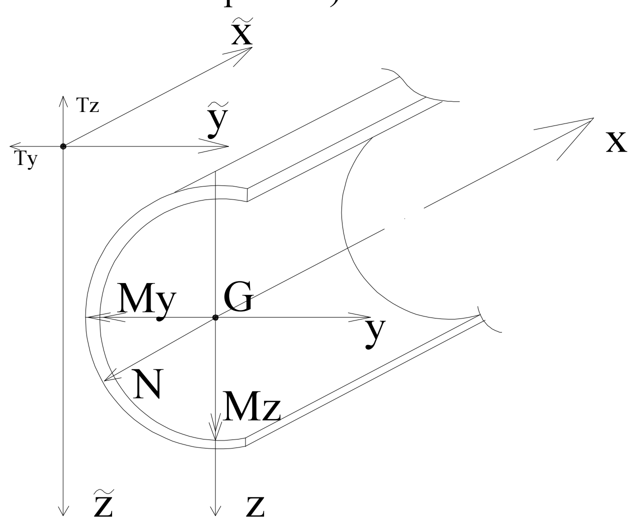
Chapter 10
BENDING AND AXIAL SOLICITATION
(Eccentric tension or compression)

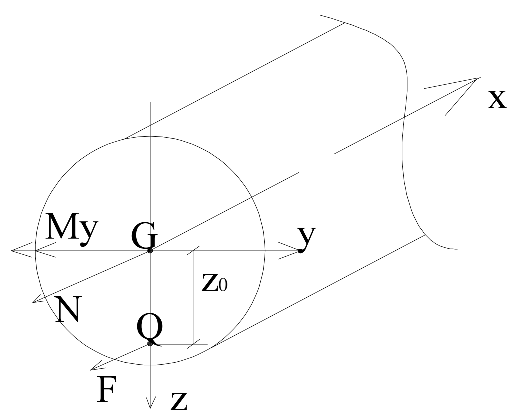
10.1 DEFINITION

A cross section is subjected to *bending with axial force (eccentric tension or compression)* if the normal (direct) stresses σ_x from section are reduced to an axial force N and two bending moments M_y and M_z (figure 10.1). In this case we discuss about *biaxial (oblique) bending with axial force*. As, in general, oblique bending is accompanied by *oblique shearing*, the shear stresses τ_x are reduced to two shear forces V_z and V_y (solicitation presented in chapter 10).



$N = F$
$M_y = F \times z_0$
$M_z = F \times y_0$

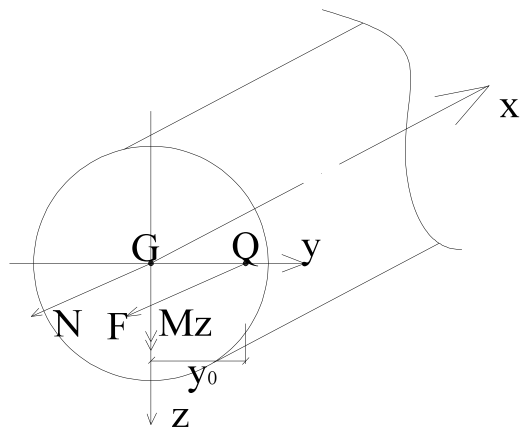
Fig.10.1



$N = F$
$M_y = F \times z_0$
$M_z = 0$

Fig.10.2.a

If the normal stresses σ_x from section are reduced to the axial force N and only bending moment M_y (figure 10.2.a, M_z =0) or M_z (figure 10.2.b, M_y=0), we discuss about *uniaxial (straight) bending with axial force*.



$N = F$
$M_y = 0$
$M_z = F \times y_0$

Fig.10.2.b

10.2 THE DIRECT (NORMAL) STRESSES σ_x IN BENDING WITH AXIAL FORCE

We admit the most general case of a cross section subjected by an axial force N and two bending moments M_y and M_z, acting positive (figure 10.3.a):

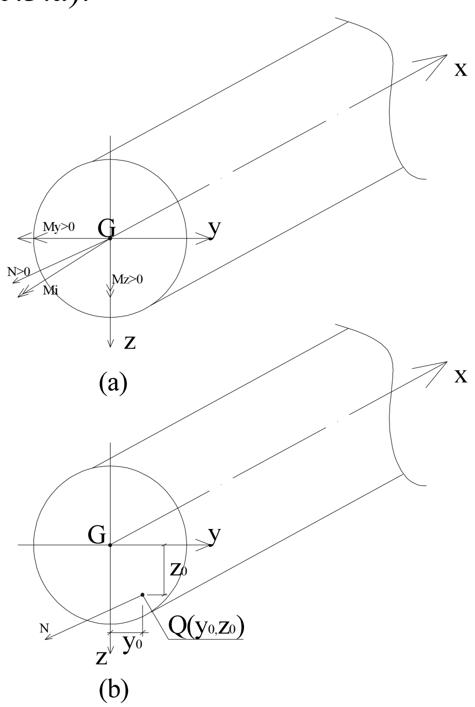


Fig.10.3

- $N > 0$ when it stretches the cross section
- $M_y > 0$ when it produces tensile stress $\sigma > 0$ in the part of the cross section of positive *z* coordinates $z > 0$
- $M_z > 0$ when it produces tensile stress $\sigma > 0$ in the part of the cross section of positive *y* coordinates $y > 0$
- With other words the moments M_y and M_z, are positive when their resultant M_i produces tensile stress $\sigma > 0$ in the first quadrant of the cross section plan

An equivalent statically image of the stresses from figure 10.3.a is given in figure 10.3.b, assuming the internal stresses *N, M_y* and *M_z* are reduced in point *Q* to an axial force *N* applied with the eccentricities y_0 and z_0. To satisfy the statically equivalence, y_0 and z_0 will be:

$$y_0 = \frac{M_z}{N} \text{ and } z_0 = \frac{M_y}{N} \tag{10.1}$$

These 3 distinct stresses acting in the cross section produce normal stress σ_x computed with the well-known relations:

- From N: $\sigma_x = \frac{N}{A}$
- From M_y: $\sigma_x = \frac{M_y}{I_y} z$
- From M_z: $\sigma_x = \frac{M_z}{I_z} y$

Taking into account the hypothesis used in Mechanics of Materials: of the small deformations and the material having a linear elastic behavior, the normal stresses σ_x will be calculated superposing the effects of the axial solicitation (from *N*) with two uniaxial bending (from *M_y* and *M_z*):

$\sigma_x = \sigma_x^N + \sigma_x^{M_y} + \sigma_x^{M_z}$, or:

$$\boxed{\sigma_x = \frac{N}{A} + \frac{M_y}{I_y} z + \frac{M_z}{I_z} y} \tag{10.2}$$

With the eccentricities of *N, y_0* and *z_0*, the bending moments are:
$M_y = N \times z_0$ and $M_z = M \times y_0$, which replaced in the formula of σ_x give:

$$\sigma_x = \frac{N}{A} + \frac{N\,z_0}{I_y} z + \frac{N\,y_0}{I_z} y$$

or: $$\sigma_x = \frac{N}{A}\left(1 + \frac{z_0 z}{\frac{I_y}{A}} + \frac{y_0 y}{\frac{I_z}{A}}\right)$$

but: $\frac{I_y}{A} = i_y^2$ and $\frac{I_z}{A} = i_z^2$, representing the square of the gyration (inertia) radius i_y and i_z

$$\sigma_x = \frac{N}{A}\left(1 + \frac{z_0 z}{i_y^2} + \frac{y_0 y}{i_z^2}\right)$$

We observe in this relation that if $y = z = 0$ (in the centroid *G*) => $\sigma_x = \frac{N}{A} \neq 0$, so in the centroid the normal stress is due only to the axial force *N*.

10.3 NEUTRAL AXIS POSITION

Let's see if there are points from the cross section where $\sigma_x = 0$. Equating to zero, as $\frac{N}{A} \neq 0$, result:

$(1+ \frac{z_0 z}{i_y^2} + \frac{y_0 y}{i_z^2}) = 0$:   **neutral axis equation**

This is the equation of a straight line, named again, the **neutral axis**. In simple bending (straight or oblique) the neutral axis always passes through the cross section centroid *G*. In this case the neutral axis equation includes a free term, so the **neutral axis doesn't pass anymore through *G*.** For this reason (Fig.10.4), the neutral axis may cut the cross section (a) and in the cross section both tension and compressive stresses occur, or it can be tangent (b) or outside (c) the cross section and only tension or compressive stresses will occur (the sign depending on the axial force N sign).

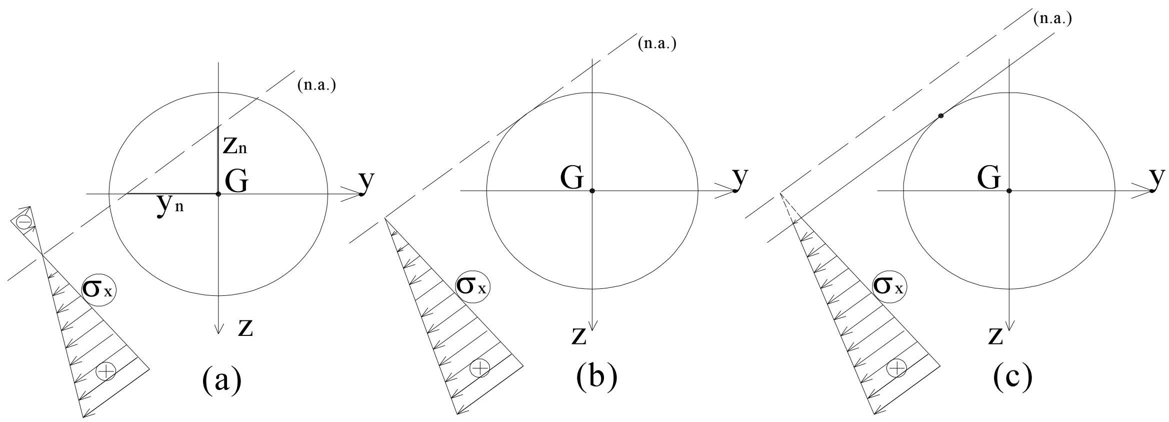


Fig. 10.4

We may determine the position of the neutral axis neutral axis by its **cuts**, considering on turn y = 0 and z = 0 in neutral axis equation. We obtain:

z = 0 => $1+ \frac{y}{\frac{i_z^2}{y_0}} = 0$ => $\mathbf{y_n = -\frac{i_z^2}{y_0}}$

respectively for y =0   => $\mathbf{z_n = -\frac{i_y^2}{z_0}}$

The neutral axis equation becomes:

$\frac{y}{y_n} + \frac{z}{z_n} = 1$, where y_n and z_n are the neutral axis coordinates, given by cuts. From their relationship: $y_n = -\frac{i_z^2}{y_0}$; $z_n = -\frac{i_y^2}{z_0}$ we may conclude that the **point of application of the eccentric force** Q (y_0, z_0) and the neutral axis (given by the cuts y_n and z_n) **are always situated on one part and the other with respect to the cross section centroid *G***. Also, the position of the neutral axis does not depend on the axial force magnitude, but only on its eccentricity.

10.4 GEOMETRICAL CORRELATIONS BETWEEN THE ECCENTRIC POINT OF APPLICATION THE AXIAL FORCE *N* (POINT Q) AND THE NEUTRAL AXIS

Point *Q* in cross section has the coordinates y_0 and z_0: $y_0 = \frac{M_z}{N}$; $z_0 = \frac{M_y}{N}$.

The neutral axis cuts, obtained before, are: $y_n = -\frac{i_z^2}{y_0}$ and $z_n = -\frac{i_y^2}{z_0}$

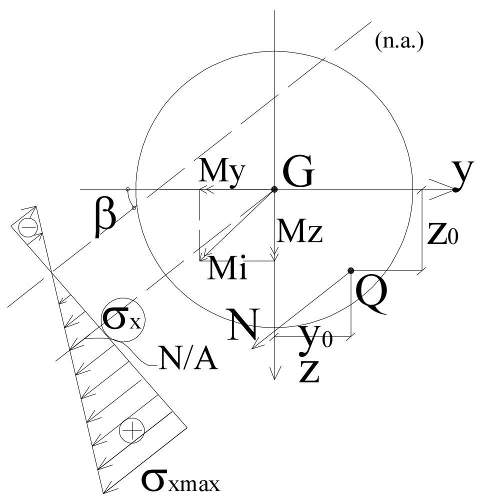


Fig.10.5

Geometrical correlations (Fig.10.5):

a) - as the neutral axis cuts y_n and z_n have different sing with respect to y_0 and z_0, neutral axis will always pass through the quadrant opposite to the one where the eccentric force *N* acts, such that the maximum normal stress σ_{xmax} is on that part of the cross section .

b) – as much the point *Q* is removed from the centroid *G*, as much the neutral axis is approaching to *G* (directly proportional).

If $y_0 = z_0 = \infty$ (N=0) $\Rightarrow$ $y_n = -\frac{i_z^2}{y_0} = 0$; $z_n = -\frac{i_y^2}{z_0} = 0$

As $y_n = z_n = 0 \Rightarrow$ neutral axis passes through *G*, and this is the case of oblique bending ($N = 0$, $M_y \neq 0$, $M_z \neq 0$)

If $y_0 = z_0 = 0$ ($M_y = M_z = 0$) => $y_n = z_n = \infty$ => the neutral axis is situated to ∞ => the case of axial solicitation ($N \neq 0$, $M_y = M_z = 0$) and a constant distribution of the normal stress σ_x.

c) – the inclination angle β between neutral axis and the principal axis *Gy*, as in the case of oblique bending, depends only on the inclination of the resultant moment M_i and the moments of inertia I_y and I_z ($tg\beta = tg\alpha \frac{I_y}{I_z} = \frac{Mz}{My}\frac{I_y}{I_z}$)

The axial force *N* existence doesn't modify its inclination, it only removes it from centroid *G*.

d) – if the point Q (y_0, z_0) is situated on a principal axis (Fig.10.6), the neutral axis is perpendicular to that axis.

$y_0 \neq 0 \Rightarrow y_n = -\frac{i_z^2}{y_0}$

$z_0 = 0 \Rightarrow z_n = \infty$ => the neutral axis doesn't intersect *Gz* axis, so it is parallel to it and perpendicular to *Gy* axis:

$\sigma_x = \frac{N}{A} + \frac{M_z}{I_z} y \quad (M_y = 0)$

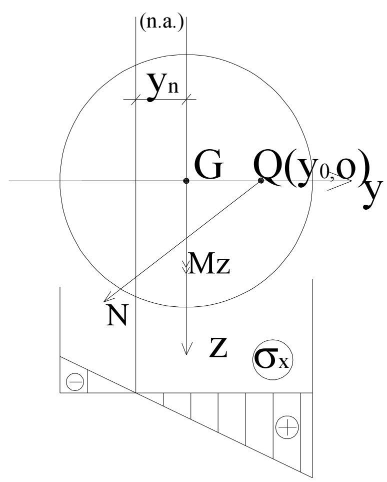


Fig.10.6

e) – If point Q (y_0, z_0), representing the point of application of the eccentric force *N*, is displaced on a straight line *d-d* (Fig.10.7), the neutral axis is rotated around a point *Q'* from the neutral axis.

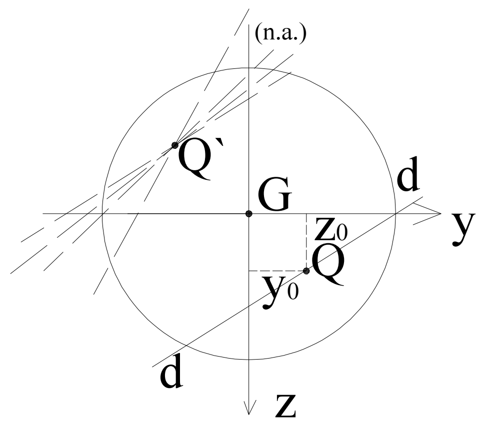


Fig.10.7

10.5 DIAGRAMS OF NORMAL STRESSES σ_x. STRENGTH VERIFICATION

With the known position of the neutral axis, the maximum values of σ_x will appear also in the extreme points from cross section (Fig.10.8).

$$\sigma_{x\,1} = \sigma_{x\,max} = \frac{N}{A} + \frac{M_y}{I_y} z_1 + \frac{M_z}{I_z} y_1 \leq R_t$$

$$\sigma_{x\,2} = \sigma_{x\,min} = \frac{N}{A} - \frac{M_y}{I_y} z_2 - \frac{M_z}{I_z} y_2 \leq R_c$$

if: $R_t = R_c = R$ (example for steel) => $\max(\sigma_{x1}, |\sigma_{x2}|) \leq R$

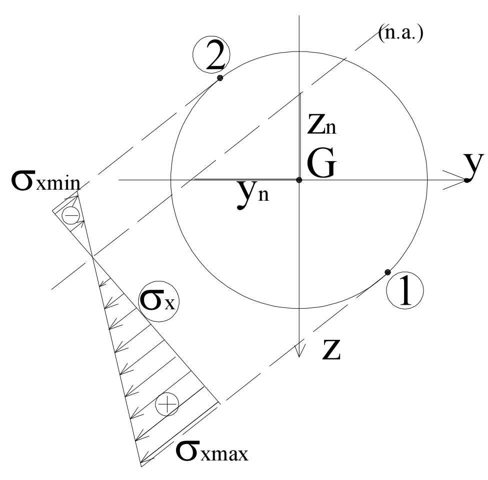


Fig.10.8

!!! Note that, if the axial force N is very big, the neutral axis can be tangent or even outside the cross section and the entire diagram σ_x has only one sign (if N>0 => σ_x>0, and inversely)

For the **rectangular cross section** (Fig.10.9):

$$\sigma_{x\,max} = \frac{N}{A} + \frac{M_y}{W_y} + \frac{M_z}{W_z} \leq R_t$$

$$\sigma_{x\,min} = \frac{N}{A} - \frac{M_y}{W_y} - \frac{M_z}{W_z} \leq R_c$$

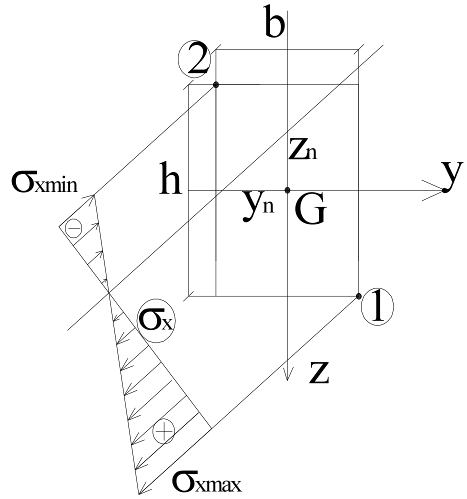
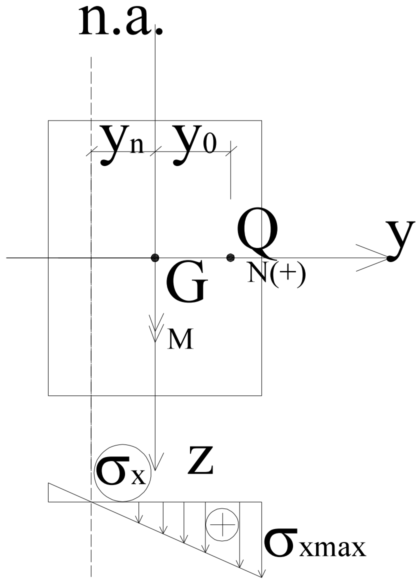


Fig.10.9

When the eccentric axial force (Fig.10.10) is situated on a principal axis (example *Gy*) the single stresses in cross section will be:



$N \neq 0$
$M_y = 0$
$M_z = N \times y_0$

Fig.10.10

The maximum normal stress will be:

$$\sigma_{x\,max} = \frac{N}{A} + \frac{M_z}{I_z}\,y$$

or: $\sigma_x = \frac{N}{A}\left(1 + \frac{y_0 y}{i_z^2}\right)$

The neutral axis position, from $\sigma_x = 0$, will be:

$$1 + \frac{y}{\frac{i_z^2}{y_0}} = 0 \qquad \Rightarrow \qquad y_n = -\frac{i_z^2}{y_0}$$

So, the neutral axis will be parallel to *Gz* axis, at the distance y_n measured from *G*. The extreme values of σ_x:

$$\sigma_{x\,max} = \frac{N}{A} + \frac{M_z}{W_z} \text{ and } \sigma_{x\,min} = \frac{N}{A} - \frac{M_z}{W_z}$$

Function the neutral axis position, σ_x diagrams will be as in Fig.10.11. Noting $y_0 = e$ (the eccentricity of *N*) and $M_z = N \times e$

$$\sigma_x = \frac{N}{A} \pm \frac{M_z}{W_z} = \frac{N}{A} \pm \frac{Ne}{\frac{b^2 h}{6}}$$

$$\boxed{\sigma_x = \frac{N}{A}\left(1 \pm \frac{6e}{b}\right)} \qquad \textbf{for rectangular cross section}$$

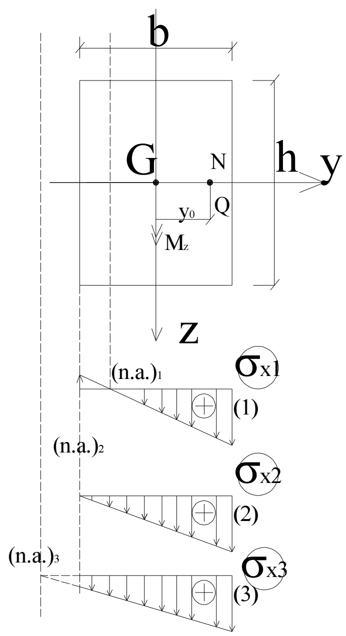


Fig.10.11

Function the eccentricity $e = y_0$ of the axial force *N* (Fig.10.11), we may conclude:

- if $e < \frac{b}{6}$ => the normal stresses σ_{xmax} and σ_{xmin} will have the same sign (situation 3)
- if $e = \frac{b}{6}$ => $\sigma_{xmax}=\frac{2N}{A}$, $\sigma_{xmin}= 0$ (situation 2)
- if $e > \frac{b}{6}$ => σ_{xmax} and σ_{xmin} will have different sing (situation 1)

10.6 ECCENTRIC COMPRESSION IN CASE OF MATERIALS WHICH CAN'T UNDERTAKE TENSION (HAVE VERY WEAK TENSILE STRENGTH)

From these final observations, we may conclude that function the position of the eccentric axial force *N* (distance *e*) and implicit the neutral axis position, the normal stresses have the same signs if neutral axis is outside the cross section (or at limit if it is tangent to cross section), or different signs if neutral axis cuts the cross section. There are elements of construction made from materials which have a weak design **tensile strength** (the simple concrete, the stone of construction, the brick masonry, the soil). That's why it is recommended that these elements should be subjected mostly in compression. For this reason, we are interested in finding a domain, from the cross section, where applying the eccentric force *N*, only compression stresses σ_x appear. This domain is called **central core**.

10.6.1 The central core

Definition: the geometric place of all points of application of the eccentric axial force *N*, corresponding to neutral axis tangent to cross section contour, is a central zone of the cross section called **central core.**
- If the point of application of *N* is inside the central core, the neutral axis is outside the cross section.
- If the point of application is on the central core contour, the neutral axis is tangent to the cross section contour.
- If the point of application is outside the central core, the neutral axis will intersect the cross section.

The **central core**, also named the cross section **kern**, will always represent an area around the cross section centroid *G*.
Some **properties** of the central core can be formulated (Fig.10.12):

1. The cross section contour and the central core contour are reciprocal figures, because to every side of the cross section corresponds a vertex of the central core

2. The central core of a convex polygonal section is also a convex polygon, in which the cross section is inscribed.
3. If the cross section presents symmetry conditions the central core presents the same symmetry condition
4. The neutral axis tangent to the cross section contour (that from the convex polygon) should not intersect the cross section.
5. The tangent neutral axis $n_i - n_i$ and the central core vertex P_i are always situated on one part and the other with respect to the cross section centroid G.

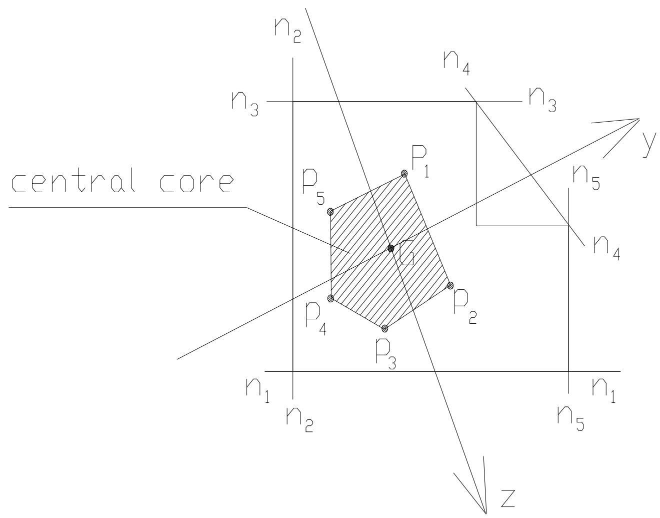


Fig.10.12

In order to represent the central core of a cross section we represent, on turn, neutral axis which are tangent to each side of the cross section, without intersecting it (Fig.10.12). To every neutral axis $n_i - n_i$ vertex P_i will correspond, having the coordinates (in the principal system of axis) $P_i(y_{vi},z_{vi})$:

$$y_{vi} = -\frac{i_z^2}{y_{ni}}; \qquad z_{vi} = -\frac{i_y^2}{z_{ni}}$$

where: $i_z^2 = \frac{I_z}{A}$; $i_y^2 = \frac{I_y}{A}$

y_{ni} and z_{ni} : are the neutral axis $n_i - n_i$ cuts, on the principal axis

10.6.2 The central core for some usual sections

a) The rectangle

The rectangular cross sections are mostly used in constructions for masonry structures and **foundations.** As the rectangle is a double symmetrical figure, it is enough to find two vertex of the core, so 2 neutral axis tangents to cross section are considered (Fig.10.13).

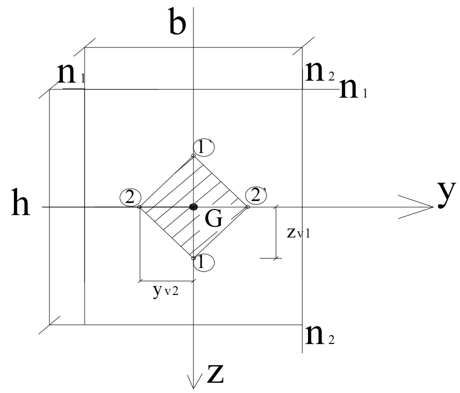


Fig.10.13

Tangent axis n_1 - n_1:

$y_{n1} = \infty$; $z_{n1} = -\frac{h}{2}$

$i_y^2 = \frac{I_y}{A} = \frac{bh^3}{12}\frac{1}{bh} = \frac{h^2}{12}$; $i_z^2 = \frac{b^2}{12}$

$y_{v1} = -\frac{i_z^2}{y_{n1}} = 0$; $z_{v1} = -\frac{i_y^2}{z_{n1}} = -\frac{h^2}{12}\left(-\frac{2}{h}\right) = \frac{h}{6}$

From symmetry, vertex 1’ will have the coordinates: 1’(0; $-\frac{h}{6}$)

Tangent axis n_2 - n_2 :

$y_{n2} = \frac{b}{2}$; $z_{n2} = \infty \Rightarrow y_{v2} = -\frac{b}{6}$; $z_{v2} = 0$

From symmetry: 2’($\frac{b}{6}$, 0)

The core for a rectangle is a **rhomb** (Fig.10.13), and it will be also a rhomb for any double symmetrical cross section inscribed in a rectangle: IPN profile (Fig.10.14), composed section made from 2 channel profiles[].

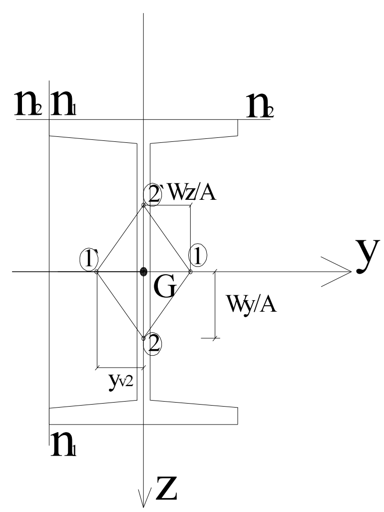


Fig.10.14

Tangent axis n_1 - n_1:

$y_{n1} = -\frac{b}{2}$; $z_{n1} = \infty$

$i_z^2 = \frac{I_z}{A}$; $i_y^2 = \frac{I_y}{A}$

$y_{v1} = -\frac{I_z}{A}\left(-\frac{2}{b}\right) = \frac{W_z}{A}$

$z_{v1} = 0$

Tangent axis n_2 – n_2:

$y_{n2} = \infty$; $z_{n2} = -\frac{h}{2}$ => $y_{v2} = 0$; $z_{v2} = \frac{W_y}{A}$

b) The circle

In case of circle, any tangent neutral axis is situated at the radius $R = \frac{D}{2}$. This means that the core will be a circle too (Fig.10.15), of radius: $\rho = \frac{i_y^2}{R} = \frac{i_z^2}{R}$

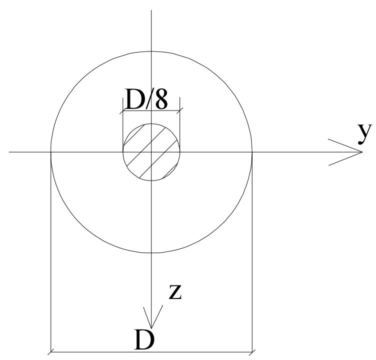


Fig.10.15

$$i_y^2 = i_z^2 = \frac{\pi D^4}{64}\frac{4}{\pi D^2} = \frac{D^2}{16} \Rightarrow \rho = \frac{D^2}{16}\frac{2}{D} = \frac{D}{8}$$

c) Channel profile UPN

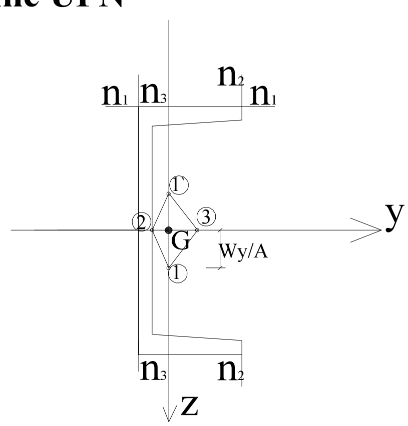


Fig.11.16

Tangent axis n_1 - n_1:

$y_{n1} = \infty;\ z_{n1} = -\frac{h}{2}$

$y_{v1} = 0;\ z_{v1} = -\frac{I_y}{A}\left(-\frac{2}{h}\right) = \frac{W_y}{A}$

Tangent axis n_2 – n_2:

$y_{n2} = b - y_s;\ z_{n2} = \infty$

$y_{v2} = \frac{W_{z2}}{A};\ z_{v2} = 0$

Tangent axis n_2 – n_2:

$y_{n3} = -y_s;\ z_{n3} = \infty$

$y_{v3} = \frac{W_{z3}}{A};\ z_{v3} = 0$

10.6.3 The active zone in case of materials with weak tensile strength

From centric compression we saw that the most dangerous section for foundations was the one at the contact between the concrete foundation and soil. But, in that case as only compressive stresses appeared, the only verification to be made was that: $\sigma_{xmax} \leq R_{soil}$

At eccentric compression, another important problem appears, because no tensile stress can be developed at the contact between foundation and soil.

In what follows it will be considered that the material can nott undertake tensile stresses. For any shape of cross section there are 2 possible distinct situations (Fig.10.17):

1. The axial compressive force *N* acts within the cross section core (a) or even at the central core limit (b). In both cases only compressive stresses will appear, as the neutral axis does not intersect the cross section (a) or at limit it is tangent to the cross section (b).

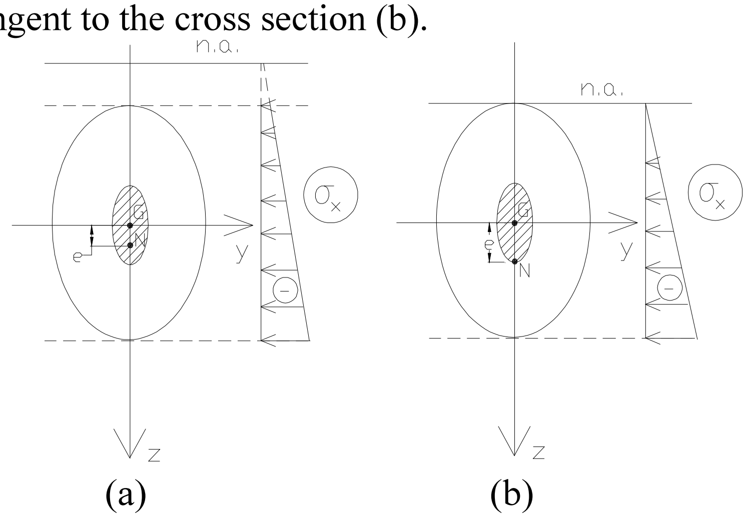


Fig.10.17

In both cases:

$$\sigma_x = \frac{N}{A} + \frac{M_y}{I_y} z$$

2. The compressive force *N* acts outside the core. Because only compression can be transmitted, the compressive stresses volume must have a resultant *C* equal and of opposite sense with respect to the axial force *N*, in order to satisfy the static equilibrium condition (Fig.10.18).

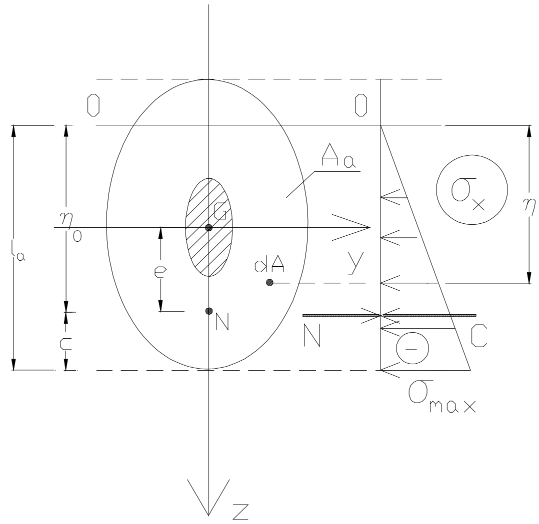


Fig.10.18

The eccentric force *N* is carried out only by the compressed part of the cross section, called **"active zone"** This area A_a of the active zone is limited by an axis called *"zero axis"* (noted in Fig.10.18 with 0-0), because in this axis $\sigma_x = 0$.

!!! ***"Zero axis" is no more identically to the neutral axis, being closer to the most compressed fiber from the cross section.***

Assuming σ has a linear variation, the normal stress σ at the level η is:

$$\frac{\sigma}{\sigma_{max}} = \frac{\eta}{l_a} \quad => \sigma = \sigma_{max} \frac{\eta}{l_a}$$

The stresses acting in cross section written on strength way with respect to "zero axis", are:

The axial eccentric force:

$$N = \int_{A_a} \sigma \, dA = \int_{A_a} \sigma_{max} \frac{\eta}{la} dA = \frac{\sigma_{max}}{la} S_0 \qquad (a)$$

where S_0: is the static moment (first moment of area) of the active area A_a with respect to "zero axis"

The bending moment:

$$N \times \eta_0 = \int_{A_a} \sigma \, dA \, \eta = \int_{A_a} \sigma_{max} \frac{\eta^2}{la} dA = \frac{\sigma_{max}}{la} I_0 \qquad (b)$$

where I_0: is the moment of inertia (second moment of area) of the active area A_a with respect to “zero axis”
Dividing (b) to (a) we may find the unknown distance:

$$\boxed{\eta_0 = \frac{I_0}{S_0}}$$

η_0 - is related to the position of the compressive force N
The maximum value of σ, σ_{max} can be found from (a) replacing the active zone length l_a by $l_a = \eta_0 + c$

$$N(\eta_0 + c) = \sigma_{max} S_0 \Rightarrow \boxed{\sigma_{max} = \frac{N}{S_0}(\eta_0 + c)}$$

For the rectangular cross section:

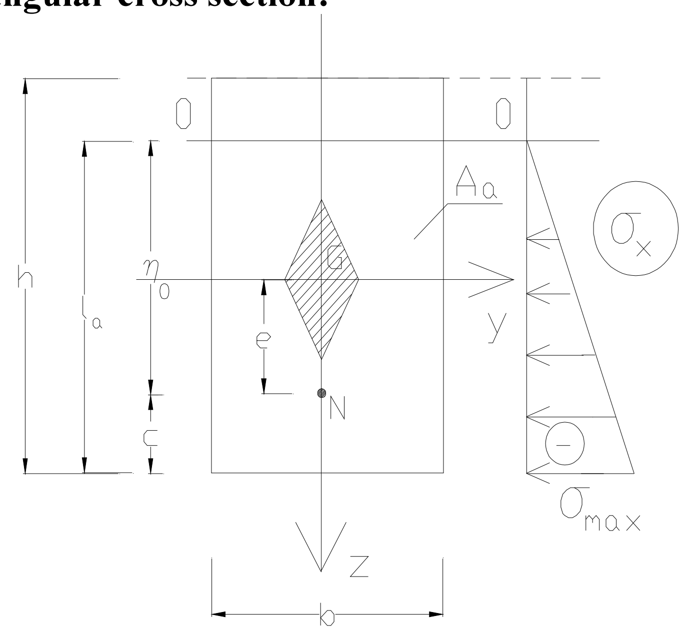


Fig.10.19

$$I_0 = \frac{b(\eta_0+c)^3}{12} + b(\eta_0 + c)\,\frac{b(\eta_0+c)^2}{4} = \frac{b(\eta_0+c)^3}{12} + \frac{b(\eta_0+c)^3}{4} = \frac{b(\eta_0+c)^3}{3}$$

$$S_0 = \frac{b(\eta_0+c)^2}{2}$$

$$\eta_0 = \frac{I_0}{S_0} = \frac{2}{3}(\eta_0 + c) \Rightarrow \boldsymbol{\eta_0 = 2c}$$

$$l_a = \eta_0 + c \Rightarrow l_a = \mathbf{3c}$$

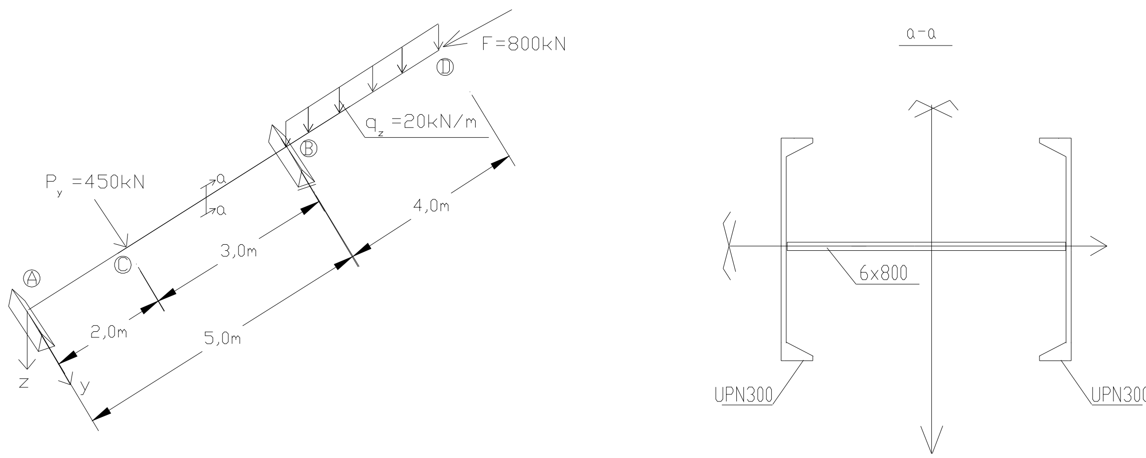
$$\boxed{\sigma_{x\,max} = \frac{2N}{3bc}}$$

10.7 APPLICATION TO BENDING WITH AXIAL SOLICITATION

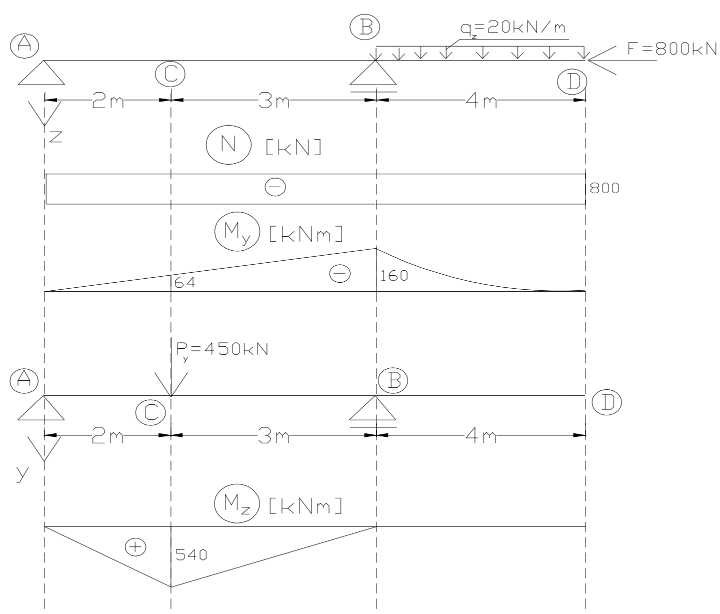
10.7.1 For the simple supported beam with the static scheme and the cross section from the figure bellow calculate:

a. diagrams of stresses

b. the strength verification ($R = 2200 daN/cm^2$). In the critical sections the neutral axis and the normal stress diagram σ_x will be represented.



The diagrams of stresses:



The critical sections, where the strength verification should be made, are:

- **section C**, where N_{max} = -800kN, M_y = -64kNm and $M_{z\ max}$ = 540kNm, so a section of biaxial bending with compression (eccentric biaxial compression)
- **section B**, where N_{max} = -800kN, $M_{y\ max}$ = -160kNm and M_z = 0, so a section of uniaxial bending with compression (eccentric uniaxial compression)

With the second moments of area $I_y = 16061cm^4$ and $I_z = 199096cm^4$ and the area $A = 165.6cm^2$, the maximum normal stresses and the neutral axis are:

- **in section C**:

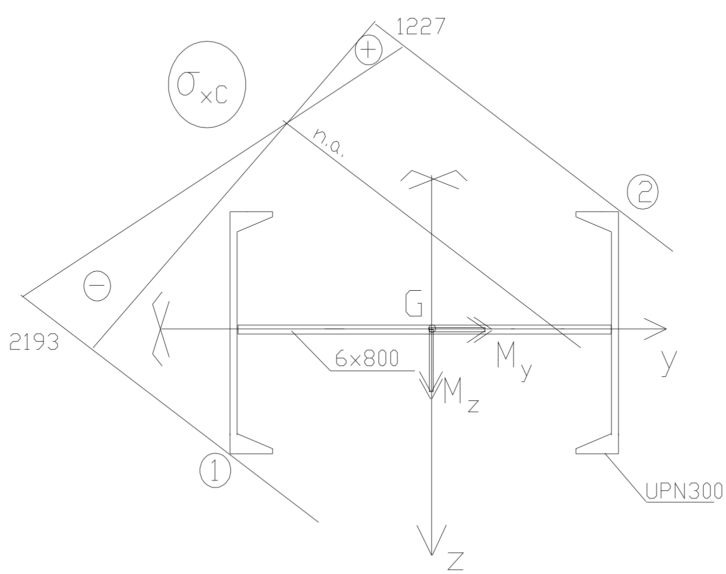
$$\sigma_{x\,max} = \frac{N}{A} + \frac{M_y}{I_y} z_1 + \frac{M_z}{I_z} y_1 =$$

$$= \frac{(-800 \times 10^2)}{165.6} + \frac{(-64 \times 10^4)}{16061} 15 + \frac{540 \times 10^4}{199096} (-41)$$

$$= -2193 \frac{daN}{cm^2} < 2200 \frac{daN}{cm^2}$$

$$\sigma_{x\,min} = \frac{(-800 \times 10^2)}{165.6} + \frac{(-64 \times 10^4)}{16061} (-15) + \frac{540 \times 10^4}{199096} 41 =$$

$$= 1227 \frac{daN}{cm^2}$$



The neutral axis cuts:

$$y_n = -\frac{i_z^2}{y_0};\quad z_n = -\frac{i_y^2}{z_0}$$

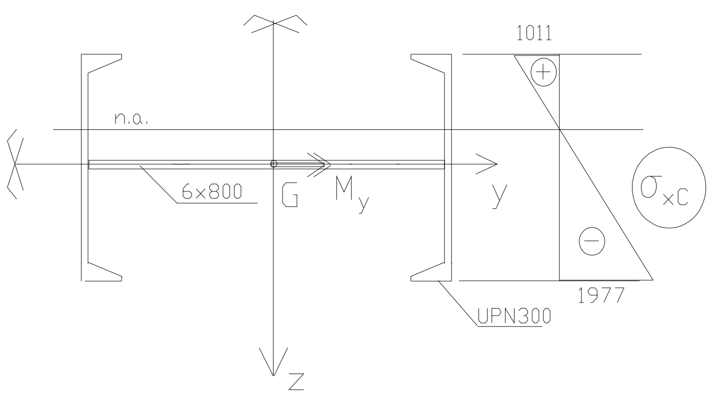
$i_z^2 = \frac{I_z}{A} = \frac{199096}{165.6} = 1202.27cm^2$
$i_y^2 = \frac{I_y}{A} = \frac{16061}{165.6} = 96.98cm^2$

$$y_0 = \frac{M_z}{N} = \frac{540 \times 10^2}{-800} = -67.5cm\ ;\ z_0 = \frac{M_y}{N} = \frac{(-64 \times 10^2)}{-800} = 8cm$$

$y_n = -\frac{i_z^2}{y_0} = -\frac{1202.27}{-67.5} = 17.81cm$
$z_n = -\frac{i_y^2}{z_0} = -\frac{96.98}{8} = -12.12cm$

- **in section B**:

$$\sigma_{x\,max} = \frac{N}{A} + \frac{M_y}{I_y}z = \frac{(-800 \times 10^2)}{165.6} + \frac{(-160 \times 10^4)}{16061}15 =$$
$$= -1977\frac{daN}{cm^2} < 2200\frac{daN}{cm^2}$$
$$\sigma_{x\,min} = \frac{N}{A} + \frac{M_y}{I_y}z = \frac{(-800 \times 10^2)}{165.6} + \frac{(-160 \times 10^4)}{16061}(-15) =$$
$$= 1011\frac{daN}{cm^2}$$



The neutral axis cuts:

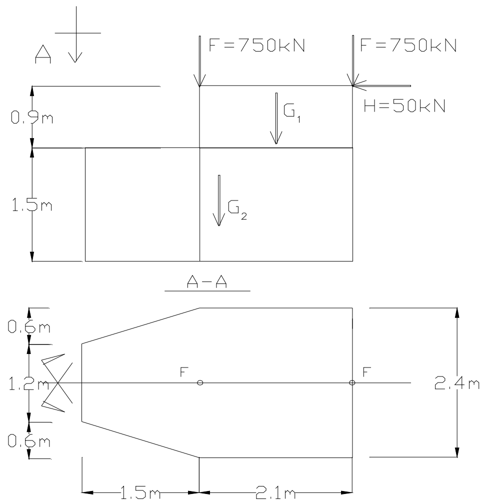
$$y_0 = \frac{M_z}{N} = \frac{0}{-800} = 0\ ;\ z_0 = \frac{M_y}{N} = \frac{(-160 \times 10^2)}{-800} = 20cm$$

$y_n = -\frac{i_z^2}{y_0} = -\frac{1202.27}{0} = \infty\ ;\ z_n = -\frac{i_y^2}{z_0} = -\frac{96.98}{20} = -4.85cm$

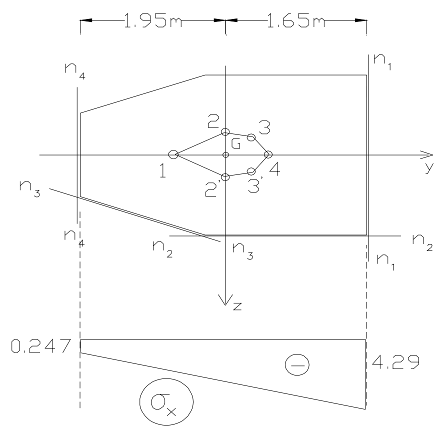
10.7.2 For the following concrete foundation with $\gamma_{concr.}$=24kN/m^3 make:

10.7.2.1 Represent the central core (kern) calculating the position of all the vertex of the core

10.7.2.2 Represent the normal stress diagram σ_x in the section from the bottom part of the foundation, inscribing the extreme values.



a.



b.

10.7.2.1

To represent the central core we have to calculate first the geometrical characteristics of the cross section. The centroid position is represented on figure b. and the other characteristics are:

$A = 7.74m^2$, $I_y = 3.2292m^4$, $I_z = 7.52085m^4$,
$i_y^2 = 0.4172m^2$, $i_z^2 = 0.9717m^2$
On turn, neutral axes n_i-n_i tangent to cross section are represented. To each neutral axis coordinates of the kern vertex are calculated:

Tangent axis n_1 - n_1:

$y_{n1} = 1.65m$; $z_{n1} = \infty$

$$y_{v1} = -\frac{i_z^2}{y_{n1}} = -\frac{0.9717}{1.65} = -0.589m \; ; \; z_{v1} = -\frac{i_y^2}{z_{n1}} = 0 \rightarrow$$

vertex 1 has the coordinates: (-0.589m ; 0)

Tangent axis n_2 - n_2 :

$y_{n2} = \infty$; $z_{n2} = 1.2m$

$$y_{v2} = -\frac{i_z^2}{y_{n2}} = 0 \; ; \; z_{v2} = -\frac{i_y^2}{z_{n2}} = -\frac{0.4172}{1.2} = -0.347m \rightarrow$$

vertex 2 has the coordinates: (0 ; -0.347m)

From symmetry, vertex 2' will have the coordinates: (0;0.347m)

Tangent axis $n_3 - n_3$: to determine the neutral axis cuts we write the equation of a line which passes through 2 points A and B of known coordinates: $\frac{y-y_A}{z-z_A} = \frac{y_B-y_A}{z_B-z_A} \rightarrow \frac{y+1.95}{z-0.6} = \frac{-0.45+1.95}{1.2-0.6} \rightarrow$ 0.6y-1.5z=-2.07 $\rightarrow$

For $z = 0 \rightarrow y_{n3} = -3.45m$; $y = 0 \rightarrow z_{n3} = 1.38m$

$y_{v3} = -\frac{i_z^2}{y_{n3}} = -\frac{0.9717}{-3.45} = 0.282m$; $z_{v3} = -\frac{i_y^2}{z_{n3}} = -\frac{0.4172}{1.38} = -0.302m \rightarrow$

vertex 3 has the coordinates: (0.282 ; -0.302m)

From symmetry, vertex 3' will have the coordinates: (0.282 ; 0.302m)

Tangent axis $n_4 - n_4$:

$y_{n4} = -1.95m$; $z_{n4} = \infty$

$y_{v4} = -\frac{i_z^2}{y_{n4}} = -\frac{0.9717}{-1.95} = 0.498m$; $z_{v4} = -\frac{i_y^2}{z_{n4}} = 0 \rightarrow$

vertex 4 has the coordinates: (0.498m ; 0)

10.7.2.2

The two weights of the foundation are:

$$G_1 = 24 \cdot 2.1 \cdot 2.4 \cdot 0.9 = 108.86kN$$

$$G_2 = 24 \cdot 7.74 \cdot 1.5 = 278.64kN$$

The axial force and the bending moment in the section from the bottom part of the foundation (calculation section) are:

$$N = -(2F + G_1 + G_2) = -1887.5kN$$

$$M_z = F \cdot 0.45 - F \cdot 1.65 + H \cdot 2.4 - G_1 \cdot 0.6 = -845.32kNm \left(M_y = 0\right)$$

The eccentricity: $e = \frac{M_z}{N} = \frac{-845.32}{-1887.5} = 0.448m < y_{v4} = 0.498m$

The extreme values of the normal stress are:

$$\sigma_{x\,max} = \frac{N}{A} + \frac{M_z}{I_z}y = \frac{-1887.5 \cdot 10^2}{7.74 \cdot 10^4} + \frac{-845.32 \cdot 10^4}{7.52085 \cdot 10^8}165 = -4.29\frac{daN}{cm^2}$$

$$\sigma_{x\,min} = \frac{-1887.5 \cdot 10^2}{7.74 \cdot 10^4} + \frac{-845.32 \cdot 10^4}{7.52085 \cdot 10^8}(-195) = -0.247\frac{daN}{cm^2}$$

The diagram of the normal stresses σ_x is represented in fig.b.