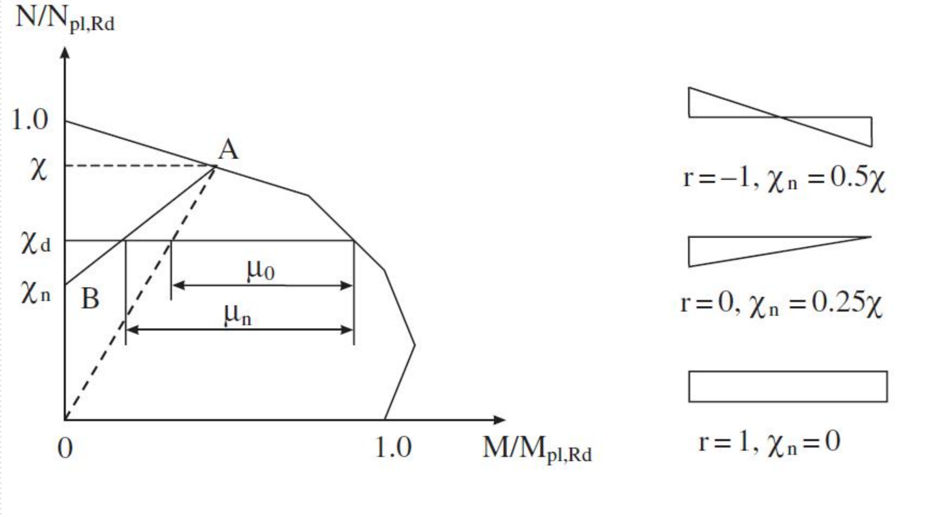
- In the figure above: if the primary bending moment distribution is uniform, the effect of the secondary bending moment is immediate and the column bending moment capacity is reduced from the beginning of loading as indicated by line OA.

# § 3.3 Combined Compression and Bending

## COLUMN BENDING MOMENTS

**Effect of Imperfections**

Figure: effect of bending moment distribution on *N–M* interaction diagram




a: Interaction diagram b: Value of $\chi_n$

- If the primary bending moment distribution is non-uniform, the effect of the secondary bending moment is delayed until a much higher load so that the available bending moment capacity is higher. This is shown in figure above by line BA.
- In consequence, the design bending moment capacity is $\mu_n$, instead of $\mu_0$. The position B, at axial compression $\chi_n$, depends on the ratio of the primary bending moments at the column ends. In Eurocode 4, $\chi_n$ is given by:

$$\chi_n = \chi\frac{1-r}{4} \quad \text{with} \quad -1 \le r \le 1$$

# § 3.3 Combined Compression and Bending

## COLUMN BENDING MOMENTS

where *r* is the ratio of the numerically smaller to the larger end bending moment.

Obs: Figure above shows values of $\chi_n$ for the three common cases of column bending moment diagram. Obviously, $\chi_n$ is higher with a higher gradient in the column bending moment distribution, giving a higher value of $\mu_n$.

### *P−δ* effect

- Due to the second order ($P-\delta$) effect, the column bending moment will be larger than that obtained from the first order analysis. The increase in the column bending moment is high for slender and more heavily loaded columns.

- Conversely, it is negligible for columns of low slenderness or for columns on which the applied compressive force is low. Under the latter conditions, the $P-\delta$ effect does not need be considered. Eurocode 4 defines columns of low slenderness as:

$$\bar{\lambda} \leq 0.2(2-r)$$

Obs: In the case of a column under transverse loading, r = 1.

# § 3.3 Combined Compression and Bending

## COLUMN BENDING MOMENTS

**$P-\delta$ effect**

- The definition of low compressive force is when the design compression load is less than 10% of the column Euler buckling load:

$$\frac{N_{Sd}}{N_{cr}} \leq 0.1$$

- In other cases, second order analysis has to be carried out to obtain the increased column bending moment. In the absence of such a refined analysis, the increased column bending moment may be approximately obtained by magnifying the first order bending moment:

*M* (design column bending moment) = *k* *M* (first order bending moment)

where the magnification factor *k* depends the column bending moment distribution, obtained using:

$$k = \frac{\beta}{1 - \frac{N_{Sd}}{N_{cr}}} \geq 1.0$$

in this equation, the values of $\beta$ are given in the following table:

| | |
|---|---|
| Column with transverse loading | β=1.0 |
| End moments | β=0.66+0.44r≥ 0.44 |

# § 3.3 Combined Compression and Bending

## UNSYMMETRICAL SECTIONS

- In this case, depending on the direction of the column bending moment, the *N–M* interaction curve of an unsymmetrical composite cross-section about the same axis of bending will be different.
- Thus, when determining the *N–M* interaction curve of a unsymmetrical cross-section, the direction of bending moment should be observed. Here, the bending moment **should be calculated with regard to the plastic centroid** of the uncracked composite cross-section.
- Moreover, the *N–M* interaction curve cannot be obtained using the simplified method but can only be obtained following the general procedure.
- Other design considerations, such as local buckling of steel and shear resistance are evaluated in the same way as for symmetrical sections.

# § 3.4 Effect of Shear

- Since a column mainly resists compression, the effect of shear force will be small and can often be neglected. In rare occasions where the effect of shear force has to be included, shear force may be assumed to be resisted by the shear area of the steel component of the composite cross-section.
- The shear area of a composite cross-section is the same as that of the steel section.

# § 3. 5 Load Introduction

- It is assumed that the applied load is properly transferred to steel and concrete to obtain composite action. This is achieved by ensuring that in the region of load introduction, the shear resistance at the interface of the steel section and the concrete is not exceeded.
- However the design approach is usually to use a construction detailing to eliminate this problem.

# § 3.5 Load Introduction

- The longitudinal shear at the interface between concrete and steel should be verified where it is caused by transverse loads and /or end moments.
- Provided that the surface of the steel section in contact with the concrete is unpainted and free from oil, grease and loose scale or rust, the values given in following table may be assumed for $\tau_{Rd}$.

| Type of cross section | $\tau_{Rd}$ (N/mm$^2$) |
|---|---|
| Completely concrete encased steel sections | 0.30 |
| Concrete filled circular hollow sections | 0.55 |
| Concrete filled rectangular hollow sections | 0.40 |
| Flanges of partially encased sections | 0.20 |
| Webs of partially encased sections | 0.00 |

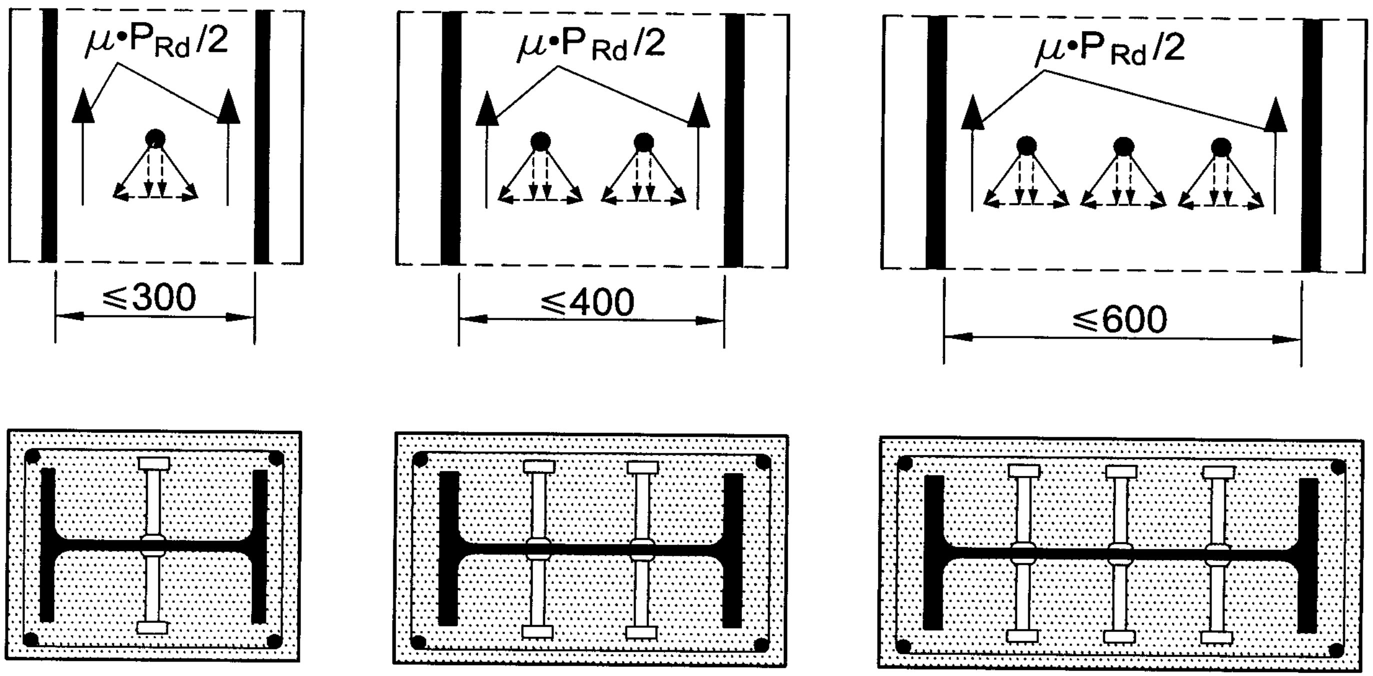
- Where the design value of longitudinal shear exceeds the design shear strength $\tau_{Rd}$ , shear connectors should be provided.

# § 3.5 Load Introduction

- The shear forces should be determined from the change of sectional forces of the steel or reinforced concrete section within the introduction length.
- The introduction length should not exceed *2d* or *L/3*, where *d* is the minimum transverse dimension of the column and *L* is column length.
- The longitudinal shear at the interface is determined using an elastic analysis, considering long term effects and cracking of concrete.
- Where stud connectors are attached to the web of a fully or partially concrete encased steel I-section or a similar section, account may be taken of the frictional forces that develop from the prevention of lateral expansion of the concrete by the adjacent steel flanges.

# § 3.5 Load Introduction

- The clear distance between the flanges should not exceed the values given in figure below.




Additional frictional forces in composite columns by use of headed studs