Universitatea Politehnica Timişoara
Facultatea de Construcţii
Departamentul de Căi de Comunicație Terestre, Fundații și Cadastru

FOUNDATIONS

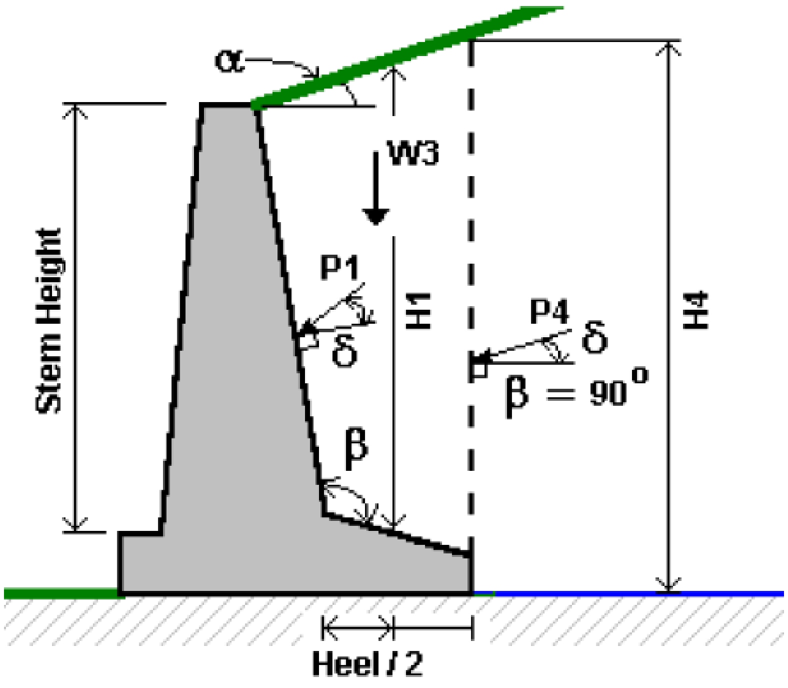
- CURS 5 -

Lateral Earth Pressure and Retaining Walls

Prof.dr.ing Adrian CIUTINA

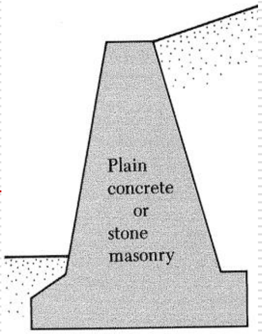
§ 5.1 Introduction

- A **retaining wall** is a wall that provides lateral support for a vertical or near vertical slope of soil.

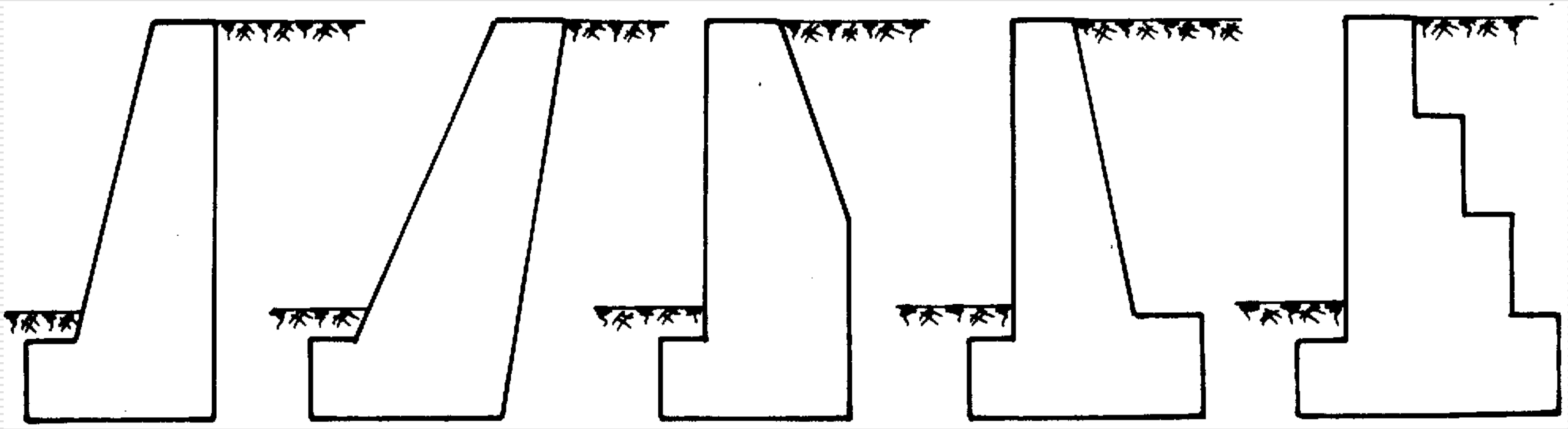


§ 5.1 Introduction

Main typologies of retaining walls



□ Gravity retaining walls



□ The Gravity Retaining Walls are constructed from plain concrete or stone masonry.

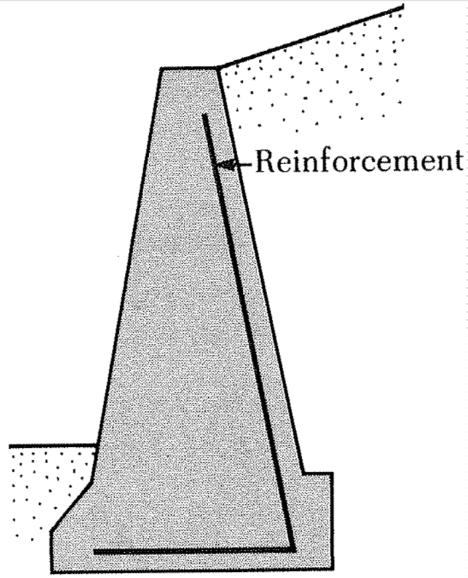
□ The stability of the wall-soil system is assured by the own weight and any soil resting on the masonry, assuring its stability.

Obs: The gravity retaining walls are un-economical for high walls. Other types are recommended in these situations.

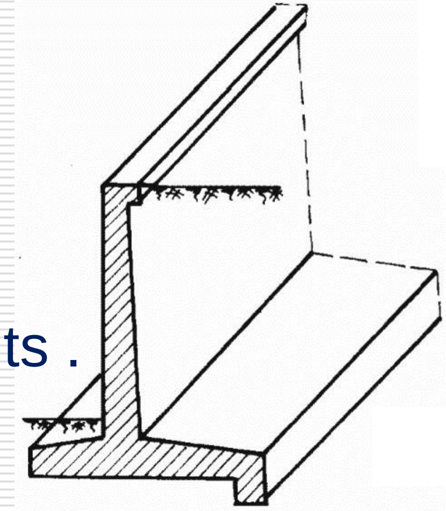
§ 5.1 Introduction

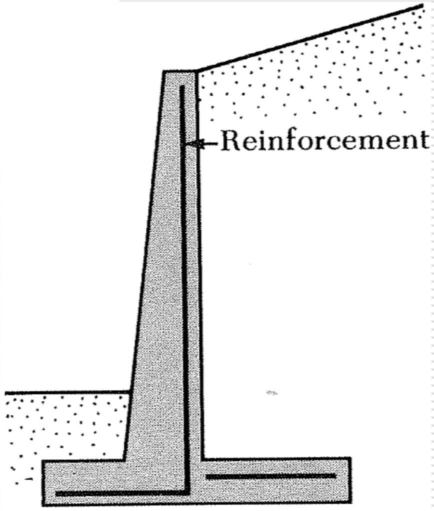
Main typologies of retaining walls

- **Semi-Gravity retaining walls**
- In some cases, in order to reduce the size of the section of gravity-walls, a small amount of reinforcement is added.
- Thus **semi-gravity retaining walls** are formed.



- **Cantilever retaining walls**
- Cantilever retaining walls are made of reinforced concrete that consists of a thin stem and a base slab.
- This type of retaining walls is economical up to heights of 8m.
- It is characterized by smaller weights .
- Their stability is assured by the soil weight over the base slab.

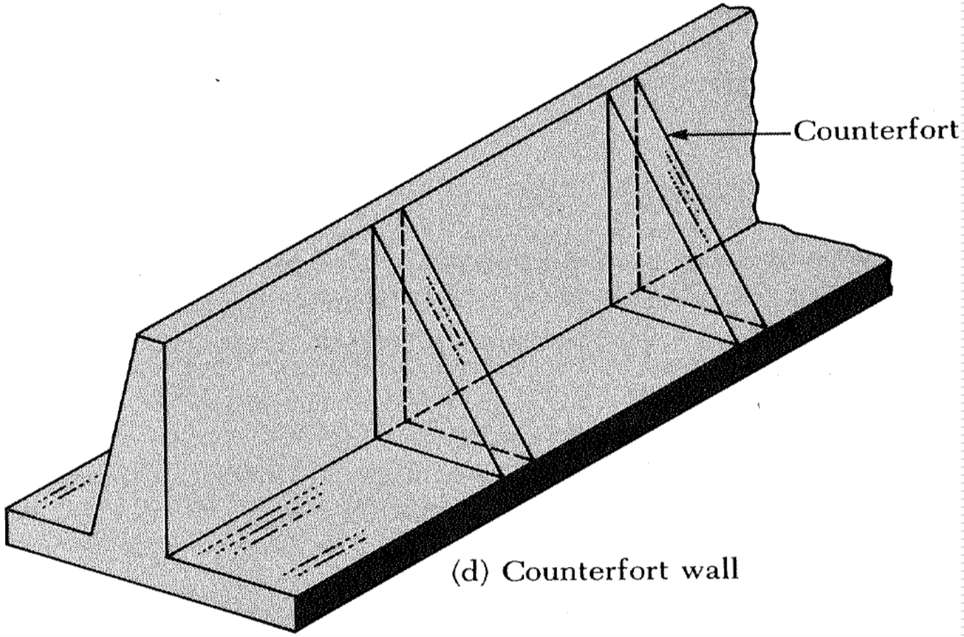




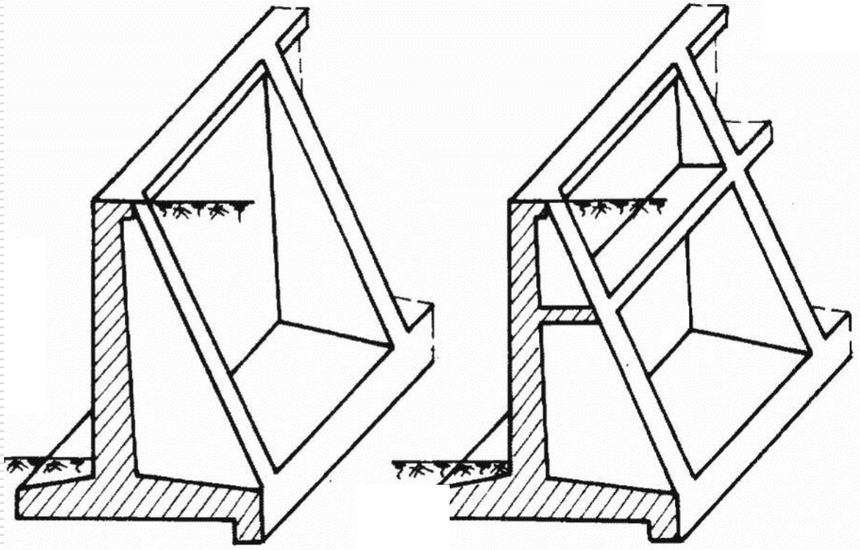
§ 5.1 Introduction

Main typologies of retaining walls

- **Counterfort retaining walls**
- Are similar to cantilever walls excepting that, at regular intervals they have thin vertical concrete slabs (counterforts) that tie the wall and the base slab.
- The purpose of the counterforts is to reduce base shear and the bending moment in the vertical wall.



(d) Counterfort wall



§ 5.1 Introduction

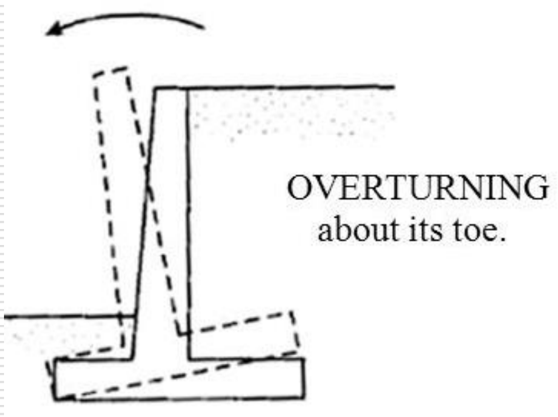
Forces acting on retaining walls

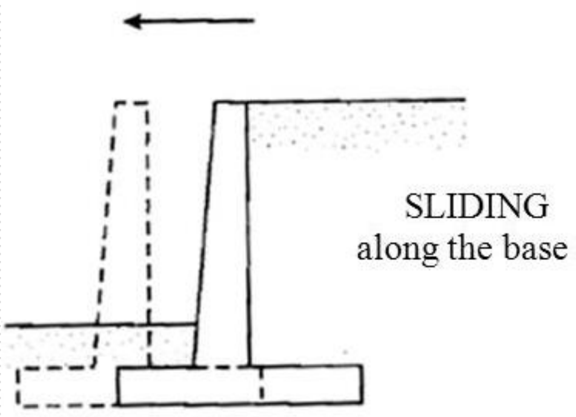
- The principal forces acting on retaining walls are:
  - Self weight of the wall
  - Additional load on the soil surface (if exists)
  - Lateral earth pressure
- The **self weight** of the wall depends on the geometrical dimensions and the building material (unit weight). The self weight acts in its centroid.
- In order to design the retaining walls, one must know the basic soil parameters for retained soil and the soil below the base slab:
  - Unit weight
  - Angle of friction
  - Cohesion
- On the basis of these parameters, the **lateral earth pressure** could be determined.

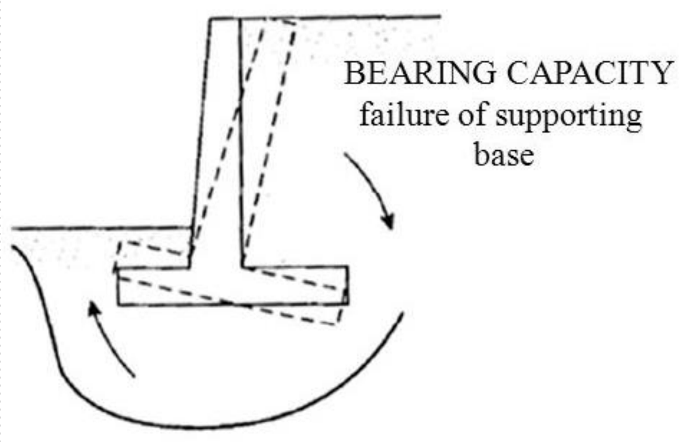
§ 5.1 Introduction

Phases of design

- There are two phases in the design of retaining walls:
- Check for **stability**, including:
  - Overturning
  - Sliding
  - Bearing capacity failure
- **Structural design** of the wall, including reinforcement.



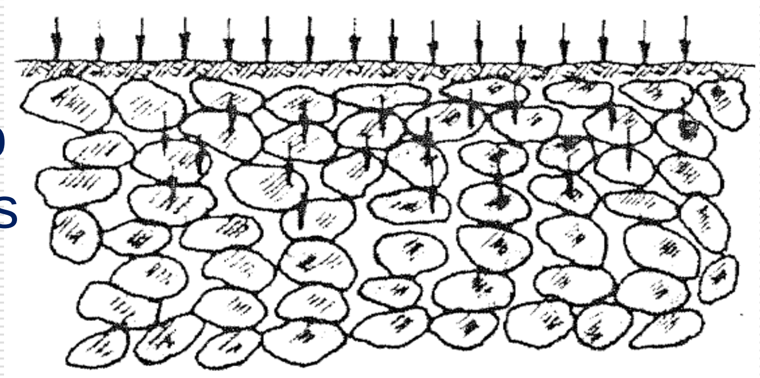


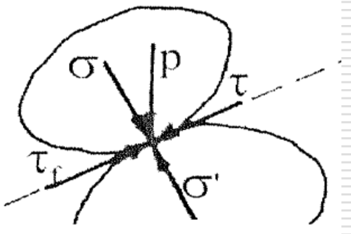


§ 5.2 Lateral Earth Pressure

Soil Shear Strength

- The application of an exterior load on a soil to the self weight of a soil will lead to development of normal and tangential stresses.
- Practically, the **normal stresses σ** will induce a **closeness** of particles or aggregates of the soil.
- On the other hand, the **tangential stresses τ** will induce a lateral relative movement between particles.
- The **tangential (shear) strength** of the soil τ_f tends to oppose the tangential stresses caused by loads.





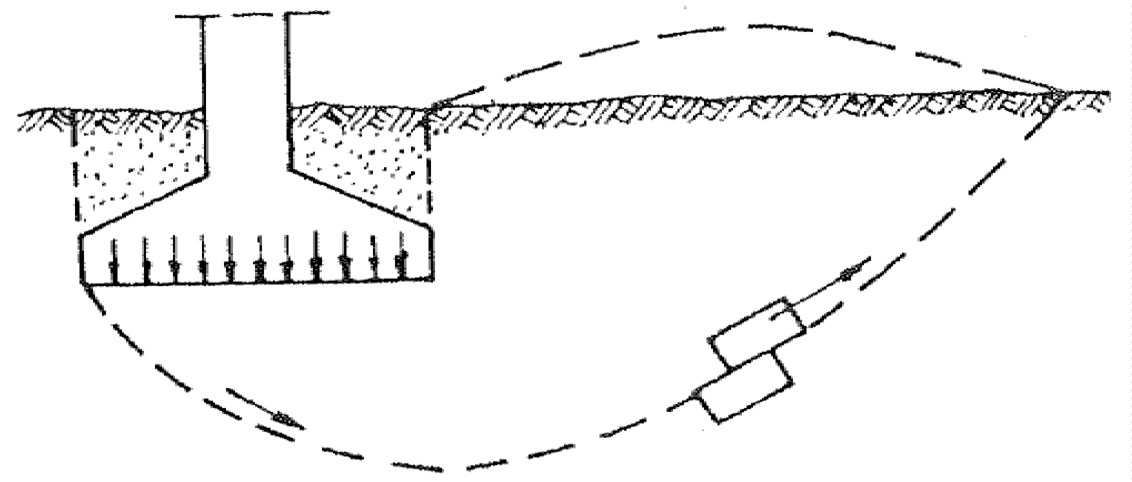
- The **shear strength** of the soil is generated by the bonding forces between the constitutive particles of the soil

§ 5.2 Lateral Earth Pressure

Soil Shear Strength

- By increasing the load on the soil, the normal **σ** and shear **τ** stresses also increase.
- However, by increasing the tangential stresses, at a certain moment the bond between particles is broke by shear and the particles start to slide one on another.
- Thus, shear failure zones are formed and the failure of the foundation – soil system is generalized:

Shear failure for a shallow foundation



§ 5.2 Lateral Earth Pressure

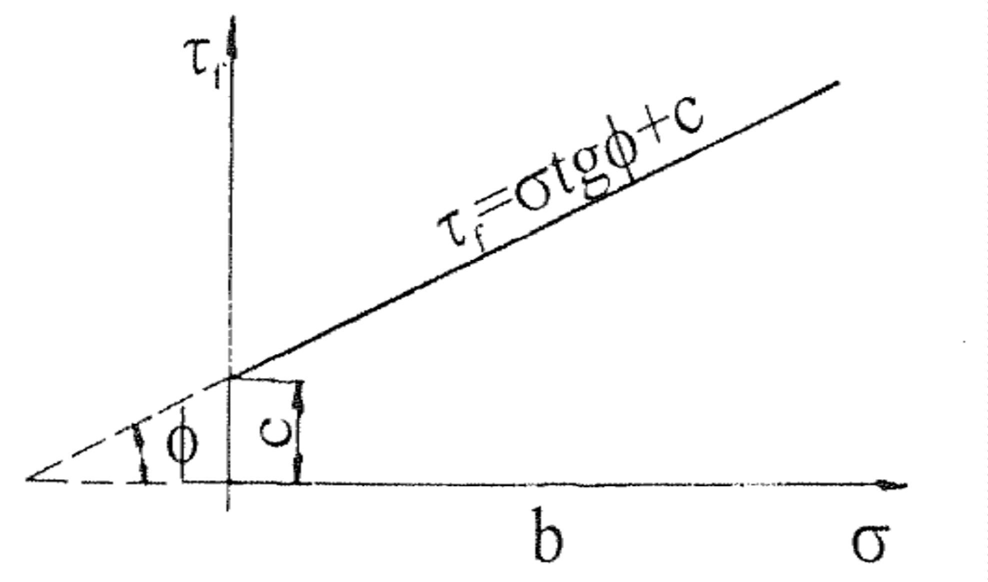
Soil Shear Strength

- Definition: The **Shear Strength** represents the strength of the soil to shear failure of the component particles. This is equal to the tangential stress that produces the failure.
- According to the Coulomb's law, the shear strength of a soil is expressed through an equation of a line passing the ordinate at a known value (equal to the **cohesion** of the soil) and has a slope equal with the **friction angle** of the soil.

Shear intrinsic line

(Coulomb's Line)

$$\tau_f = \sigma \cdot tan\Phi + c$$



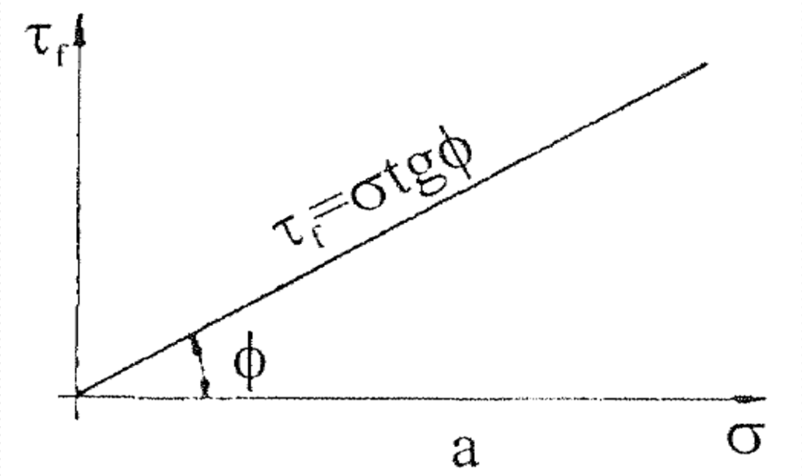
§ 5.2 Lateral Earth Pressure

Soil Shear Strength

- For granular soils (cohesionless), the Coulomb's line is passing through the origin of the system:

Shear intrinsic line (Coulomb's Line) for granular soils

$$\tau_f = \sigma \cdot tan\Phi$$



- Characteristics of the Coulomb's lines:
  - the slope of the line represents the **angle *Φ* of the internal friction** of the soil (it is a constant of the soil)
  - The ordinate at origin represents the **soil cohesion *c*** (also a constant of the soil)

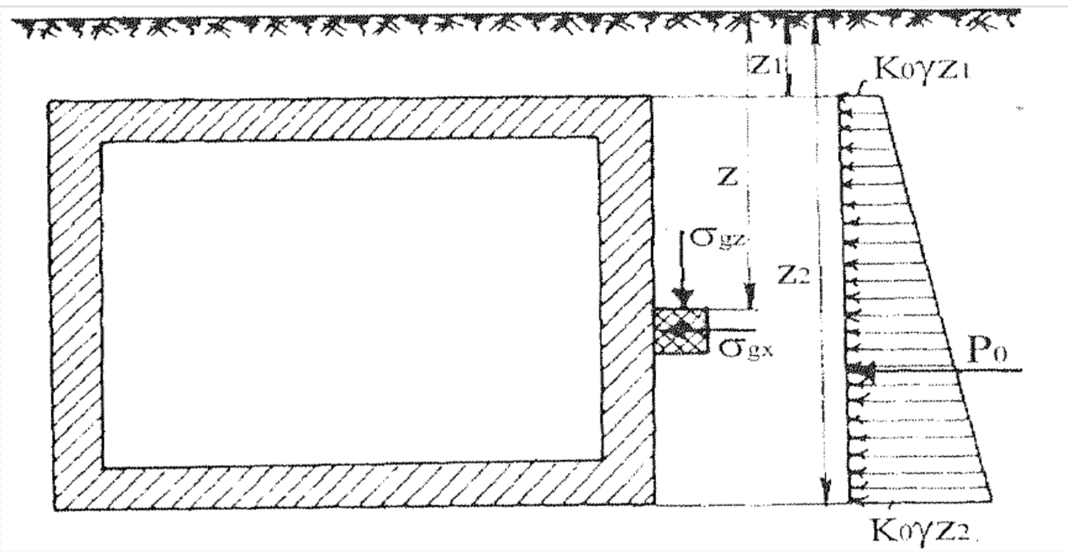
§ 5.2 Lateral Earth Pressure

Lateral Earth Pressure at Rest

- The soil action on a rigid retaining body (which could be considered as fixed) is named **Earth Pressure at Rest**.

- Considering a vertical wall behind a soil mass. The vertical stress acting on a differential element located at depth *z* below the soil surface is:

$$\sigma_{gz} = \gamma \cdot z$$



- Due to the vertical pressure, the differential element will be compressed on vertical direction, trying to expand laterally.

- Thus, the vertical pressure will generate an horizontal pressure σ_{gx} expressed by:

$$\sigma_{gx} = K_0 \cdot \sigma_{gz} = K_0 \cdot \gamma \cdot z$$

§ 5.2 Lateral Earth Pressure

Lateral Earth Pressure at Rest

$$\sigma_{gx} = K_0 \cdot \sigma_{gz} = K_0 \cdot \gamma \cdot z$$

- This formula results by applying the Poisson's law.

In the formula:

K_0 is the coefficient of at-rest earth pressure, given in function of the friction angle Φ.

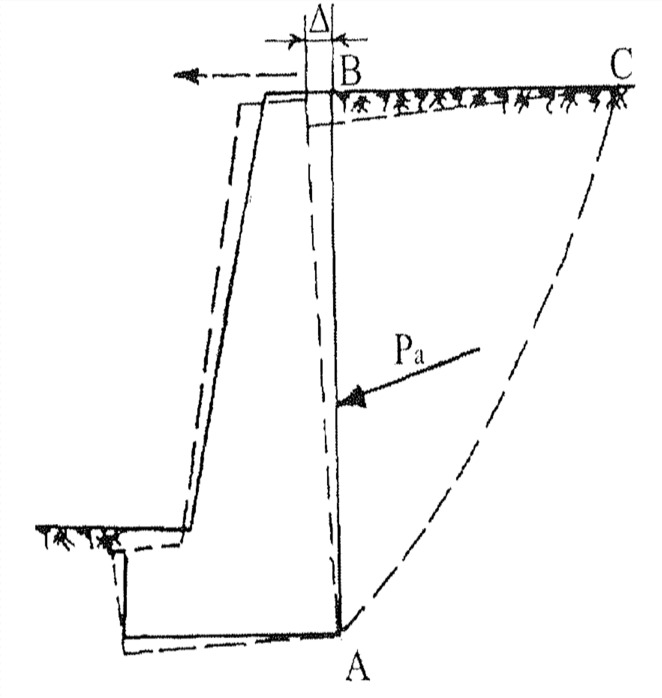
- For normally consolidated granular soils: $K_0 \approx 1 - \sin\Phi$
- For normally consolidated clay: $K_0 \approx 0.95 - \sin\Phi$

Obs: If the soil is loaded at the upper surface by a distributed load *q*, this will be mathematically added to the formula of σ_{gz}.

§ 5.2 Lateral Earth Pressure

Active Earth Pressure

- When the retaining wall suffers a displacement or rotation, moving away from the soil, with a distance Δ, the soil pressure on the wall, at any depth will decrease as compared to the Pressure of Earth at Rest.
- For a certain movement of the retaining wall, a slipping prismatic surface (ABC) is dislocated and is moving together with the wall:



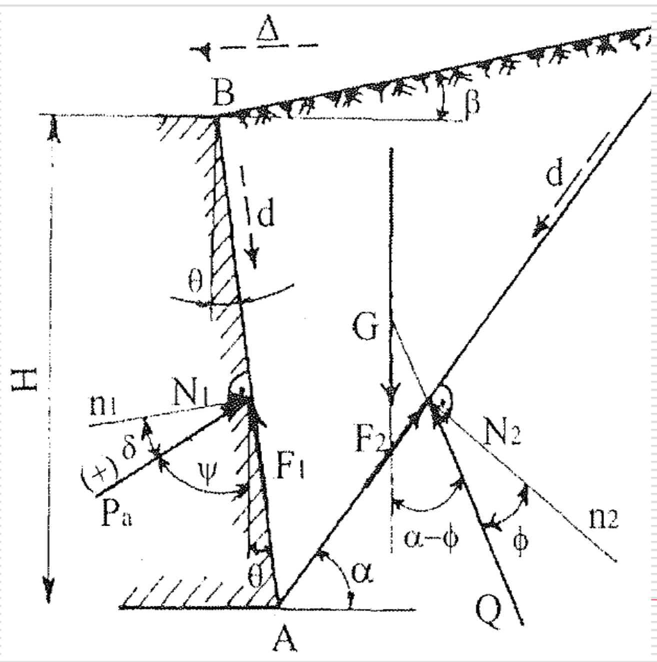
- This movement starts when the AC surface is sliding, by creating a shear failure surface.
- The **Active Earth Pressure** represents the action exerted on the retaining wall by the sheared soil mass ABC.

Obs: Experimentally was demonstrated that the displacement Δ, required for the formation of a shearing surface AC is about 0.001 H (where H is the height of the wall)

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Coulomb's approach

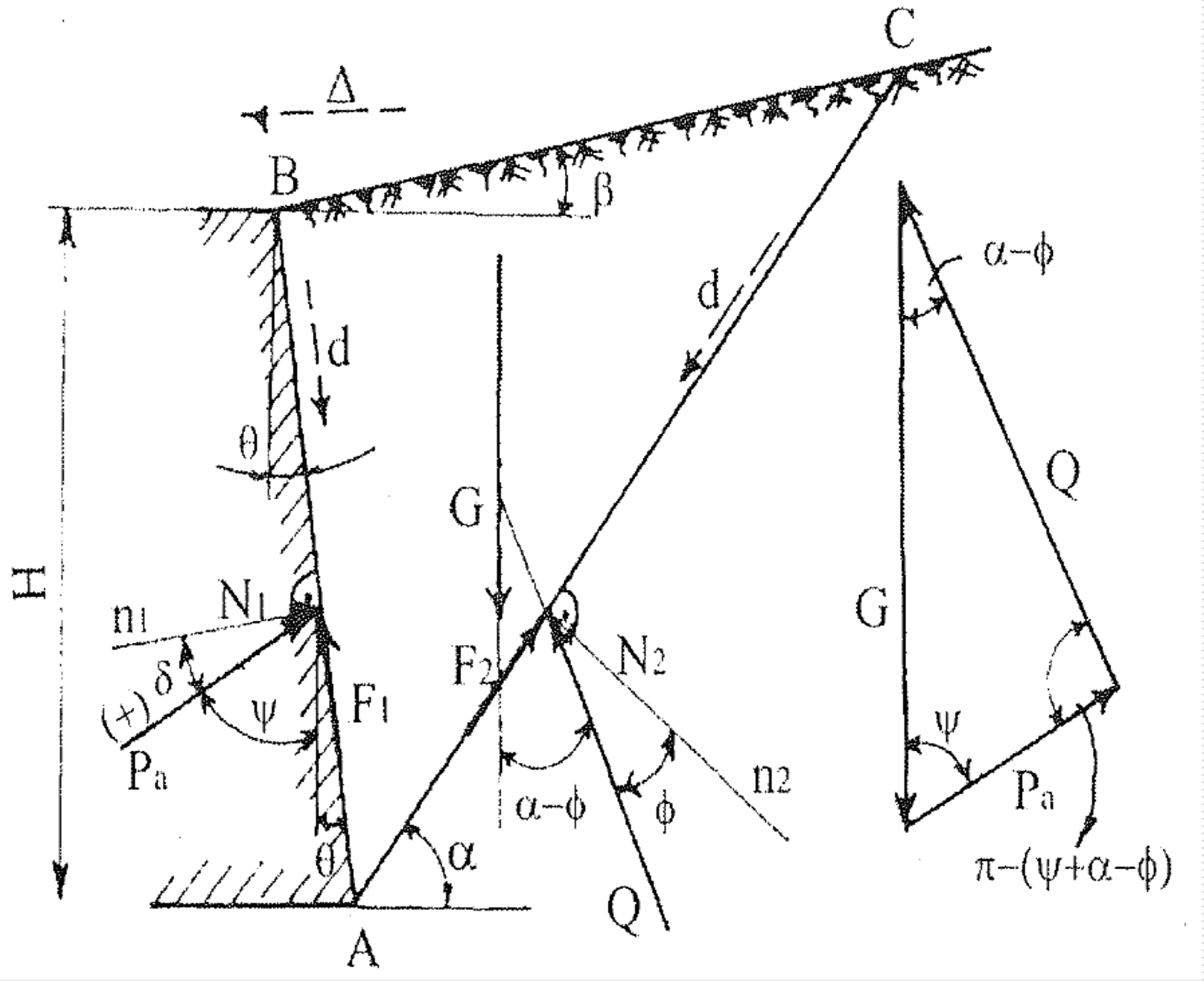
- Two simplifying hypothesis for the determination of the active earth pressure were proposed by Coulomb:
  - The AC sliding surface is plane
  - The soil is homogeneous and cohesionless.
- Considering a retaining wall, making an angle *θ* with vertical, for a certain plain surface making an angle *α* with horizontal, there are the following forces acting on the wall-soil system:



  - The weight of the wedge G. For this force it is known both the magnitude and the application point (wedge centroid);
  - Reaction on the AB side, equal in magnitude with the active pressure P_a but opposed to this. Its direction could be determined at equilibrium;
  - Reaction Q on the sliding surface AC. Its direction could be established at equilibrium.

§ 5.2 Lateral Earth Pressure

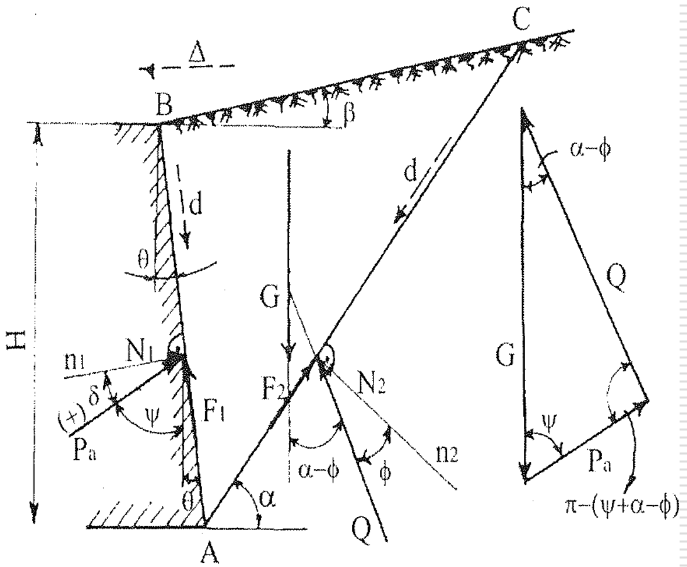
Active Earth Pressure – Coulomb's approach



- The Coulomb's approach resumes at estimation of Pa in the triangle of forces created.
- The plane problem is solved by considering one meter on wall length.

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Coulomb's approach



- The weight of the soil edge is given by:

$$G = \gamma \cdot S_{ABC} = \gamma \cdot f1(H, \theta, \beta, \alpha)$$

where:

H is wall height;

θ is angle of the retaining wall with the vertical;

β angle of the backfill with the horizontal;

α is angle of the shear plane with the horizontal.

- The soil edge is considered to move downward, creating the following friction forces:
  - F_1, between the wall and the soil: $F_1 = N_1 \cdot tan\delta$
  - F_2, between soil wedge and the non-moving soil: $F_2 = N_2 \cdot tan\Phi$

where:

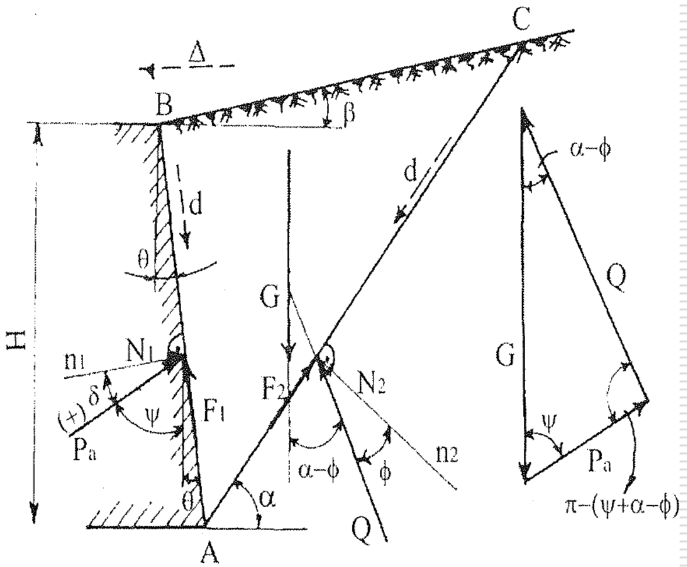
δ is friction angle between the soil and the wall;

Φ is angle of internal friction of the retained soil.

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Coulomb's approach

- The reaction P_a, equal in magnitude with the active earth pressure represents the resultant of the N_1 and F_1 forces.
- The reaction P_a acts at an angle δ made with the normal N_1 on the wall.
- The reaction Q, represents the resultant of the N_2 and F_2 forces.



- The reaction Q acts at an angle α made with the normal n_2 on the AC surface. Its value could be found by applying the geotechnical laws.
- Considering that the system of forces is in equilibrium, the active earth pressure could be found by applying the sinus theorem in the triangle of forces:

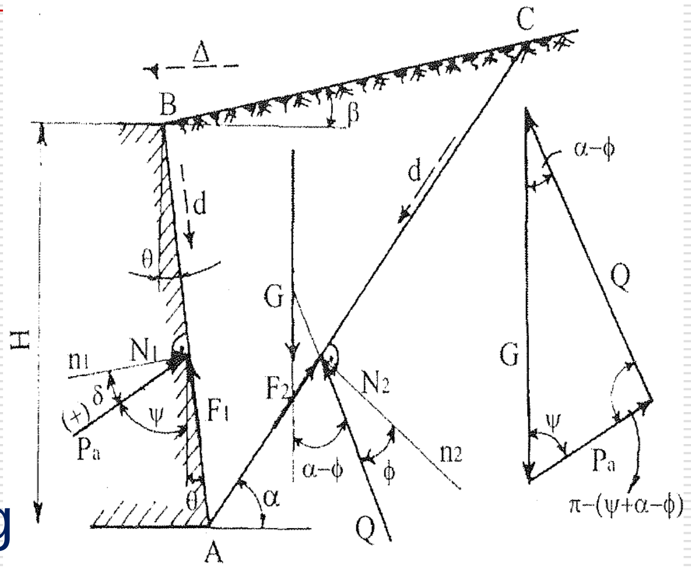
$$\frac{P_a}{\sin(\alpha-\Phi)}=\frac{G}{\sin[\pi-(\psi+\alpha-\Phi)]}=\frac{G}{\sin(\psi+\alpha-\Phi)}$$

- or: $$P_a=\frac{G\cdot\sin(\alpha-\Phi)}{\sin(\psi+\alpha-\Phi)}=\gamma\cdot f_2(H,\theta,\beta,\Phi,\delta,\alpha)$$ where: $$\psi=\frac{\pi}{2}-\delta-\Phi$$

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Coulomb's approach

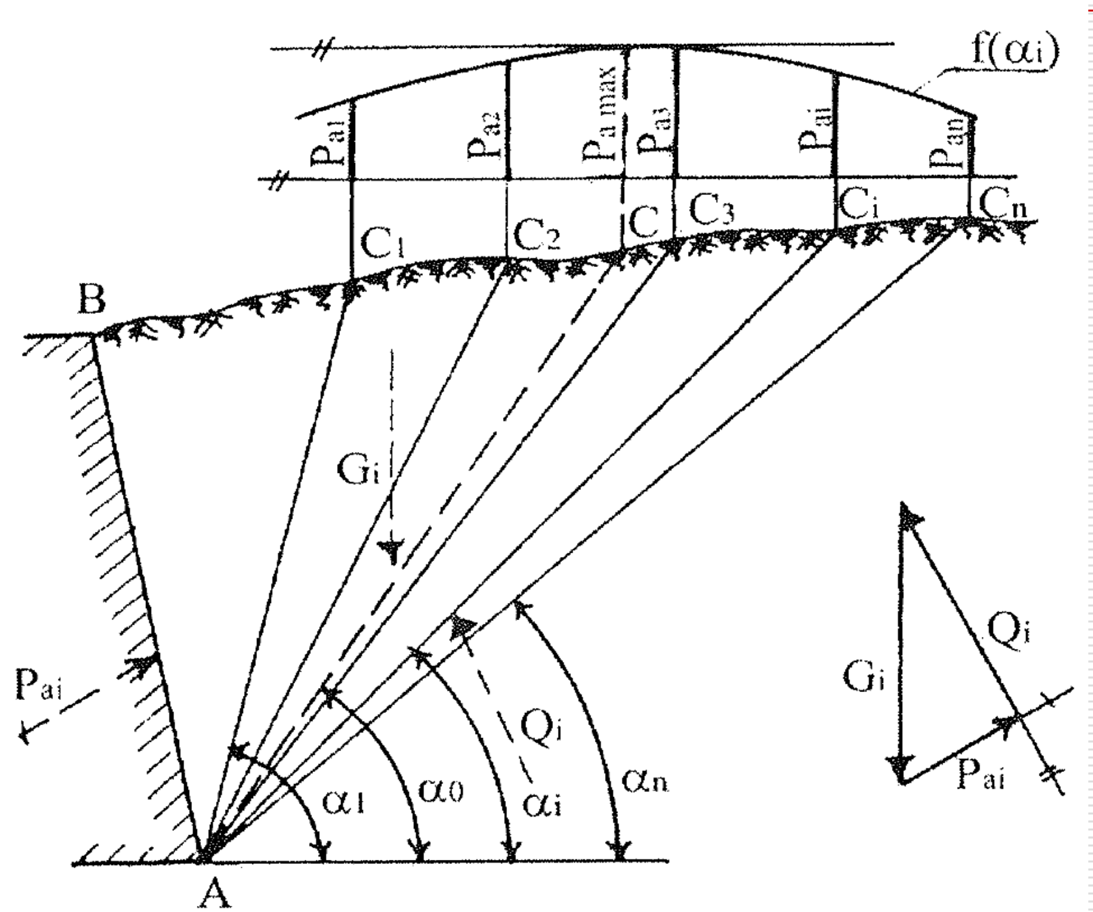
$$P_a = \gamma \cdot f_2(H, \theta, \beta, \Phi, \delta, \alpha)$$



- For certain given data: soil, wall geometrical dimensions etc. as known, the value of the active pressure depends only on the angle α.
- The maximum value of the active pressure could be graphically determined by considering several failure planes: AC_1, AC_2, …, AC_n.
- For each case a triangle of G-N_i-Q forces is built and the values of the active pressure P_{ai} are determined.
- The values obtained are plot for the points C_1, C_2, …, C_n, obtaining thus the variation of the P_{ai} in function of angle α.
- The maximum value of $P_{a,max}$ results and the failure angle α_0 , the most dangerous for the given data.

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Coulomb's approach

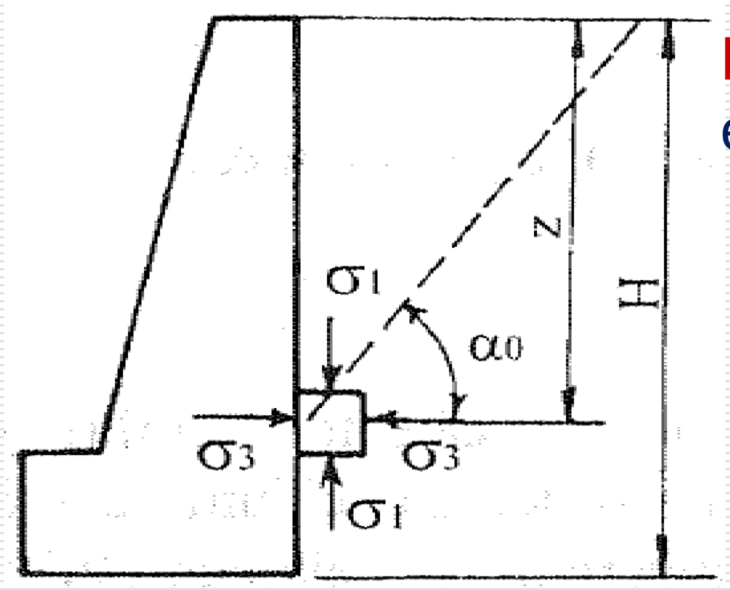


- Also, the maximum of the function P_a could be found by writing the condition: $\frac{dP_a}{d\alpha}=0$
- Thus it results the angle α_0 for which the active pressure is maximum.

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Rankine's approach

- Rankine established a relationship for the Active Earth pressure starting from the stress state in the soil mass.
- Considering a differential soil volume located at the boundary *AB* of the retaining wall at depth *z*.
- By neglecting the friction between the wall and the soil, then in the vertical plane of the AB surface become a principal plane. On the same considerations. the horizontal plane is principal, too.



- The stresses acting on the differential element are:

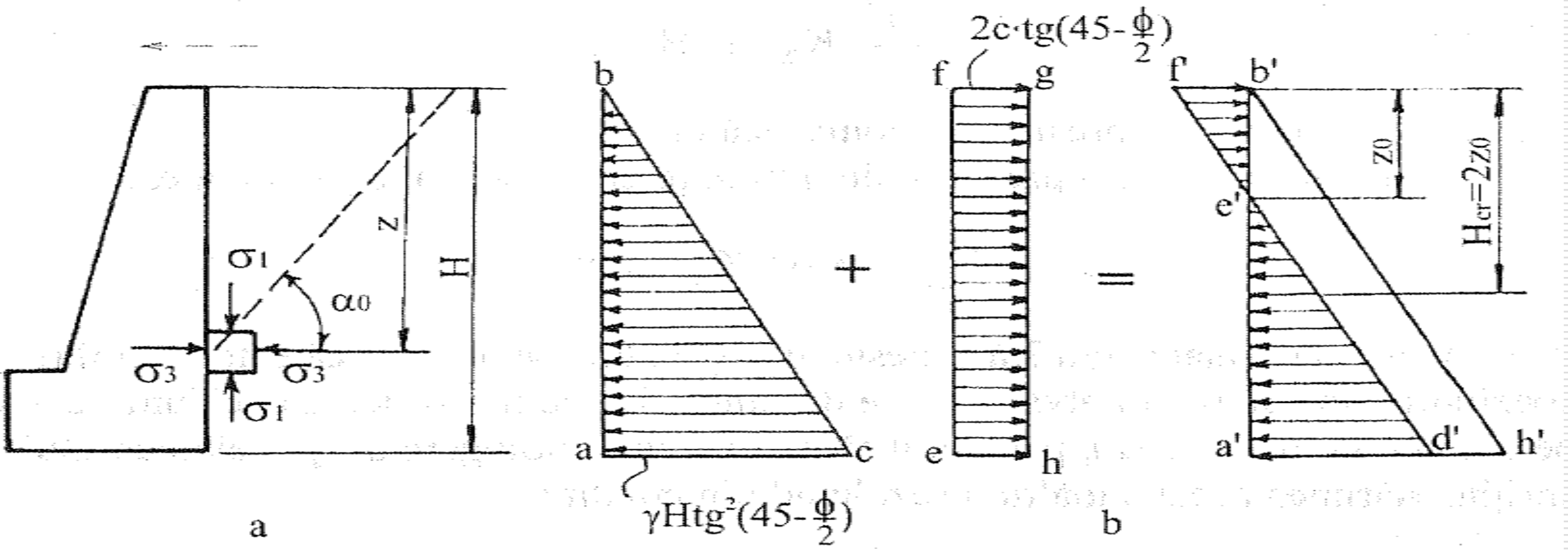
$$\sigma_1 = \sigma_{gz} = \gamma \cdot z$$

$$\sigma_3 = \sigma_{gx} = K_0 \cdot \sigma_{gz} = K_0 \cdot \gamma \cdot z$$

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Rankine's approach

- By the displacement of the retaining surface from the retaining surface with thrust, there exist a separation of the wall from the soil which leads to lowering the principal stress σ_3 while keeping constant the other principal stress σ_1.



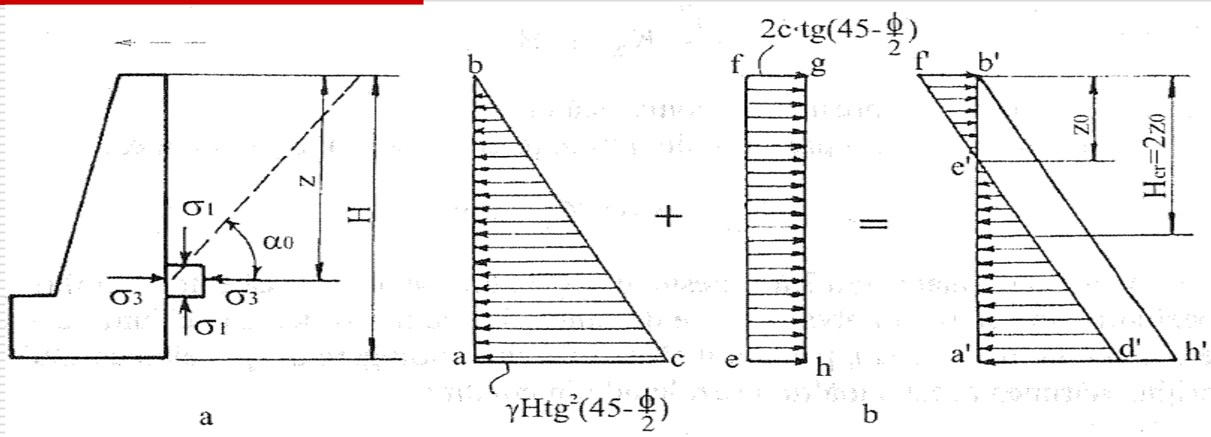
- By reaching the limiting lateral displacement, a wedge of soil is dislocated.

§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Rankine's approach

- The angle between the slip surface (assumed as plane) with the horizontal is:

$$\alpha = \alpha_0 = \frac{\pi}{4} + \frac{\Phi}{2}$$



- The minimum value of the stress diagram σ_3 corresponding to the formation of the shear plane represents the active pressure acted by the soil on the retaining element.

- The maximum $\sigma_3 = \gamma \cdot H$ stress is at the maximum depth (*z*=*H*) and belongs to both the wall and the shear failure plane.

- In consequence, here the failure condition is fulfilled:

$$sin\Phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 + 2c \cdot ctan\Phi}$$

§ 5.2 Lateral Earth Pressure

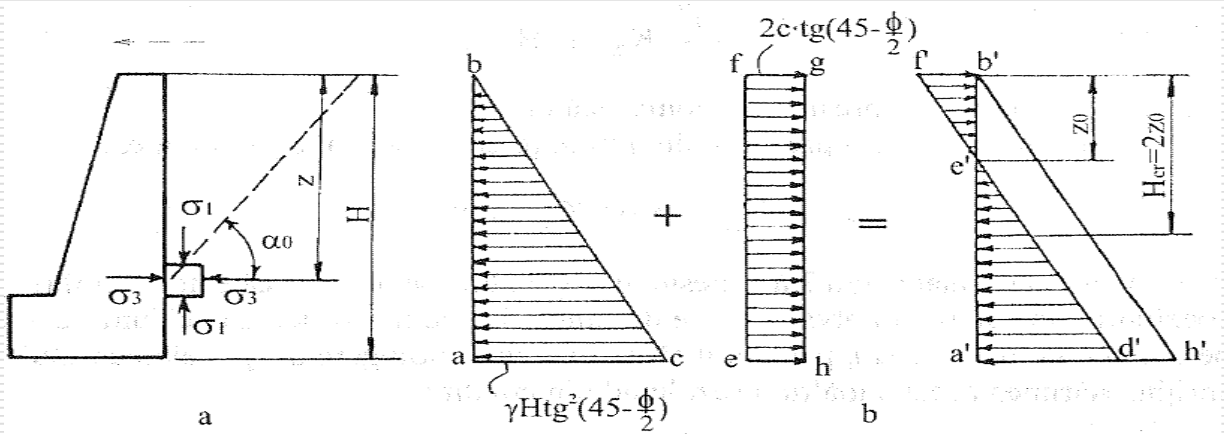
Active Earth Pressure – Rankine's approach

- Considering that

$\sigma_3 = pa_z$, it results: $p_{az} = \gamma \cdot z \cdot \tan^2\left(\frac{\pi}{4} - \frac{\Phi}{2}\right)$-$2c\cdot\tan\left(\frac{\pi}{4} - \frac{\Phi}{2}\right)$

- The real pressure diagram is the sum of the geotechnical pressure and the reactive pressure.
- The depth z on which the null pressure and tension stresses are formed is given by:

$$z_0 = \frac{2c}{\gamma \cdot \tan\left(\frac{\pi}{4} - \frac{\Phi}{2}\right)} = \frac{2c}{\gamma} \cdot \text{ctan}\left(\frac{\pi}{4} - \frac{\Phi}{2}\right)$$



§ 5.2 Lateral Earth Pressure

Active Earth Pressure – Rankine's approach

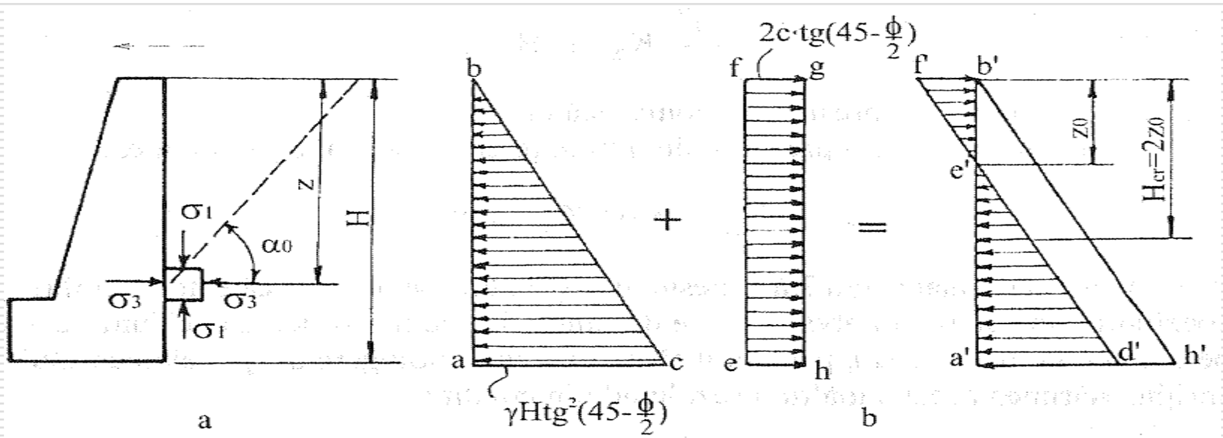
- Due to the fact that in reality tension stresses cannot exist in soils, the real diagram is considered only the triangle *a'e'd'*:

$$P_a = S_{abc} - S_{efgh} + S_{b'e'f'}$$

$$= \frac{1}{2} \cdot \gamma \cdot H^2 \cdot \tan^2\left(\frac{\pi}{4} - \frac{\Phi}{2}\right) - 2c \cdot H \cdot \tan\left(\frac{\pi}{4} - \frac{\Phi}{2}\right) + \frac{1}{2} \cdot 2c \cdot \tan\left(\frac{\pi}{4} - \frac{\Phi}{2}\right) \cdot \frac{2c}{\gamma \cdot \tan\left(\frac{\pi}{4} - \frac{\Phi}{2}\right)}$$

Or:

$$P_a = \frac{1}{2} \cdot \gamma \cdot H^2 \cdot \tan^2\left(\frac{\pi}{4} - \frac{\Phi}{2}\right) - 2c \cdot H \cdot \tan\left(\frac{\pi}{4} - \frac{\Phi}{2}\right) + \frac{2c^2}{\gamma}$$



Obs: The Rankine's approach could be applied for the cases when:

- The bottom surface of the retaining wall is horizontal;
- The wall is vertical;
- There is no friction between the wall and the soil.