10.1 DEFINITION

A cross section is subjected to bending with axial force (eccentric tension or compression) if the normal (direct) stresses $\sigma_x$ from section are reduced to an axial force $N$ and two bending moments $M_y$ and $M_z$ (figure 10.1). In this case we discuss about biaxial (oblique) bending with axial force. As, in general, oblique bending is accompanied by oblique shearing, the shear stresses $\tau_x$ are reduced to two shear forces $V_z$ and $V_y$ (solicitation presented in chapter 10).

![Fig.10.1](image1)

$$N = F$$
$$M_y = F \times z_0$$
$$M_z = 0$$

![Fig.10.2.a](image2)
If the normal stresses $\sigma_x$ from section are reduced to the axial force $N$ and only bending moment $M_y$ (figure 10.2.a, $M_z = 0$) or $M_z$ (figure 10.2.b, $M_y = 0$), we discuss about uniaxial (straight) bending with axial force.

$N = F$
$M_y = 0$
$M_z = F \times y_0$

Fig. 10.2.b

10.2 THE DIRECT (NORMAL) STRESSES $\sigma_x$ IN BENDING WITH AXIAL FORCE

We admit the most general case of a cross section subjected by an axial force $N$ and two bending moments $M_y$ and $M_z$, acting positive (figure 10.3.a):

- $N > 0$ when it stretches the cross section
- $M_y > 0$ when it produces tensile stress $\sigma > 0$ in the part of the cross section of positive $z$ coordinates $z > 0$
- $M_z > 0$ when it produces tensile stress $\sigma > 0$ in the part of the cross section of positive $y$ coordinates $y > 0$
- With other words the moments $M_y$ and $M_z$, are positive when their resultant $M_i$ produces tensile stress $\sigma > 0$ in the first quadrant of the cross section plan

Fig. 10.3
An equivalent statically image of the stresses from figure 10.3.a is given in figure 10.3.b, assuming the internal stresses $N$, $M_y$ and $M_z$ are reduced in point $Q$ to an axial force $N$ applied with the eccentricities $y_0$ and $z_0$. To satisfy the statically equivalence, $y_0$ and $z_0$ will be:

$$y_0 = \frac{M_y}{N} \quad \text{and} \quad z_0 = \frac{M_z}{N}$$

(10.1)

These 3 distinct stresses acting in the cross section produce normal stress $\sigma_x$ computed with the well-known relations:

- From $N$: $\sigma_x = \frac{N}{A}$
- From $M_y$: $\sigma_x = \frac{M_y}{I_y} z$
- From $M_z$: $\sigma_x = \frac{M_z}{I_z} y$

Taking into account the hypothesis used in Mechanics of Materials: of the small deformations and the material having a linear elastic behavior, the normal stresses $\sigma_x$ will be calculated superposing the effects of the axial solicitation (from $N$) with two uniaxial bending (from $M_y$ and $M_z$):

$$\sigma_x = \sigma_x^N + \sigma_x^{M_y} + \sigma_x^{M_z} \quad \text{or:}$$

$$\sigma_x = \frac{N}{A} + \frac{M_y}{I_y} z + \frac{M_z}{I_z} y$$

(10.2)

With the eccentricities of $N$, $y_0$ and $z_0$, the bending moments are:

$M_y = N \times z_0$ and $M_z = M \times y_0$, which replaced in the formula of $\sigma_x$ give:

$$\sigma_x = \frac{N}{A} + \frac{N z_0}{I_y} z + \frac{N y_0}{I_z} y$$

or:

$$\sigma_x = \frac{N}{A} \left(1 + \frac{z_0 z}{i_y^2} + \frac{y_0 y}{i_z^2}\right)$$

but: $\frac{I_y}{A} = i_y^2$ and $\frac{I_z}{A} = i_z^2$, representing the square of the gyration (inertia) radius $i_y$ and $i_z$:

$$\sigma_x = \frac{N}{A} \left(1 + \frac{z_0 z}{i_y^2} + \frac{y_0 y}{i_z^2}\right)$$

We observe in this relation that if $y = z = 0$ (in the centroid $G$) $\Rightarrow \sigma_x = \frac{N}{A} \neq 0$, so in the centroid the normal stress is due only to the axial force $N$. 


10.3 NEUTRAL AXIS POSITION

Let’s see if there are points from the cross section where $\sigma_x = 0$. Equating to zero, as $\frac{N}{A} \neq 0$, result:

$$(1 + \frac{z_0^2}{i_y^2} + \frac{y_0 y}{i_z^2}) = 0 : \text{neutral axis equation}$$

This is the equation of a straight line, named again, the neutral axis. In simple bending (straight or oblique) the neutral axis always passes through the cross section centroid $G$. In this case the neutral axis equation includes a free term, so the neutral axis doesn’t pass anymore through $G$. For this reason (Fig.10.4), the neutral axis may cut the cross section (a) and in the cross section both tension and compressive stresses occur, or it can be tangent (b) or outside (c) the cross section and only tension or compressive stresses will occur (the sign depending on the axial force N sign).

We may determine the position of the neutral axis neutral axis by its cuts, considering on turn $y = 0$ and $z = 0$ in neutral axis equation. We obtain:

$$z = 0 \implies 1 + \frac{y}{i_y} = 0 \implies y_n = -\frac{i_y^2}{y_0}$$

respectively for $y = 0$ $\implies z_n = -\frac{i_z^2}{y_0}$

The neutral axis equation becomes:

$$\frac{y}{y_n} + \frac{z}{z_n} = 1,$$

where $y_n$ and $z_n$ are the neutral axis coordinates, given by cuts.

From their relationship: $y_n = -\frac{i_y^2}{y_0}$; $z_n = -\frac{i_z^2}{z_0}$ we may conclude that the point of application of the eccentric force $Q (y_0, z_0)$ and the neutral axis (given by
the cuts $y_n$ and $z_n$) are always situated on one part and the other with respect to the cross section centroid $G$. Also, the position of the neutral axis does not depend on the axial force magnitude, but only on its eccentricity.

10.4 GEOMETRICAL CORRELATIONS BETWEEN THE ECCENTRIC POINT OF APPLICATION THE AXIAL FORCE $N$ (POINT Q) AND THE NEUTRAL AXIS

Point $Q$ in cross section has the coordinates $y_0$ and $z_0$: $y_0 = \frac{M_z}{N}$; $z_0 = \frac{M_y}{N}$.

The neutral axis cuts, obtained before, are: $y_n = -\frac{i_y}{y_0}$ and $z_n = -\frac{i_y}{z_0}$

![Geometrical correlations](image)

Fig.10.5

Geometrical correlations (Fig.10.5):

a) - as the neutral axis cuts $y_n$ and $z_n$ have different sing with respect to $y_0$ and $z_0$, neutral axis will always pass through the quadrant opposite to the one where the eccentric force $N$ acts, such that the maximum normal stress $\sigma_{\text{ymax}}$ is on that part of the cross section.

b) – as much the point $Q$ is removed from the centroid $G$, as much the neutral axis is approaching to $G$ (directly proportional).

If $y_0 = z_0 = \infty$ $(N=0)$ => $y_n = -\frac{i_y}{y_0} = 0$; $z_n = -\frac{i_y}{z_0} = 0$

As $y_n = z_n = 0$ => neutral axis passes through $G$, and this is the case of oblique bending $(N = 0, M_y \neq 0, M_z \neq 0)$
If $y_0 = z_0 = 0$ ($M_y = M_z = 0$) $\Rightarrow$ $y_n = z_n = \infty \Rightarrow$ the neutral axis is situated to $\infty \Rightarrow$ the case of axial solicitation ($N \neq 0$, $M_y = M_z = 0$) and a constant distribution of the normal stress $\sigma_x$.

c) – the inclination angle $\beta$ between neutral axis and the principal axis $G_y$, as in the case of oblique bending, depends only on the inclination of the resultant moment $M_i$ and the moments of inertia $I_y$ and $I_z$

$$\tan \beta = \tan \alpha \frac{I_y}{I_z} = \frac{M_z}{M_y \cdot I_z}$$

The axial force $N$ existence doesn’t modify its inclination, it only removes it from centroid $G$.

d) – if the point $Q$ $(y_0, z_0)$ is situated on a principal axis (Fig.10.6), the neutral axis is perpendicular to that axis.

$y_0 \neq 0 \Rightarrow y_n = -\frac{I_y}{y_0}$

$z_0 = 0 \Rightarrow z_n = \infty \Rightarrow$ the neutral axis doesn’t intersect $G_z$ axis, so it is parallel to it and perpendicular to $G_y$ axis:

$$\sigma_x = \frac{N}{A} + \frac{M_z}{I_z} y \quad (M_y = 0)$$

![Fig.10.6](image)

e) – If point $Q$ $(y_0, z_0)$, representing the point of application of the eccentric force $N$, is displaced on a straight line $d-d$ (Fig.10.7), the neutral axis is rotated around a point $Q'$ from the neutral axis.
10.5 DIAGRAMS OF NORMAL STRESSES $\sigma_x$. STRENGTH VERIFICATION

With the known position of the neutral axis, the maximum values of $\sigma_x$ will appear also in the extreme points from cross section (Fig.10.8).

\[
\sigma_{x1} = \sigma_{x\max} = \frac{N}{A} + \frac{M_y}{I_y} z_1 + \frac{M_z}{I_z} y_1 \leq R_t
\]

\[
\sigma_{x2} = \sigma_{x\min} = \frac{N}{A} - \frac{M_y}{I_y} z_2 - \frac{M_z}{I_z} y_2 \leq R_c
\]

if: $R_t = R_c = R$ (example for steel) $\Rightarrow \max(\sigma_{x1}, |\sigma_{x2}|) \leq R$
Note that, if the axial force $N$ is very big, the neutral axis can be tangent or even outside the cross section and the entire diagram $\sigma_x$ has only one sign (if $N>0 \Rightarrow \sigma_x>0$, and inversely).

For the rectangular cross section (Fig.10.9):

$$\sigma_{x_{\text{max}}} = \frac{N}{A} + \frac{M_y}{W_y} + \frac{M_z}{W_z} \leq R_l$$

$$\sigma_{x_{\text{min}}} = \frac{N}{A} - \frac{M_y}{W_y} - \frac{M_z}{W_z} \leq R_c$$

When the eccentric axial force (Fig.10.10) is situated on a principal axis (example $Gy$) the single stresses in cross section will be:

$$N \neq 0$$
$$M_y = 0$$
$$M_z = N \times y_0$$
The maximum normal stress will be:

\[ \sigma_{x\text{ max}} = \frac{N}{A} + \frac{M_z}{I_z} y \]

or:

\[ \sigma_x = \frac{N}{A} (1 + \frac{y_0 y}{i_z^2}) \]

The neutral axis position, from \( \sigma_x = 0 \), will be:

\[ 1 + \frac{y}{i_z^2} = 0 \quad => \quad y_n = -\frac{i_z^2}{y_0} \]

So, the neutral axis will be parallel to \( Gz \) axis, at the distance \( y_n \) measured from \( G \). The extreme values of \( \sigma_x \):

\[ \sigma_{x\text{ max}} = \frac{N}{A} + \frac{M_z}{W_z} \quad \text{and} \quad \sigma_{x\text{ min}} = \frac{N}{A} - \frac{M_z}{W_z} \]

Function the neutral axis position, \( \sigma_x \) diagrams will be as in Fig.10.11. Noting \( y_0 = e \) (the eccentricity of \( N \)) and \( M_z = N \times e \)

\[ \sigma_x = \frac{N}{A} \pm \frac{M_z}{W_z} = \frac{N}{A} \pm \frac{Ne}{b^2 h} \]

\[ \sigma_x = \frac{N}{A} (1 \pm \frac{6e}{b}) \quad \text{for rectangular cross section} \]

![Diagram of rectangular cross section with stress distributions](image)

Function the eccentricity \( e = y_0 \) of the axial force \( N \) (Fig.10.11), we may conclude:
- if \( e < \frac{b}{6} \) => the normal stresses \( \sigma_{xmax} \) and \( \sigma_{xmin} \) will have the same sign (situation 3)
- if \( e = \frac{b}{6} \) => \( \sigma_{xmax} = \frac{2N}{A} \), \( \sigma_{xmin} = 0 \) (situation 2)
- if \( e > \frac{b}{6} \) => \( \sigma_{xmax} \) and \( \sigma_{xmin} \) will have different sign (situation 1)

10.6 ECCENTRIC COMPRESSION IN CASE OF MATERIALS WHICH CAN’T UNDERTAKE TENSION (HAVE VERY WEAK TENSIILE STRENGTH)

From these final observations, we may conclude that function the position of the eccentric axial force \( N \) (distance \( e \)) and implicit the neutral axis position, the normal stresses have the same signs if neutral axis is outside the cross section (or at limit if it is tangent to cross section), or different signs if neutral axis cuts the cross section. There are elements of construction made from materials which have a weak design tensile strength (the simple concrete, the stone of construction, the brick masonry, the soil). That’s why it is recommended that these elements should be subjected mostly in compression. For this reason, we are interested in finding a domain, from the cross section, where applying the eccentric force \( N \), only compression stresses \( \sigma \) appear. This domain is called central core.

10.6.1 The central core

**Definition:** the geometric place of all points of application of the eccentric axial force \( N \), corresponding to neutral axis tangent to cross section contour, is a central zone of the cross section called central core.
- If the point of application of \( N \) is inside the central core, the neutral axis is outside the cross section.
- If the point of application is on the central core contour, the neutral axis is tangent to the cross section contour.
- If the point of application is outside the central core, the neutral axis will intersect the cross section.

The central core, also named the cross section kern, will always represent an area around the cross section centroid \( G \).

Some properties of the central core can be formulated (Fig.10.12):

1. The cross section contour and the central core contour are reciprocal figures, because to every side of the cross section corresponds a vertex of the central core.
2. The central core of a convex polygonal section is also a convex polygon, in which the cross section is inscribed.
3. If the cross section presents symmetry conditions the central core presents the same symmetry condition.
4. The neutral axis tangent to the cross section contour (that from the convex polygon) should not intersect the cross section.
5. The tangent neutral axis $n_i - n_i$ and the central core vertex $P_i$ are always situated on one part and the other with respect to the cross section centroid $G$.

In order to represent the central core of a cross section we represent, on turn, neutral axis which are tangent to each side of the cross section, without intersecting it (Fig.10.12). To every neutral axis $n_i - n_i$ vertex $P_i$ will correspond, having the coordinates (in the principal system of axis) $P_i(y_{vi}, z_{vi})$:

$$y_{vi} = - \frac{i_x^2}{y_{ni}}; \quad z_{vi} = - \frac{i_y^2}{z_{ni}}$$

where: $i_x^2 = \frac{I_x}{A}$; $i_y^2 = \frac{I_y}{A}$

$y_{ni}$ and $z_{ni}$: are the neutral axis $n_i - n_i$ cuts, on the principal axis.

10.6.2 The central core for some usual sections

**a) The rectangle**

The rectangular cross sections are mostly used in constructions for masonry structures and **foundations**. As the rectangle is a double
symmetrical figure, it is enough to find two vertex of the core, so 2 neutral axis tangents to cross section are considered (Fig.10.13).

From symmetry, vertex 1’ will have the coordinates: 1’(0; $-\frac{h}{6}$)

**Tangent axis $n_1$ - $n_1$:**

$y_{n1} = \infty$ ; $z_{n1} = -\frac{h}{2}$

$i_y = \frac{I_y}{A} = \frac{bh^3}{12} \frac{1}{bh} = \frac{h^2}{12}$ ; $i_x = \frac{b^2}{12}$

$y_{v1} = -i_y z_{n1} = -\frac{h}{12} \left( -\frac{2}{h} \right) = \frac{h}{6}$

**Tangent axis $n_2$ - $n_2$:**

$y_{n2} = \frac{b}{2}$ ; $z_{n2} = \infty \Rightarrow y_{v2} = -\frac{b}{6}$ ; $z_{v2} = 0$

From symmetry: 2’($\frac{b}{6}$, 0)

The core for a rectangle is a rhomb (Fig.10.13), and it will be also a rhomb for any double symmetrical cross section inscribed in a rectangle: IPN profile (Fig.10.14), composed section made from 2 channel profiles[].

![Fig.10.13](image)

![Fig.10.14](image)
**Tangent axis \( n_1 - n_1 \):**

\[ y_{n1} = -\frac{b}{2}; \quad z_{n1} = \infty \]

\[ i_z^2 = \frac{I_z}{A}; \quad i_y^2 = \frac{I_y}{A} \]

\[ y_{v1} = -\frac{I_z}{A} \left( \frac{2}{b} \right) = \frac{W_y}{A} \]

\[ z_{v1} = 0 \]

**Tangent axis \( n_2 - n_2 \):**

\[ y_{n2} = \infty; \quad z_{n2} = -\frac{h}{2} \Rightarrow y_{v2} = 0; \quad z_{v2} = \frac{W_y}{A} \]

**b) The circle**

In case of circle, any tangent neutral axis is situated at the radius \( R = \frac{D}{2} \). This means that the core will be a circle too (Fig.10.15), of radius: \( \rho = \frac{i_y^2}{R} = \frac{i_z^2}{R} \)

![Fig.10.15](image)

\[ i_y^2 = i_z^2 = \frac{\pi D^4}{64} \cdot \frac{4}{\pi D^2} = \frac{D^2}{16} \Rightarrow \rho = \frac{D^2}{16} \frac{2}{D} = \frac{D}{8} \]

**c) Channel profile UPN**

![Fig.11.16](image)
Tangent axis $n_1 - n_1$:

$y_{n1} = \infty$; $z_{n1} = -\frac{h}{2}$

$y_{v1} = 0$; $z_{v1} = \frac{l_y(-\frac{2}{h})}{A} = \frac{W_y}{A}$

Tangent axis $n_2 - n_2$:

$y_{n2} = b - y_s$; $z_{n2} = \infty$

$y_{v2} = \frac{W_{zz}}{A}$; $z_{v2} = 0$

Tangent axis $n_2 - n_2$:

$y_{n3} = -y_s$; $z_{n3} = \infty$

$y_{v3} = \frac{W_{zz}}{A}$; $z_{v3} = 0$

10.6.3 The active zone in case of materials with weak tensile strength

From centric compression we saw that the most dangerous section for foundations was the one at the contact between the concrete foundation and soil. But, in that case as only compressive stresses appeared, the only verification to be made was that: $\sigma_{\text{max}} \leq R_{\text{soil}}$

At eccentric compression, another important problem appears, because no tensile stress can be developed at the contact between foundation and soil.

In what follows it will be considered that the material cannot undertake tensile stresses. For any shape of cross section there are 2 possible distinct situations (Fig.10.17):

1. The axial compressive force $N$ acts within the cross section core (a) or even at the central core limit (b). In both cases only compressive stresses will appear, as the neutral axis does not intersect the cross section (a) or at limit it is tangent to the cross section (b).
In both cases:

\[ \sigma_x = \frac{N}{A} + \frac{M_y}{I_y} Z \]

2. The compressive force \( N \) acts outside the core. Because only compression can be transmitted, the compressive stresses volume must have a resultant \( C \) equal and of opposite sense with respect to the axial force \( N \), in order to satisfy the static equilibrium condition (Fig.10.18).

![Fig.10.18](image)

The eccentric force \( N \) is carried out only by the compressed part of the cross section, called **active zone**. This area \( A_a \) of the active zone is limited by an axis called **zero axis** (noted in Fig.10.18 with 0-0), because in this axis \( \sigma_x = 0 \).

**“Zero axis” is no more identically to the neutral axis, being closer to the most compressed fiber from the cross section.**

Assuming \( \sigma \) has a linear variation, the normal stress \( \sigma \) at the level \( \eta \) is:

\[ \frac{\sigma}{\sigma_{\text{max}}} = \frac{\eta}{I_a} \Rightarrow \sigma = \sigma_{\text{max}} \frac{\eta}{I_a} \]

The stresses acting in cross section written on strength way with respect to **zero axis**, are:

**The axial eccentric force:**

\[ N = \int_{A_a} \sigma \, dA = \int_{A_a} \sigma_{\text{max}} \frac{\eta}{I_a} \, dA = \frac{\sigma_{\text{max}}}{I_a} S_0 \quad \text{(a)} \]

where \( S_0 \) is the static moment (first moment of area) of the active area \( A_a \) with respect to **zero axis**

**The bending moment:**

\[ N \times \eta_0 = \int_{A_a} \sigma \, dA \, \eta = \int_{A_a} \sigma_{\text{max}} \frac{\eta^2}{I_a} \, dA = \frac{\sigma_{\text{max}}}{I_a} I_0 \quad \text{(b)} \]
where \( I_0 \): is the moment of inertia (second moment of area) of the active area \( A_a \) with respect to “zero axis”

Dividing (b) to (a) we may find the unknown distance:

\[ \eta_0 = \frac{l_0}{S_0} \]

\( \eta_0 \) - is related to the position of the compressive force \( N \)

The maximum value of \( \sigma, \sigma_{\text{max}} \) can be found from (a) replacing the active zone length \( l_a \) by \( l_a = \eta_0 + c \)

\[ N(\eta_0 + c) = \sigma_{\text{max}} S_0 \Rightarrow \sigma_{\text{max}} = \frac{N}{S_0}(\eta_0+c) \]

For the rectangular cross section:

\[ I_0 = \frac{b(\eta_0+c)^3}{12} + b(\eta_0+c) \frac{b(\eta_0+c)^2}{4} = \frac{b(\eta_0+c)^3}{12} + \frac{b(\eta_0+c)^3}{4} = \frac{b(\eta_0+c)^3}{3} \]

\[ S_0 = \frac{b(\eta_0+c)^2}{2} \]

\[ \eta_0 = \frac{l_0}{S_0} = \frac{2}{3}(\eta_0 + c) \Rightarrow \eta_0 = 2c \]

\[ l_a = \eta_0 + c \Rightarrow l_a = 3c \]

\[ \sigma_{x \text{ max}} = \frac{2N}{3bc} \]
10.7 APPLICATION TO BENDING WITH AXIAL SOLICITATION

10.7.1 For the simple supported beam with the static scheme and the cross section from the figure below calculate:

a. diagrams of stresses

b. the strength verification \((R = 2200 \text{daN/cm}^2)\). In the critical sections the neutral axis and the normal stress diagram \(\sigma_x\) will be represented.

The diagrams of stresses:
The critical sections, where the strength verification should be made, are:

- **section C**, where \( N_{\text{max}} = -800 \text{kN} \), \( M_y = -64 \text{kNm} \) and \( M_z = 540 \text{kNm} \), so a section of biaxial bending with compression (eccentric biaxial compression)

- **section B**, where \( N_{\text{max}} = -800 \text{kN} \), \( M_{y\text{max}} = -160 \text{kNm} \) and \( M_z = 0 \), so a section of uniaxial bending with compression (eccentric uniaxial compression)

With the second moments of area \( I_y = 16061 \text{cm}^4 \) and \( I_z = 199096 \text{cm}^4 \) and the area \( A = 165.6 \text{cm}^2 \), the maximum normal stresses and the neutral axis are:

- **in section C**:
  \[
  \sigma_{x\text{ max}} = \frac{N}{A} + \frac{M_y}{I_y}z_1 + \frac{M_z}{I_z}y_1 = \frac{(-800 \times 10^2)}{165.6} + \frac{(-64 \times 10^4)}{16061}(15) + \frac{540 \times 10^4}{199096}(-41) = -2193 \frac{daN}{cm^2} \]
  \[
  \sigma_{x\text{ min}} = \frac{(-800 \times 10^2)}{165.6} + \frac{(-64 \times 10^4)}{16061}(-15) + \frac{540 \times 10^4}{199096}41 = 1227 \frac{daN}{cm^2}
  \]

The neutral axis cuts:

\[
y_n = -\frac{i_z}{y_0}; \quad z_n = -\frac{i_y}{z_0}
\]
\[ i_z^2 = \frac{I_z}{A} = \frac{199096}{165.6} = 1202.27 \text{cm}^2 \]
\[ i_y^2 = \frac{I_y}{A} = \frac{16061}{165.6} = 96.98 \text{cm}^2 \]
\[ y_0 = \frac{M_z}{N} = \frac{540 \times 10^2}{-800} = -67.5 \text{cm} \quad z_0 = \frac{M_y}{N} = \frac{(-64 \times 10^2)}{-800} = 8 \text{cm} \]

\[ y_n = -\frac{i_z^2}{y_0} = -\frac{1202.27}{-67.5} = 17.81 \text{cm} \]
\[ z_n = -\frac{i_y^2}{z_0} = -\frac{96.98}{8} = -12.12 \text{cm} \]

- In section B:

\[ \sigma_{\text{max}} = \frac{N}{A} + \frac{M_y}{I_y} z = \frac{(-800 \times 10^2)}{165.6} + \frac{(-160 \times 10^4)}{16061} (15) = \]
\[ = -1977 \frac{daN}{cm^2} < 2200 \frac{daN}{cm^2} \]
\[ \sigma_{\text{min}} = \frac{N}{A} + \frac{M_y}{I_y} z = \frac{(-800 \times 10^2)}{165.6} + \frac{(-160 \times 10^4)}{16061} (-15) = \]
\[ = 1011 \frac{daN}{cm^2} \]

The neutral axis cuts:

\[ y_0 = \frac{M_z}{N} = \frac{0}{-800} = 0 \quad z_0 = \frac{M_y}{N} = \frac{(-160 \times 10^2)}{-800} = 20 \text{cm} \]

\[ y_n = -\frac{i_z^2}{y_0} = -\frac{1202.27}{0} = \infty \quad z_n = -\frac{i_y^2}{z_0} = -\frac{96.98}{20} = -4.85 \text{cm} \]
10.7.2 For the following concrete foundation with $\gamma_{\text{concr.}} = 24\,\text{kN/m}^3$ make:

10.7.2.1 Represent the central core (kern) calculating the position of all the vertex of the core

10.7.2.2 Represent the normal stress diagram $\sigma_x$ in the section from the bottom part of the foundation, inscribing the extreme values.

10.7.2.1

To represent the central core we have to calculate first the geometrical characteristics of the cross section. The centroid position is represented on figure b. and the other characteristics are:

- $A = 7.74\,\text{m}^2$, $I_y = 3.2292\,\text{m}^4$, $I_z = 7.52085\,\text{m}^4$
- $i_y^2 = 0.4172\,\text{m}^2$, $i_z^2 = 0.9717\,\text{m}^2$

On turn, neutral axes $n_1$-$n_2$ tangent to cross section are represented. To each neutral axis coordinates of the kern vertex are calculated:

**Tangent axis $n_1$ - $n_1$:**

- $y_{n1} = 1.65\,\text{m}$; $z_{n1} = \infty$
- \[ y_{v1} = -\frac{i_z^2}{y_{n1}} = -\frac{0.9717}{1.65} = -0.589\,\text{m}; \quad z_{v1} = -\frac{i_y^2}{z_{n1}} = 0 \rightarrow \]

vertex 1 has the coordinates: ($-0.589\,\text{m}; 0$)

**Tangent axis $n_2$ - $n_2$:**

- $y_{n2} = \infty$; $z_{n2} = 1.2\,\text{m}$
- \[ y_{v2} = -\frac{i_z^2}{y_{n2}} = 0; \quad z_{v2} = -\frac{i_y^2}{z_{n2}} = -\frac{0.4172}{1.2} = -0.347\,\text{m} \rightarrow \]

vertex 2 has the coordinates: ($0; -0.347\,\text{m}$)
From symmetry, vertex 2’ will have the coordinates: (0;0.347m)

**Tangent axis n₃ – n₃**: to determine the neutral axis cuts we write the equation of a line which passes through 2 points A and B of known coordinates:

\[ \frac{y-A_y}{z-A_z} = \frac{B_y-A_y}{B_z-A_z} \]

\[ \frac{y+1.95}{z-0.6} = \frac{-0.45+1.95}{1.2-0.6} \]

\[ 0.6y-1.5z=-2.07 \]

For z = 0 \( y_{n₃} = -3.45 \) m; y = 0 \( z_{n₃} = 1.38 \) m

\[ y_{v₃} = \frac{i_z^2}{y_{n₃}} = \frac{0.9717}{-3.45} = 0.282 \text{m}; z_{v₃} = \frac{i_y^2}{z_{n₃}} = \frac{0.4172}{1.38} = -0.302 \text{m} \]

vertex 3 has the coordinates: (0.282 ; -0.302m)

From symmetry, vertex 3’ will have the coordinates: (0.282 ; 0.302m)

**Tangent axis n₄ – n₄**:

\[ y_{n₄} = -1.95 \text{m}; z_{n₄} = \infty \]

\[ y_{v₄} = \frac{i_z^2}{y_{n₄}} = \frac{0.9717}{-1.95} = 0.498 \text{m}; z_{v₄} = \frac{i_y^2}{z_{n₄}} = 0 \]

vertex 4 has the coordinates: (0.498m ; 0)

10.7.2.2

The two weights of the foundation are:

\[ G_1 = 24 \cdot 2.1 \cdot 2.4 \cdot 0.9 = 108.86 \text{kN} \]

\[ G_2 = 24 \cdot 7.74 \cdot 1.5 = 278.64 \text{kN} \]

The axial force and the bending moment in the section from the bottom part of the foundation (calculation section) are:

\[ N = -(2F + G_1 + G_2) = -1887.5 \text{kN} \]

\[ M_z = F \cdot 0.45 - F \cdot 1.65 + H \cdot 2.4 - G_1 \cdot 0.6 = -845.32 \text{kNm} \] \( (M_y = 0) \)

The eccentricity: \( e = \frac{M_z}{N} = \frac{-845.32}{-1887.5} = 0.448 \text{m} < y_{v₄} = 0.498 \text{m} \)

The extreme values of the normal stress are:

\[ \sigma_{x\ max} = \frac{N}{A} + \frac{M_z}{I_z} y = \frac{-1887.5 \cdot 10^2}{7.74 \cdot 10^4} + \frac{-845.32 \cdot 10^4}{7.52085 \cdot 10^8} 165 = -4.29 \text{ daN/cm}^2 \]

\[ \sigma_{x\ min} = \frac{-1887.5 \cdot 10^2}{7.74 \cdot 10^4} + \frac{-845.32 \cdot 10^4}{7.52085 \cdot 10^8} (-195) = -0.247 \text{ daN/cm}^2 \]

The diagram of the normal stresses \( \sigma_x \) is represented in fig.b.