Buletinul Ştiinţific al Universităţii "POLITEHNICA" din Timişoara

Seria HIDROTEHNICA

TRANSACTIONS on HYDROTECHNICS

Tom 56(70), Fascicola 1, 2011

A MATHEMATICAL APPROACH TO SOLVE RIVER FLOODING PROBLEMS

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Abstract: Floods have accompanied mankind throughout its entire history. Although the causes for this have varied, e.g. extreme changes in the river catchments and material deposition along the riverbed, the impact is the same. Floods are dangerous to people's lives and vital interests. Floods are now the main hydrological topics worldwide. The sensitivity of environment and national economy to the impact of floods is becoming ever more pronounced. In certain developing countries such as Indonesia, the land use in the catchment area changes continuously and it is very difficult to be restrained. No wonder that flood damage in the past that was caused by discharges having a return period of 100 years, can today be the results of a 20-year maximum discharge.

Keywords: floods, mathematical modelling in rivers, discharge, flow resistance

1.1 NEED FOR MODELING RIVER

Therefore the prediction of stage, discharge, time of occurrence and duration of the flood, especially of peak discharge at a specified point on a stream is absolutely necessary. These activity is known as flood forecasting, whereas flood warning is defined as the provision of advance notice that a flood may occur in the near future at a certain station or in a certain river basin (WMO,1974).

When a region is affected, the system of flood control service is activated and operates according to previously drafted flood plans. An effective flood warning system needs to be based on accurate timely flow forecasts.

There are two main reasons to undertake numerical modelling of floodplain flow: first as an alternative to laboratory experiments or field data to improve understanding of the processes involved in floodplain flow; and • second to obtain predictions of quantities useful for the management of floodplain systems, e.g. discharge, water surface elevation, inundation extent and flow velocity.

In this context a model consists of a user's best estimate of the processes that are perceived to be relevant to the application, and may be tested by comparison to analytical solutions, scale models or field data. Physical realism is of utmost importance in the first class of application, whereas for flood management the emphasis may be on computational efficiency.

Compound channel flows are fully turbulent over a wide range of space scales and unsteady in time, but it is computationally prohibitive to simulate flows with this level of complexity. Fortunately, the processes perceived by modellers to be relevant to the accurate simulation of floodplain flow for a particular purpose are typically a small subset of the known physical mechanisms. The key step in selecting an appropriate numerical modelling framework for floodplain flows is therefore to identify those processes that are relevant to a particular modelling problem and decide how these can be discretized and parameterised in the most computationally efficient manner.

Naturally rivers flow in the lowest areas in a given topography with their discharges flowing inbank and this results in identifiable river channels. However it happens that sometimes hydrological conditions vary with high rainfall and thus higher discharges occur that cause the channel to flow in an overbank condition, resulting in an increased flow area, depth and width.

In an inbank flow condition flows may be treated as if they were predominantly one-dimensional flows in the streamwise direction. However, overbank flows must be treated differently since three-dimensional processes begin to be especially important, particularly at the interaction between the main channel and the floodplain.

Water flow in natural channels is almost always unsteady. It is a complex phenomenon and cannot be understood in all details (Miller, 1975). That is why in certain cases unsteady flow is sometimes approximated by steady flow, particularly when the change of discharge with time is very gradual. In hydraulic engineering problems it is important to recognise when an unsteady flow may properly be treated as a steady flow. The mathematical treatment of unsteady open channel flow is an important but relatively difficult problem. The difficulty exist, basically because many variables enter into the functional relationship and because the differential equations cannot be integrated in closed forms except under very simplified conditions (Mahmoud, 1975) For engineering purposes most of the solutions of unsteady flow equations are numerical with a great number and variety of techniques.

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Mathematical modelling in rivers is the simulation of flow conditions based on the formulation and solution of mathematical relationships expressing hydraulic principles. Advanced mathematical treatment of unsteady flow in open channels was started with the development of two partial differential equations. Although there were many attempts to modify and to improve them, the equations remain substantially unchanged. The equations resulted from these various attempts are more complete and sophisticated but reduce to the basic de Saint-Venant equations whenever simplified for practical use (Mahmoud, 1975. It is not possible to solve de Saint-Venant equation analytically, hence the need for numerical solution. The finite difference schemes are the practical method used to deal with natural channels, where the watercourse is simulated by a series of computational points (Cunge, 1975). This method was developed because of the limitations imposed on time step, Δt, when using explicit schemes (Ligget, 1975). Each computational points represents an elementary reach corresponding to the space step, Δx, in which each point corresponds to the cross section. This section should be selected to represent all important topographical and hydraulical features of the reach.

The flow structure of a river that is either straight or meandering channel can be represented mathematically by use of equations of fluid flow. The following section explains in brief the reduction of the mathematical equations from 3D to 2D depth averaged and to 1D flow in a river section.

2.2. FUNDAMENTAL EQUATIONS

2.2.1 Three dimensional flow

The three dimensional Reynolds averaged Navier Stokes equations describe the general motion of turbulent flow. Taking flow in one co-ordinate, as in the case of a river, the equation can be written as in equation 2.1 for a small cross sectional area for an open channel.

$$\rho\left[\frac{\partial UV}{\partial y}+\frac{\partial UW}{\partial z}\right]=\rho g S_0+\frac{\partial}{\partial y}\left(-\rho\overline{uv}\right)+\frac{\partial}{\partial z}\left(-\rho\overline{uw}\right) \tag{2.1}$$

Where A is the secondary flow term, B is the Weight component term C and D are the Reynolds stresses for vertical and horizontal planes respectively and x, y, and z are the streamwise, lateral and vertical directions respectively. U,V,W are temporal mean velocity components in the {xyz} directions, ρ is fluid density, S_0 is channel bed slope, g is gravitational acceleration and *yx* τ and *zx* τ are Reynolds stresses on planes perpendicular to the y and z directions, respectively.

Figure 2-1 shows the essential difficulty in analysing even a simple steady uniform flow in a prismatic channel due to the various forces involved at any given point. The Navier-Stokes equations apply at a single point in the fluid such as at point J. The driving gravity force is balanced by the two Reynolds stress terms which control the vertical and lateral shearing processes arising from friction forces on the channel bed and sides and also the secondary flows traverse to the mean streamwise direction of flow with velocity components V and W.

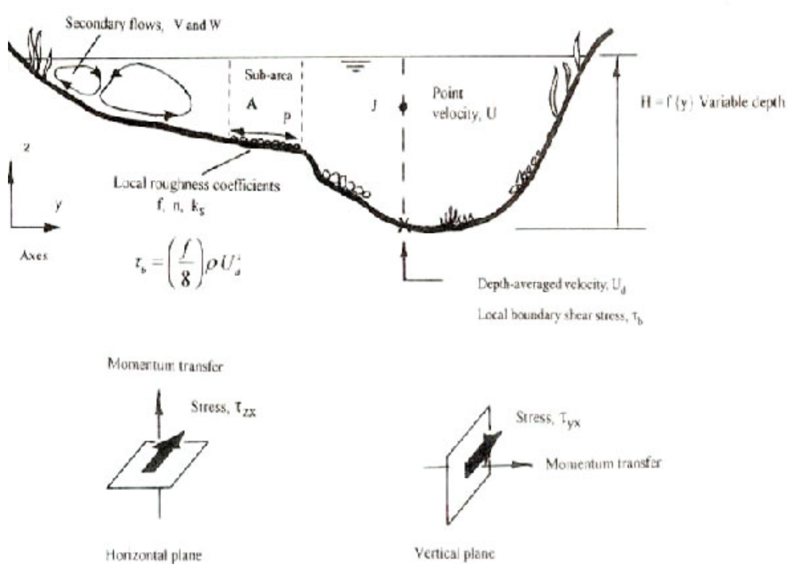

Figure 2-1: Flow in a channel (after Knight & Shiono, 1996).

2.2.2 Two dimensional flow

Usually river engineers are only concerned with the parameters at the boundaries and therefore equation 2-1 above has to be integrated over the depth, width or area. This means that the resulting 3D flow fields in figure 2.2 below has to be simplified.

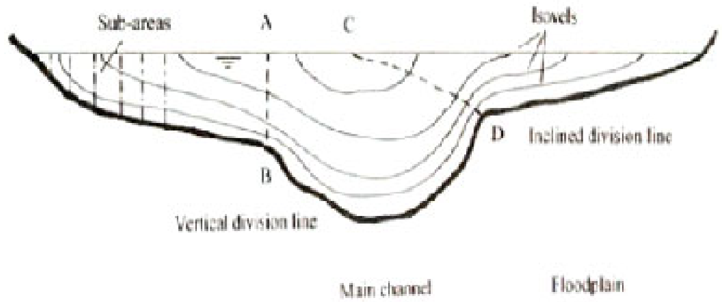

Figure 2-2: Channel subdivision methods for calculation of discharge (after Knight & Shiono, 1996)

Usually lateral distributions are of importance in rivers and due to this integration over the depth is undertaken resulting to a simplified depth-averaged form of the equation 2.2

$$\rho gHS_0 - \tau_b\left(1+\frac{1}{s^2}\right)^{\frac{1}{2}} + \frac{\partial}{\partial y}\left\{H\overline{\tau}_{yx}\right\} = \Gamma \qquad (2.2)$$

Where f, λ and Γ are the local friction factor, dimensionless eddy viscosity and secondary flow parameters respectively

2.2.3 One dimensional flow

The flow of water in channels is governed by the Navier-Stokes equations. A one dimensional version of these equations are known as St. Venant equations. The resistance laws which are generally adopted for open channel flow are those based on steady flow and include the Darcy-Weisbach, Manning and Chezy formulae. These resistance laws essentially relate the conveyance capacity of the channel to the cross-sectional shape, longitudinal bed slope and resistance parameters.

The resistance to flow in a river channel can be subdivided into the following components that are partially interconnected:

- bed grain roughness,
- form resistance associated with large-scale bed undulations,
- flow resistance associated with irregular and asymmetric cross-sectional shape,
- roughness height of flexible vegetation,
- flow resistance of stiff vegetation,
- flow resistance caused by the momentum exchange between the main channel and the floodplain,
- flow resistance caused by the momentum exchange between vegetated and non vegetated section,
- sinuosity,
- large obstructions, e.g. rocks and woody debris, and
- ice cover.

Instead of integrating over the depth, the equation 2-3 is integrated over the cross-sectional area of the channel. For instance the Manning (1857) equation is expressed as:

$$Q = \left(AR^{2/3}S_f^{1/2}\right)/n \qquad (2.3)$$

Where n is the resistance coefficient, A is the cross sectional area, R is the hydraulic radius and S_f is the frictional slope.

2.2.4 Compound Channels Flows

A compound channel is generally visualized as a two-stage channel consisting of a main channel and a wider overbank flow channel usually referred to as a floodplain which inundates during high flows, see figure 2-3. In an attempt to compute discharge conveyed in such a compound channel it is realised that it is very complex because of the change in the resistance material from the main channel, to the floodplain this varies because in the floodplain vegetation of even buildings could be expected as compared to the main channel where boulders or even in highly maintained rivers sand and small stones could be found. Again, the lateral momentum transfer between the main channel and the floodplain does decrease the discharge in the main channel and increase the discharge on the floodplain. The irregularities in the topography which result in cross-sectional irregularities further make it difficult to compute compounded channel conveyance.

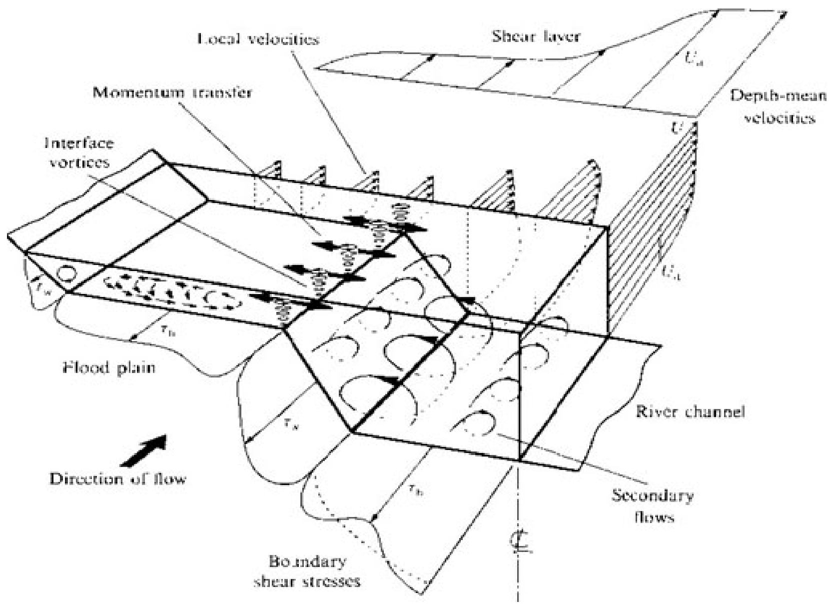

Figure 2-3: Flow structures in a straight two-stage channel (after Kinght & Shiono, 1996).

However, various studies have been carried out in an attempt to understand the compound channel conveyance and their interrelated factors that affect the flow. In Sellin,1964 study it was realised that when a river rose above bank-full discharge the overbank flow reduced the velocities of the flow contained within the main river channel due to an intensive vortex shedding at the boundary of the main channel and the floodplain and that maximum average velocities were present in near bank-full stage.

In Pasche & Rouvé (1985) observations were made that when there is no floodplain vegetation, the slope of the bank between the main channel and the floodplain especially the width of the floodplain has a significant effect on the shear stress at the interface; but when the floodplain is vegetated, the slope has no significant influence on the shear stress, although the width of the floodplain has, especially when the vegetation is very dense.

Thornton *et al.* (2000) found out that apparent shear stress at the interface of the main channel and the floodplain cannot only be quantified as a function of the local turbulence at the interface but also it was realised that it is influenced by the velocities, flow depth and vegetation density on the main channel and floodplain.

Knight (2006) explains that when discharge in a river exceeds bankfull discharge, it changes from inbank to overbank flow, a significant change in the complexity of the flow behavior results due to differences in velocities between the main channel and the floodplain flows which produce strong lateral shear layers, which lead to generation of organized plan form vortices induced by inflection point instability.

When overbank flows occur, there are major changes in the river which result and require special considerations the abrupt change at the bankfull stage, major interactions between main river and floodplain flows. The proportion of flow between sub-areas, roughness differences between river and floodplains i.e global, zonal and local friction factors, significant variation of resistance parameters with depth and flow regime and flood routing parameters basically the wave speed and attenuation among others.

In flood problem discharge and stage or water level are the two primary parameters. Knight, 2006, shows from laboratory and field stage-discharge curves for overbank, that in general Q increases with depth H, but once bankfull is reached under certain circumstances there is an actual reduction in Q despite a larger flow area.Q increases significantly due to increased flow area, with the slope of the h versus Q curve decreasing as the width of the floodplain increases.

2.2.5 Flood RoutingCunge, Holly & Verwey,1980 derived and showed that for unsteady one-dimensional flow in an open channel, the principles of mass and momentum conservation lead to the St. Venant equations, equation 2.4 and 2.5, The following assumptions are taken into account in developing the momentum and continuity equations:

Velocity is constant, and the water surface is horizontal across any channel section.

- All flow is gradually varied, with hydrostatic pressure prevailing at all points in the flow. Thus vertical accelerations can be neglected.
- No lateral, secondary circulation occurs.
- Channel boundaries are fixed; erosion and deposition do not alter the shape of a channel cross section.
- Water is incompressible (uniform density), resistance to flow can be described by empirical formulas, such as Manning's and Chezy's equation.

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{2.4}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + gA\left(\frac{\partial h}{\partial x} + s_f - s_0\right) = 0 \tag{2.5}$$

Where
Q =discharge,
A =cross sectional area of flow
q = lateral inflow/outflow per unit length.

For a momentum correction coefficient, β, equal to 1.0, the momentum correction coefficient, equation may be expressed in terms of the section mean velocity, u, to give the friction slope, S_f, as in equation 2.6:

$$\underbrace{\underbrace{\overbrace{S_f = S_0}^{\text{Steady uniform flow}} - \frac{\partial h}{\partial x}}_{\text{Steady non-uniform flow}} - \frac{U}{g}\frac{\partial U}{\partial x} - \frac{1}{g}\frac{\partial U}{\partial t}}_{\text{Unsteady non-uniform flow}} \tag{2.6}$$

Flow categories can be defined according to the number of terms used in the equation above. Steady Uniform flow will imply that the weight force balances the resisting shear force applied around the boundary wetted perimeter. Under these conditions Manning or Darcy-Weisbach equations apply. In steady uniform flow s_f=s_0 combining the equation above with resistance law e.g. manning equation yields the relationship between the unsteady Q and unsteady discharge Qn as in equation 2.7:

$$Q = Q_n\left[\underbrace{\underbrace{\overbrace{1}^{\text{Kinematic wave}} - \frac{1}{s_0}\frac{\partial h}{\partial x}}_{\text{Diffusion wave}} - \frac{U}{gS_0}\frac{\partial U}{\partial x} - \frac{1}{gS_0}\frac{\partial U}{\partial t}}_{\text{Full dynamic wave}}\right]^{1/2} \tag{2.7}$$

Where, these terms are grouped to indicate different levels of flood routing model i.e. Kinematic, diffusive and fully dynamic wave.

The convective-diffusion equation

The diffusion model results from combining equation 2.4 and 2.7 to give the convective –diffusion equation, which is represented as

$$\frac{\partial Q}{\partial t} + C\frac{\partial Q}{\partial x} = D\frac{\partial^2 Q}{\partial x^2} \tag{2.8}$$

Where C is Kinematic wave speed and D is the diffusion coefficient given by:

$$C = \frac{1}{B}\frac{\partial Q}{\partial h} \tag{2.9}$$

$$D = \frac{Q}{(2BS_0)} \tag{2.10}$$

Discharge in a channel during a flood event has characteristics of a wave that translates and attenuates; however in river engineering C and D are functions of discharge Q, as shown by equation 2.9 and 2.10. The gradient of the stage discharge curve is related to the kinematic wave speed by equation 2-9 and it indicates that during a flood C will vary with Q as dQ/dh and B change with time.

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