Buletinul Ştiinţific al Universităţii POLITEHNICA Timişoara

Seria HIDROTEHNICA

TRANSACTIONS on HYDROTECHNICS

Volume 59(73), Issue 2, 2014

DISCUSSION ABOUT THE SHEAR STRESS DISTRIBUTION TERM IN OPEN CHANNEL FLOW

David Ioan [1] Sumălan Ioan[1] Ștefănescu Camelia[1]

Abstract: The modelling of open channel flows as 1D scheme is usually performed by advanced numerical models developed in the last years, in which an essential problem is the approach of the shear stress at the channel bed. Starting from fundamental equations of the hydrodynamics, in the paper, the basic equation for 1D open channel flow will be obtained and discussed concerning the approaches of the term which contain the shear stress for both literature approach and numerical models.

Keywords: open channel flow, fundamental equations, shear stress, hydraulic radius, resistance radius

1. INTRODUCTION

In the modelling of the flow in the open channels the theory is based on the fundamental equations developed for a current tube in the 1 D case, with the mean velocities on cross sections. Based on these equations, the later development has a general character in the sense that the term referring to shear stress at the rigid walls of the current tube is not specified. For this reason there are significant differences to appreciate the term representing shear stress by different authors. These differences are encountered even in the advanced developed numerical models on all over the world, for example HEC-RAS or Mike 11.

The paper aims a discussion about the term referring the shear stress at the rigid wall of the current tube in the open channel flow by comparing approaches from the specialty literature used especially in the advanced numerical models existing on all over the world.

2. THE DYNAMICS EQUATION FOR STREAM TUBE

Technically, the pipe flow and the open channel flow are stream tubes. The hydrodynamic equation for such a stream tube can be obtained from the basic equation of the hydrodynamics for a control volume.

This equation in its turn is obtained on the base of the linear momentum balance (dynamics equation) for a moving fluid body and shows that the time rate of change of the total linear momentum of a fluid body which at the time t occupied the spatial volume V_t is equal to the resultant force acting on the considered body [1], [2]:

$$\frac{d}{dt}\int_{V_t} \rho\, \vec{v}\, dv = \int_{V_t} \rho\, \vec{f}\, dv + \int_{S_t} \vec{t}\, dS \qquad (1)$$

where ρ is the water density, $\vec{f}$ is the intensity of the mass forces (e.g. gravity forces $\vec{f}=\vec{g}$, $\vec{v}(\vec{r},t)$ is the fluid velocity in the spatial point $\vec{r}(t)$, $\vec{t}$ is the external *stress vector* (i.e. surface traction force per unit area which acts on the boundary S_t) and V_t is the moving fluid body as material system which at the time t occupied the space volume V_t limited by his boundary surface S_t.

Using the Reynolds Transport Theorem i.e. the derivation under the integral sign over V_t from (1) the following dynamic equation will be obtained [1], [2]:

$$\int_{V_t} \frac{\partial}{\partial t}(\rho\vec{v})dV + \int_{S_t} \rho\vec{v}\cdot\vec{n}dS = \int_{V_t} \rho\vec{f}dV + \int_{S_t} \vec{t}dS \qquad (2)$$

$\vec{n}$ is the outward unit-normal vector on the boundary surface S_t.

In the applied hydrodynamics like open channel flow it is quiet useful to use specific spatial region called *control volume* denoted V_C bounded with its S_C *called control surface*. If the control volume V_C which coincides at the time t with the space volume V_t (i.e. $V_C \equiv V_t$) the dynamics equation (2) maintain their mathematical form [1], [2]:

$$\frac{\partial}{\partial t}\int_{V_C} \rho\vec{v}dV + \int_{S_C} \rho\vec{v}_{fa}\cdot\vec{n}dS = \int_{V_C} \rho\vec{f}dV + \int_{S_C} \vec{t}dS \qquad (3)$$

In this equation $\vec{v}_{fa}$ represents the absolute velocity of the fluid crossing the boundary surface *Sc*.

The dynamics equation (3) can be extended for a deformable spatial control volume V_{Ct} which at the time t is occupied from the moving fluid (i.e. $V_{Ct} \equiv V_t$)

[1] Politehnica University of Timisoara, Department of Hydrotechnical Engineering, G.Enescu Street, no.1A, Timisoara, Romania, e-mail address:Ioan.David@gmx.net; ioan.sumalan@upt.ro; achim_camelia@yahoo.co.uk

and whose boundary S_{Ct} (i.e. the control surface) moves at a given speed $\vec{v}_{S_C}$ which can differed from the fluid velocity $\vec{v}_{fa}$:

$$\frac{\partial}{\partial t}\int_{V_{Ct}} \rho\vec{v}dV + \int_{S_{Ct}} \rho\vec{v}_r \cdot \vec{n}dS = \int_{V_{Ct}} \rho\vec{f}dV + \int_{S_{Ct}} \vec{t}dS \quad (4)$$

where $\vec{v}_{fr}$ is the elative velocity of the fluid crossing the moving boundary S_{Ct} of the deformable control volume V_{Ct}.:

$$\vec{v}_{fr} = \vec{v}_{fa} - \vec{v}_{Sc} \qquad (5)$$

In this case in equation (4), different from equation (3): the partial derivative with respect to time in the first term ($\frac{\partial}{\partial t}$) refers to the time dependence through the velocity $\vec{v}(\vec{r},t)$ and through the moving control volume $V_{Ct}(t)$ without considering the indirect variation respect to time t through $\vec{r}(t)$ via velocity $\vec{v}(\vec{r}(t),t)$, like it the total derivative ($\frac{d}{dt}$) would considered.. In [3] is given the same form where the first term, incorrect, as material (or total) derivative is considered. One can prove that this form leads in some case to errors. Such a case is that of the basic equations (Sant-Venant) for open channel flow.

To obtain the *dynamic equation for open channel* flow the general equation the momentum equation (4) will be applied for a stream tube portion which schematized the open channel or river flow (Fig. 1). As control volume an elementary stream tube portion in the vicinity of a current cross section will be considered which is delimited by flux surfaces A_l and A_{l+dl} (i.e. a stream tube portion having an elementary length of *dl*).

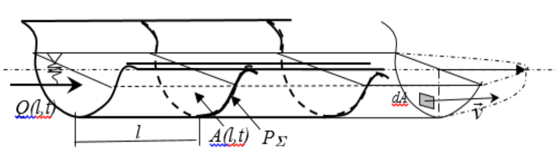

Figure1.Stream tube scheme for open channel flow

The equation (4) will be integrated for the elementary control volume and projected on the stream tube axis obtaining by this way the so called Saint-Venant Equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial l}\left(\frac{\beta Q^2}{A}\right) + gA\frac{\partial}{\partial l}\left(\frac{p}{\rho g} + z\right) = \frac{1}{\rho}\int_{P_\Sigma} \tau_\Sigma dP_\Sigma \quad (6)$$

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial l}\left(\frac{\beta Q^2}{A}\right) + \frac{\partial}{\partial l}\left(\frac{p}{\rho g} + z\right) = \frac{1}{\rho g A}\int_{P_\Sigma} \tau_\Sigma dP_\Sigma \qquad (6')$$

where β is the momentum coefficient:

$$\beta = \frac{1}{A}\int_{P_\Sigma}\left(\frac{v^2}{v}\right)dA \qquad (7)$$

Usually in the praxis can be take β=1.

Follow up remain to make explicit the integral from the right side of the equation (6) which represent the resistance effect of the wetted wall of the open channel or of the pipe determined by shear stress τ_Σ which for an arbitrary flow section is unknown.

To obtain a global physical meaning of the resistance term, steady flow regime will be considered when

$$\frac{\partial Q}{\partial t} = 0 \text{ and } \frac{\partial A}{\partial t} = 0 \qquad (8)$$

In this case under consideration of continuity equation (i.e. Q= vA=const.) the basic equation (6) can be written in the form:

$$\frac{1}{\rho g A}\int_{P_\Sigma} \tau_\Sigma dP_\Sigma = I_E \qquad (9)$$

where I_E is the energy sloop which represent the hydraulic energy loss per unit length of the stream tube:

$$I_E = \frac{\partial}{\partial l}\left(\frac{v^2}{2g} + \frac{p}{\rho g} + z\right) \qquad (10)$$

The relation (9) represent for the practically use a very important connection between the shear stress τ_Σ and the global hydraulic energy loss I_E expressed.

Through introduction the hydraulic radius defined as ratio between flow area and wetted perimeter

$$R_h = \frac{A}{P_\Sigma} \qquad (11)$$

relation (9) can be expressed in an equivalent form

$$\frac{1}{\rho g P_E}\int_{P_\Sigma}\tau_\Sigma dP_\Sigma = R_h I_E \qquad (12)$$

The most important problem for the practically use of the basic equation (6) for modelling is the evaluation of the resistance term because the shear stress distribution along the stream tube wall is generally unknown. This aspect will be discussed in the next paragraph.

3. EXPLANATIONS OF THE HYDRAULIC RESISTANCE TERM

3.1 PIPES WITH CIRCULAR FLOW SECTION

As mentioned above an important problem for the practically use of the basic equation (6) is the evaluation of the resistance term because the shear stress distribution τ_Σ along the stream tube wall is generally unknown. As basis we consider the simplest case i.e. pipes with circular flow section because in this case the shear stress τ_Σ is constant $\tau_\Sigma = \tau_0$. In this case from relation (12) we obtain the resistance term in an explicit form:

$$\frac{1}{\rho g P_\Sigma}\int_{P_\Sigma}\tau_\Sigma dP_\Sigma = \frac{\tau_0}{\rho g} = R_h I_E \qquad (13)$$

To approach the resistance term there are several empiric procedures which can be used. The most usually approach is *the Chezy proposal* only valid for quadratic turbulent flow regime:

$$\frac{\tau_0}{\rho g} = \frac{v^2}{C^2} \qquad (14)$$

with C the Chezy cofficient. From (13) and (14) we obtain the much known Chezy formula for on the flow cross section averaged velocity:

$$v = C\sqrt{R_h I_E} \qquad (15)$$

and the for the total discharge respectively:

$$Q = Av = AC\sqrt{R_h I_E} \qquad (16)$$

The Chezy C coefficient usually is given after Manning-Strickler:

$$C = \frac{1}{n} R_h^{1/6} \qquad (17)$$

with n the Manning roughness coefficient.

Another approach is the *Darcy-Weissbach proposal* using a coefficient λ which for circular pipe is expressed with the Colebrook-White formula [2]:

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2,71}{R_e\sqrt{\lambda}} + \frac{k}{3,71D}\right) \qquad (18)$$

where k is the wall roughness, D is the pipe diameter and R_e the Reynolds number defined as

$$R_e = \frac{vD}{\nu} \qquad (19)$$

where ν is the kinematic viscosity coefficient.
In opposite to the Chezy approach the Darcy approach is general valid for all flow regime.
For quadratic turbulent flow regime can be use a simplified formula:

$$\frac{1}{\sqrt{\lambda}} = -2.03 lg\left(\frac{k_s}{14.84\,Re}\right) \qquad (18')$$

The connection between the two coefficients is expressed in the following relation:

$$C^2 = \frac{8g}{\lambda} \qquad (20)$$

For on the flow cross section averaged velocity can be expressed in term of Darcy coefficient as follow:

$$v = \sqrt{\frac{1}{\lambda} 8g\ R_h I_E} \qquad (21)$$

3.2 OPEN CHANNEL FLOW (RIVERS, CHANNELS)

In the case of open channels the shear stress τ_Σ is not more constant (Fig. 2) and so the integral expression of hydraulic resistance in the form of (13) is not more valid.

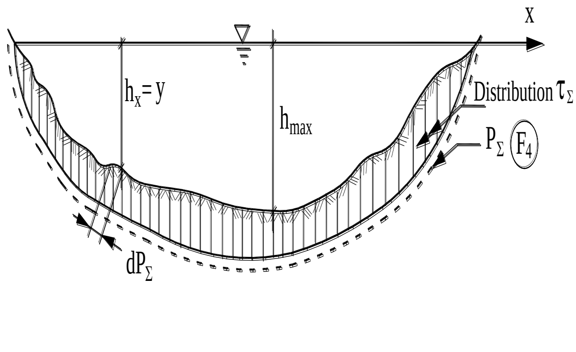

Figure 2. Share stress distribution in open channel flow

In this case the shear stress distribution τ_Σ is unknown, no more constant, depending from the local depth y. To prove that we can apply the simplified basic equation of steady flow (12) for an elementary flow section considered in a current point x of the entire flow cross section (see Fig. 2 and Fig.3).

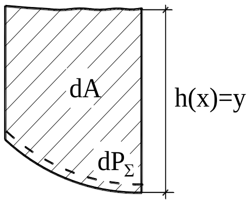

Figure 3. Elementary cross section

Equation (12) applied to elementary section, by neglecting the lateral shear stress and taking into account that the hydraulic radius R_h=h(x), has the following expression:

$$\frac{1}{\rho g dP_E}\int_{dP_\Sigma}\tau_\Sigma(x)dP_\Sigma = h(x)I_E \qquad (22)$$

Along the elementary wetted perimeter dP_Σ the shear stress τ_Σ is practically constant so that integral in (21) can be performed. Consequently we obtain the friction stress distribution along flow cross section boundary:

$$\frac{\tau_\Sigma}{\rho g} = h\left(x\right)I_E = yI_E \qquad (23)$$

Consequently the estimation of the global friction should be investigated.

4. SHARE STRESS AND FRICTION TERM EVALUATION VARIANTS FOR OPEN CHANNEL FLOW

4.1 SIMPLIFIED ESTIMATION AS UNIQUE FLOW CROSS SECTION

The shear stress τ_Σ is considered like in the case of the circular pipes, by introducing an *averaged value* over the cross section noted $\tilde{\tau}_\Sigma$. With this assumption the friction term in the basic equation can be expressed like at the pipe presented above resulting:

$$\frac{1}{\rho}\int_{P_\Sigma}\tau_\Sigma dP_\Sigma = \frac{\tilde{\tau}_\Sigma}{\rho}P_\Sigma \qquad (24)$$

This means that by accepting for τ_Σ an average values τ_0 remain valid the considerations discussed in paragraph 3.1, including both approach Chezy and Darcy and the formulas (15) – (20).

It is quite common at modelling open channel flow to use the concept of conveyance K defined as:

$$K = CA\sqrt{R_h} \qquad (25)$$

With the conveyance the flow discharge Q can be expressed as:

$$Q = CA\sqrt{R_h I_E} = K\sqrt{I_E} \qquad (26)$$

with I_E the energy slope.

In this case of *simplified approach* for a structured/composed cross section represented in (Figure 7) the hydraulic radius is the ratio of the total area and total wetted perimeter of the flow section:

$$R_h = \frac{A_1 + A_2 + A_3}{P_1 + P_2 + P_3} \qquad (27)$$

It will be shown in the next paragraph that this simplified approach can lead to huge errors in comparison with an approach which take into account different flow conditions in the channel and floodplain.

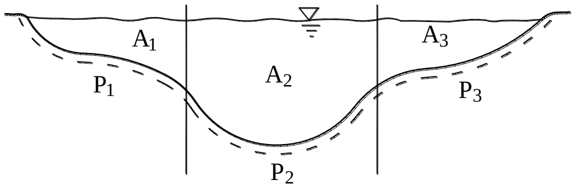

Figure 4. Flow conditions in the certain composed flow area

4.2 STRUCTURED FLOW CROSS-SECTION

In reality the flow area in not unique, generally being composed one (Fig.7). Taking into account the different flow condition in the main channel and both flood plane the following relationships can be set:

$$Q_i = K_i\sqrt{I_{Ei}}\ ;\ i=1,2,3 \qquad (28)$$

where Q_i are the flow discharges in the main channel (i=2) and in the flood plans (i=1,3) (Fig.4), K_i are the conveyances and I_{Ei} are the energy sloops.
Using the continuity equation the total flow discharge of the channel can be obtained:

$$Q = Q_1 + Q_2 + Q_3 \qquad (29)$$

With the realistic assumption that the energy slopes are equal in all parts of the flow cross sections i.e.:

$$I_{Ei} = I_E;\ i=1,2,3 \tag{30}$$

So it is useful to introduce an equivalent Conveyance K for the entire flow section A. Using the continuity equation we obtain the following equation for conveyances:

$$K = K_1 + K_2 + K_3 \tag{31}$$

with:

$$K_i = C_i A_i \sqrt{R_{hi}}\ ,\qquad R_{hi} = \frac{A_i}{P_i}\ ,\quad i=1,2,3 \tag{32}$$

Consequently for the calculation of the total flow rate can be used the same formula (25) and (26) but *don't use the same hydraulic radius* calculated with the formula (27) according to *simplified approach*. In this case the conveyance will be calculated form (31) which takes into account the different flow conditions in main channel and flood plans (Figure 7). Replacing in (31) the conveyances K and K_i is obtained the following formula for calculation of the hydraulics radius

$$R_h = \left(\frac{n_e}{A} \sum_{i=1}^{3} \frac{A_i}{n_i} R_{hi}^{2/3} \right)^{3/2}, \tag{33}$$

where n_i are the Manning coefficients of the wetted perimeter of the sub cross sections and n_e an equivalent Manning coefficients for the total wetted perimeter. If n_i=n=constant

For the coefficient n_e there are different formula in technical literature. The simplest is the weighted average on wetted perimeter:

$$n_e = \frac{\sum_{i=1}^{3} n_i P_i}{P} \tag{34}$$

In HECRAS software is used []:

$$n_e = \left(\frac{\sum_{i=1}^{3} n_i^{3/2} P_i}{P} \right)^{2/3} \tag{35}$$

In both cases if n_i=n results n_e=n.

In order to show the differences between the two conceptions and is considered a relevant example of compound flow area (rectangular) shown in Figure 5, for which the conveyance in both variants calculated and compared.

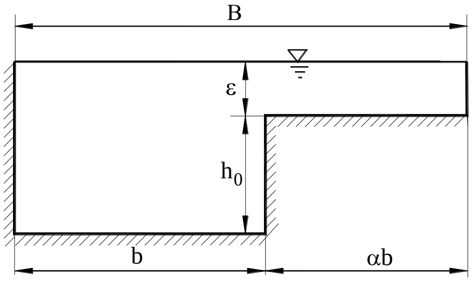

Figure 5. Compound rectangular flow area

In the unique flow area concept (variant I) applied formulas (25), (27) we obtain:

$$K_I = \frac{1}{n} \frac{b\, h_0^{5/3}}{\left(1+\alpha\right)^{2/3}} \tag{36}$$

The terms containing multiplication of ε are neglected.

By considering o compounded flow area (variant II) the following relationships will be obtained:

$$\begin{aligned} A_2 &= b\left(h_0 + \varepsilon\right) \cong b h_0 \\ P_2 &= b + 2h_0 \cong b \\ A_3 &= \alpha b \varepsilon \\ P_3 &= \alpha b + 2\varepsilon \end{aligned} \tag{37}$$

and for the conveyance in this variant is obtained:

$$\begin{aligned} K_{II} &= K_2 + K_3 \\ K_{II} &= \frac{1}{n} b h_0^{5/3} + \frac{1}{n} \\ \alpha b \varepsilon_0^{5/3} &\cong \frac{1}{n} b h_0^{5/3} \end{aligned} \tag{38}$$

It is observing than in two different variants of conception (unique flow area and compounded one respectively) the conveyance is not the same. The parameter αb represents the floodplain width (Figure 5.

The ratio of both conveyances put in evidence the difference between these variants of modeling approaches:

$$\frac{K_I}{K_{II}} = \frac{1}{\left(1+\propto\right)^{2/3}} \tag{39}$$

The numerical values are represented in Figure 6.

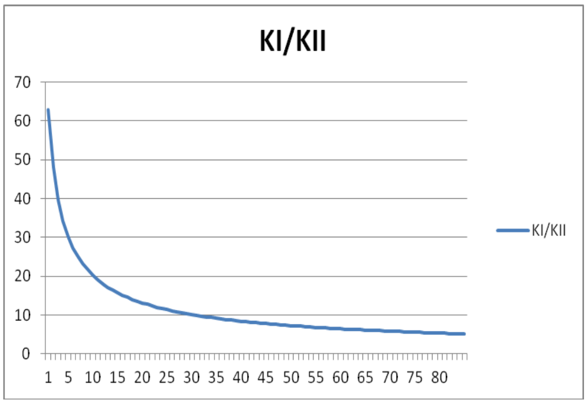

Figure 6. The graph of KI/KII % in terms of α parameter

REMARKS:

It can be observed that the unique flow area variant can be only used in the cases in which the value of KI*/KII* is being limited at an acceptable relative error of 3-5% percentage;

It also can be mentioned that high error occurs only at the start of the floodplain inundation (small values of ε compared with water depth h_0);

If the value of ε rises at comparable values with water depth in the channel, the differences between KI and KII is reduced.

CONCLUSIONS

The friction term in the basic equation is usually approximated using the global conveyance for the flow cross section and the Manning-Chezy approach. The various programs in specialized literature use various methods for both Manning coefficient and for the hydraulic radius. As examples we mention the most used flood flow modeling software in rivers, like the commercial software HECRAS and MIKE 11. It can be seen that the modelling results are generally good, although a relatively coarse simplification was accepted.

Problems can arise, however, in the case of flows with structured flow cross sections, when in addition to the main channel exist the left and right overbank flow. In this case the determination of total conveyance for the entire cross-section requires the subdivision into units for which the velocity is relative uniformly distributed is and so an averaging on these cross-section units is possible.

In the paper it is shown that accepting a single section, which is a simple sum of the parts of subdivisions can lead to considerable errors especially for relatively small depths riverbed (early flooding major riverbed). For this reason it is recommended to determine of total conveyance using weighted averaging.

REFERENCES

[1] Truesdell, C. and Toupin, R.; The Classical Field Theories, in S. Flugge(ed.), "Encyclopedia of Physics," Vol III, Pt. 1, Sppringer-Verlag OHG, Berlin, 1960.

[2] David, I., Hidraulica, Politehnica University of Timisoara, Timisoara, Vol. I, 1982, 1990

[3] White, M., F., Fluid Mechanics, McGraw-Hill, 2003

[4] Mohammadi, M., Shape effects and definition of hydraulic radius in Manning's equation in open channel flow, International Journal of Engineering, Vol.10, No.3, 1997, pp127-141

[5] Chow, V.T., Open-Channel Hydraulics, McGraw-Hill Book, Co., New York, 1959

[6] DEFRA Environmental Agency, Reducing Uncertaining in River Flood Conveance. Review of Methods for Estimating Conveyance. Project W5A-057, 2003

[7] http://www.dhigroup.com/