

FOUNDATIONS

- CURS 3 -

Bearing Capacity of Foundations

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CHAPTER III – BEARING CAPACITY OF FOUNDATIONS

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

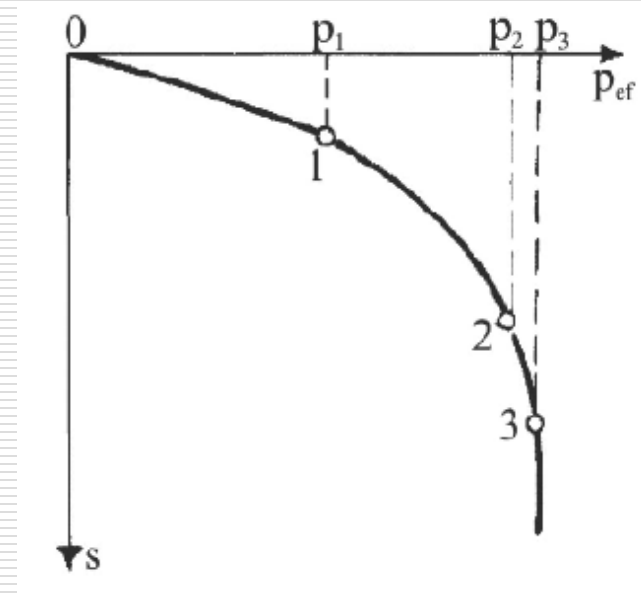
In different design situations we may use as bearing capacity:

- Conventional pressure p_{conv}
- Plastic pressure p_{pl}
- Critical pressure p_{cr}

□ The conventional pressure p_{conv} is used for usual “prescriptive method” used in NP112-2014

□ The plastic pressure p_{pl} is used in serviceability limit state design, according to NP112-2014 and EN 1997-1.

□ The critical pressure p_{cr} is used for usual “hybrid model method” used in NP112-2014



§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Conventional pressure p_{conv}

□ The conventional pressure of a soil represents the bearing capacity of the soil by considering its linear behavior (interval 0-1).

□ The conventional pressure is obtained empirically (empirically = through experimental tests and interpretation of experimental data, without theoretical support).

□ NP112 offers the following formula for computing p_{conv}

$$p_{conv} = \overline{p_{conv}} + C_B + C_D$$

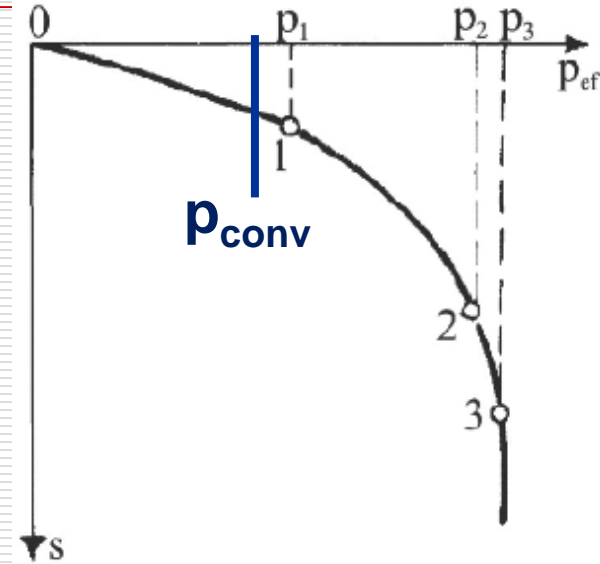
where:

$\overline{p_{conv}}$ is the base value of the conventional soil pressure (given in tables)

C_B is the width correction

C_D is the depth correction

□ The $\overline{p_{conv}}$ value is given in NP 112 for different soils and foundation basis having the width $B=1.0\text{m}$ and the foundation depth $D=2.0\text{m}$.



§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Conventional pressure p_{conv}

Compact soils

Tabelul D.1

Denumirea terenului de fundare		\bar{p}_{conv} [kPa]
Roci stâncoase		1000 ÷ 6 000
Roci semi-stâncoase	Marne, marne argiloase și argile marnoase compacte	350 ÷ 1100
	Șisturi argiloase, argile șistoase și nisipuri cimentate	600 ÷ 850

Coarse granular soils

Tabelul D.2

Denumirea terenului de fundare		\bar{p}_{conv} [kPa]
Pământuri foarte grosiere	Blocuri și bolovănișuri cu interspațiile umplute cu nisip și pietriș	750
	Blocuri cu interspațiile umplute cu pământuri argiloase	350 ÷ 600 ¹⁾
Pământuri grosiere	Pietrișuri curate (din fragmente de roci cristaline)	600
	Pietrișuri cu nisip	550
	Pietrișuri din fragmente de roci sedimentare	350
	Pietrișuri cu nisip argilos	350 ÷ 500 ¹⁾

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Conventional pressure p_{conv}

Fine granular soils

Tabelul D.3

Denumirea terenului de fundare		Îndesate ¹⁾	Îndesare medie ¹⁾	
		$\overline{p_{conv}}$ [kPa]		
Pământuri grosiere	Nisip mare	700	600	
	Nisip mijlociu	600	500	
	Nisip fin	uscăt sau umed	500	350
		foarte umed sau saturat	350	250
	Nisip fin prăfos	uscăt	350	300
		umed	250	200
foarte umed sau saturat		200	150	

□ The $\overline{p_{conv}}$ values are obtained by laboratory interpretation of some soil physical properties:

- Granulometry
- Void ratio (e)
- Plasticity index (I_P)
- Consistency index (I_C)
- Density index (I_D)

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Conventional pressure \bar{p}_{conv}

Fine particle soils

Denumirea terenului de fundare	Indicele porilor ¹⁾ e	Consistența ^{1,2)}			
		$I_C = 0,5$	$I_C = 0,75$	$I_C = 1$	
\bar{p}_{conv} [kPa]					
Pământuri fine	Cu plasticitate redusă: ($I_p \leq 10\%$): nisipuri argiloase, prafuri nisipoase și prafuri, având $e < 0,7$				
	• $I_C \geq 0,75$	0,5		325	350
		0,7		285	300
	• $0,5 < I_C < 0,75$	0,5	300	325	
		0,7	275	285	
	Cu plasticitatea mijlocie: ($10\% < I_p \leq 20\%$): nisipuri argiloase, prafuri nisipoase-argiloase, având $e < 1,0$				
	• $I_C \geq 0,75$	0,5		325	350
		0,7		285	300
		1,0		225	250
	• $0,5 < I_C < 0,75$	0,5	300	325	
		0,7	275	285	
		1,0	200	225	
Cu plasticitate mare ($I_p > 20\%$): argile nisipoase, argile prăfoase și argile, având $e < 1,1$					
• $I_C \geq 0,75$	0,5		600	650	
	0,6		485	525	
	0,8		325	350	
	1,1		260	300	
• $0,5 < I_C < 0,75$	0,5	550	600		
	0,6	450	485		
	0,8	300	325		
	1,1	225	260		

□ The \bar{p}_{conv} values are obtained by laboratory interpretation of some soil physical properties:

- Granulometry
- Void ratio (e)
- Plasticity index (I_p)
- Consistency index (I_C)
- Density index (I_D)

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Conventional pressure p_{conv}

□ Width correction C_B

where:

$$\text{for } B \leq 5\text{m: } C_B = \overline{p_{conv}} K_1 (B - 1)$$

$$\text{for } B > 5\text{m: } C_B = 0.4 \overline{p_{conv}} \text{ for granular soils}$$

$$C_B = 0.2 \overline{p_{conv}} \text{ for compact soils}$$

B is the foundation width

$K_1 = 0.1$ for granular soils

$K_1 = 0.05$ for compact soils

□ Depth correction C_D

$$\text{for } D \leq 2\text{m: } C_D = \overline{p_{conv}} (D - 2) / 4$$

$$\text{for } D > 2\text{m: } C_D = \bar{\gamma} (D - 2) / 4$$

where:

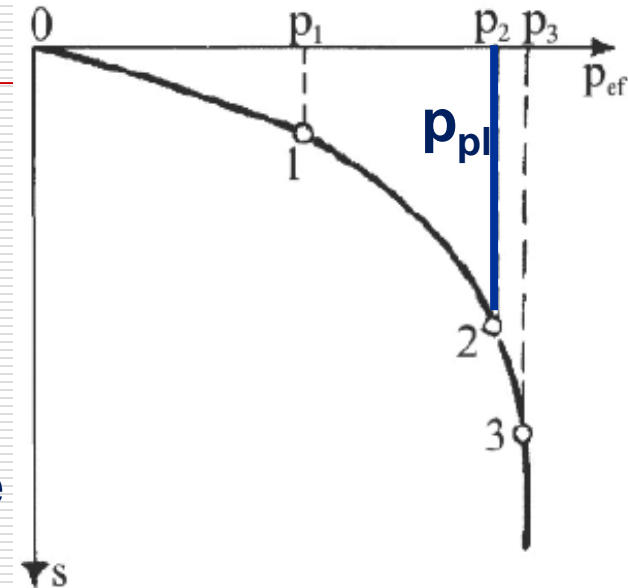
D is the foundation depth

$\bar{\gamma}$ is the volume weight of the layers located above foundation foot

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

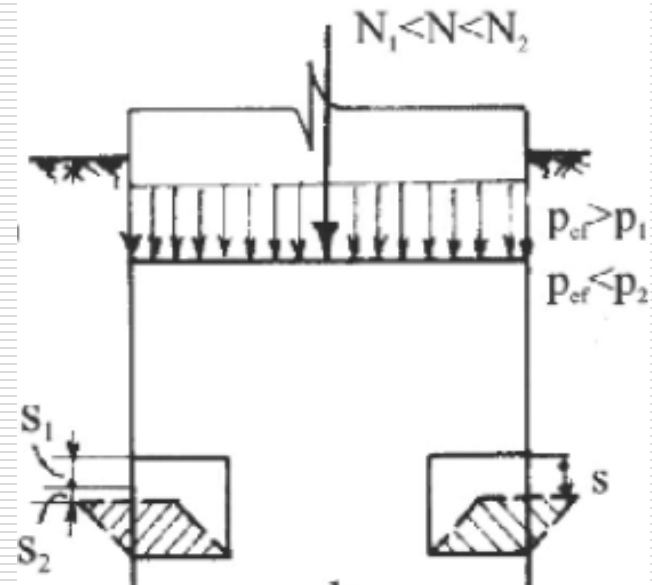
Plastic pressure p_{pl}

- Overpassing the point 1 on p - s soil response curve, the soil passes in linear-plastic behavior.
- The **plastic pressure** represents the bearing capacity of the soil corresponding to a limit state that considers the allowable settlement of the foundation soil under acting loads.



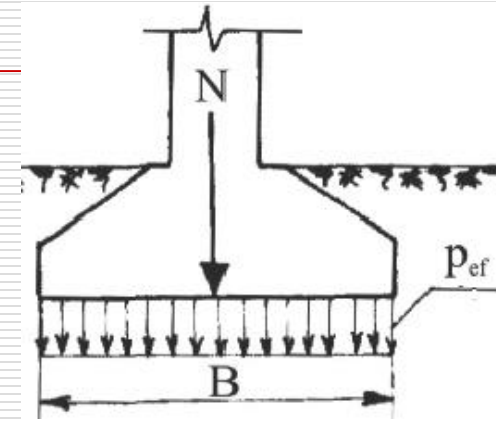
Obs: Foundation loads = structural loads (fundamental combination) + foundation weight + long-term loads

- Conventionally, p_{pl} is computed for the following limit state: the depth of the plastic zones reaches $B/4$.



§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}



□ Considering a continuous foundation of width B at a depth D_f transmitting under the foundation the pressure p .

□ The geologic pressure p_g under the foundation is: $p_a = \gamma \cdot D_f$

□ The net pressure given by foundation (subtracting the geologic pressure) is: $p_{net} = p - p_a = p - \gamma \cdot D_f$

□ In a point M of the foundation soil, in which the angle created between M and the foundation limits is $2\beta_0$, the stresses are generated by both the geological pressure and the foundation:

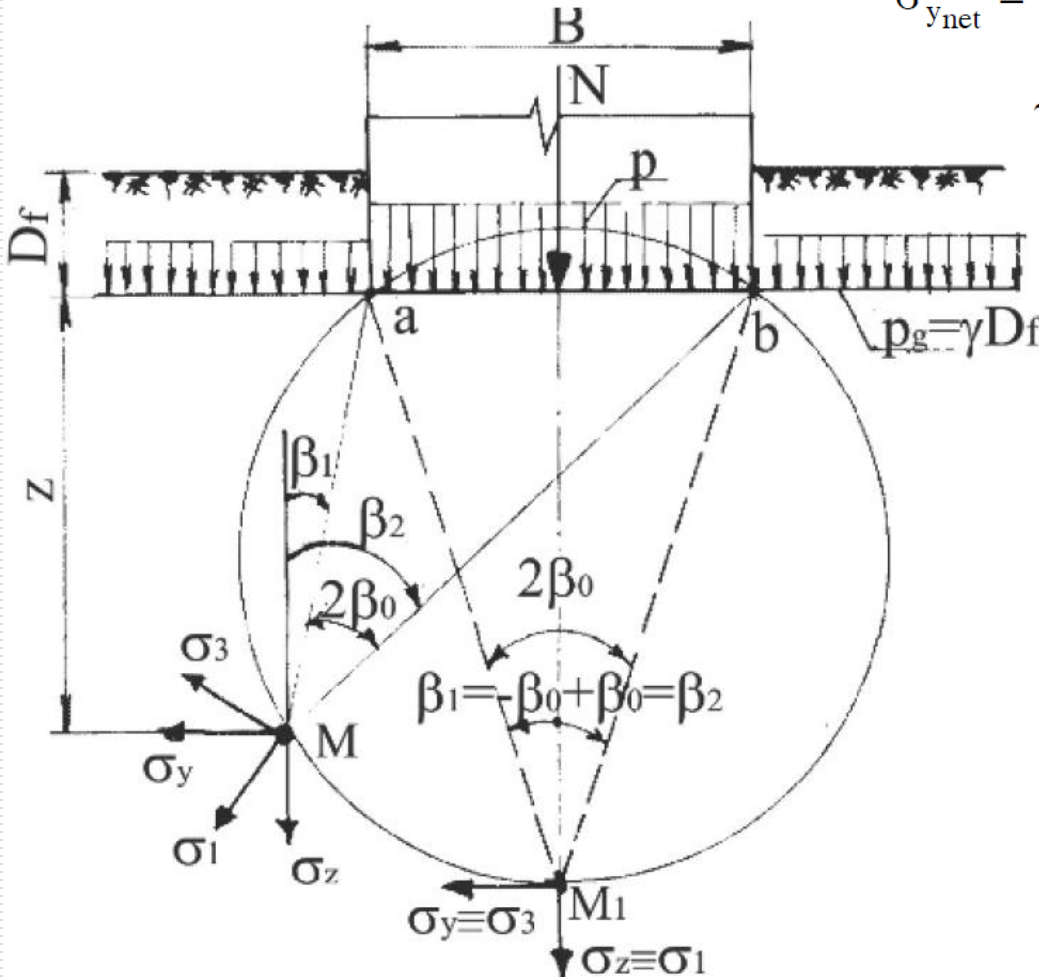
$$\sigma_{z_{net}} = \frac{p - \gamma \cdot D_f}{\pi} [(\beta_2 - \beta_1) + \sin(\beta_2 - \beta_1) \cdot \cos(\beta_2 + \beta_1)]$$

$$\sigma_{y_{net}} = \frac{p - \gamma \cdot D_f}{\pi} [(\beta_2 - \beta_1) - \sin(\beta_2 - \beta_1) \cdot \cos(\beta_2 + \beta_1)]$$

$$\tau_{net} = \frac{p - \gamma \cdot D_f}{\pi} \cdot \sin(\beta_2 - \beta_1) \cdot \sin(\beta_2 + \beta_1)$$

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}



$$\sigma_{z_{net}} = \frac{p - \gamma \cdot D_f}{\pi} [(\beta_2 - \beta_1) + \sin(\beta_2 - \beta_1) \cdot \cos(\beta_2 + \beta_1)]$$

$$\sigma_{y_{net}} = \frac{p - \gamma \cdot D_f}{\pi} [(\beta_2 - \beta_1) - \sin(\beta_2 - \beta_1) \cdot \cos(\beta_2 + \beta_1)]$$

$$\tau_{net} = \frac{p - \gamma \cdot D_f}{\pi} \cdot \sin(\beta_2 - \beta_1) \cdot \sin(\beta_2 + \beta_1)$$

□ In the point $M1$, located on the axis of symmetry of the foundation and on the circle described by points M and a and b (edges of the foundation), the stresses σ_y and σ_z become principal stresses:

$$\sigma_{1net} = \frac{p - \gamma \cdot D_f}{\pi} (2\beta_0 + \sin 2\beta_0)$$

$$\sigma_{3net} = \frac{p - \gamma \cdot D_f}{\pi} (2\beta_0 - \sin 2\beta_0)$$

$$\tau = 0$$

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

□ These relations show that the principal stresses under a foundation are dependent on the value of the pressure p generated by the foundation and the angle $2\beta_0$.

$$\sigma_{1net} = \frac{p - \gamma \cdot D_f}{\pi} (2\beta_0 + \sin 2\beta_0)$$
$$\sigma_{3net} = \frac{p - \gamma \cdot D_f}{\pi} (2\beta_0 - \sin 2\beta_0)$$
$$\tau = 0$$

□ For a given p value, in all points for which the angle created with the foundation edges is $2\beta_0$, the principal stresses have the same values.

□ The geometrical locus of these points is the circle passing through the foundation edges and the points M .

□ The total stresses in points M are obtained by adding to the pressure given by pressure p the geologic pressures:

$$\sigma_{1g} = \gamma \cdot D_f + \gamma \cdot z = \gamma \cdot (z + D_f)$$

$$\sigma_{3g} = K_0 \cdot \sigma_{1g} = K_0 \cdot \gamma \cdot (z + D_f)$$

With $K_0=0$ for the soils in plastic zones

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

□ The total stresses are expressed by:

$$\sigma_1 = \frac{p - \gamma \cdot D_f}{\pi} (2\beta_0 + \sin 2\beta_0) + \gamma \cdot (z + D_f)$$

$$\sigma_3 = \frac{p - \gamma \cdot D_f}{\pi} (2\beta_0 - \sin 2\beta_0) + \gamma \cdot (z + D_f)$$

□ These relations show that the principal stresses under a foundation located on a circle creating a sectorial angle $2\beta_0$ between the edges of the foundation are not constant but depends on the height z of the considered point.

□ For having some limiting conditions on the development of the plastic zones, is important to know the depth z on which plastifications develop under the principal stresses σ_1 and σ_3 .

□ The development of plastic deformations in foundation soils assumes the fulfilment of conditions for soil fracture, which in case of cohesive soils is expressed through:

$$\sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 + 2 \cdot c \cdot \operatorname{ctg} \phi}$$

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

- Replacing σ_1 and σ_3 in the above condition one can find the depth z in function of the angle $2\beta_0$.

$$z = \frac{p - \gamma \cdot D_f}{\gamma \cdot \pi} \cdot \left(\frac{\sin 2\beta_0}{\sin \phi} - 2\beta_0 \right) - D_f - \frac{c}{\gamma} \operatorname{ctg} \phi = f(2\beta_0)$$

- Practically, in the foundation soil there can exist several points creating a sectorial angle $2\beta_0$ with the edges of the foundation for which there could be produced plastic deformations under a given load p produced by the foundation.
- For determining the maximum depth z_{max} to which the plastic zones can be extended in the foundation soil, one set the condition of the maximum for the function $z=f(2\beta_0)$:

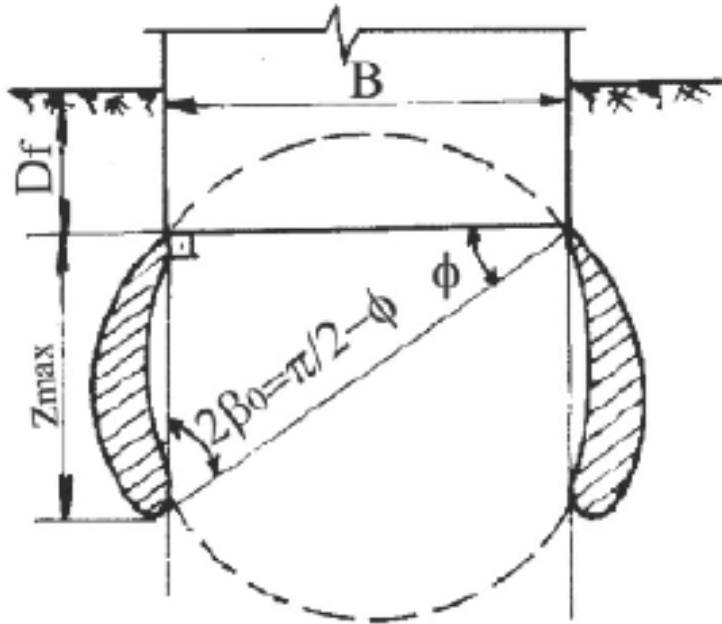
$$\frac{dz}{d\beta_0} = \frac{p - \gamma \cdot D_f}{\gamma \cdot \pi} \cdot \left(\frac{2 \cos 2\beta_0}{\sin \phi} - 2 \right) = 0 \quad \text{Resulting} \quad 2\beta_0 = \frac{\pi}{2} - \phi$$

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

□ Replacing the value $2\beta_0$ in the relationship of z , it results the maximum depth of the plastic zone as:

$$z_{\max} = \frac{p - \gamma \cdot D_f}{\gamma \cdot \pi} \cdot \left(\operatorname{ctg}\phi - \frac{\pi}{2} + \phi \right) - D_f - \frac{c}{\gamma} \operatorname{ctg}\phi$$



N.N. Maslov proposes the existence of plastic zones only outside of the vertical foundation borders ($z_{\max} = B \cdot \operatorname{tg}\phi$) and:

$$p = p_1 = \frac{\gamma \pi \cdot \left(B \cdot \operatorname{tg}\phi + D_f + \frac{c}{\gamma} \cdot \operatorname{ctg}\phi \right)}{\operatorname{ctg}\phi - \frac{\pi}{2} + \phi} + \gamma \cdot D_f$$

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

□ The Romanian norm NP112 limits the extension of the plastic zones in the foundation soil at $z_{max}=B/4$:

$$p = p_1 = \frac{\gamma \pi \cdot \left(\frac{B}{4} + D_f + \frac{c}{\gamma} \cdot \text{ctg} \phi \right)}{\text{ctg} \phi - \frac{\pi}{2} + \phi} + \gamma \cdot D_f$$

□ By grouping the terms in the above formula, the relation can be written as:

$$p_{pl} = \bar{\gamma} \cdot B \cdot N_1 + q \cdot N_2 + c \cdot N_3$$

□ In normative formula, a coefficient of the working conditions is applied (formula given for isolated foundations without basement):

$$p_{pl} = m_i \cdot (\bar{\gamma} \cdot B \cdot N_1 + q \cdot N_2 + c \cdot N_3)$$

OBS: A similar formula can be derived for structures with basements.

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

$$p_{pl} = m_i \cdot (\bar{\gamma} \cdot B \cdot N_1 + q \cdot N_2 + c \cdot N_3)$$

where:

m_i is the working conditions coefficient ($m_i=1.1 \dots 2$);

$\bar{\gamma}$ - average weight density of soil layers in height $B/4$ (kN/m³);

q – design overload (geologic pressure) at the level of the footing;

c – design value of specific cohesion of the soil layer beneath the footing

N_1, N_2, N_3 – dimensionless coefficients (bearing capacity coefficients), given in tables in function of the friction angle ϕ of the soil layer beneath the footing

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

$$p_{pl} = m_i \cdot (\bar{\gamma} \cdot B \cdot N_1 + q \cdot N_2 + c \cdot N_3)$$

Values of N_1 , N_2 , N_3 coefficients

ϕ grade	N_1	N_2	N_3	ϕ grade	N_1	N_2	N_3
0	0,00	1,00	3,14	24	0,72	3,87	6,45
2	0,03	1,12	3,32	26	0,84	4,37	6,90
4	0,06	1,25	3,51	28	0,98	4,93	7,40
6	0,10	1,39	3,71	30	1,15	5,59	7,95
8	0,14	1,55	3,93	32	1,34	6,35	8,55
10	0,18	1,73	4,17	34	1,55	7,21	9,21
12	0,23	1,94	4,42	36	1,81	8,25	9,98
14	0,29	2,17	4,69	38	2,11	9,44	10,80
16	0,36	2,43	5,00	40	2,46	10,84	11,73
18	0,43	2,72	5,31	42	2,87	12,50	12,77
20	0,51	3,06	5,66	44	3,37	14,48	13,96
22	0,61	3,44	6,04	45	3,66	15,64	14,64

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Plastic pressure p_{pl}

$$p_{pl} = m_i \cdot (\bar{\gamma} \cdot B \cdot N_1 + q \cdot N_2 + c \cdot N_3)$$

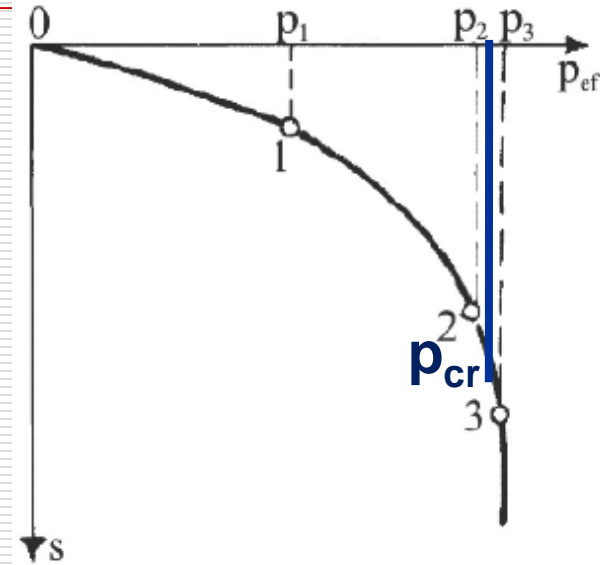
Values of m_i coefficients

Nr. crt.	Denumirea terenului de fundare	m_i
1.	Bolovănișuri cu interspațiile umplute cu nisip, pietrișuri și nisipuri cu excepția nisipurilor fine și prăfoase	2,0
2.	Nisipuri fine: - uscate sau umede ($S_r \leq 0,8$)	1,7
	- foarte umede sau saturate ($S_r > 0,8$),	1,6
3.	Nisipuri prăfoase: - uscate sau umede ($S_r \leq 0,8$)	1,5
	- foarte umede sau saturate ($S_r > 0,8$) r	1,3
4.	Bolovănișuri și pietrișuri cu interspațiile umplute cu pământuri coezive cu $I_c \geq 0,5$	1,3
5.	Pământuri coezive cu $I_c \geq 0,5$	1,4
6.	Bolovănișuri și pietrișuri cu interspațiile umplute cu pământuri coezive cu $I_c < 0,5$	1,1
7.	Pământuri coezive cu $I_c < 0,5$	1,1

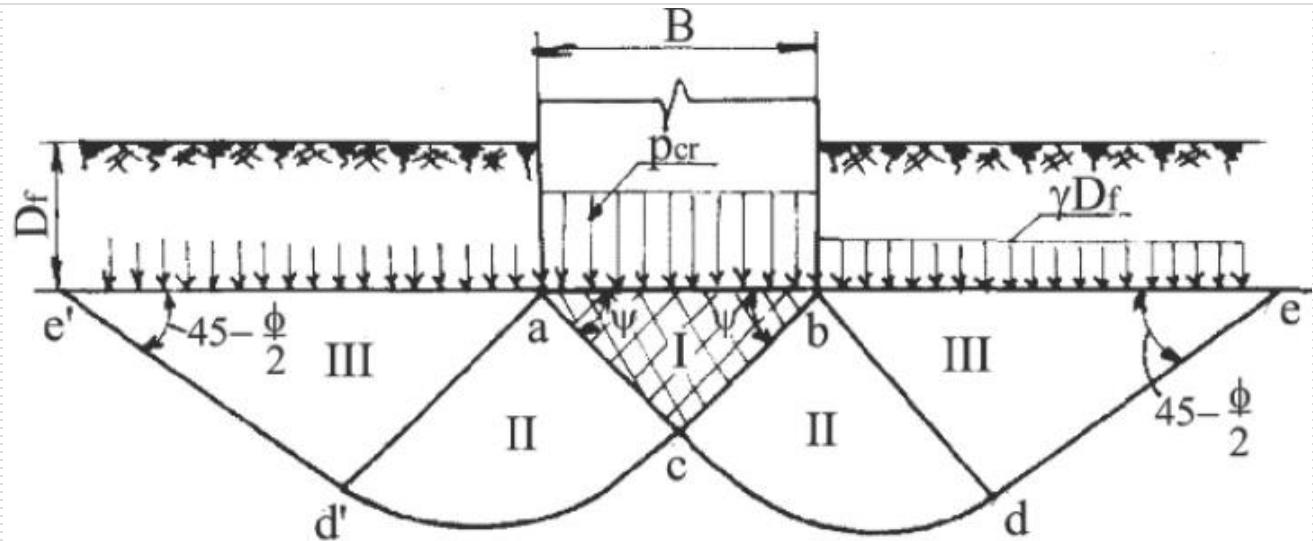
§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Critical pressure p_{cr}

- By increasing the soil pressure beyond point 2 we can arrive to the failure condition of the foundation soil.
- In this stage, the failure mechanism produced below the foundation are due to increased settlements by soil shear.



Failure of a foundation soil



§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Critical pressure p_{cr}

□ K Terzaghi divided the soil foundation in three zones:

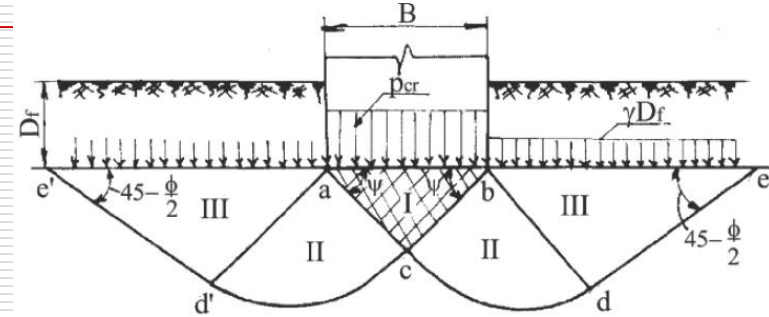
□ **zone I** – a prismatic zone under the foundation, acting as a rigid body (**thrusting zone**)

□ **zone II** represents sheared volumes of soil (**radial shearing zones**), presenting shear failure boundaries under the form of a circular arch or logarithmic spiral.

□ **zone III (passive soil zones)** represents passive resistance soil volumes, loaded only by geologic pressure and the thrust.

□ The theoretical models for finding p_{cr} are based on equilibrium condition on the foundation soil.

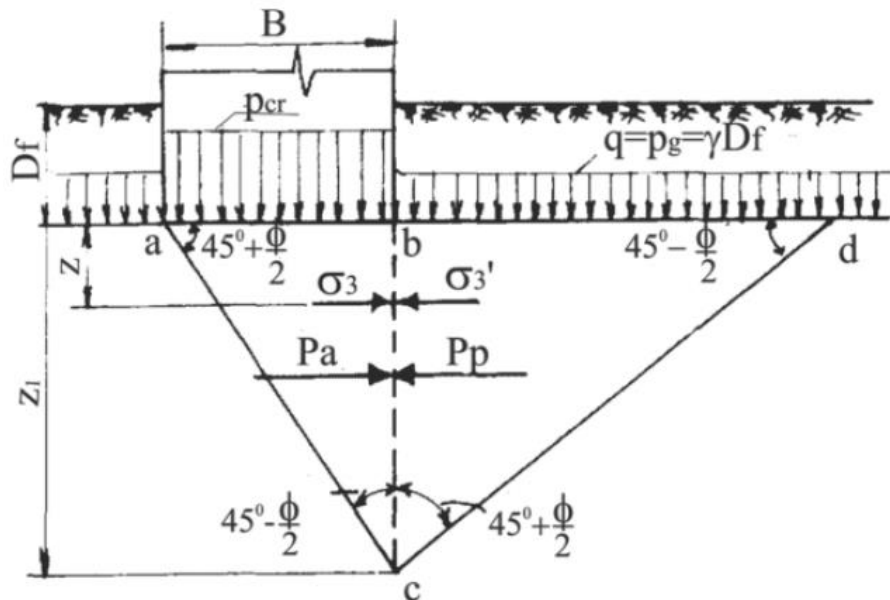
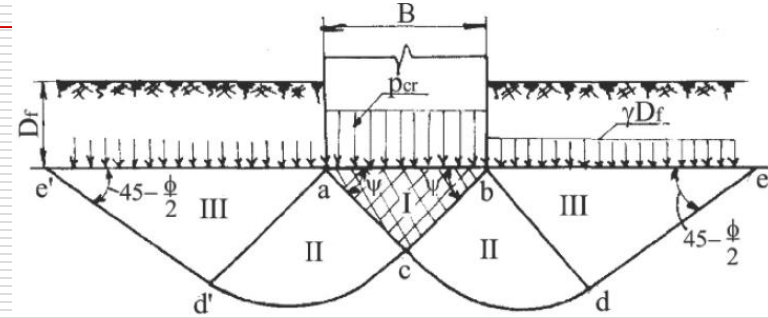
OBS: Karl Terzaghi was a pioneer searching theories about soil behavior under the applied loads. By his studies made possible the study of consolidation of soils and also calculation of bearing capacities of foundation soils.



§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Critical pressure p_{cr}

Considering a continuous foundation of width B , transmitting to the soil an uniform pressure $p_{ef} = p_{cr}$, the failure and the sliding of the foundation soil takes place by considering $a-c$ and $c-d$ planes, at angles $45-\phi/2$ and respectively $45+\phi/2$ (Rankine' hypothesis).



At depth z below the foundation, the principal vertical stresses σ_1 ($a-c-d$ volume) for the active zone and σ_1' for the passive zone ($b-c-d$ volume) are:

$$\sigma_1 = p_{cr} + \gamma \cdot z$$

$$\sigma_1' = \gamma \cdot D_f + \gamma \cdot z = \gamma \cdot (D_f + z)$$

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Critical pressure p_{cr}

□ The principal horizontal stresses can be expressed from failure condition, in function of the principal vertical stress σ_1 by:

$$\sigma_3 = \sigma_1 \cdot \frac{1 - \sin \phi}{1 + \sin \phi} - 2 \cdot c \cdot \frac{\cos \phi}{1 + \sin \phi}$$

□ Replacing σ_1 and considering trigonometric transformations, σ_3 and σ'_3 becomes:

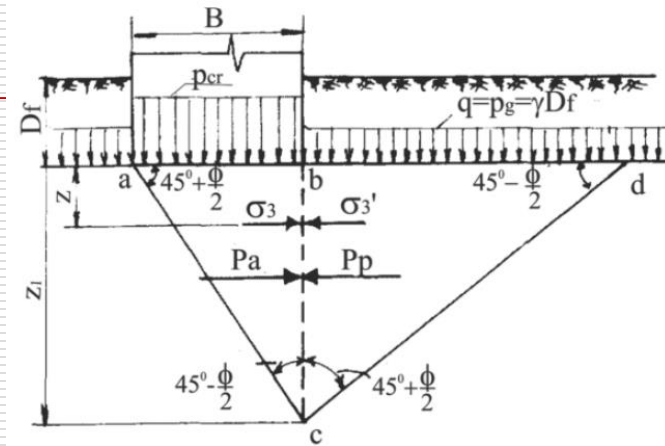
$$\sigma_3 = \sigma_1 \cdot K_a - 2c\sqrt{K_a} = (p_{cr} + \gamma \cdot z) \cdot K_a - 2c\sqrt{K_a}$$

$$\sigma'_3 = \sigma'_1 \cdot K_p + 2c\sqrt{K_p} = \gamma \cdot (D_f + z) \cdot K_p + 2c\sqrt{K_p}$$

where:

K_a is the coefficient of active thrust: $K_a = \operatorname{tg}^2(45^\circ - \frac{\phi}{2})$

K_p is the coefficient of passive resistance: $K_p = \operatorname{tg}^2(45^\circ + \frac{\phi}{2})$



§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Critical pressure p_{cr}

□ Based on the described theory, the norm NP112 (following the EN 1997-1) uses the following formula for the bearing capacity

$$(p_{cr}=R/A'): \quad R/A' = c' N_c b_c s_c i_c + q' N_q b_q s_q i_q + 0,5 \gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma$$

where:

N_γ , N_q and N_c are bearing capacity coefficients.

$$N_q = e^{\pi \tan \bar{\phi}'} \tan^2 (45 + \phi'/2)$$

$$N_c = (N_q - 1) \cot \phi'$$

$$N_\gamma = 2 (N_q - 1) \tan \phi', \text{ where } \delta \geq \phi'/2 \text{ (rough base)}$$

s_q , s_γ , and s_c shape factors:

$$s_q = 1 + (B' / L') \sin \phi', \text{ for a rectangular shape;}$$
$$s_q = 1 + \sin \phi', \text{ for a square or circular shape;}$$

$$\text{— } s_\gamma = 1 - 0,3 (B'/L'), \text{ for a rectangular shape;}$$
$$s_\gamma = 0,7, \text{ for a square or circular shape}$$

$$\text{— } s_c = (s_q \cdot N_q - 1) / (N_q - 1) \text{ for rectangular, square or circular shape,}$$

§ 3.3 Bearing capacities according to Romanian norm NP 112-2014

Critical pressure p_{cr}

$$R/A' = c' N_c b_c s_c i_c + q' N_q b_q s_q i_q + 0,5 \gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma$$

where:

b_γ , b_q and b_c are inclination of the foundation base coefficients.

$$b_c = b_q - (1 - b_q) / (N_c \tan \varphi')$$

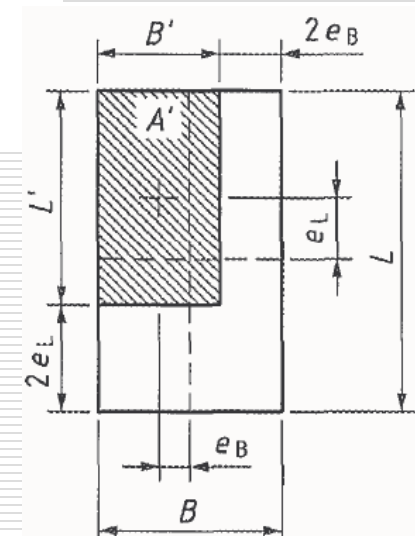
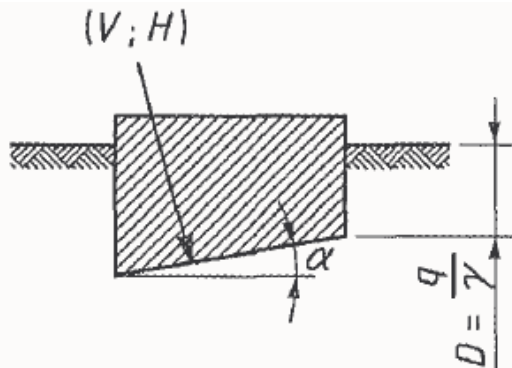
$$b_q = b_\gamma = (1 - \alpha \cdot \tan \varphi')^2$$

i_q , i_γ , and i_c are load inclination coefficients:

$$i_c = i_q - (1 - i_q) / (N_c \cdot \tan \varphi');$$

$$i_q = [1 - H/(V + A'c'\cot \varphi')]^m;$$

$$i_\gamma = [1 - H/(V + A'c'\cot \varphi')]^{m+1}.$$



Used notations

Text and adaptation from:

Sisteme de fundare a Construcțiilor,

Mirea Monica, Marin Marin, Editura Orizonturi Universitare, Timișoara 2011