Chapter 5

CENTRIC TENSION OR COMPRESSION

(AXIAL LOADING)

5.1 DEFINITION

A construction member is subjected to ***centric (axial) tension or compression*** if in any cross section the single distinct stress is ***the axial force N.***

If the axial force N is ***positive*** we discuss about ***centric tension*** (Fig.5.1), respectively if it is ***negative*** we have ***centric compression*****:**

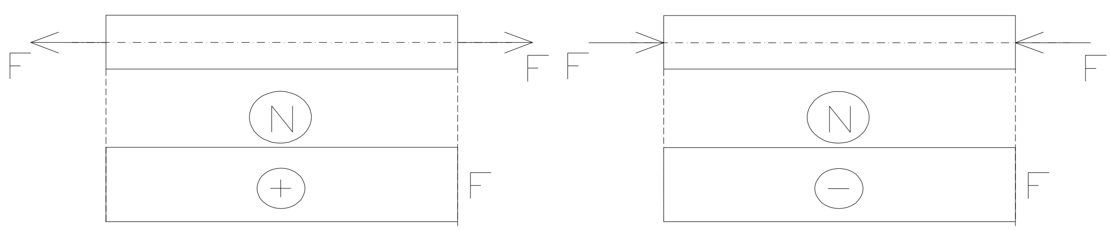
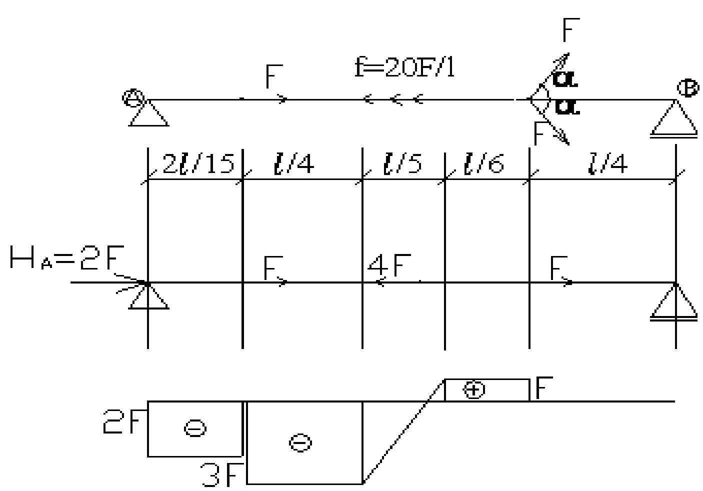


Fig.5.1

Example of a bar subjected only to **axial loading:**



5.2 GEOMETRICAL ASPECT

We study this aspect on a rubber model, with a rectangular cross section (Fig.5.2). On the lateral surface, parallel and equidistant lines are traced.

The model is subjected to centric tension by two equal forces *F* at each end, the bar reaching a deformed elongated shape.

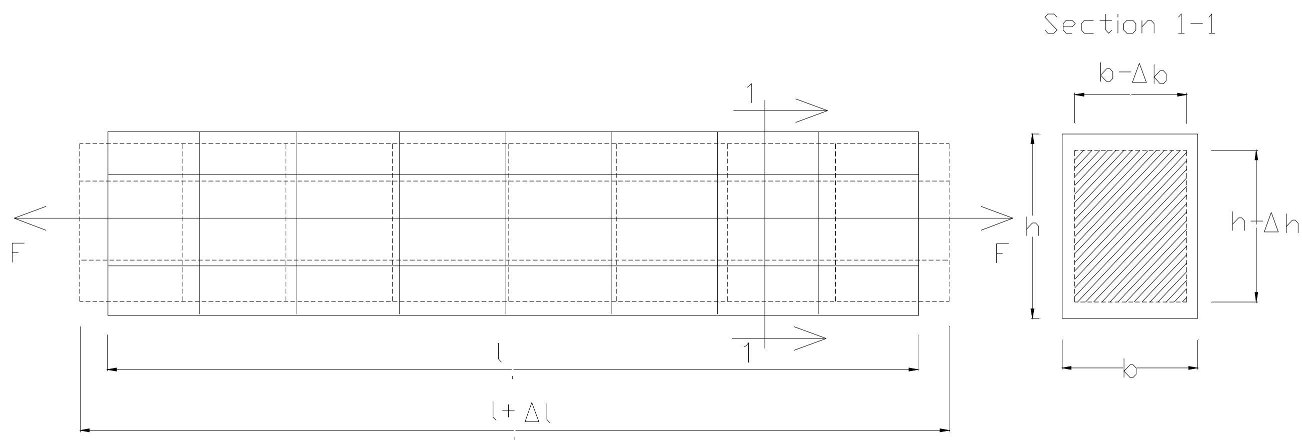


Fig.5.2

After deformation we observe:

- the initial straight lines remain straight and parallel, respectively perpendicular, to the longitudinal axis of the model.

- the longitudinal lines become longer, all being increased with the same quantity.

- the transversal lines remain parallel and become shorter with the same quantitie.

From these observations we may say that the initial cross sections, plane and parallel on longitudinal axis *Gx* before deformation remain perpendicular and plane after deformation (Bernoulli's hypothesis is valid). So the lengthening is produced by the relative displacement of the cross sections. The lengthening along the longitudinal fibers is uniformly distributed on cross section and the specific elongations ε_x are also uniformly distributed on cross section:

$$\varepsilon_x = \text{const.} = \frac{\Delta l}{l}$$

The same observation can be made for the other two specific strains:

$$\varepsilon_y = \frac{\Delta b}{b} = -\mu\, \varepsilon_x\, ; \qquad \varepsilon_z = \frac{\Delta h}{h} = -\mu\, \varepsilon_x$$

μ : Poisson's ratio: $\mu = 0\ldots..0{,}3$ (ex.: for steel $\mu = 0{,}3$)

As in practice ε_x is very small, we may neglect ε_y and ε_z, the single important specific strain remaining $\boldsymbol{\varepsilon_x}$.

If we observe a rectangle from model, we can see that after deformation the initial straight angles remain also straight, so the specific sliding is null on all directions:

$$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$$

In conclusion, the single strain different from zero is the specific elongation $\boldsymbol{\varepsilon_x}$ which is constant on cross section.

5.3 PHYSICAL ASPECT

Considering the case of a linear-elastic material, we may write Hooke's law:

$$\sigma = E \cdot \varepsilon \quad \text{and} \quad \tau = G \cdot \gamma$$

For our solicitation these relations become:

$$\sigma_x = E \cdot \varepsilon_x = \text{const.}$$

$$\sigma_y = \sigma_z = 0\, ; \; \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

!!!!! The normal stress σ_x, for the axial solicitation (loading), is always constant on cross section (Fig.5.3).

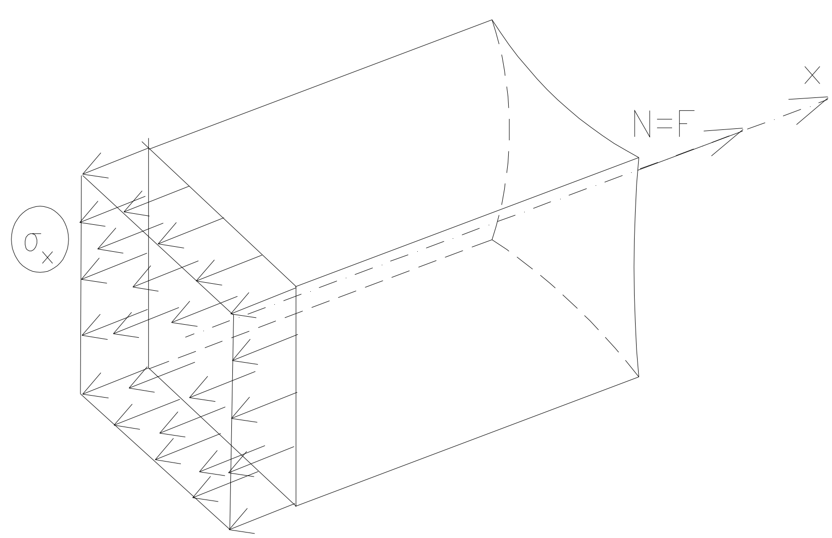


Fig.5.3

5.4 THE STATIC ASPECT

For our model, the single stress in cross section is the axial force:

$N = F$ (written from exterior)

Writing from interior, the axial force is (3.6):

$$N = \int_A \sigma_x dA = \sigma_x \int_A dA = \sigma_x \cdot A$$

$$\boxed{\sigma_x = \frac{N}{A}} \qquad (5.1)$$

Relation (5.1) represents the formula of the normal stress σ_x due only to the axial force N, so from axial loading.

From Hook's law, the specific strain:

$$\varepsilon_x = \frac{\sigma_x}{E} \text{, or replacing } \sigma_x \Rightarrow \varepsilon_x = \frac{N}{E \cdot A}$$

From the geometrical aspect explained in chapter 2, the elongation of a differential element *dx* is:

$$\varepsilon_x = \frac{\Delta dx}{dx} \quad \text{(par. 2.1.2)}$$

$$\Rightarrow \Delta dx = \varepsilon_x \cdot dx = \frac{N}{E \cdot A} dx$$

The **total elongation** of a bar of length *l*, is:

$$\Delta l = \int_0^l \frac{N}{E \cdot A} dx$$

If *N, E* and *A* are constant on the entire length *l*, the total elongation will be:

$$\boxed{\Delta l = \frac{Nl}{E \cdot A}} \qquad (5.2)$$

Otherwise: $\Delta l = \sum \frac{N_i \cdot l_i}{E_i \cdot A_i}$

The factor **(EA)** is called ***modulus of rigidity at centric tension or compression***

The main problems regarding the strength calculation are:

a) The strength verification (checking):

$$\sigma_{x\,max} = \frac{N^d_{max}}{A_{min}} \leq R \text{ in Ultimate Limit State} \qquad (5.3)$$

where: $N^d = n \cdot N^k$: is the design axial force

N^k : is the characteristic axial force

n: is the partial safety factor for actions

Usually: $R - 5\% \leq \sigma_{x\,max} \leq R + 3\%$

Observing the above relations, the maximum normal stress $\sigma_{x\,max}$ will appear in the *critical (dangerous) sections*, which are:

- sections of maximum axial force : N_{max}
- sections of minimum (net) area (Fig.5.4): $A_{min} = A_{net} = A_{gross} - \Delta A$

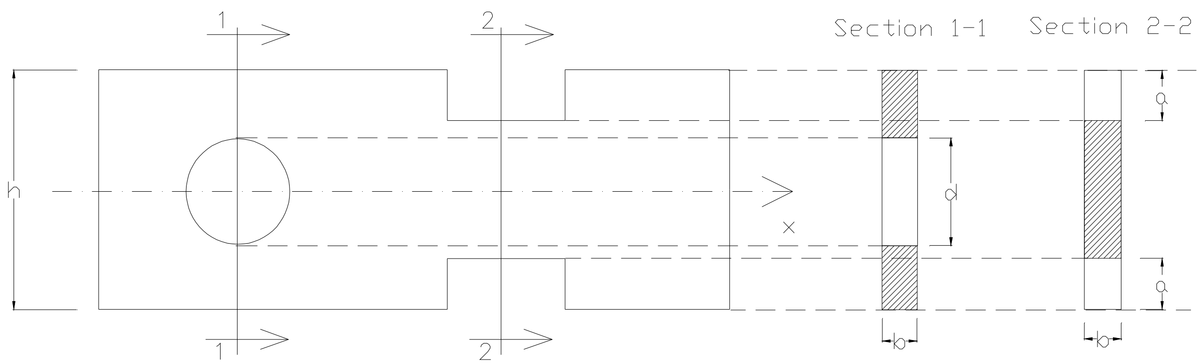
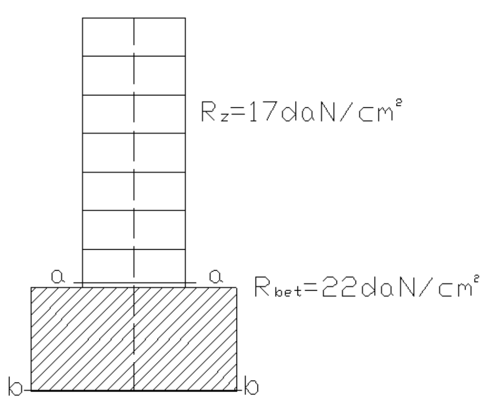


Fig.5.4

$A_{net_{1-1}} = bh - bd$

$A_{net_{2-2}} = bh - 2ab$

- sections of contact between 2 materials of different resistances (Fig.5.5):



Sections a-a and b-b are dangerous sections in the strength computing

Fig.5.5

b) The bar dimensioning

$$A_{nec} \geq \frac{N^d_{max}}{R} \tag{5.4}$$

c) The capable (maximum) axial force (the bearing capacity):

$$N^d_{max} \leq A_{net}R \tag{5.5}$$

5.5 CENTRIC TENSION OR COMPRESSION, WHEN WE TAKE INTO ACCOUNT THE EFFECT OF THE MASS FORCES (THE SELF-WEIGHT)

Let's assume a cantilever is axially compressed on vertical direction, and the length *l* is big enough so that its own weight can't be neglected (Fig.5.6).

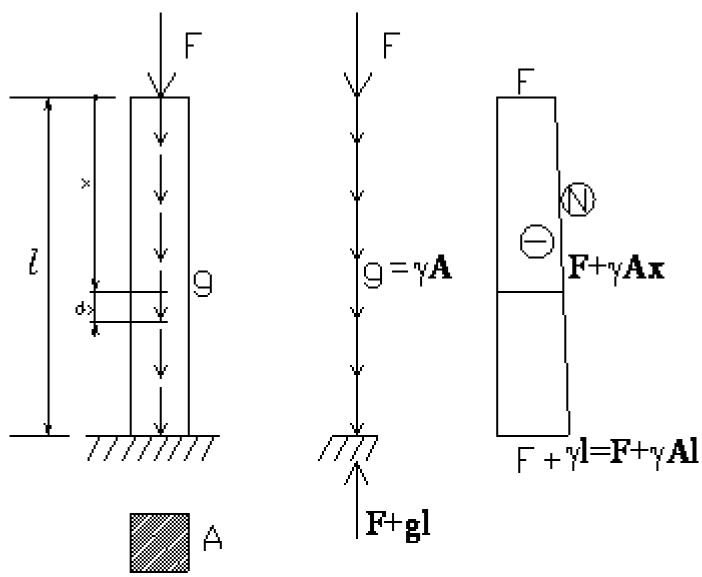


Fig.5.6

Besides the external force *F* the bar is subjected by the self-weight *g*, which is a uniformly distributed load along the longitudinal axis of the bar *x*. The normal stress σ_x in a section at level *x* is:

$$\sigma_x = \frac{N(x)}{A} = \frac{F + A\gamma x}{A} = \frac{F}{A} + \gamma \cdot x$$

The maximum value of σ_x is, for x = l:

$$\sigma_{x\,max} = \frac{F}{A} + \gamma \cdot l = \frac{F+G}{A}$$

G: the total weight of the bar

The total elongation Δl:

$$\Delta l = \int_0^l \frac{N(x)}{EA}\,dx = \int_0^l \frac{F+A\gamma\, x}{EA}\,dx = \frac{Fl}{EA} + \frac{A\gamma}{EA}\frac{x^2}{2}\Big|_0^l = \frac{Fl}{EA} + \frac{1}{2}\,\frac{Gl}{EA}$$

Relation of Δl may be written shorter, for bars with different N, E and A:

$$\Delta l = \sum \Delta l_i = \sum\left(\frac{N_i\, l_i}{E_i\, A_i} + \frac{1}{2}\,\frac{G_i\, l_i}{E_i A_i}\right)$$

We note: $N_i\, l_i + \frac{1}{2}\, G_i\, l_i = \Omega_{Ni}$: the area of the characteristic axial force diagram. Δl will be then:

$$\Delta l = \sum \frac{\Omega_{Ni}}{E_i\, A_i} \qquad (5.6)$$

The relation will be written separately for each part of the bar for which *N*, *E* and *A* are constant, the total elongation Δl being finally their sum $\sum \Delta l_i$.

5.6 STRESSES ON AN OBLIQUE PLANE

Let's consider a tensioned bar and a cross section 1-1 which is perpendicular to the longitudinal bar axis (Fig.5.7). Removing one part, in the cross section 1-1 of the other part the effect of the removed part is introduced by distributed internal forces measured by unit stresses σ_x.

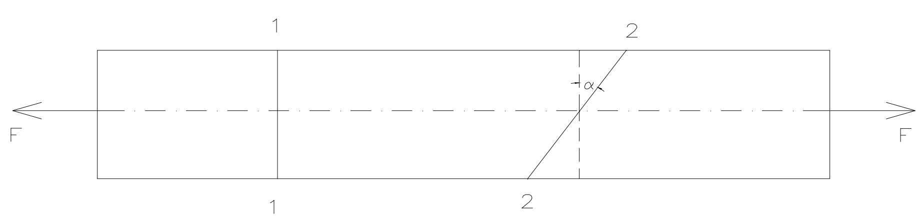


Fig.5.7

In section 1-1 the resultant of the internal forces is the axial force:

$$N = \sigma_x \cdot A$$

Now we consider another section 2–2, which will be inclined with an angle α with respect to section 1-1 (Fig.5.8).

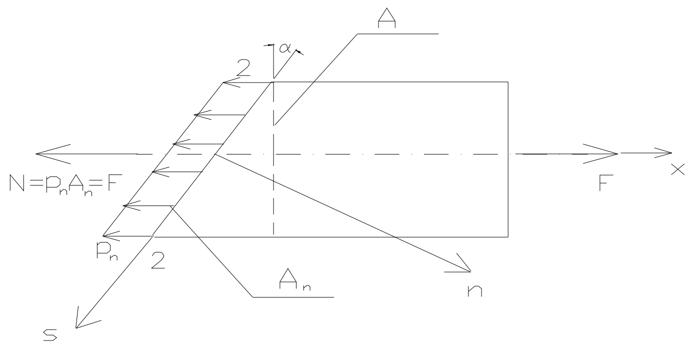


Fig.5.8

Similarly, removing one part of the bar, the other part will be in equilibrium if in the inclined section 2 -2 it is introduced the effect of the removed part also by internal distributed forces.

These two sections 1-1 (transversal) and 2-2 (oblique) are fictitious sections and they can't modify the state of stresses from bar, meaning the axial force and the direction of the internal forces (σ_x respectively p_n). Therefore through the cross section 1-1 the same axial force will be transmitted as through the inclined section 2-2, and the unit stresses σ_x and p_n will be uniformly distributed on both sections. We may write then:

$$N = \sigma_x \cdot A = p_n \cdot A_n$$

But: $\cos\alpha = \frac{A}{A_n}$ => $A_n = \frac{A}{\cos\alpha}$

Replacing: $\sigma_x \cdot A = p_n \cdot \frac{A}{\cos\alpha}$ => $p_n = \sigma_x \cdot \cos\alpha$

Decomposing p_n (Fig.5.9), we obtain the normal, respective tangential component of the unit stress p_n in the inclined section:

$$\sigma_n = p_n \cdot \cos\alpha = \sigma_x \cdot \cos^2\alpha = \frac{\sigma_x}{2}(1 + \cos 2\alpha)$$

$$\tau_{ns} = - p_n \cdot \sin\alpha = - \sigma_x \cdot \sin\alpha \cdot \cos\alpha = - \frac{\sigma_x}{2} \cdot \sin 2\alpha$$

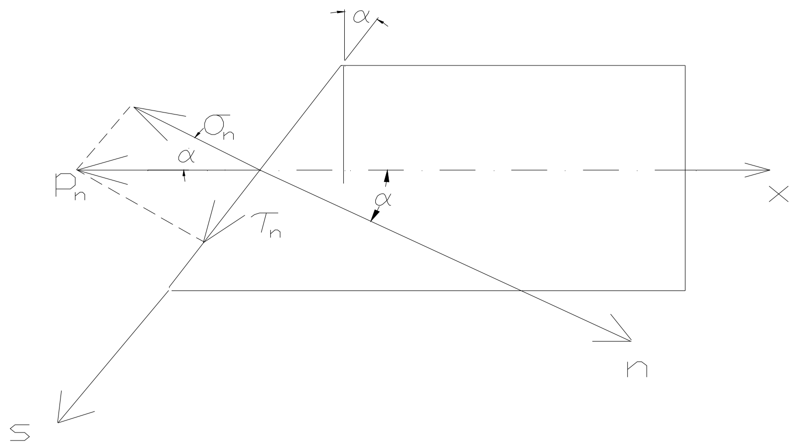


Fig.5.9

The sign (-) from the shear stress τ_{ns} appear from a reason of positive conventional orientation of the tangential stress τ in the system of axis *n–s* from section 2-2.

For different values of the angle α , different stresses σ_n and τ_{ns} are obtained. We are interested in the extreme values of σ_n and τ_{ns} which are obtained for the trigonometrically values $cos2\alpha = \pm 1$ and $sin2\alpha = \pm 1$:

$$\cos 2\alpha = 1 \Rightarrow \alpha = 0 \Rightarrow A_n = A \Rightarrow \begin{cases} \sigma_n = \sigma_x = \sigma_{max} \\ \tau_{ns} = 0 \end{cases}$$

$$\cos 2\alpha = -1 \Rightarrow \alpha = \frac{\pi}{2} = 90^0 \Rightarrow A_n = A_z \Rightarrow \begin{cases} \sigma_n = \sigma_z = 0 \\ \tau_{ns} = 0 \end{cases}$$

$$\sin 2\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4} = 45^0 \Rightarrow A_n = A_{n_{45}} \Rightarrow \begin{cases} \sigma_n = \frac{\sigma_x}{2} \\ \tau_{ns} = -\frac{\sigma_x}{2} = \tau_{min} \end{cases}$$

$$\sin 2\alpha = -1 \Rightarrow \alpha = \frac{3\pi}{4} = 135^0(-45^0) \Rightarrow A_n = A_{n_{135}} \Rightarrow \begin{cases} \sigma_n = \frac{\sigma_x}{2} \\ \tau_{ns} = \frac{\sigma_x}{2} = \tau_{max} \end{cases}$$

In conclusion, the maximum normal stresses $\sigma_{x\ max}$ appear in the cross section, while the extreme shear stresses $\tau_{max,min}$ appear in the sections inclined with 45^0.

To have a view of the state of stresses around an interior point K from bar, these results can be represented in a plan, called ***the plan of the unit stresses*** (Fig.5.10)**.**

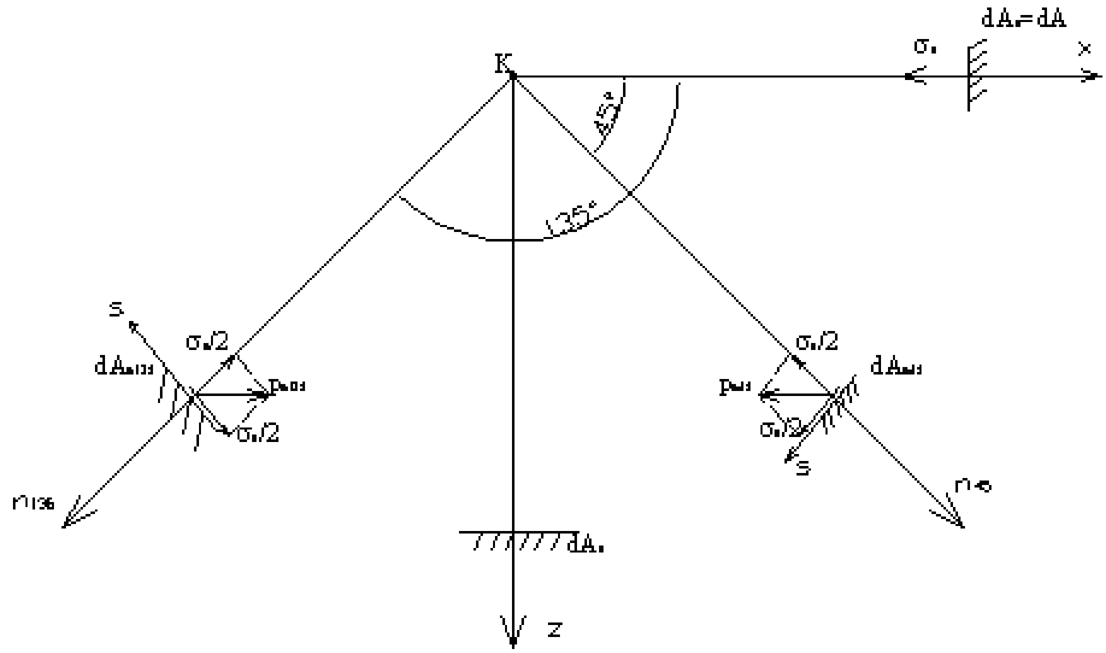


Fig.5.10

5.7 UNDETERMINED STATIC STRUCTURES (HYPERSTATIC SYSTEMS) SUBJECTED TO AXIAL SOLICITATION

Structures for which internal forces and reactions cannot be determined from statics alone are said to be ***statically indeterminate***.

A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.

In these structures, we have more unknowns than equations, that's why we introduce other relationships written in deformations in order to find the state of stresses from structure.

5.7.1 Double fixed bar actioned by a concentrate force

Let's draw the diagram of the axial force for a double fixed bar (Fig.5.11), having a constant rigidity EA.

In the fixed support the vertical reactions are introduced, R_{VA} and R_{VB}, reactions which assure the static equilibrium:

$$R_{VA} + R_{VB} = P$$

To obtain the reaction R_{VA} and R_{VB} we have to write a supplementary condition in deformations, writing the total elongation of the bar $\Delta l = 0$:

$$\Delta l = \Delta l_{A-C} + \Delta l_{C-B} = \frac{R_{VA} \cdot b}{EA} + \frac{(R_{VA} - P)a}{EA} = 0 \Rightarrow \quad R_{VA} = P\,\frac{a}{l} \text{ and } R_{VB} = P\frac{b}{l}$$

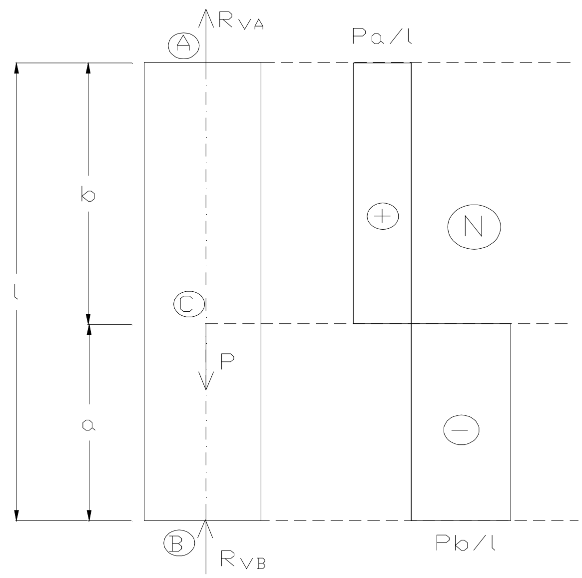


Fig.5.11

5.7.2 Undetermined system of parallel bars.

We consider a very rigid bar (bar of infinite rigidity), suspended horizontally by 3 tyrants (tie rods) made from different materials and with different areas (E_1A_1, E_2A_2, E_3A_3), so with different rigidities (Fig.5.12). The rigid bar ABCDE is hinged in A, and solicitated by a concentrate force P in E.

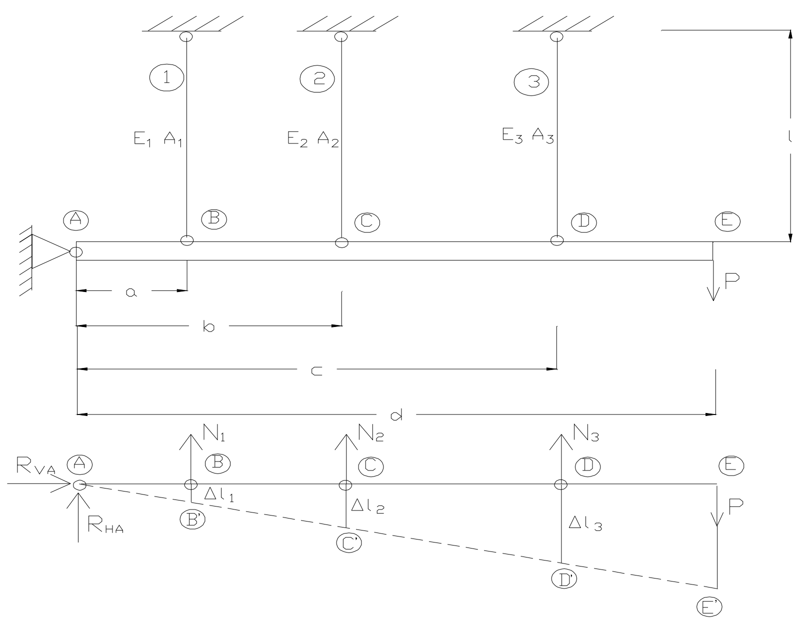


Fig.5.12

Let's compute the axial stresses N_i from rods. First, an equation of moment about the hinged support is written:

(a) $(\boldsymbol{\Sigma} M)_A = 0 : N_1 \cdot a + N_2 \cdot b + N_3 \cdot c = P \cdot d$

Equations in deformations are written considering that the rigid bar remains rectilinear after the system deformation:

(b) $\frac{\Delta l_3}{\Delta l_1} = \frac{c}{a}$ $\qquad \frac{\Delta l_2}{\Delta l_1} = \frac{b}{a}$ $\qquad \frac{\Delta l_3}{\Delta l_2} = \frac{c}{b}$

But: $\Delta l_1 = \frac{N_1 l}{E_1 A_1}$ $\qquad \Delta l_2 = \frac{N_2 l}{E_2 A_2}$ $\qquad \Delta l_3 = \frac{N_3 l}{E_3 A_3}$

(b') $\frac{N_3 a}{E_3 A_3} = \frac{N_1 c}{E_1 A_1}$ $\qquad \frac{N_2 a}{E_2 A_2} = \frac{N_1 b}{E_1 A_1}$

=> $N_3 = N_1 \frac{c}{a} \frac{E_3 A_3}{E_1 A_1}$ and $N_2 = N_1 \frac{b}{a} \frac{E_2 A_2}{E_1 A_1}$

(a') $N_1 a + N_1 \frac{b^2}{a} \frac{E_2 A_2}{E_1 A_1} + N_1 \frac{c^2}{a} \frac{E_3 A_3}{E_1 A_1} = Pd$

$$N_1 (a^2 E_1 A_1 + b^2 E_2 A_2 + c^2 E_3 A_3) = PdaE_1 A_1$$

If: $a^2 E_1 A_1 + b^2 E_2 A_2 + c^2 E_3 A_3 = \lambda$

=> $N_1 = P\,d\,\frac{a}{\lambda} E_1 A_1$; $\quad N_2 = P\,d\,\frac{b}{\lambda} E_2 A_2$; $\quad N_3 = P\,d\,\frac{c}{\lambda} E_3 A_3$

From equations of vertical and horizontal equilibrium we may calculate also the reactions from A.

5.7.3. Undetermined system of concurrent bars.

Three concurrent tie rods are subjected to tension by the concentrate force P (Fig.5.13). The inclined rods are identically (the same length and rigidity E_1A_1), while the vertical bar has the rigidity E_2A_2. Let's calculate the stresses from rods.

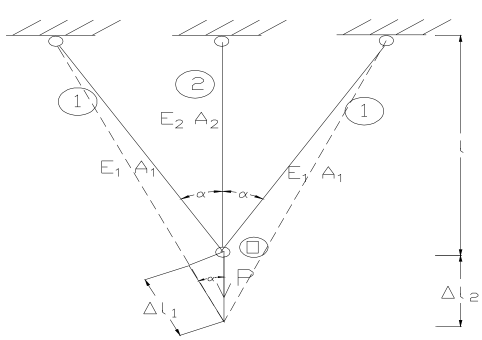
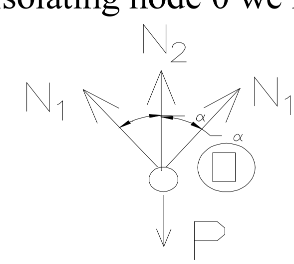


Fig.5.13

Isolating node 0 we have:



The horizontal equilibrium is an identity from symmetry reason (from geometrical and rigidity point of view), also the moment equilibrium is an identity (all forces pass through point O). The single equation which can be written is the vertical equilibrium equation:

$$2N_1\cos\alpha + N_2 = P \qquad \text{(a)}$$

After deformation the new angle $\alpha' \cong \alpha$, as the lengthening Δl_1 and Δl_2 are very small. We may write a second equation in deformations:

$$\Delta l_1 = \Delta l_2 \cos\alpha \qquad \text{(b)}$$

or: $$\frac{N_1 l_1}{E_1 A_1} = \frac{N_2 l_2}{E_2 A_2}\cos\alpha$$

but: $l_2 = l = l_1 \cos\alpha$

$$\Rightarrow \quad \frac{N_1 l_1}{E_1 A_1} = \frac{N_2 l_1 \cos\alpha}{E_2 A_2}\cos\alpha \qquad \Rightarrow \qquad N_1 = N_2\frac{E_1 A_1}{E_2 A_2}\cos^2\alpha$$

From (a): $$2N_2\frac{E_1 A_1}{E_2 A_2}\cos^3\alpha + N_2 = P$$

$$N_2 = \frac{P}{\frac{2E_1A_1}{E_2A_2}\cos^3\alpha+1} \qquad\qquad N_2 = \frac{P}{\frac{2E_1A_1}{E_2A_2}\cos^3\alpha+1}\,\frac{E_1A_1}{E_2A_2}\cos^2\alpha$$

5.7.4 The effect of the temperature variation in undetermined systems.

We'll study the same system from paragraph 5.7.3, but unloaded by the force P. After assembling the rods, the temperature in rods grows with Δt. As a consequence of temperature variation the bars will present lengthening (from dilatation), but as this lengthening isn't free, axial stress will be produced in rods. To simplify the calculus we admit the rods have the same rigidity EA (Fig.5.14).

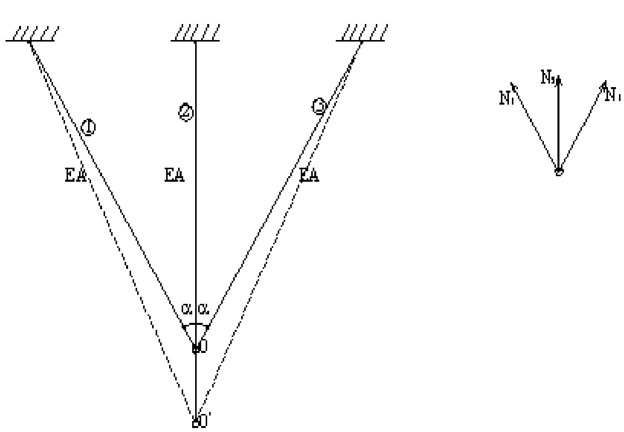


Fig.5.14

From vertical equilibrium equation:

$$2N_1\cos\alpha + N_2 = 0 \qquad \text{(a)}$$

The condition in displacements will be written also:

$$\Delta l_1 = \Delta l_2 \cos\alpha \qquad \text{(b)}$$

But in the expression of Δl we must add a term that takes into account the temperature variation:

$\Delta l_t = \alpha l \Delta t$ α: thermal expansion coefficient ($1/^{o}C$)

Equation (b) will be then:

$$\frac{N_1 l_1}{EA} + \alpha l_1 \Delta t = \left(\frac{N_2 l_2}{EA} + \alpha l_2 \Delta t\right) \cos\alpha \qquad \text{(b')}$$

With $l_2 = l_1 \cos\alpha$ =>

$$\frac{N_1}{EA} + \alpha\,\Delta t = \left(\frac{N_2}{EA} + \alpha\Delta t\right) \cos^2\alpha$$

$$\frac{1}{EA}(N_1 - N_2 \cos^2\alpha) = \alpha\,\Delta t(\cos^2\alpha - 1) \qquad \text{(b'')}$$

From (a): $N_2 = -2N_1\cos\alpha$ which introduced in (b'') give the stresses:

$$N_1 = EA \cdot \alpha\,\Delta t\,\frac{\cos^2\alpha - 1}{1 + 2\cos^3\alpha}$$

$$N_2 = -2EA \cdot \alpha\,\Delta t\,\frac{\cos\alpha\,(\cos^2\alpha - 1)}{1 + 2\cos^3\alpha}$$

5.7.5 Bars with unhomogene sections, subjected to centric tension or compression

In all previous examples of undetermined structures, we admitted that section is homogene and the unit stresses σ have uniform distribution on section. In practice there are also bars with unhomogene cross sections, as: columns from reinforced concrete, rods from copper or aluminum with steel core, etc.

If we admit that the axial force N which acts in a section of such bar is known, let's determine the repartition of the normal stresses σ in that section (Fig.5.15).

Both materials from rod will overtake a fraction from the axial force N = F, the copper: N_{cu}, the steel: $N_{ol.}$

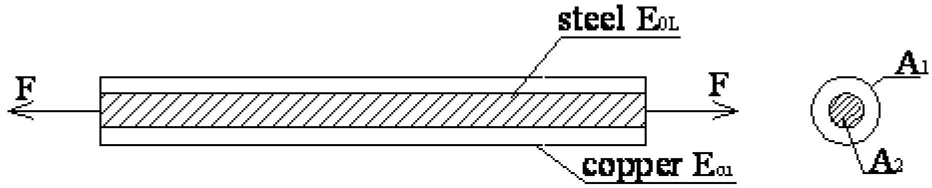


Fig.5.15

Obviously:

$N_{cu} + N_{ol} = N$ (a)

We need also a condition written in deformations. We admit that the materials are solidarized between them, so their deformations must be equal. We write this condition in specific deformations ε, from Hook's law ($\sigma = E\,\varepsilon$):

$$\varepsilon = \frac{\sigma_{ol}}{E_{ol}} = \frac{\sigma_{cu}}{E_{cu}} \qquad \text{(b)}$$

We multiply each fraction with the correspondent area A_i and writing that these fractions are equal to the sum of numerators divided to the sum of denominators, we obtain:

$$\frac{\sigma_{ol}}{E_{ol}} = \frac{\sigma_{cu}}{E_{cu}} = \frac{\sigma_{cu}A_{cu}}{E_{cu}A_{cu}} = \frac{\sigma_{ol}A_{ol}}{E_{ol}A_{ol}} = \frac{\sigma_{ol}A_{ol}+\sigma_{cu}A_{cu}}{E_{ol}A_{ol}+E_{cu}A_{cu}} = \frac{N_{ol}+N_{cu}}{E_{ol}A_{ol}+E_{cu}A_{cu}} = \frac{N}{E_{ol}A_{ol}+E_{cu}A_{cu}}$$

$$\Rightarrow \quad \sigma_{cu} = \frac{E_{cu}}{E_{ol}A_{ol}+E_{cu}A_{cu}} \quad \text{and} \quad N_{cu} = \sigma_{cu}A_{cu}$$

$$\sigma_{ol} = \frac{E_{ol}}{E_{ol}A_{ol}+E_{cu}A_{cu}} \quad \text{and} \quad N_{ol} = \sigma_{ol}A_{ol}$$

5.8 APPLICATIONS

5.8.1 A steel column sits on a concrete foundation by a metal base plate (Fig.5.16). The following information are known: the load parameters are $n_F = 1.5$ and $n_\gamma = 1.35$; the design strength of the materials are $R_{steel} = 2100$ daN/cm^2, $R_{concrete} = 55$ daN/cm^2, $R_{soil} = 3.2$ daN/cm^2 ; Young modulus of the materials are $E_{steel} = 2.1\times10^6$ daN/cm^2, $E_{concrete} = 240000$ daN/cm^2 ; specific weight of concrete $\gamma_{concrete}$=25kN/m^3. Make the strength check and dimension the steel plate (l=?). Then compute the total shortening Δl.

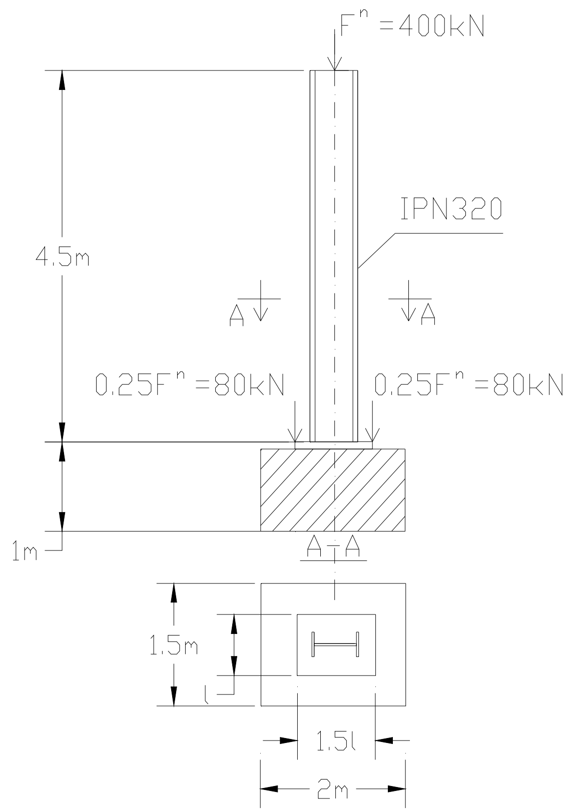


Fig.5.16

The characteristic weight of the foundation is: $G_f^k = 75\ kN$
The design weight of the foundation is: $G_f^d = 75\ \cdot 1.35 = 101.25\ kN$
The axial force diagram (both characteristic and design) is presented in fig.5.16.1.
We shall make the strength check in section 2-2 and from section 1-1 we shall dimension the steel plate. Both calculations are strength calculations made with the design axial forces N^d

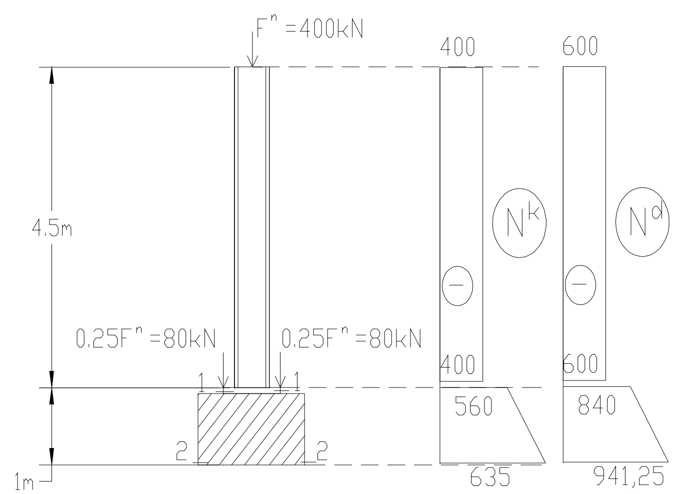


Fig.5.16.1

<u>Section 1-1:</u>

$$\sigma_x = \frac{N^d}{A} = \frac{600 \cdot 10^2}{1.5 l^2} \leq R_{concrete} = 55\ \mathrm{daN/cm^2} \rightarrow l \geq 26.96 cm \rightarrow l = 27 cm$$

Section 2-2:

$$\sigma_x = \frac{N^d}{A} = \frac{941.25 \cdot 10^2}{200 \cdot 150} = 3.13 \text{ daN/cm}^2 < R_{soil} = 3.2 \text{ daN/cm}^2$$

The total shortening Δl is calculated with formula 5.6 and using the characteristic axial force N^k diagram:

$$\Delta l = -\left[\frac{400 \cdot 10^2 \cdot 450}{2.1 \cdot 10^6 \cdot 77.8} + \frac{(560 + 635) \cdot 10^2 \cdot 100}{2 \cdot 240000 \cdot 200 \cdot 150}\right] = -0.1184cm$$

5.8.2 The bar of infinite rigidity ABC is fixed in points B and C by 2 tie rods of different rigidities (Fig.5.17). Knowing: $a = 1.2m$, $A_2 = 2A_1$, $A_1 = A_0 = 5cm^2$, calculate:

1. The axial forces N_1 and N_2 in the two rods.

2. The load parameter q from the condition that in both rods the maximum normal stress $\sigma_{x\,max} = 2000daN/cm^2$ should not be exceeded

3. If the tie rods areas $A_1 = A_2 = A_0$ calculate the vertical displacement of point D ($E = 2.1 \times 10^6$ daN/cm^2)

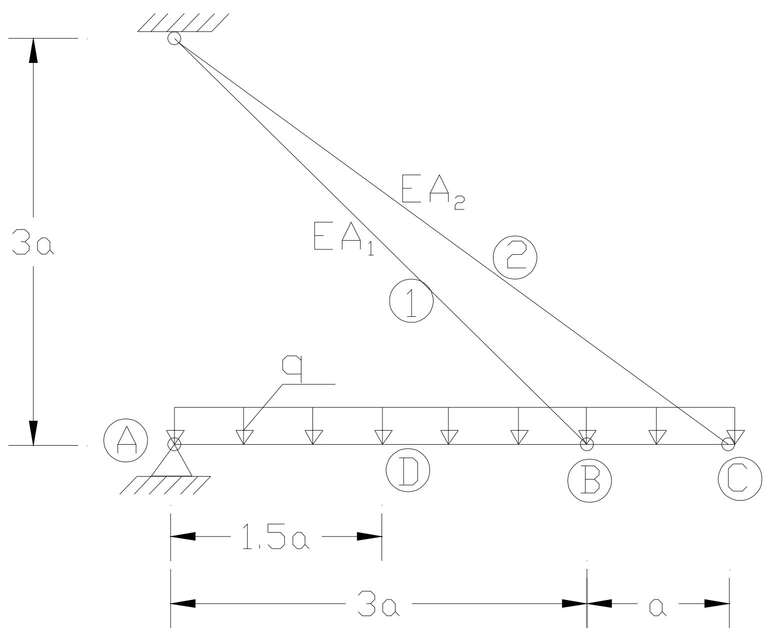


Fig.5.17

1.

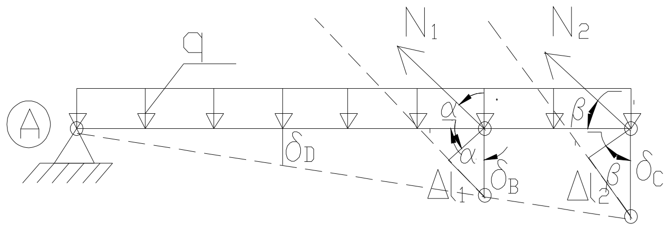


Fig.5.18

From the geometry of the structure (Fig.5.17 and 5.18):

$$l_1 = \frac{3a}{sin\alpha}$$
$$l_2 = \frac{3a}{sin\beta}$$

As the bar ABC has infinite rigidity it can only move (remaining straight), rotating around the hinge from point A (Fig.5.18). After displacement the rigid bar reaches the dotted position from Fig.5.18. Point B is displaced with δ_B, while point C is displaced with δ_C.

In points B and C from Fig.5.18 the axial forces from rods N_1 and N_2 are introduced. The moment equilibrium equation is:

$$\left(\sum M\right)_A = 0: q \cdot 4a \cdot 2a - N_1 sin\alpha \cdot 3a - N_2 sin\beta \cdot 4a = 0 \rightarrow$$

$$3N_1 \sin\alpha + 4N_2 \sin\beta = 8qa \quad (1)$$

From Fig.5.18 we may write a supplementary condition in deformations:

$\frac{\delta_B}{\delta_C} = \frac{3a}{4a}$ (2)

But:

$\delta_B = \frac{\Delta l_1}{sin\alpha}$ and $\delta_C = \frac{\Delta l_2}{sin\beta}$. With these equation (2) becomes:

$4\frac{\Delta l_1}{sin\alpha} = 3\frac{\Delta l_2}{sin\beta} \rightarrow 4\frac{N_1 l_1}{EA_1 sin\alpha} = 3\frac{N_2 l_2}{EA_2 sin\beta}$ (2') $\rightarrow N_1 = \frac{3N_2 sin^2\alpha}{8sin^2\beta}$

Replacing N_1 in equation (1) we get: $N_2 = \frac{9{,}6q}{\frac{9sin^3\alpha}{8sin^2\beta}+4sin\beta}$

Function the load parameter *q* the axial forces from rods will be:

N_2=2.739q and N_1=1.426q

2. From the condition that $\sigma_{x\,max}=2000daN/cm^2$ in both rods:

$$\sigma_{x_1} = \frac{1.426\text{q} \cdot 10^2}{5} \leq 2000 \rightarrow q \leq 70.12kN/m$$

$$\sigma_{x_2} = \frac{2.739\text{q} \cdot 10^2}{10} \leq 2000 \rightarrow q \leq 73.02kN/m$$

The load parameter q is the minimum between the two above values, so:

$\boxed{q = 70.12 \text{ kN/m}}$

3. If $A_1 = A_2 = A_0=5cm^2$, we must re-calculate the axial forces N_1 and N_2.

From the previous equation (2'') : $4\frac{N_1 l_1}{EA_1 sin\alpha} = 3\frac{N_2 l_2}{EA_2 sin\beta} \rightarrow N_1 = \frac{3N_2 sin^2\alpha}{4 sin^2\beta}$

Inserting N_1 in equation (1) and with the load parameter q= 70.12 kN/m, the axial forces from rods will be:

N_2=62.47 kN and N_1=65.07 kN

From Fig.5.18 we may write:

$$\frac{\delta_B}{\delta_D} = \frac{3a}{1.5a} \quad \rightarrow \quad \delta_D = \frac{\delta_B}{2} = \frac{N_1\frac{3a}{sin\alpha}}{2EA_0 sin\alpha} = 0.223cm$$