Suport de curs

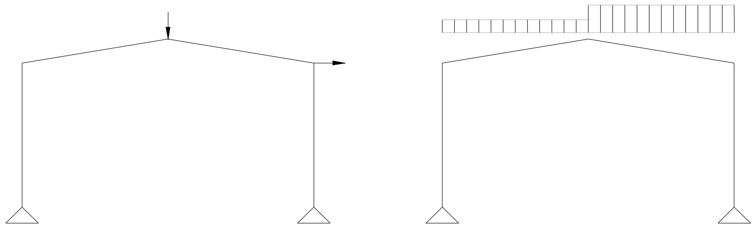
„Statica constructiilor – Structural analysis”

Inginerie civila
Grupa ICE
An de studiu III

2013

1. **Flexibility Method**

Portal frame – 2 load cases



Load case 1

Degree of static indeterminacy

$$d = l_i + r - 3e$$

l_i -number of internal links

r – number of external links (reactions)

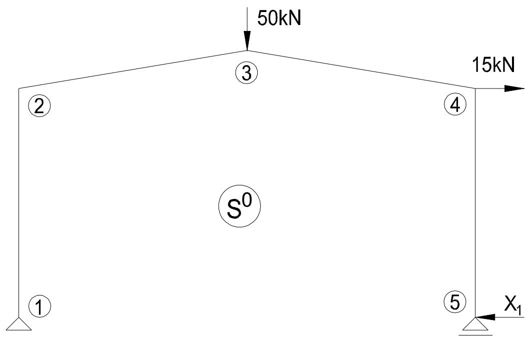
e – number of elements

$d = 9 + 4 - 3 \cdot 4 = 1$ (considering 4 elements rigidly connected, with 3 internal links between elements)

or

$d = 0 + 4 - 3 \cdot 1 = 1$ (the 4 elements rigidly connected may be seen as a single body, therefore there are no internal links)

Primary structure



Compatibility equation

$$D_{1P} = d_{11} \cdot X_1 + D_{1P}^0 = 0$$

d_{11} – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by the unit value of X_1, acting on the primary structure;

$d_{11} \cdot X_1$ – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by X_1, acting on the primary structure;

D^0_{1P} – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by the external loads, acting on the primary structure;

D_{1P} – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by X_1 and by the external loads, acting on the primary structure; it must be zero, , because in this point and on the direction of X1, on the real structure exists a link which does not allow this displacement (pinned support).

Bending moment on the primary structure due to external loads

$$\left(\sum H\right) = 0 \qquad R^P_{H1} = 15\text{kN}$$

$$\left(\sum M\right)_1 = 0 \qquad 50 \cdot 6 + 15 \cdot 6 - R^P_{V5} \cdot 12 = 0$$

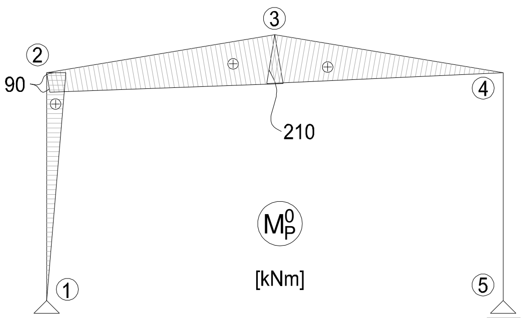
(Clockwise is considered positive)

$$\Longrightarrow R^P_{V5} = 32.5\text{kN}$$

$$\left(\sum M\right)_5 = 0 \qquad R^P_{V1} \cdot 12 - 50 \cdot 6 + 15 \cdot 6 = 0 \qquad R^P_{V1} = 17.5\text{kN}$$

Verification

$$\left(\sum V\right) = 0 \qquad R^P_{V1} + R^P_{V5} - 50\text{kN} = 0$$



Bending moment on the primary structure due to X_1=1

$$\left(\sum H\right) = 0 \qquad R^1_{H1} = 1$$

$$\left(\sum M\right)_1 = 0 \qquad R^1_{V1} = 0$$

$$\left(\sum M\right)_5 = 0 \qquad R^1_{V5} = 0$$

$$\text{EI} \cdot d_{11} = 2\left[\frac{1}{1} \cdot \frac{6 \cdot (-6)}{2} \cdot \frac{2}{3} \cdot (-6) + \frac{1}{2} \cdot \frac{6.083 \cdot (-6)}{2} \cdot \left(\frac{2}{3} \cdot (-6) + \frac{1}{3} \cdot (-7)\right) + \frac{1}{2} \cdot \frac{6.083 \cdot (-7)}{2} \cdot \left(\frac{2}{3} \cdot (-7) + \frac{1}{3} \cdot (-6)\right)\right]$$

$$\mathrm{EI} \cdot \mathrm{d}_{11} = 401.514$$

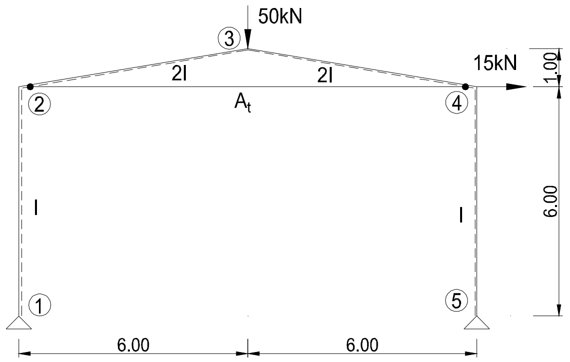
$$\mathrm{EI} \cdot \mathrm{D}_{1P}^{0} = \frac{1}{1} \cdot \frac{6 \cdot 90}{2} \cdot \frac{2}{3} \cdot (-6) + \frac{1}{2} \cdot \frac{6.083 \cdot 90}{2} \cdot \left(\frac{2}{3} \cdot (-6) + \frac{1}{3} \cdot (-7)\right) +$$

$$+ \frac{1}{2} \cdot \frac{6.083 \cdot 210}{2} \cdot \left(\frac{2}{3} \cdot (-7) + \frac{1}{3} \cdot (-6)\right) + \frac{1}{2} \cdot \frac{6.083 \cdot 210}{2} \cdot \left(\frac{2}{3} \cdot (-7) + \frac{1}{3} \cdot (-6)\right)$$

$$\mathrm{EI} \cdot \mathrm{D}_{1P}^{0} = -6204.93$$

$$\mathrm{X}_1 = -\frac{\mathrm{D}_{1P}^{0}}{\mathrm{d}_{11}} = 15.454 \text{ kN}$$

Portal frame with tie

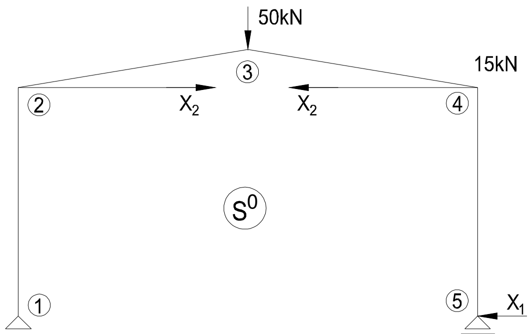


$$\mathrm{E} \cdot \mathrm{I} = 1.512 \cdot 10^5 \mathrm{kNm}^2$$
$$\mathrm{E}_t \cdot \mathrm{A}_t = 1.4845 \cdot 10^5 \mathrm{kN}$$

Degree of static indeterminacy (the frame may be considered as having 4 elements rigidly connected each-other by 3 internal links + the tie, hinged to the frame, thus resulting 2 internal links on each end of the tie):

$$\mathrm{d} = \mathrm{l_i} + \mathrm{r} - 3\mathrm{e} = 13 + 4 - 3 \cdot 5 = 2$$

Primary structure



Compatibility equations

$$\begin{cases} D_{1P} = \mathrm{d}_{11} \cdot \mathrm{X}_1 + \mathrm{d}_{12} \cdot \mathrm{X}_2 + \mathrm{D}_{1P}^{0} = 0 \\ D_{2P} = \mathrm{d}_{21} \cdot \mathrm{X}_1 + \mathrm{d}_{22} \cdot \mathrm{X}_2 + \mathrm{D}_{2P}^{0} = -\Delta \mathrm{l}_2 \end{cases}$$

d_{11} – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by the unit value of X_1, acting on the primary structure;

d_{22} – linear displacement at the point and in the direction of the redundant axial force X_2, caused by the unit value of X_2, acting on the primary structure;

$d_{11} \cdot X_1$ – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by X_1, acting on the primary structure;

$d_{22} \cdot X_2$ – linear displacement at the point and in the direction of the redundant axial force X_2, caused by X_2, acting on the primary structure;

d_{12} – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by the unit value of X_2, acting on the primary structure;

d_{21} – linear displacement at the point and in the direction of the redundant axial force X_2, caused by the unit value of X_1, acting on the primary structure;

$d_{12} \cdot X_2$ – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by X_2, acting on the primary structure

$d_{21} \cdot X_1$ – linear displacement at the point and in the direction of the redundant axial force X_2, caused by X_1, acting on the primary structure;

D_{1P}^0 – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by the external loads, acting on the primary structure

D_{2P}^0 – linear displacement at the point and in the direction of the redundant axial force X_2, caused by the external loads, acting on the primary structure;

D_{1P} – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by X_1, X_2 and by the external loads, acting on the primary structure; it must be zero, because in this point and on the direction of X_1, on the real structure exists a link which does not allow this displacement;

D_{2P} – linear displacement at the point and in the direction of the redundant axial force X_2, caused by X_1, X_2 and by the external loads, acting on the primary structure. This displacement is equal to the relative displacement between points 2 and 4 (Δl_2) on the statically indeterminate structure. The "-" sign indicates that the displacement produced by the axial force in the tie (X_2) on the statically indeterminate structure, is equal and opposite to the displacement from the same force X_2 acting on the primary structure.

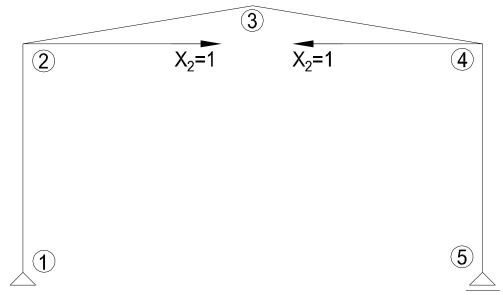
$$\Delta \mathrm{l}_2 = \frac{\mathrm{l}_{24}}{\mathrm{E_t} \cdot \mathrm{A_t}} \cdot \mathrm{X}_2$$

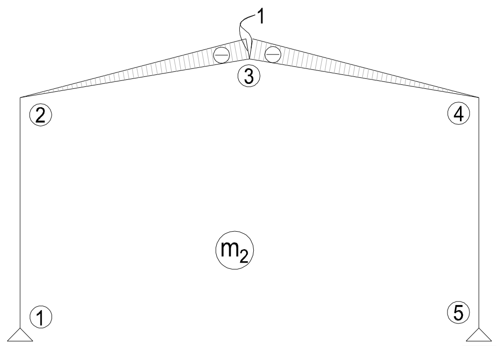
where:

l_{24} – length of the tie

$E_t \cdot A_t$ – axial rigidity of the tie

Bending moment on the primary structure due to X_2=1





$$EI \cdot d_{11} = 2\left[\frac{1}{1}\cdot\frac{6\cdot(-6)}{2}\cdot\frac{2}{3}\cdot(-6)+\frac{1}{2}\cdot\frac{6.083\cdot(-6)}{2}\cdot\left(\frac{2}{3}\cdot(-6)+\frac{1}{3}\cdot(-7)\right)+\frac{1}{2}\cdot\frac{6.083\cdot(-7)}{2}\cdot\left(\frac{2}{3}\cdot(-7)+\frac{1}{3}\cdot(-6)\right)\right]$$

$$EI \cdot d_{11} = 401.514$$

$$EI \cdot d_{12} = 2\left[\frac{1}{2}\cdot\frac{6.083\cdot(-6)}{2}\cdot\left(\frac{1}{3}\cdot(-1)\right)+\frac{1}{2}\cdot\frac{6.083\cdot(-7)}{2}\cdot\left(\frac{2}{3}\cdot(-1)\right)\right] = 20.277$$

$$EI \cdot d_{22} = 2\left[\frac{1}{2}\cdot\frac{6.083\cdot(-1)}{2}\cdot\left(\frac{2}{3}\cdot(-1)\right)\right] = 2.03$$

$$EI \cdot D_{1P}^{0} = \frac{1}{1}\cdot\frac{6\cdot 90}{2}\cdot\frac{2}{3}\cdot(-6)+\frac{1}{2}\cdot\frac{6.083\cdot 90}{2}\cdot\left(\frac{2}{3}\cdot(-6)+\frac{1}{3}\cdot(-7)\right)+\frac{1}{2}\cdot\frac{6.083\cdot 210}{2}\cdot\left(\frac{2}{3}\cdot(-7)+\frac{1}{3}\cdot(-6)\right)+\frac{1}{2}\cdot\frac{6.083\cdot 210}{2}\cdot\left(\frac{2}{3}\cdot(-7)+\frac{1}{3}\cdot(-6)\right)$$

$$EI \cdot D_{1P}^{0} = -6204.93$$

$$EI \cdot D_{2P}^{0} = \frac{1}{2}\cdot\frac{6.083\cdot 90}{2}\cdot\left(\frac{1}{3}\cdot(-1)\right)+\frac{1}{2}\cdot\frac{6.083\cdot 210}{2}\cdot\left(\frac{2}{3}\cdot(-1)\right)+\frac{1}{2}\cdot\frac{6.083\cdot 210}{2}\cdot\left(\frac{2}{3}\cdot(-1)\right)$$

$$EI \cdot D_{2P}^{0} = -471.433$$

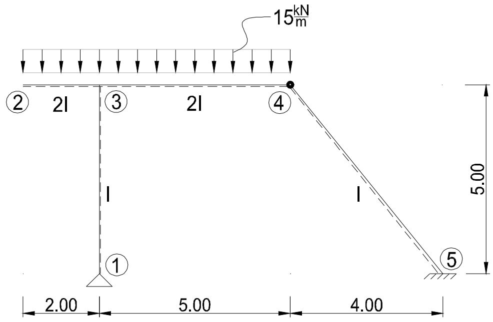
$$\begin{cases} D_{1P} = \dfrac{401.514}{EI} \cdot X_1 + \dfrac{20.277}{EI} \cdot X_2 - \dfrac{6204.93}{EI} = 0 \\ D_{2P} = \dfrac{20.277}{EI} \cdot X_1 + \dfrac{2.03}{EI} \cdot X_2 - \dfrac{471.433}{EI} = -\dfrac{12.0}{E_t A_t} \cdot X_2 \end{cases}$$

$$\begin{cases} D_{1P} = 401.514 \cdot X_1 + 20.277 \cdot X_2 - 6204.93 = 0 \\ D_{2P} = 20.277 \cdot X_1 + 2.03 \cdot X_2 - 471.433 = -12 \dfrac{EI}{E_t A_t} \cdot X_2 \end{cases}$$

$$\frac{EI}{E_t A_t} = \frac{1.512 \cdot 10^5 \text{kNm}^2}{E_t \cdot A_t = 1.4845 \cdot 10^5 \text{kN}} = 1.0165 \text{m}^2$$

$X_1 = 14.852$ kN
$X_2 = 11.95$ kN

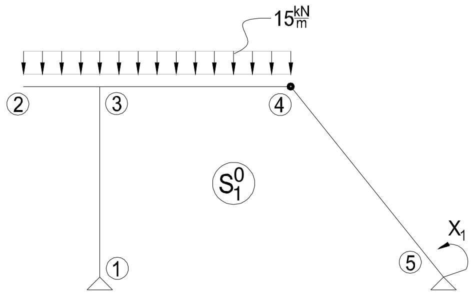
Portal frame with inclined elements



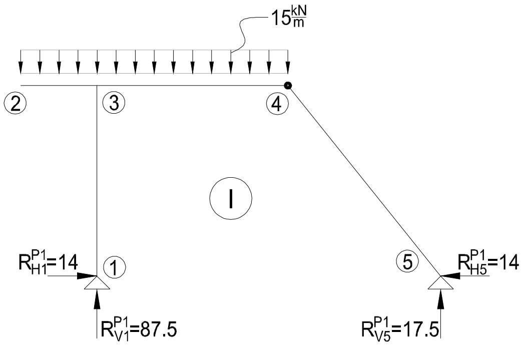
Degree of static indeterminacy (the structure may be seen as two bodies connected by a hinge, thus having 2 internal links):

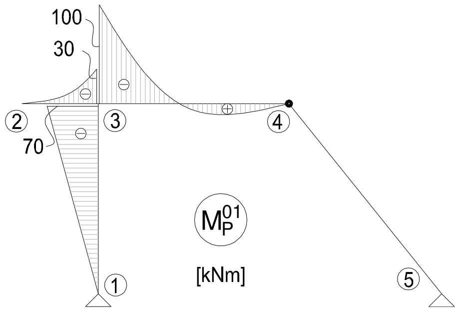
$$d = l_i + r - 3e = 2 + 5 - 3 \cdot 2 = 1$$

- In the first case, the unknown is the moment reaction in point 5:



Bending moment on the primary structure due to external loads





Compatibility equation

$$D_{1P} = d_{11} \cdot X_1 + D_{1P}^0 = 0$$

The terms in the compatibility equation represent:

d_{11} – rotation at the point and in the direction of the redundant moment reaction X1, caused by the unit value of X1, acting on the primary structure

D_{1P}^0 – rotation at the point and in the direction of the redundant moment reaction X1, caused by the external loads, acting on the primary structure

D_{1P} – rotation at the point and in the direction of the redundant moment reaction X1, caused by X1 and by the external loads, acting on the primary structure; it must be zero, because in this point and on the direction of X1, on the real structure exists a link which does not allow this rotation (fixed support).

The expression for bending moment between nodes 3 and 4 is:

$$\begin{aligned} M_P^0 = R_{V1}^P \cdot x - p \cdot (x+2) \cdot \left(\frac{x+2}{2}\right) - R_{H1}^0 \cdot 5 \\ = 87.5 \cdot x - 7.5 \cdot (x^2 + 4x + 4) - 70 \\ = -7.5 \cdot x^2 + 57.5 \cdot x - 100 \end{aligned}$$

$$m_1 = R_{V1}^1 \cdot x - R_{H1}^1 \cdot 5 = 0.111 \cdot x - 0{,}555$$

$$d_{11} = \int \frac{m_1 \cdot m_1}{EI} dx$$

$$EI \cdot d_{11} = \frac{1}{1} \cdot \frac{0.555 \cdot 5}{2} \cdot 0.37 + \frac{1}{2} \cdot \frac{0.555 \cdot 5}{2} \cdot 0.37 + \frac{1}{1} \cdot \frac{1 \cdot 6.403}{2} \cdot 0.667 = 2{,}904$$

$$D_{1P}^0 = \int \frac{m_1 \cdot M_p^0}{EI} dx$$

$$EI \cdot D_{1P}^0 = \frac{1}{1} \cdot \frac{-70 \cdot 5}{2} \cdot \frac{2}{3} \cdot (-0{,}555) + \frac{1}{2} \cdot \int_0^5 (0.111 \cdot x - 0{,}555) \cdot (-7.5 \cdot x^2 + 57.5 \cdot x - 100)dx$$

$$EI \cdot D_{1P}^0 = 64.75 + \frac{1}{2} \cdot \int_0^5 (-0.8325 \cdot x^3 + 6.3825 \cdot x^2 - 11.1 \cdot x + +4.162 \cdot x^2 - 31.9125 \cdot x + 55.5)dx$$

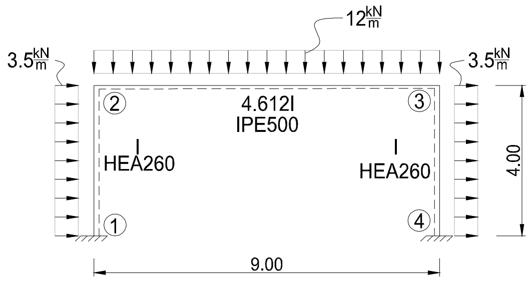
$$EI \cdot D_{1P}^0 = 64.75 + \frac{1}{2} \cdot \int_0^5 (-0.8325 \cdot x^3 + 10.545 \cdot x^2 - 43.0125 \cdot x + 55.5)dx$$

$$EI \cdot D_{1P}^0 = 64.75 + \frac{1}{2} \cdot \left(-0.8325 \cdot \frac{x^4}{4} + 10.545 \cdot \frac{x^3}{3} - 43.0125 \cdot \frac{x^2}{2} + 55.5 \cdot x\right)\Bigg|_0^5$$

$$EI \cdot D_{1P}^0 = 64.75 + 24.57 = 89.32$$

$$\Rightarrow X_1 = -\frac{D_{1P}^0}{d_{11}} = -30.75 \text{ kNm}$$

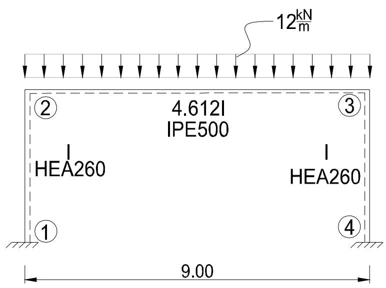
Symmetric portal frame

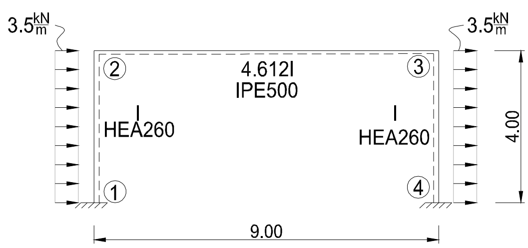


Degree of static indeterminacy

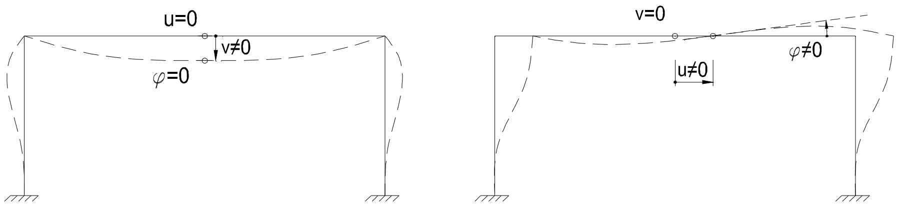
$$d = l_i + r - 3e = 6 + 6 - 3 \cdot 3 = 3$$

The structure is symmetric. The load may be decomposed in two load cases, which will be treated separately: one symmetric and one antisymmetric.



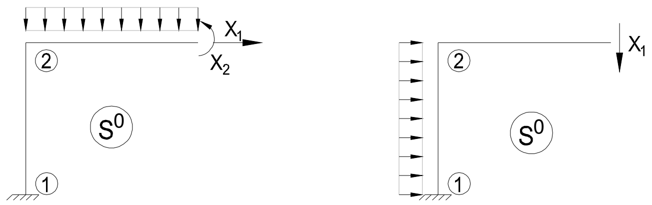


The deformed shapes of the structure for the two load cases are symmetric and antisymmetric, respectively. For each load case, in the point situated in the middle of the beam (in the axis of symmetry of the structure), some displacements are zero:



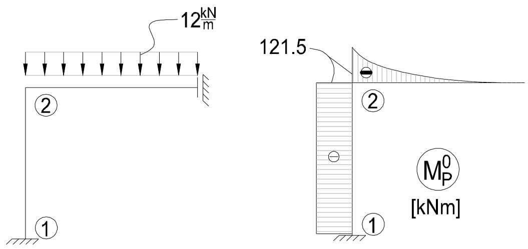
In order to simplify the calculation, half of structure may be considered for both load cases. For the point in the middle of the beam, external links corresponding to the zero displacements from figures above may be considered, by means of appropriate supports:

Consequently, the following primary structures may be considered for the two structures:

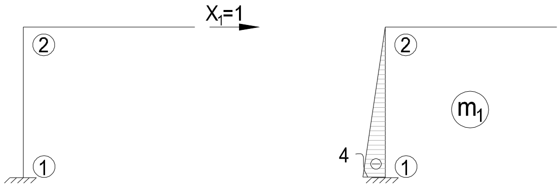


The symmetric structure

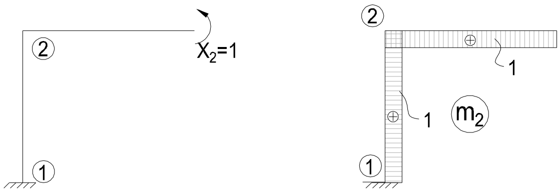
Bending moment on the primary structure due to external loads



Bending moment on the primary structure due to X_1=1



Bending moment on the primary structure due to X_2=1



Compatibility equations

$$\begin{cases} D_{1P} = d_{11} \cdot X_1 + d_{12} \cdot X_2 + D_{1P}^0 = 0 \\ D_{2P} = d_{21} \cdot X_1 + d_{22} \cdot X_2 + D_{2P}^0 = 0 \end{cases}$$

$$EI \cdot d_{11} = \frac{1}{1} \cdot \frac{4 \cdot (-4)}{2} \cdot \frac{2}{3} \cdot (-4) = 21.333$$

$$EI \cdot d_{12} = \frac{1}{1} \cdot \frac{4 \cdot (-4)}{2} \cdot 1 = -8$$

$$EI \cdot d_{22} = \frac{1}{1} \cdot (4 \cdot 1) \cdot 1 + \frac{1}{4.612} \cdot (4.5 \cdot 1) \cdot 1 = 4.9757$$

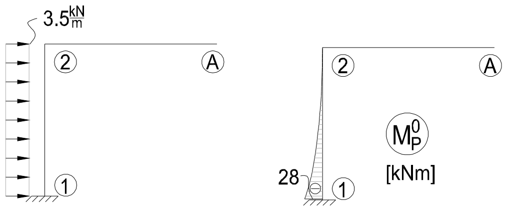
$$EI \cdot D_{1P}^0 = \frac{1}{1} \cdot [4 \cdot (-121.5)] \cdot \frac{1}{2} \cdot (-4) = 972$$

$$EI \cdot D_{2P}^0 = \frac{1}{1} \cdot [4 \cdot (-121.5)] \cdot 1 + \frac{1}{4.612} \cdot \left[\frac{1}{3} \cdot 4.5 \cdot (-121.5)\right] \cdot 1 = -525.516$$

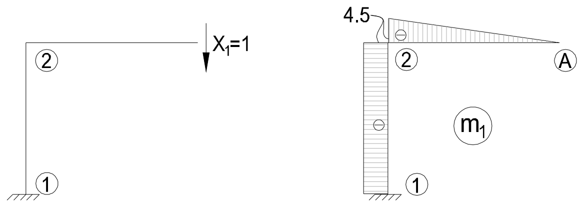
$X_1 = -15.001$ kN
$X_2 = 81.497$ kNm

The antisymmetric structure

Bending moment on the primary structure due to external loads



Bending moment on the primary structure due to X_1=1



Compatibility equations

$$D_{1P} = d_{11} \cdot X_1 + D_{1P}^0 = 0$$

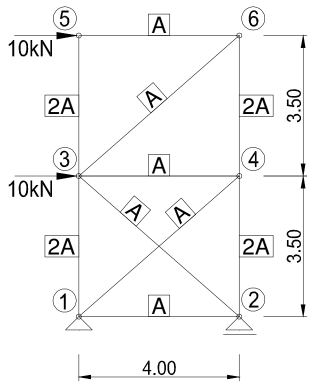
$$EI \cdot d_{11} = \frac{1}{1} \cdot [4 \cdot (-4.5)] \cdot (-4.5) + \frac{1}{4.612} \cdot \frac{4.5 \cdot (-4.5)}{2} \cdot \frac{2}{3} \cdot (-4.5) = 87.586$$

$$EI \cdot D_{1P}^0 = \frac{1}{1} \cdot \left[\frac{1}{3} \cdot 4 \cdot (-28)\right] \cdot (-4.5) = 168$$

$$X_1 = -1.918 \text{ kN}$$

2. <u>TRUSSES</u>

Problem 1

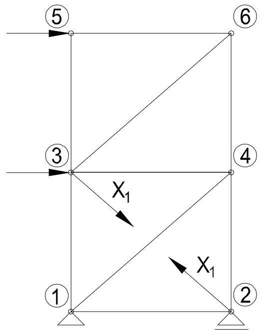


Degree of static indeterminacy

$d = b + r - 2 \cdot j = 10 + 3 - 2 \cdot 6 = 1$

- b - number of bars
- r - number of external links (reactions)
- j - number of joints

Primary structure



Compatibility equation

$$D_{1P} = d_{11} \cdot X_1 + D_{1P}^0 = -\Delta l_1$$

d_{11} – linear displacement at the point and in the direction of the redundant axial force X_1, caused by the unit value of X_1, acting on the primary structure;

D_{1P}^0 – linear displacement at the point and in the direction of the redundant axial force X_1, caused by the external loads, acting on the primary structure;

D_{1P} – displacement at the point and in the direction of the redundant axial force X_1, caused by X_1 and by the external loads, acting on the primary structure;

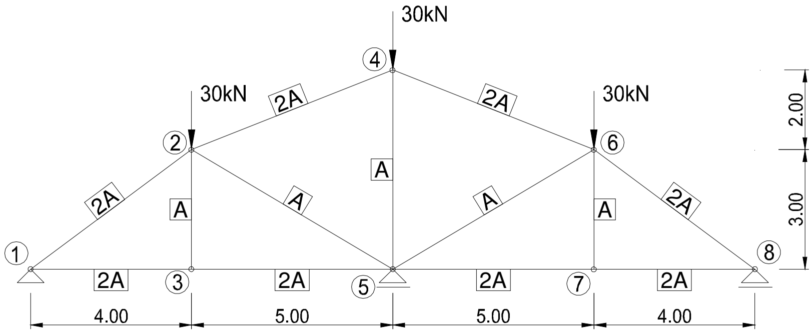
Δl_1 – relative displacement of joints 2 and 3 on the real structure. The "-" sign indicates that the displacement produced by the axial force X_1 on the statically indeterminate structure is equal and opposite to the displacement from the same force acting on the primary structure.

$$\Delta l_1 = \frac{l_{23}}{\mathrm{EA}} \cdot X_1$$

where:

l_{23} – length of the bar 2-3
EA_{23} – axial rigidity of the bar 2-3

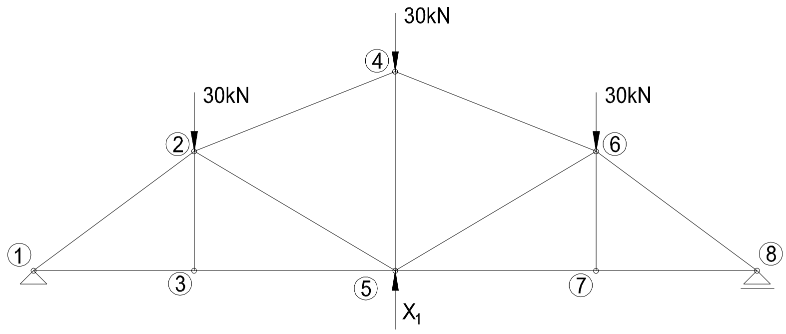
Problem 2



Degree of static indeterminacy

$$d = b + r - 2 \cdot j = 13 + 4 - 2 \cdot 8 = 1$$

Primary structure



Compatibility equation

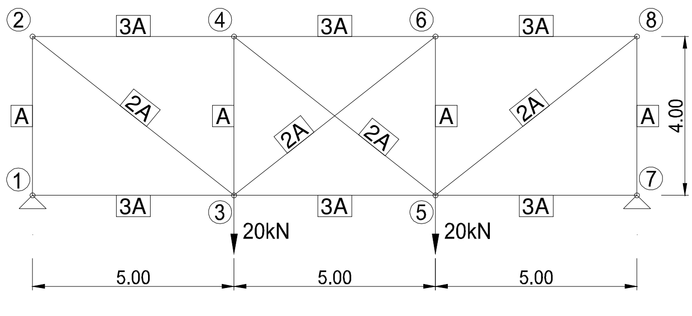
$$D_{1P} = d_{11} \cdot X_1 + D_{1P}^0 = 0$$

d_{11} – linear displacement at the point and in the direction of the redundant vertical reaction X_1, caused by the unit value of X_1, acting on the primary structure;

D_{1P}^0 – linear displacement at the point and in the direction of the redundant vertical reaction X_1, caused by the external loads, acting on the primary structure;

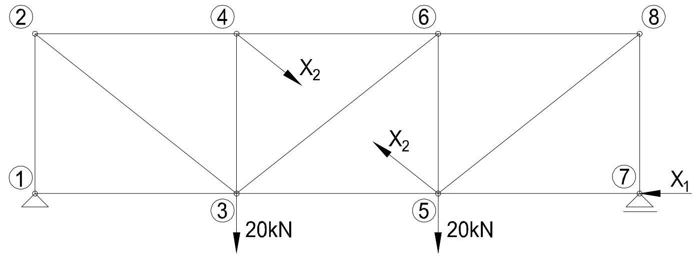
D_{1P} – displacement at the point and in the direction of the redundant vertical reaction X_1, caused by X_1 and by the external loads, acting on the primary structure; it must be zero, because in this point, on the real structure exists a link which prevents this vertical displacement (roller support).

Problem



$$d = b + r - 2 \cdot j = 14 + 4 - 2 \cdot 8 = 2$$

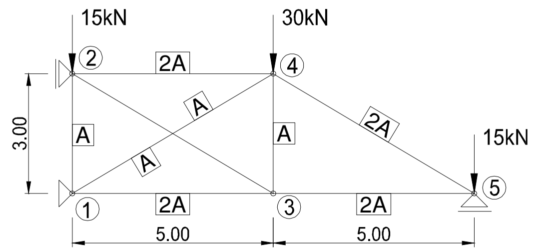
Primary structure



Compatibility equations

$$\begin{cases} D_{1P} = d_{11} \cdot X_1 + d_{12} \cdot X_2 + D_{1P}^0 = 0 \\ D_{2P} = d_{21} \cdot X_1 + d_{22} \cdot X_2 + D_{2P}^0 = -\Delta l_2 \end{cases}$$

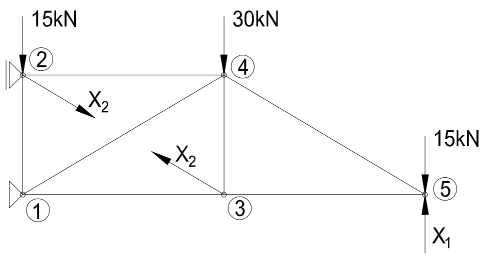
Problem



Degree of static indeterminacy

$d = b + r - 2 \cdot j = 8 + 4 - 2 \cdot 5 = 2$

Primary structure



Compatibility equations

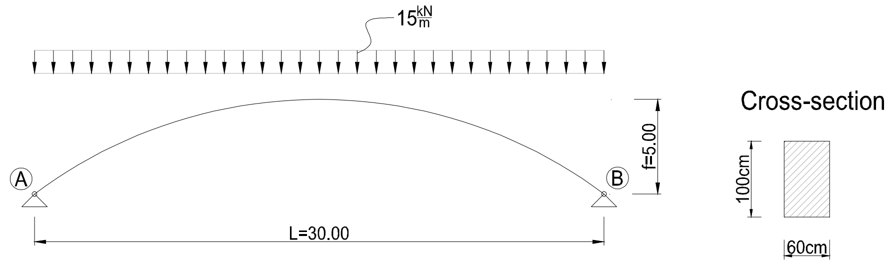
$$\begin{cases} D_{1P} = d_{11} \cdot X_1 + d_{12} \cdot X_2 + D_{1P}^0 = 0 \\ D_{2P} = d_{21} \cdot X_1 + d_{22} \cdot X_2 + D_{2P}^0 = -\Delta l_2 \end{cases}$$

ARCHES

Problem

Circular arch with constant cross-section

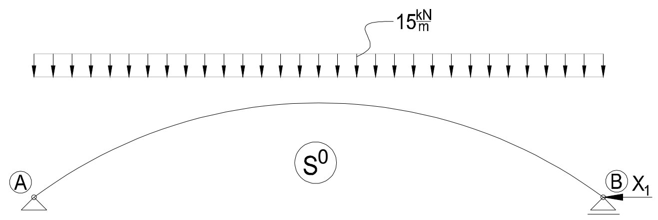
$$\frac{f}{L} = \frac{1}{6} < \frac{1}{5}$$



Degree of static indeterminacy

$$d = l_i + r - 3e = 0 + 4 - 3 \cdot 1 = 1$$

Primary structure



Compatibility equation

$$D_{1P} = d_{11} \cdot X_1 + D_{1P}^0 = 0$$

d_{11} – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by the unit value of X_1, acting on the primary structure;

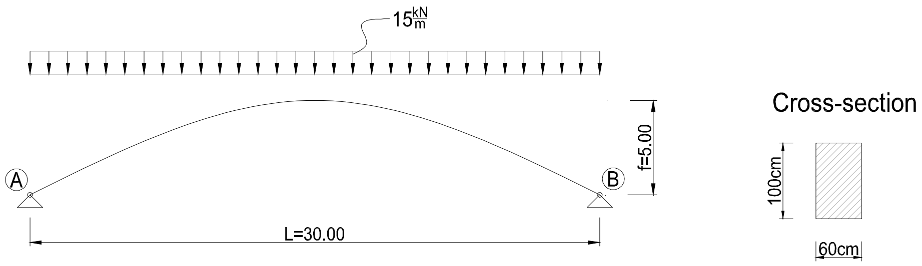
D_{1P}^0 – linear displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by the external loads, acting on the primary structure;

D_{1P} – displacement at the point and in the direction of the redundant horizontal reaction X_1, caused by X_1 and by the external loads, acting on the primary structure; it must be zero, because in this point and on the direction of X_1, on the real structure exists a link which does not allow this displacement (pinned support).

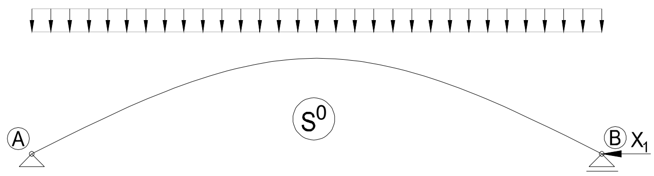
Problem

Parabolic arch with the same span, rise, cross-section and external load as for the structure from problem 1.9.

$$y(x) = \frac{4 \cdot f \cdot x}{l^2} \cdot (l - x)$$



Primary structure



Compatibility equation

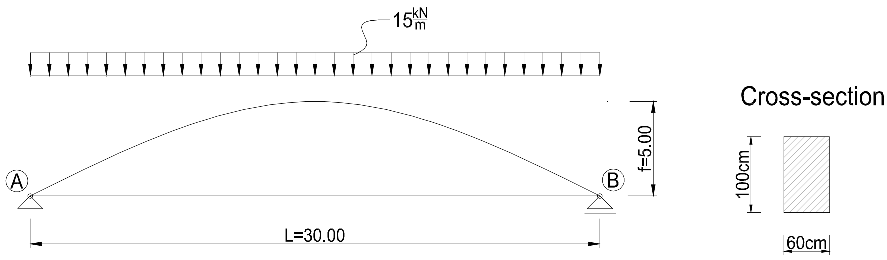
$$D_{1\text{P}} = d_{11} \cdot X_1 + D_{1P}^0 = 0$$

Problem

Tied parabolic arch with the same span, rise, cross-section and load as for the arch from Problem 1.10.

$$\text{E} = 2.1 \cdot 10^4 \frac{\text{N}}{\text{mm}^2}, \quad \text{E}_\text{t} = 2.1 \cdot 10^5 \frac{\text{N}}{\text{mm}^2}, \quad \text{A}_\text{t} = 6362 \text{ mm}^2$$

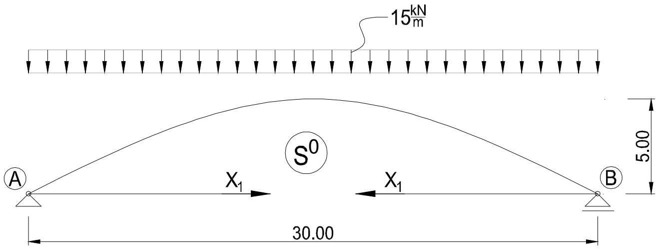
$$\text{y(x)} = \frac{4 \cdot \text{f} \cdot \text{x}}{\text{l}^2} \cdot (\text{l} - \text{x})$$



Degree of static indeterminacy

$$\text{d} = \text{l}_\text{i} + \text{r} - 3\text{e} = 4 + 3 - 3 \cdot 2 = 1$$

Primary structure



Compatibility equation

$$D_{1P} = d_{11} \cdot X_1 + D_{1P}^0 = -\Delta l_1$$

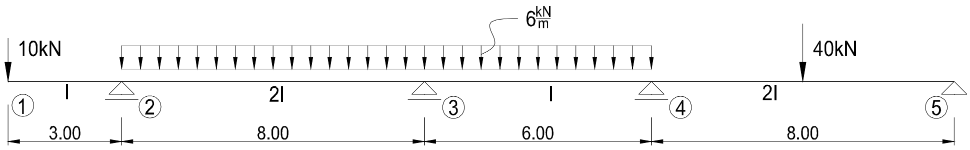
d_{11} – linear displacement at the point and in the direction of the redundant axial force X_1, caused by the unit value of X_1, acting on the primary structure;

D_{1P}^0 – displacement at the point and in the direction of the redundant axial force X_1, caused by the external loads, acting on the primary structure;

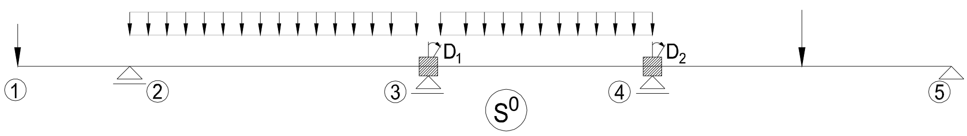
D_{1P} – displacement at the point and in the direction of the redundant axial force X_1, caused by X_1 and by the external loads, acting on the primary structure, equal to the relative horizontal displacement of the supports on the real statically indeterminate arch Δl_1 . The "-" sign indicates that the displacement produced by the axial force in the tie (X_1) on the statically indeterminate arch, is equal and opposite to the displacement of the same force X_1 acting on the primary structure.

Displacement method

Problem



Primary structure



Equilibrium equations

$$\begin{cases} R_{1P} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + R_{1P}^0 = 0 \\ R_{2P} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + R_{2P}^0 = 0 \end{cases}$$

r_{11} – moment reaction in the rotational restraint 1, caused by the unit rotation of D_1 imposed on the primary structure;

$r_{11} \cdot D_1$ – moment reaction in the rotational restraint 1, caused by D_1 imposed on the primary structure;

r_{22} – moment reaction in the rotational restraint 2, caused by the unit rotation of D_2 imposed on the primary structure;

$r_{22} \cdot D_2$ – moment reaction in the rotational restraint 2, caused by D_2 imposed on the primary structure;

r_{12} – moment reaction in the rotational restraint 1, caused by the unit rotation of D_2 imposed on the primary structure;

$r_{12} \cdot D_2$ – moment reaction in the rotational restraint 1, caused by D_2 imposed on the primary structure;

r_{21} – moment reaction in the rotational restraint 2, caused by the unit rotation of D_1 imposed on the primary structure;

$r_{21} \cdot D_1$ – moment reaction in the rotational restraint 2, caused by D_1 imposed on the primary structure;

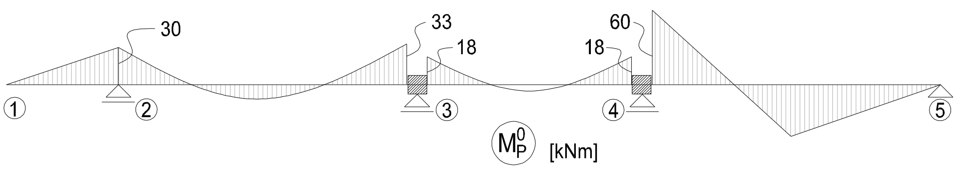
R_{1P}^{0} – moment reaction in the rotational restraint 1, caused by the external loads on the primary structure;

R_{2P}^{0}– moment reaction in the rotational restraint 2, caused by the external loads on the primary structure;

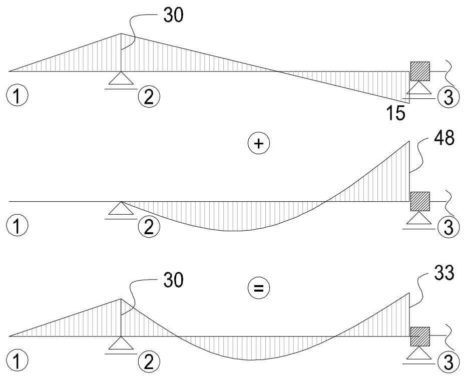
$R_{1\mathrm{P}}$ – moment reaction in the rotational restraint 1, caused by the rotations D_1, D_2 and the external loads on the primary structure; it must be zero, because on the real structure, joint (3) is free to rotate on the direction of D_1;

$R_{2\mathrm{P}}$ – moment reaction in the rotational restraint 2, caused by the rotation D_1, D_2 and the external loads on the primary structure; it must be zero, because on the real structure joint (4) is free to rotate on the direction of D_2.

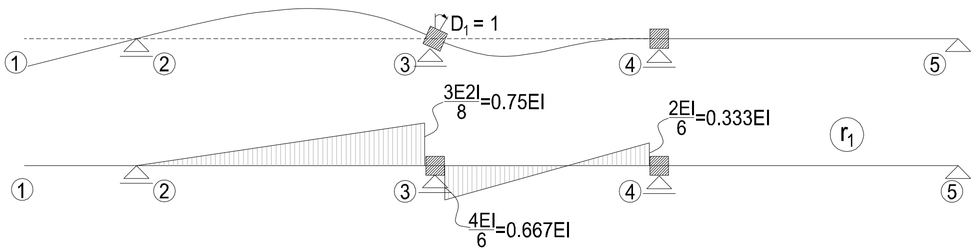
Fixed end moments due to external loads



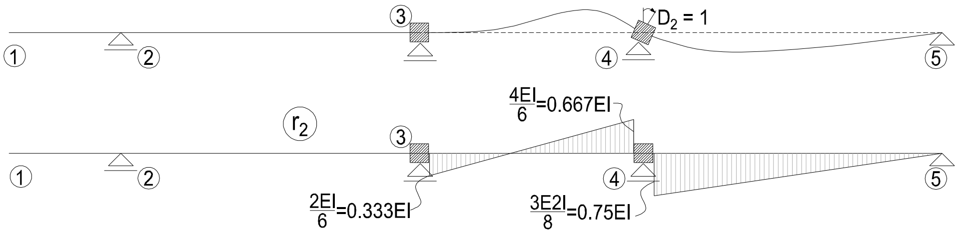
The bending moment diagram between point 1 and 3 is the result of adding the bending moment due to the point load (30kN) and the bending moment due to the uniform distributed load as shown in the figure bellow:



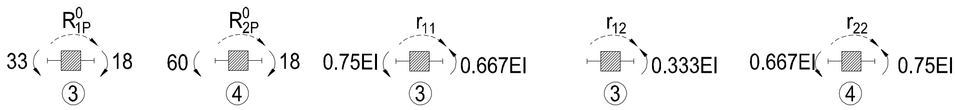
Fixed end moments due to unit rotation D_1=1



Fixed end moments due to the unit rotation D_2=1



Moment reactions in the restrained joints

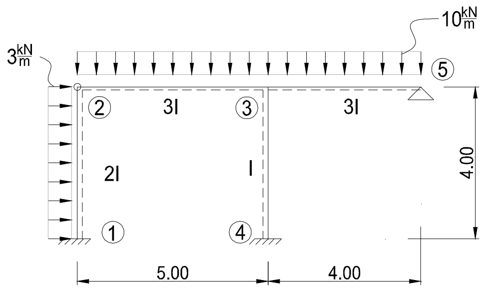


$$R^0_{1P} + 18 + 15 - 48 = 0 \implies R^0_{1P} = 15\ \mathrm{kNm}$$
$$r_{11} = 0.667 \cdot EI + 0.75 \cdot EI \implies r_{11} = 1.417 \cdot EI$$
$$R^0_{2P} + 60 - 18 = 0 \implies R^0_{2P} = -42\ \mathrm{kNm}$$
$$r_{12} - 0.333 \cdot EI = 0 \implies r_{12} = 0.333 \cdot EI = r_{21}$$
$$r_{22} = 1.417EI$$

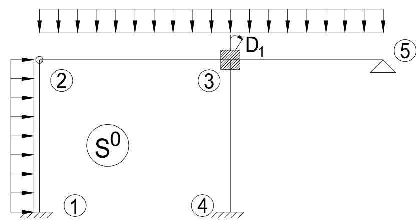
$$\begin{cases} 1.417 \cdot EI \cdot D_1 + 0.333 \cdot EI \cdot D_2 + 15 = 0 \\ 0.333 \cdot EI \cdot D_1 + 1.417 \cdot EI \cdot D_2 - 42 = 0 \end{cases}$$

$$\Rightarrow D_1 = -\frac{18.576}{EI}$$
$$\Rightarrow D_2 = \frac{34}{EI}$$

Problem



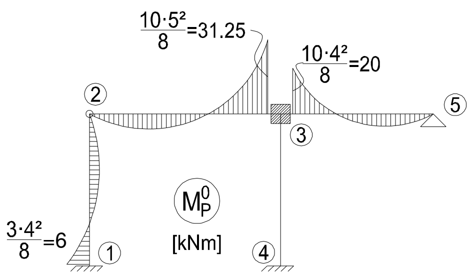
Primary structure



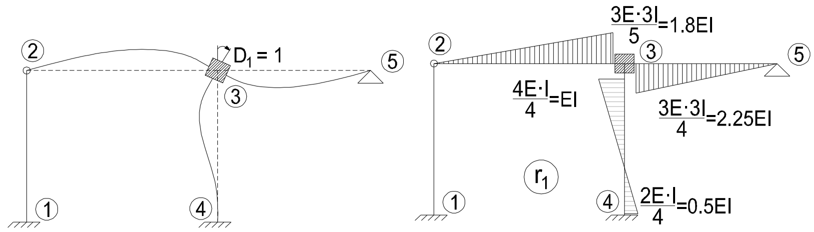
Equilibrium equation

$$R_{1P} = r_{11} \cdot D_1 + R_{1P}^0 = 0$$

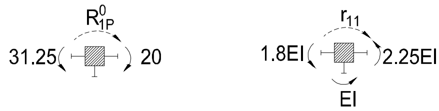
Fixed end moments due to external loads



Fixed end moments due to unit rotation D_1=1



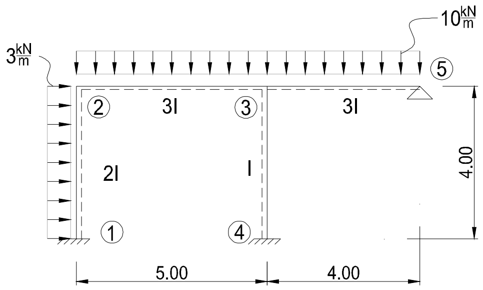
Moment reactions in the restrained joint 3



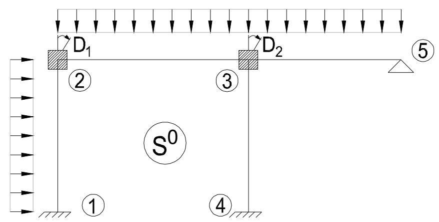
$R^0_{1P} + 20 - 31.25 = 0 \;\; \Rightarrow R^0_{1P} = 11.25$
$r_{11} - 2.25EI - EI - 1.8EI = 0 \;\; \Rightarrow r_{11} = 5.05\ EI$
$5.05EI \cdot D_1 + 11.25 \; = 0$
$D_1 = -\dfrac{2,228}{EI}$

Problem

The structure is similar to the one from problem 1.13, but a rigid joint is considered in point 2.



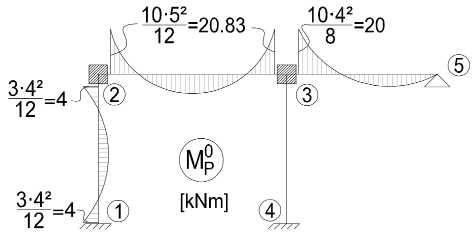
Primary structure



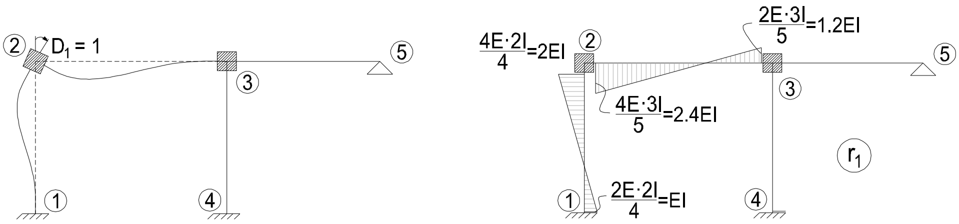
Equilibrium equations

$$\begin{cases} R_{1P} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + R^0_{1P} = 0 \\ R_{2P} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + R^0_{2P} = 0 \end{cases}$$

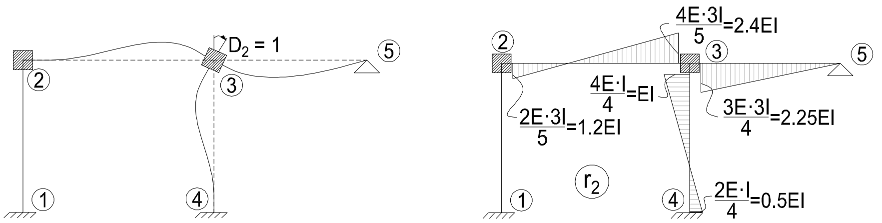
Fixed end moments due to external loads



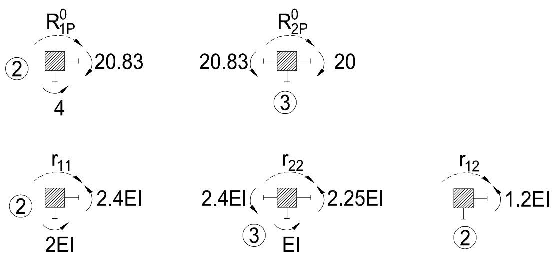
Fixed end moments due to unit rotation D_1=1



Fixed end moments due to unit rotation D_2=1



Moment reactions in the restrained joints



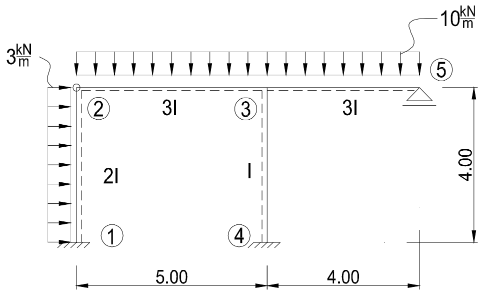
$$R_{1P}^0 + 20.83 - 4 = 0 \implies R_{1P}^0 = -16.83\text{kNm}$$
$$R_{2P}^0 + 20 - 20.83 = 0 \implies R_{2P}^0 = 0.83\text{kNm}$$
$$r_{11} - 2.4\text{EI} - 2\text{EI} = 0 \implies r_{11} = 4.4\text{EI}$$
$$r_{22} - 2.25\text{EI} - \text{EI} - 2{,}4 \cdot \text{EI} = 0 \implies r_{22} = 5.65\text{EI}$$
$$r_{12} - 1.2\text{EI} = 0 \implies r_{12} = 1.2\text{EI} = r_{21}$$

$$\begin{cases} 4.4 \cdot D_1 + 1.2 \cdot D_2 - 16.83 = 0 \\ 1.2 \cdot D_1 + 5.65 \cdot D_2 + 0.83 = 0 \end{cases}$$

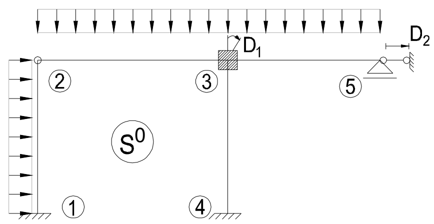
$$D_1 = \frac{4.103}{EI}$$
$$D_2 = -\frac{1.018}{EI}$$

Problem

The structure is similar to the one from problem 1.13, but a roller is considered in point 5.



Primary structure



Equilibrium equations

$$\begin{cases} R_{1P} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + R_{1P}^0 = 0 \\ R_{2P} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + R_{2P}^0 = 0 \end{cases}$$

r_{11} – moment reaction in the rotational restraint 1, caused by the unit rotation D_1 imposed on the primary structure;

r_{22} – force reaction in the linear displacement restraint 2, caused by the unit linear displacement D_2 imposed on the primary structure;

r_{12} – moment reaction in the rotational restraint 1, caused by the unit linear displacement D_2 imposed on the primary structure;

r_{21} – force reaction in the linear displacement restraint 2, caused by the unit rotation D_1 imposed on the primary structure;

R_{1P}^0 – moment reaction in the rotational restraint 1, caused by the external loads on the primary structure;

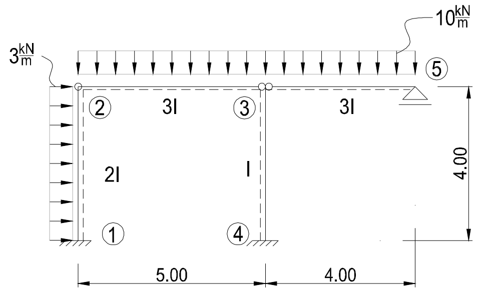
R_{2P}^0 – force reaction in the linear displacement restraint 2, caused by the external loads on the primary structure;

R_{1P} – moment reaction in the rotational restraint 1 caused by the rotation D_1, the linear displacement D_2 and the external loads on the primary structure; it must be zero, because on the real structure joint (3) is free to rotate in the direction of D_1;

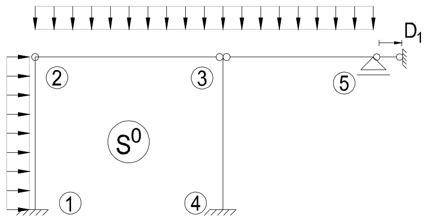
R_{2P}– force reaction in the linear displacement restraint 2, caused by the rotation D_1, the linear displacement D_2 and the external loads on the primary structure; it must be zero, because on the real structure the roller in point (5) allows the lateral displacement.

Problem

The structure is similar to the one from problem1.15, but the beams 2-3 and 3-5 are hinged to the column 3-4.



Primary structure



Equilibrium equations

$$R_{1P} = r_{11} \cdot D_1 + R_{1P}^0 = 0$$

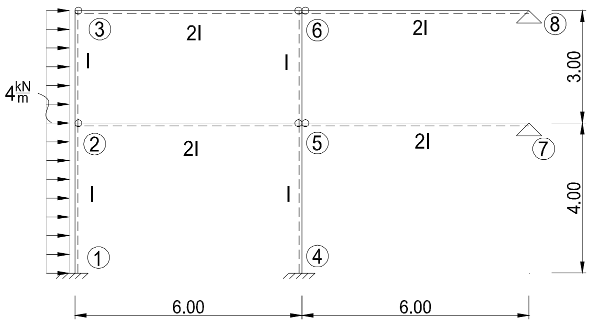
r_{11} – force reaction in the linear displacement restraint 1, caused by the unit displacement D_1 imposed on the primary structure;

R_{1P}^0 – horizontal reaction in the linear displacement restraint 1, caused by the external loads on the primary structure;

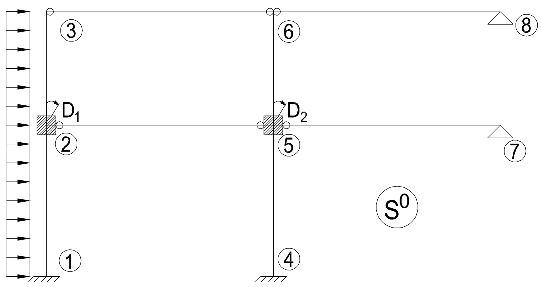
R_{1P} – force reaction in the linear displacement restraint 1, caused by the linear displacement D_1 and the external loads on the primary structure

Fixed end moments due to external loads

Problem



Primary structure

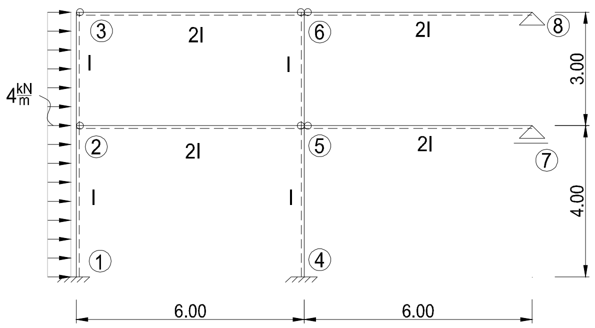


Equilibrium equations

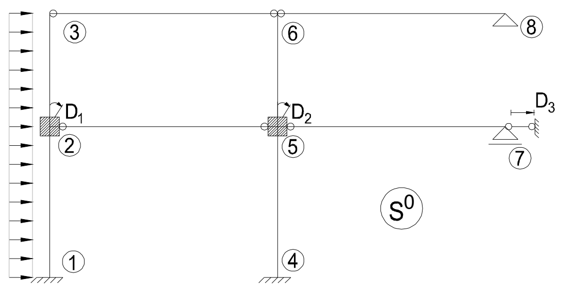
$$\begin{cases} R_{1P} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + R_{1P}^0 = 0 \\ R_{2P} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + R_{2P}^0 = 0 \end{cases}$$

Problem

The structure is similar to the one from problem 1.17, but a roller is considered in point 7. The degree of kinematic indeterminacy becomes higher than for the structure from problem 1.17, but the degree of static indeterminacy is lower. It may be observed that this structure is statically determinate, but it still may be analysed using the displacement method.



Primary structure

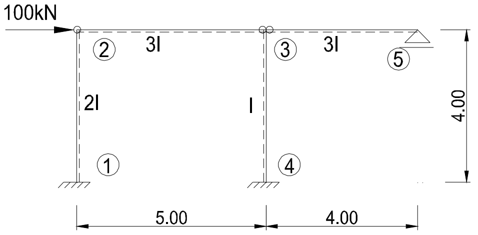


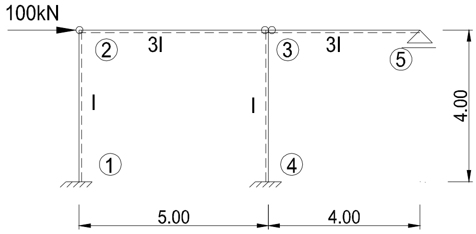
Equilibrium equations

$$\begin{cases} R_{1P} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + r_{13} \cdot D_3 + R_{1P}^0 = 0 \\ R_{2P} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + r_{23} \cdot D_3 + R_{2P}^0 = 0 \\ R_{3P} = r_{31} \cdot D_1 + r_{32} \cdot D_2 + r_{33} \cdot D_3 + R_{3P}^0 = 0 \end{cases}$$

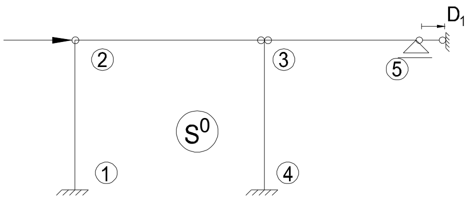
Problem

The geometry of the two structures bellow is similar, excepting for the bending rigidity of column 1-2.

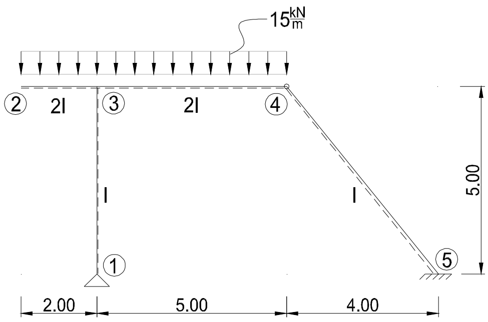




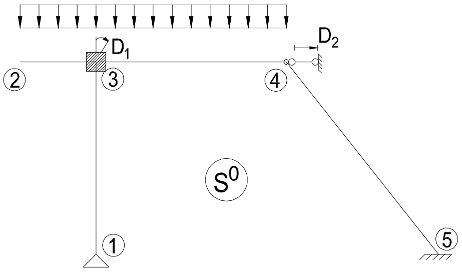
Primary structure (the same for both structures)



Problem



Primary structure



Equilibrium equations

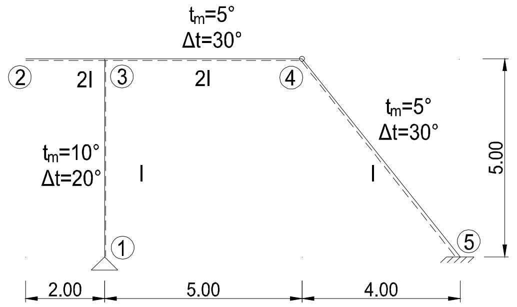
$$\begin{cases} R_{1P} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + R_{1P}^0 = 0 \\ R_{2P} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + R_{2P}^0 = 0 \end{cases}$$

3. STATICALLY INDETERMINATE STRUCTURES SUBJECTED TO TEMPERATURE VARIATION

2.1 Flexibility Method

Problem

The structure is the same as the one from problem 1.3



$EI = 0.448 \cdot 10^5\ kNm^2$

$h_{13} = 40\ cm$

$h_{34} = 50\ cm$

$h_{45} = 40\ cm$

$\alpha = 10^{-5}$

$\tan\alpha = \frac{5}{4}$

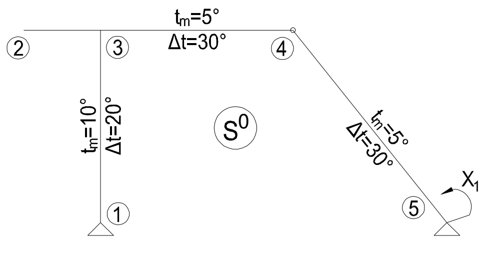
$\alpha = 51.34^\circ$

$\sin\alpha = 0.781$

$\cos\alpha = 0.625$

The degree of static indeterminacy of structure is 1, while the degree of kinematic indeterminacy is 2. Therefore, for this problem it is easier to use the flexibility method (1 unknown) instead of the displacement method (2 unknowns- one rotation and one linear displacement)

Primary structure



Compatibility equation

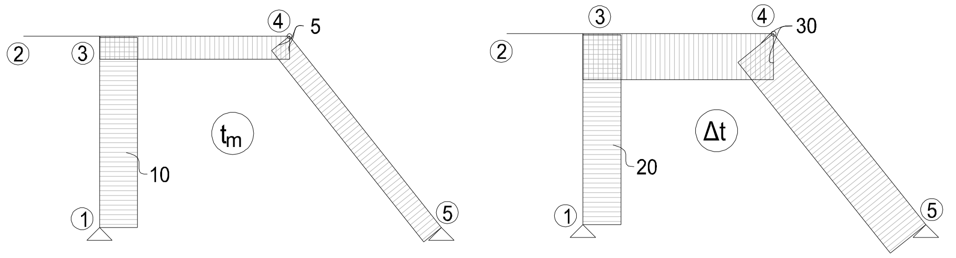
$$D_{1t} = d_{11} \cdot X_1 + D_{1t}^0 = 0$$

d_{11} – rotation at the point and in the direction of the redundant moment reaction X_1, caused by the unit value of X_1, acting on the primary structure

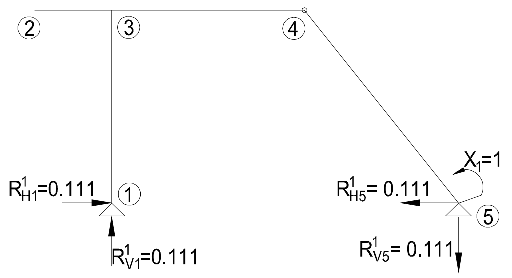
D_{1t}^0 – rotation at the point and in the direction of the redundant moment reaction X_1, caused by the temperature variation on the primary structure;

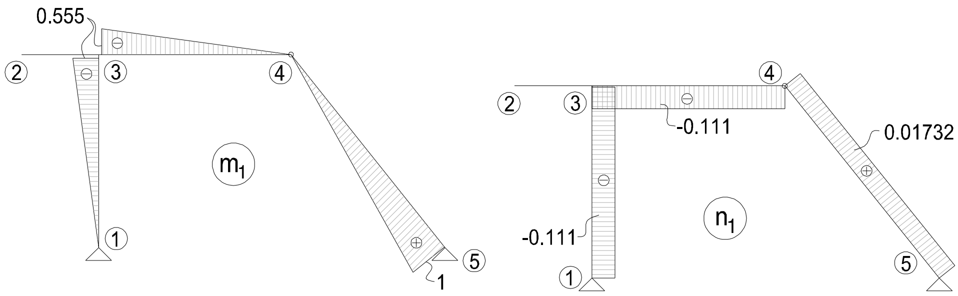
D_{1t} – rotation at the point and in the direction of the redundant moment reaction X_1, caused by X_1 and by temperature variation on the primary structure; it must be zero, because in this point and on the direction of X_1, on the real structure exists a link which does not allow this displacement (fixed support)

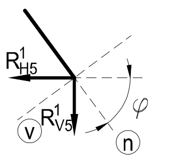
Temperature diagrams



Bending moment and axial force due to unit rotation X_1=1







$$n_{54} = R^1_{V5} \cdot \sin\alpha - R^1_{H5} \cdot \cos\alpha = 0.111 \cdot 0.781 - 0.111 \cdot 0.625 = 0.01732$$

$$n_{13} = -R^1_{V1} = -0.111$$

$$n_{34} = -R^1_{H1} = -0.111$$

The value for d_{11} is the same as for the structure from Problem 1.3

$$d_{11} = \frac{2.904}{EI}$$

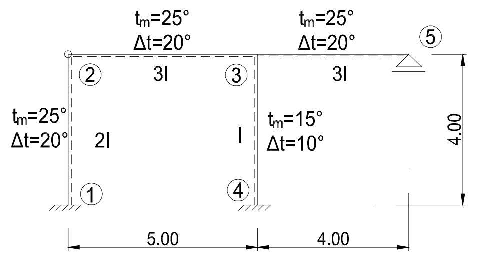
$$D_{1t}^0 = \sum \int n_1 \cdot \alpha \cdot t_m \, dx + \int m_1 \cdot \frac{\alpha \cdot \Delta t}{h} \, dx$$
$$= \alpha \cdot (-0.111 \cdot 5 \cdot 10 - 0.111 \cdot 5 \cdot 5 + 0.01732 \cdot 6.403 \cdot 5) +$$
$$+\alpha \cdot \left(\frac{20}{0.4} \cdot \frac{1}{2} \cdot (-0.555) \cdot 5 + \frac{30}{0.5} \cdot \frac{1}{2} \cdot (-0.555) \cdot 5 + \frac{30}{0.4} \cdot \frac{1}{2} \cdot 1 \cdot 6.403\right)$$
$$= \alpha \cdot (-7.771 + 87.49) = 79.72 \cdot \alpha$$

$$\frac{2.904}{EI} \cdot X_1 + 79.72 \cdot \alpha = 0$$
$$X_1 = -27.4518 \cdot EI \cdot \alpha = -12.30 kNm$$

Displacement Method

Problem

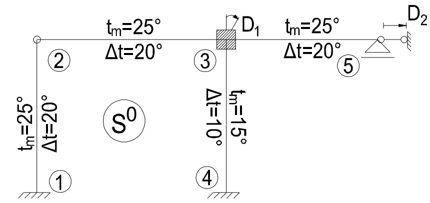
The structure is the same as the one from problem 1.15



$EI = 1.134 \cdot 10^5 \; kNm^2$
$h_{12} = 50 \; cm$
$h_{34} = 40 \; cm$
$h_{23} = h_{35} = 60 \; cm$
$\alpha = 10^{-5}$

The degree of static indeterminacy of the structure is 3, while the degree of kinematic indeterminacy is 2. Therefore, for this problem, it is easier to use the displacement method (2 unknowns – one rotation and one linear displacement) instead of the flexibility method (3 unknowns).

Primary structure



Equilibrium equations

$$\begin{cases} R_{1t} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + R_{1t}^0 = 0 \\ R_{2t} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + R_{2t}^0 = 0 \end{cases}$$

r_{11} – moment reaction in the rotational restraint 1, caused by the unit rotation of D_1 imposed on the primary structure;

r_{22} – force reaction in the linear displacement restraint 2, caused by the unit displacement of D_2 imposed on the primary structure;

r_{12} – moment reaction in the rotational restraint 1, caused by the unit displacement of D_2 imposed on the primary structure;

r_{21} – force reaction in the linear displacement restraint 2, caused by the unit rotation of D_1 imposed on the primary structure;

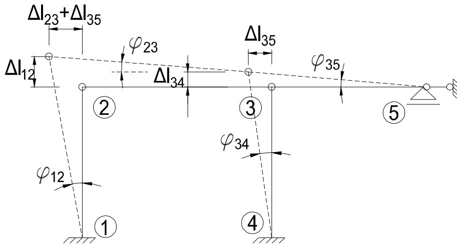
R^0_{1t} – moment reaction in the rotational restraint 1, caused by the temperature variation on the primary structure;

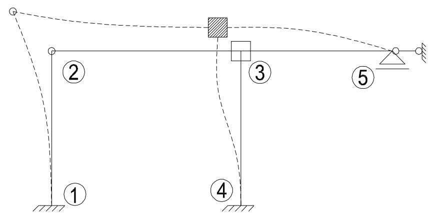
R^0_{2t} – force reaction in the linear displacement restraint 2 caused by the temperature variation on the primary structure;

R_{1t} – moment reaction in the rotational restraint 1, caused by the rotation D_1, linear displacement D_2 and the temperature variation on the primary structure;

R_{2t} – force reaction in the linear displacement restraint 2, caused by the rotation D_1, linear displacement D_2 and the temperature variation on the primary structure.

Fixed end moments on the primary structure due to t_m





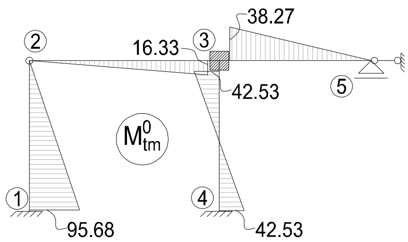
$$\begin{cases} \Delta l_{12} = \alpha \cdot t_{m12} \cdot l_{12} = \alpha \cdot 25 \cdot 4 = 100 \cdot \alpha \\ \Delta l_{23} = \alpha \cdot t_{m24} \cdot l_{24} = \alpha \cdot 25 \cdot 5 = 125 \cdot \alpha \\ \Delta l_{34} = \alpha \cdot t_{m34} \cdot l_{34} = \alpha \cdot 15 \cdot 4 = 60 \cdot \alpha \\ \Delta l_{35} = \alpha \cdot t_{m45} \cdot l_{45} = \alpha \cdot 25 \cdot 4 = 100 \cdot \alpha \end{cases}$$

$$M^{tm}_{12} = 3 \cdot \frac{E \cdot 2I}{(l_{12})^2} \cdot (\Delta l_{23} + \Delta l_{35}) = 95.68\ \text{kNm}$$

$$M^{tm}_{32} = 3 \cdot \frac{E \cdot 3I}{(l_{23})^2} \cdot (\Delta l_{12} - \Delta l_{34}) = 16.33\ \text{kNm}$$

$$M^{tm}_{43} = M^{tm}_{34} = \frac{6 \cdot EI}{(l_{34})^2} \cdot \Delta l_{35} = 42.53\ \text{kNm}$$

$$M^{tm}_{35} = 3 \cdot \frac{E \cdot 3I}{(l_{35})^2} \cdot \Delta l_{34} = 38.27\ \text{kNm}$$



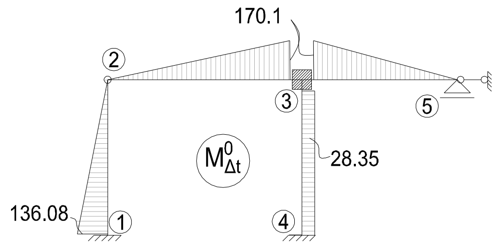
Fixed end moments on the primary structure due to Δt

$$M^{\Delta t}_{12} = \frac{3}{2} \cdot \alpha \cdot \frac{\Delta t}{h_{12}} \cdot E \cdot 2I = \frac{3}{2} \cdot 10^{-5} \cdot \frac{20}{0.5} \cdot 2 \cdot 1.134 \cdot 10^5 = 136.08$$

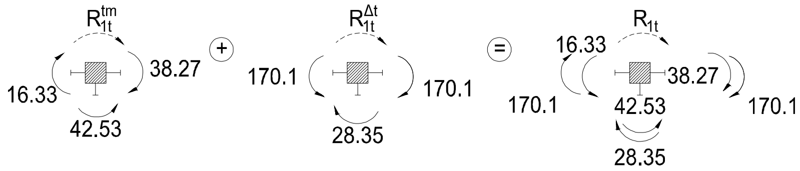
$$M_{32}^{\Delta t} = \frac{3}{2} \cdot \alpha \cdot \frac{\Delta t}{h_{24}} \cdot E \cdot 3I = \frac{3}{2} \cdot 10^{-5} \cdot \frac{20}{0.6} \cdot 3 \cdot 1.134 \cdot 10^{5} = 170.1$$

$$M_{35}^{\Delta t} = \frac{3}{2} \cdot \alpha \cdot \frac{\Delta t}{h_{45}} \cdot E \cdot 3I = \frac{3}{2} \cdot 10^{-5} \cdot \frac{20}{0.6} \cdot 3 \cdot 1.134 \cdot 10^{5} = 170.1$$

$$M_{34}^{\Delta t} = M_{43}^{\Delta t} = \alpha \cdot \frac{\Delta t}{h} \cdot E \cdot I = 10^{-5} \cdot \frac{10}{0.4} \cdot 1.134 \cdot 10^{5} = 28.35$$



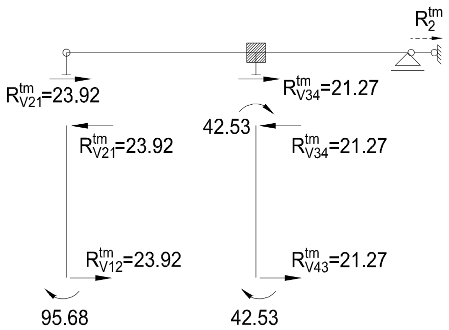
Moment reaction in the restrained joint



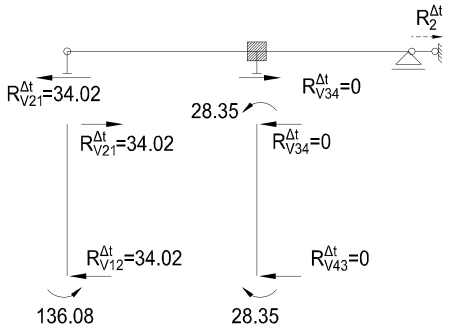
$$R_{1t}^{tm} + 38.27 - 42.53 + 16.33 = 0 \quad \Rightarrow R_{1t}^{tm} = -12.07$$
$$R_{1t}^{\Delta t} + 170.1 + 28.35 - 170.1 = 0 \quad \Rightarrow R_{1t}^{\Delta t} = -28.35$$

$$R_{1t}^{0} = R_{1t}^{tm} + R_{1t}^{\Delta m} = -40.42\ \mathrm{kNm}$$

Force reaction in the restrained joint



$$\left(\sum M\right)_1 = 0 \quad - R_{V21}^{tm} \cdot 4 + 95.68 = 0 \qquad R_{V21}^{tm} = 23.92\mathrm{kN}$$
$$\left(\sum M\right)_3 = 0 \quad - R_{V43}^{tm} \cdot 4 + 42.53 \cdot 2 = 0 \qquad R_{V43}^{tm} = 21.27\mathrm{kN}$$
$$R_2^{tm} + 23.92 + 21.27 = 0 \quad R_2^{tm} = -45.19\ \mathrm{kN}$$



$$\left(\sum M\right)_1 = 0 \quad R_{V21}^{\Delta t} \cdot 4 - 136.02 = 0 \quad R_{V21}^{\Delta t} = 34.02\text{kN}$$

$$\left(\sum M\right)_3 = 0 \quad -R_{V21}^{\Delta t} \cdot 4 + 28.35 - 28.35 = 0 \quad R_{V21}^{\Delta t} = 0$$

$$\mathrm{R}_2^{\Delta t} - 34.02 = 0 \quad \mathrm{R}_2^{\Delta t} = 34.02$$

$$\mathrm{R}_{2t}^0 = \mathrm{R}_2^{tm} + \mathrm{R}_2^{\Delta t} = -11.17\text{kN}$$

The fixed end moments due to unit rotation D_1=1 and unit linear displacement D_2=1 are the same as for the structure from problem 1.16

$$\mathrm{r}_{11} = 5.05 \cdot \mathrm{EI}$$
$$\mathrm{r}_{12} = -0.375 \cdot \mathrm{EI} = \mathrm{r}_{21}$$
$$\mathrm{r}_{22} = 0.2814 \cdot \mathrm{EI}$$
$$\begin{cases} 5.05 \cdot \mathrm{D}_1 + (-0.375) \cdot \mathrm{D}_2 + (-40.42) = 0 \\ -0.375 \cdot \mathrm{D}_1 + 0.2814 \cdot \mathrm{D}_2 + (-11.17) = 0 \end{cases}$$

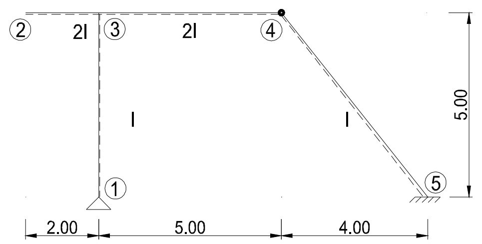
$$\mathrm{D}_1 = \frac{12.154}{\mathrm{EI}}$$
$$\mathrm{D}_2 = \frac{55.892}{\mathrm{EI}}$$

4. STATICALLY INDETERMINATE STRUCTURES SUBJECTED TO SUPPORT SETTLEMENTS

Flexibility Method

Problem

The structure is the same as the one from problem 1.3 and 2.1



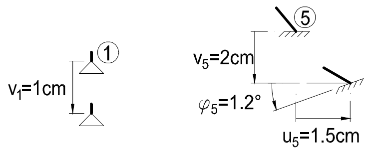
$EI = 4.48 \cdot 10^4\ kNm^2$

$\tan\alpha = \frac{5}{4}$

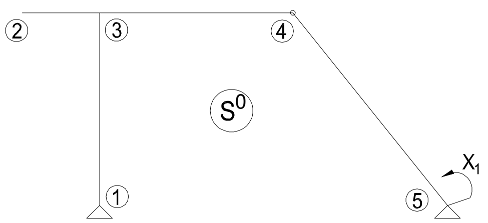
$\alpha = 51.34^\circ$

$\sin\alpha = 0.781$

$\cos\alpha = 0.625$



Primary structure



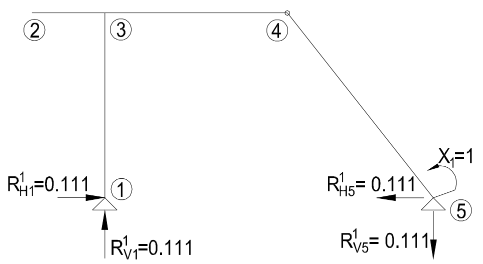
Compatibility equation

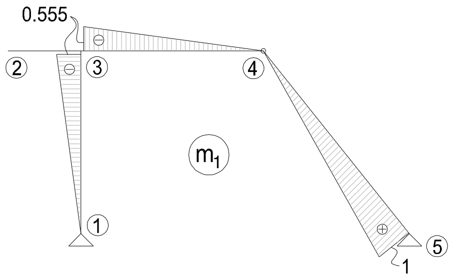
$$D_{1s} = d_{11} \cdot X_1 + D_{1s}^0 = +\varphi_5$$

d_{11} – rotation at the point and in the direction of the redundant moment reaction X_1, caused by the unit value of X_1, acting on the primary structure;

D_{1s}^0 – rotation at the point and in the direction of the redundant moment reaction X_1, caused by the displacements of the supports on the primary structure;

D_{1s} – rotation at the point and in the direction of the redundant moment reaction X_1, caused by X_1 and by the displacements of the supports on the primary structure; this must be identical to the rotation of the support on the real structure (φ_5).





$$d_{11} = \frac{2.904}{EI}$$

$$\begin{aligned} D^0_{1s} &= -[R^1_{V1} \cdot (-v_1) + (-R^1_{V5}) \cdot (-v_5) + (-R^1_{H5}) \cdot u_5] \\ &= 0.111 \cdot (-0.01) + 0.111 \cdot (0.02) + (-0.111) \cdot (0.015) \\ &= 0.000555 \end{aligned}$$

$$\varphi_5 = 1.2^\circ = 1.2 \cdot \frac{\pi}{180} \text{rad} = 0.020944$$

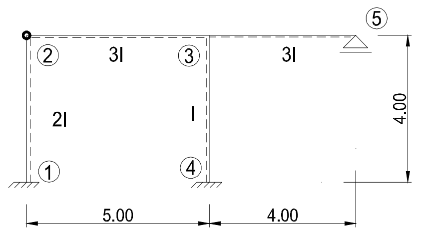
$$D_{1s} = \frac{2.904}{EI} X_1 + 0.000555 = 0.020944$$

$$X_1 = 70.21 \cdot 10^{-4} EI = 314.54 \text{ kNm}$$

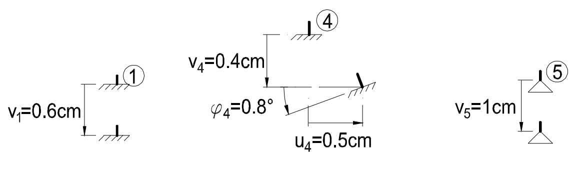
<u>3.2 Displacement Method</u>

Problem

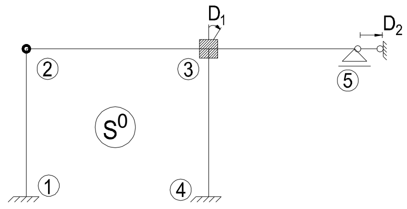
The structure is the same as the one from problem 1.15 and 2.2.



$$EI = 1.134 \cdot 10^5 \, kNm^2$$



Primary structure



Equilibrium equations

$$\begin{cases} R_{1s} = r_{11} \cdot D_1 + r_{12} \cdot D_2 + R_{1s}^0 = 0 \\ R_{2s} = r_{21} \cdot D_1 + r_{22} \cdot D_2 + R_{2s}^0 = 0 \end{cases}$$

r_{11} – moment reaction in the rotational restraint 1, caused by the unit rotation of D_1 imposed on the primary structure;

r_{22} – force reaction in the linear displacement restraint 2, caused by the unit displacement of D_2 imposed on the primary structure;

r_{12} – moment reaction in the rotational restraint 1, caused by the unit displacement of D_2 imposed on the primary structure;

r_{21} – force reaction in the linear displacement restraint 2, caused by the unit rotation of D_1 imposed on the primary structure;

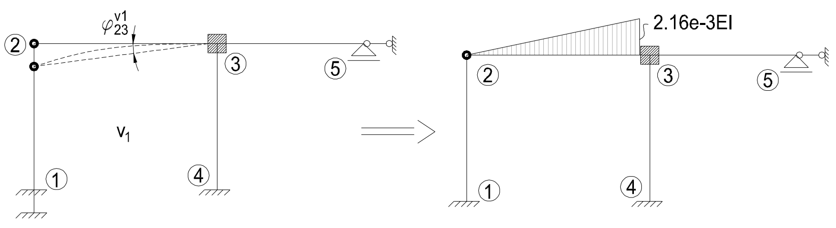
R_{1s}^0 – moment reaction in the rotational restraint 1, caused by the displacement of the support on the primary structure;

R_{2s}^0 – force reaction in the linear displacement restraint 2, caused by the displacement of the support on the primary structure;

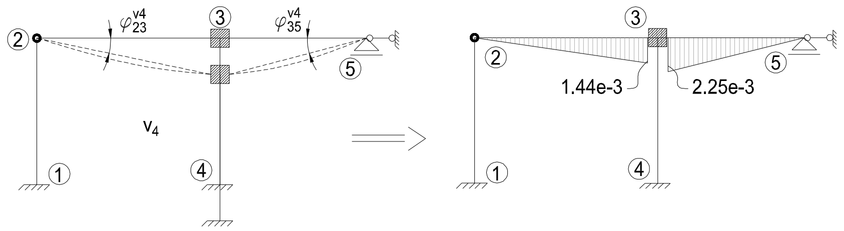
R_{1s} – moment reaction in the rotational restraint 1, caused by the rotation D_1, linear displacement D_2 and the displacement of the support on the primary structure;

R_{2s} – force reaction in the linear displacement restraint 2, caused by the rotation D_1, linear displacement D_2 and the displacement of the support on the primary structure.

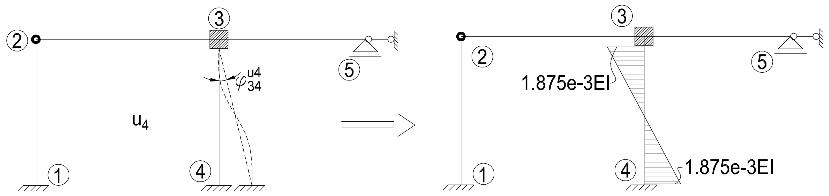
Fixed end moments caused by support displacement



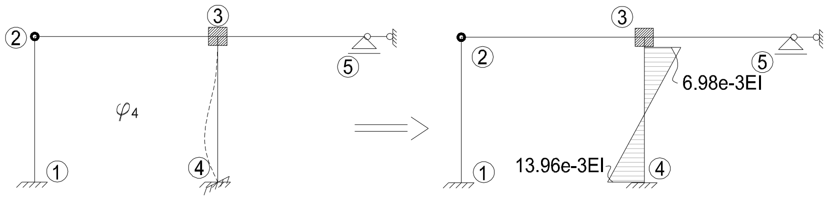
$$\varphi_{23}^{v1} = \frac{0.006}{5} \qquad M_{32}^{v1} = \frac{3 \cdot E \cdot 3I}{5} \cdot \frac{0.006}{5} = 0.00216 \cdot EI$$



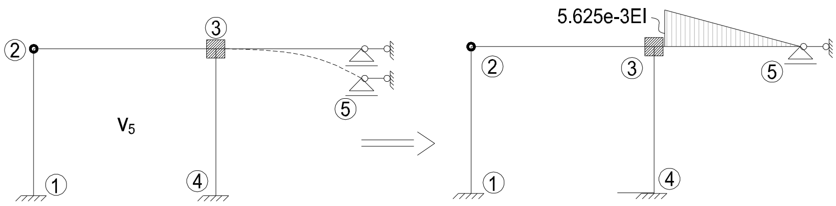
$$\varphi_{23}^{v4} = \frac{0.004}{5} \qquad M_{32}^{v3} = \frac{3 \cdot E \cdot 3I}{5} \cdot \frac{0.004}{5} = 0.00144 \cdot EI$$
$$\varphi_{35}^{v4} = \frac{0.004}{4} \qquad M_{35}^{v3} = \frac{3 \cdot E \cdot 3I}{4} \cdot \frac{0.004}{4} = 0.00225 \cdot EI$$



$$\varphi_{34}^{u4} = \frac{0.005}{4} \qquad M_{43}^{u3} = M_{34}^{u3} = \frac{6 \cdot EI}{4} \cdot \frac{0.005}{4} = 0.001875 \cdot EI$$

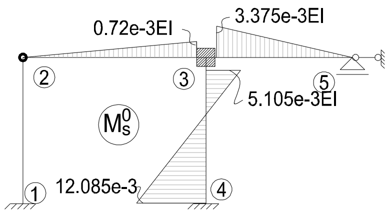


$$\varphi_4 = 0.8^{\circ} = 0.8 \cdot \frac{\pi}{180}$$
$$M_{34}^{\varphi_4} = \frac{4 \cdot EI}{4} \cdot 0.8 \cdot \frac{\pi}{180} = 13.96 \cdot 10^{-3} \cdot EI$$
$$M_{43}^{\varphi_4} = \frac{2 \cdot EI}{4} \cdot 0.8 \cdot \frac{\pi}{180} = 6.98 \cdot 10^{-3} \cdot EI$$

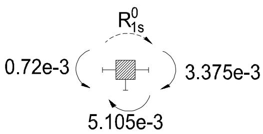


$$\varphi_{35}^{v5} = \frac{0.01}{4}$$
$$M_{45}^{v5} = \frac{3 \cdot E \cdot 3I}{4} \cdot \frac{0.01}{4} = 5.625 \cdot 10^{-3} \cdot EI$$

Final diagram for fixed end moments caused by support displacements

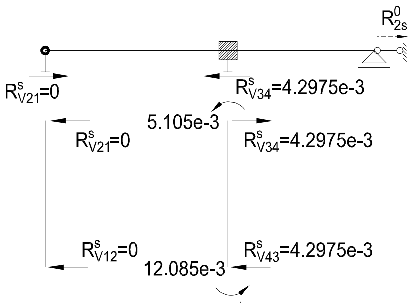


Moment reaction in the restrained joint



$$R_{1s}^{0} + 3.375 \cdot 10^{-3} \cdot EI + 5.105 \cdot 10^{-3} \cdot EI - 0.72 \cdot 10^{-3} \cdot EI = 0$$
$$R_{1s}^{0} = -7.76 \cdot 10^{-3} \cdot EI = 880kNm$$

Force reaction in the restrained joint



$$\left(\sum M\right)_3 = 0 \quad R_{V34}^{s} \cdot 4 + (-5.105 - 12.085) \cdot 10^{-3} \cdot EI = 0$$
$$R_{V34}^{s} = \frac{17.19}{4} \cdot 10^{-3} \cdot EI = 4.2975 \cdot 10^{-3} \cdot EI$$

$$R_{2s}^{0} - R_{V34}^{s} = 0 \quad R_{2s} = R_{V34}^{s} = 4.2975 \cdot 10^{-3} \cdot EI$$

The fixed end moments due to unit displacement D_1 and unit linear displacement D_2 are the same as for the structure from problem 1.15.

$$r_{11} = 5.05EI$$
$$r_{12} = -0.375EI$$
$$r_{22} = 0.2814EI$$

$$\begin{cases} 5.05EI \cdot D_1 - 0.375EI \cdot D_2 - 7.76 \cdot 10^{-3} \cdot EI = 0 \\ -0.375EI \cdot D_1 + 0.2813EI \cdot D_2 + 4.275 \cdot 10^{-3} \cdot EI = 0 \end{cases}$$

$$D_1 = 0.4462 \cdot 10^{-3}$$
$$D_2 = -14.683 \cdot 10^{-3}$$