Chapter 4

THE PHYSICAL ASPECT OF SOLICITATION

4.1 INITIAL NOTIONS ABOUT THE MATERIALS BEHAVIOR

To understand the materials behavior we have to establish a connection between the member deformation and the internal stresses. This connection is achieved experimentally by standardized tests made on construction materials (steel, wood, concrete, etc.).

From these tests a very important connection between unit stresses and specific deformations is established, connection which can be represented by the graphs (Fig.4.1):

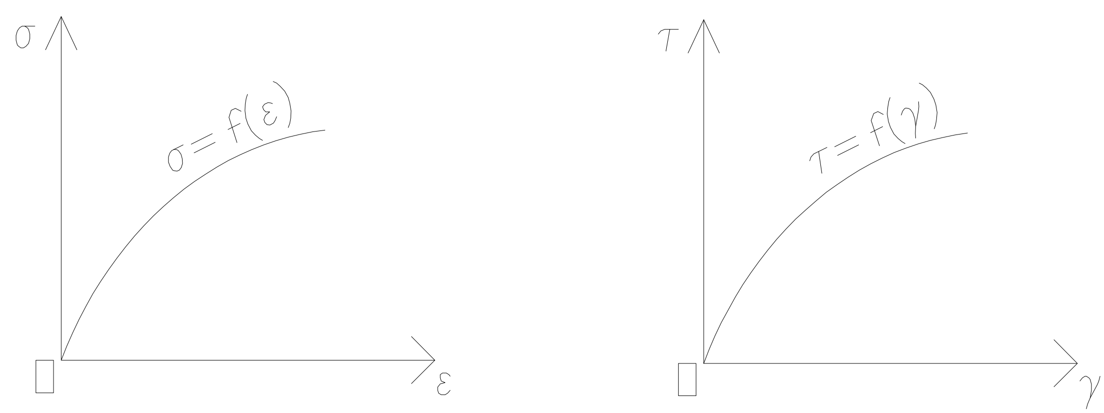


Fig.4.1

These graphs are called characteristic curves of the materials, because they characterize the behavior of materials, under loads. They depend on many factors, an essential factor being *the physical nature of the deformation*. Depending on the *nature* of the deformations, they can be:

- elastic deformations

- plastic deformations

- viscous deformations

4.1.1 Elastic deformation

For many construction elements it is a *deformation* with *small values* and it is *reversible* (when unload the tested element it returns to the initial position).

The characteristic curve is represented by a straight line passing through origin, which express the proportionality between unit stresses and specific deformations (Fig.4.2).

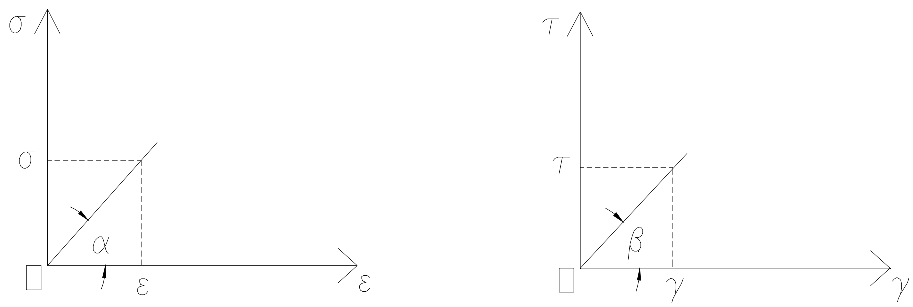


Fig.4.2

The relations (4.1) and (4.2) express ***Hooke's laws*** for simple deformations:

$$\boxed{\sigma = E \cdot \varepsilon} \qquad (4.1)$$

$$\boxed{\tau = G \cdot \gamma} \qquad (4.2)$$

$$\operatorname{tg} \alpha = \frac{\sigma}{\varepsilon} = E$$

$$\operatorname{tg} \beta = \frac{\tau}{\gamma} = G$$

Thomas Young (1773-1829)

In Hooke's laws, E and G represent:

E: modulus of longitudinal elasticity or ***Young modulus***

G: modulus of transverse elasticity or ***Shear modulus***

Ex: for steel: E =2.1×10^6daN/cm^2 $\mu = 0.3$

G= 8.1×10^5 daN/cm^2

Between E and G the following relation exists:

$$G=\frac{E}{2(1+\mu)}$$

In the above relation μ represents the coefficient of transverse contraction, known as **Poisson's coefficient (ratio).**

Siméon Denis Poisson (1781-1840)

The elongation in x-direction ε_x is accompanied by a contraction ε_y and ε_z in the other directions y and z. Assuming that the material is isotropic (no directional dependence): $\varepsilon_y = \varepsilon_z \neq 0$. With these **Poisson's ratio is** $\mu = \frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$

4.1.2 Plastic deformation

It is a *permanent* deformation, *much bigger* than the elastic deformation and it takes place without generating any internal stresses.

To illustrate the plastic deformation, we shall analyze the characteristic curve of an ideal elastic-plastic material (mild steel), known as **Prandtl's curve** (Fig.4.3).

Ludwig Prandtl (1875-1953)

After the linear elastic zone, limited by the **yield limit** (σ_c), a perfect plastic zone follows, where σ is constant, zone called **yield plateau.**

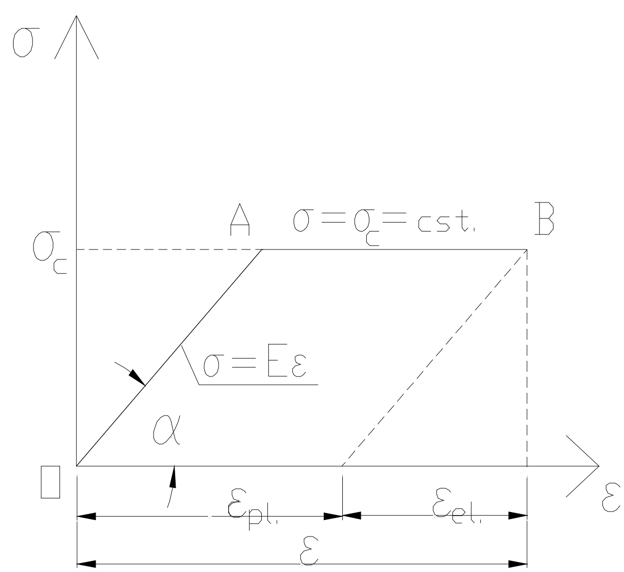


Fig.4.3

<u>O - A:</u>

$$\sigma = E \cdot \varepsilon$$

E = tg α

<u>A:</u>

$$\sigma = \sigma_c$$

<u>A - B:</u>

$$\sigma = \sigma_c = \text{constant}$$

E= $tg0^o = 0$

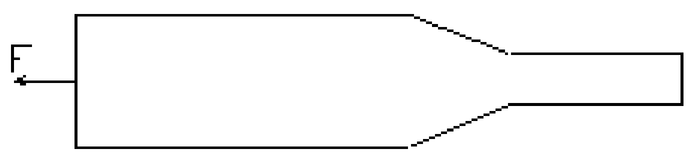
4.2. THE MECHANICAL TESTS OF THE CONSTRUCTION MATERIALS

4.2.1 Generals

Any material used in construction should be rigorously checked by mechanical tests. The tests should determine materials proprieties in direct dependence on the external factors of influence, conditions of solicitation, temperature. That's why the mechanical proprieties are relative and we discuss for any material about ***three main mechanical states*****:**

a) The ***brittle*** or ***frail state*****:** characterized by *small resistance to rupture* and *small deformations*. Example of materials which present a ***brittle rupture***: hardened steel, iron, concrete, glass, plaster, stone.

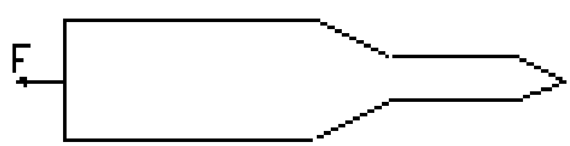
The ***fractured brittle specimen*** has this shape:



The failure surface is perpendicular to the direction of the load. This indicates that failure in brittle materials is produced mainly due to ***normal stress σ*****.**

b) The ***ductile*** or ***tenacious state*****:** characterized by *big resistance* and *big deformations*. Example of materials which present a ***ductile rupture***: common steel, aluminum alloy, copper.

The ***fractured ductile specimen*** has this shape:



We observe a cone-shaped failure surface, in which the sides of the cone make an angle of approximately 45°. This shape indicates that failure in ductile materials takes place mainly by ***shear stress τ*****.**

c) The ***plastic state*:** characterized by *small resistance* and *big deformations*. Example: the plastics.

The mechanical state of a material is influenced by:

- ***Geometrical factors:*** the specimens dimension
- ***Physical factors*:** the temperature, humidity. Example: at high temperatures the common (mild) steel passes from the ductile state to a plastic state, respectively at very low temperature it passes to a brittle state
- ***Mechanical factors*:** the type and the speed of loading, the nature of loading: statics or dynamics, the duration of loading

4.2.2 The tensile test of mild steel (Stress-strain test)

The test is realized applying to a specimen a gradually and slowly increasing tensile load F. Due to the increasing tensile load, the specimen is continuously stretched. Tensile load and specimen elongation are continuously recorded by the operator (Fig.4.5). This process is made until the specimen is fractured.

The specimen has a standard shape of circular section (Fig.4.4), with known dimensions, like length and cross-sectional area.

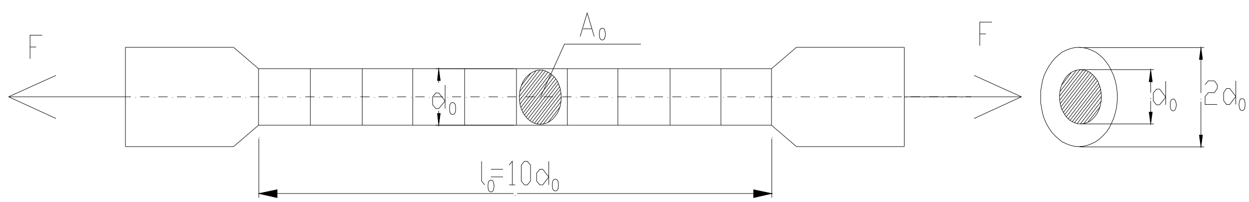


Fig.4.4

During the test, the ends of the specimen are fixed in the grips of a usually hydraulic machine (Fig.4.5), which applies a tensile force to the specimen at a prescribed loading speed.

The lengthening of the specimen, between the final marks, is measured mechanically and recorded simultaneously with the force.

Fig.4.5

If $\Delta l = l - l_0$ is the elongation in any moment of the test, the ***force-displacement diagram*** (Fig.4.6) will be:

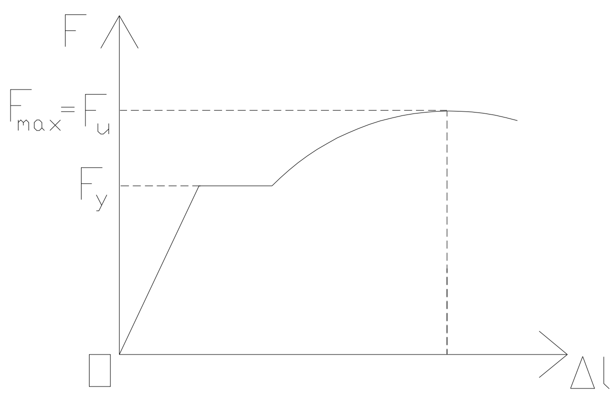


Fig.4.6

Two distinctive points can be identified on the curve (Fig.4.6):

F_y: the yield force

F_u: the ultimate (rupture) or maximum force

If we divide the quantities from the diagram ordinates by the initial geometrical characteristics of the specimen l_0 and A_0, we obtain the elongation ε and the unit stress σ:

$$\varepsilon = \frac{\Delta l}{l_0} \qquad \sigma = \frac{F}{A_0}$$

With these a new curve may be represented (Fig.4.7), named: the **characteristic curve of the structural steel (the stress-strain diagram)**.

Zone OA (Fig.4.7) present a linear variation, what shows that is this zone the normal stresses σ are proportional to the elongations ε. This shows that the material obeys Hooke's law ($\sigma = E \cdot \varepsilon$). Slope of this straight line OA provides information on the Young's modulus of the material. Point A corresponds to the ***proportionality limit*** $\boldsymbol{\sigma_p}$.

In ***zone AB*** (Fig.4.7) σ and ε are no longer proportional, but the material has still an elastic behaviour (if downloaded from point B the specimen fully recovers to the initial length l_0). Point B, an elastic limit point very close to point A, corresponds to the ***elasticity limit*** $\boldsymbol{\sigma_e}$ **.** The material is said to behave *elastically***.**

Zone BC (Fig.4.7) is an elasto-plastic zone. Point C, upper yield point of the material, corresponds to the ***yield strength (limit)*** $\boldsymbol{\sigma_y}$: $\sigma_y = \frac{F_y}{A_0}$, which is the load at upper yield point over the cross-sectional area.

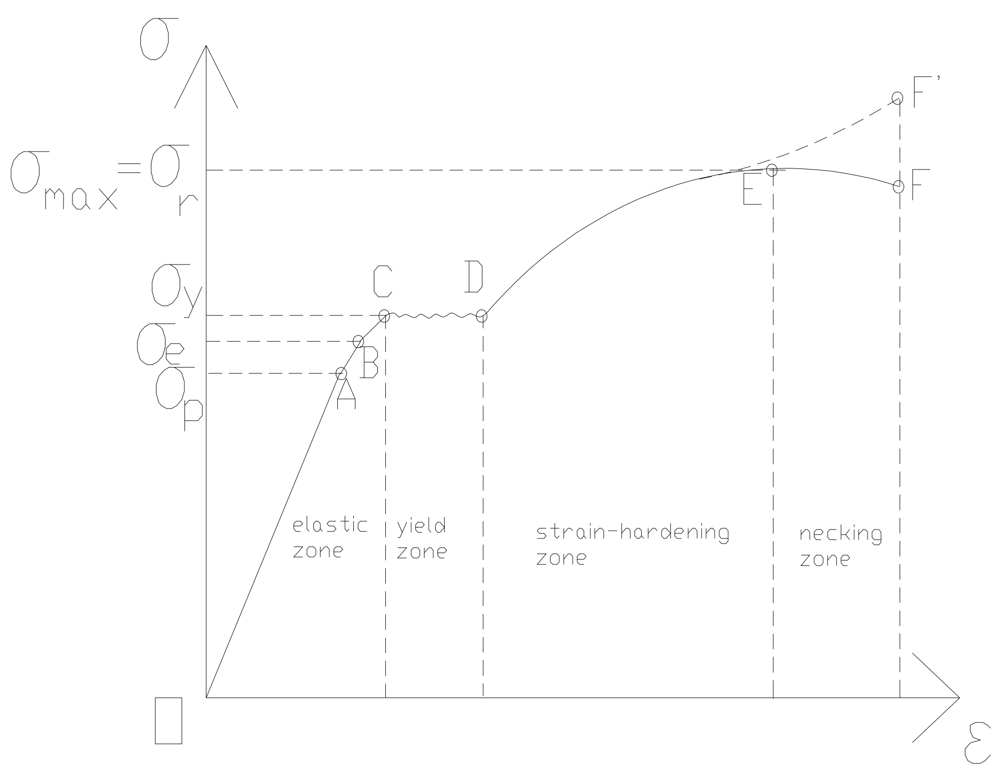


Fig.4.7

Zone CD (Fig.4.7) is called ***yield plateau,*** when the specimen presents an appreciable increase in strain, with practically no increase in stress. We say the material behave *plastically,* the material ***yield.***

Zone DE (Fig.4.7) corresponds to a ***hardening zone.*** Both σ and ε grow. Point E, *point of strain hardening,* corresponds to the ***tensile strength*** $\boldsymbol{\sigma_{max}}$ which is the ***rupture strength*** $\boldsymbol{\sigma_r}$: $\sigma_{max} = \sigma_r = \frac{F_{max}}{A_0}$.
To explain the specimen rupture, during the test it can be observed that in the vicinity of the ultimate load (F_{max}) the specimen no longer deforms uniformly along its length, but rather the deformation is concentrated on a certain region along the length, the phenomenon (Fig.4.8) being called ***necking*** (weaker section of the bar, the diameter being heavily reduced) .

The cross section presents in this portion a significant reduction, the breaking (rupture) occurring (Fig.4.8) where the specimen is weakened. In point F (Fig.4.7) the specimen breaks.

In this point F, the curve presents an apparent (Fig.4.7) decreasing of the normal stress σ, but only because the stress σ is still related to the initial area A_0 ($\sigma_{max} = \frac{F_{max}}{A_0}$). In reality, the final area of the broken specimen (Fig.4.8) is A_u, and the curve presents the dotted position (point F').

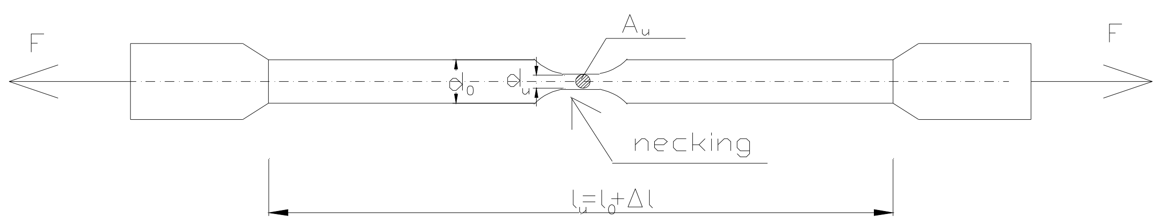


Fig.4.8

If, after rupture we join the two broken pieces we may measure the ultimate length l_u and the diameter at the neck d_u. With these, other two important mechanical properties may be determined:

- ***Percentage elongation***:
  $\delta_n = \frac{l_u - l_0}{l_0} \times 100\ (\%)$ $\qquad$ $l_u = l_0 + \Delta l$
- ***Percentage reduction in area***:
  $Z = \frac{A_0 - A_u}{A_0} \times 100\ (\%)$ $\qquad$ A_u: the ultimate area $\quad A_u = \frac{\pi d_u^2}{4}$

These two properties δ_n and Z indicate ductility of the material.