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Methods for Obtaining the Intersection Curves for the Cylindrical Surfaces

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Abstract: This paper, based on the descriptive and analytical geometry, shows the trace elements of the unfoldings various intersections of geometric corps, the mathematical relations of calculation, necessary to determine some characteristic points.

The paper presents some considerations on the theory of unfolding the cylindrical surfaces, solved using the descriptive geometry and mathematical approach of the problem. As an application, three cylinders of different diameters, and inclined at an angle are considered.

Keywords: cylinder, intersection curve, unfolding, methods.

1. THEORETICAL CONSIDERATIONS

The variety and the big frequency of the calculation and construction problems of the corps unfoldings, used in various industrial installations, made by wrapping sheet, requires a graphical and analytical solving of the encountered cases.

The paper presents some considerations on the theory of unfolding the cylindrical surfaces, solved using the descriptive geometry and mathematical approach of the problem.

The rapid introduction of modern methods to perform various problems in practice, give the possibility of using the computers to solve the unfolding problems.

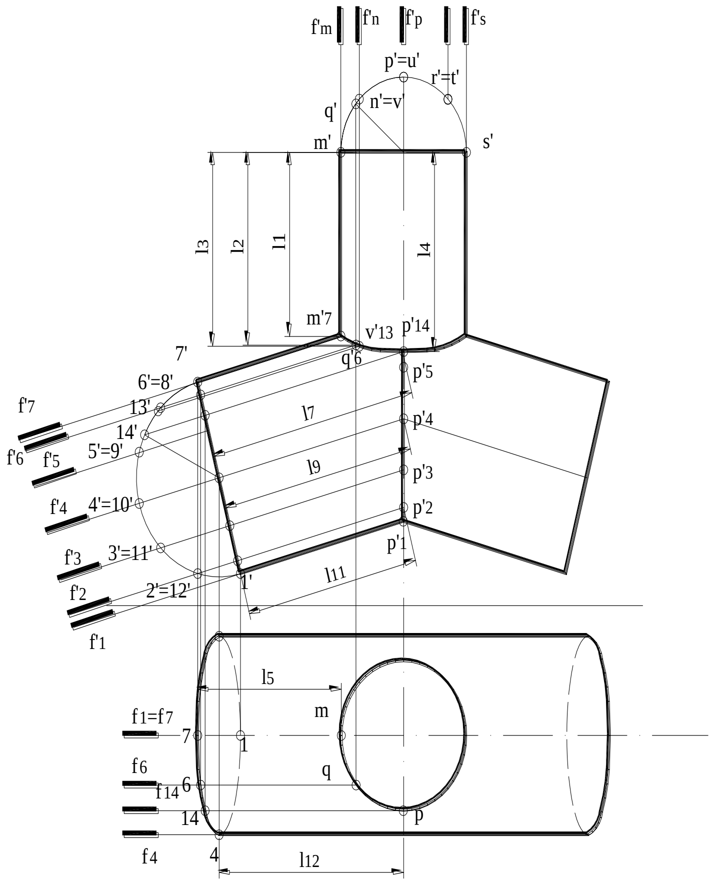

Fig. 1. Connections

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It is considered that application encountered in practice, the intersection of three cylinders: 1 cylinder C, having the diameter **D = 30 mm** and 2 cylinders C_1, having the diameters $D_1 = 40\text{mm}$ and the axes inclined to 15 ^{0} (Fig.1).

2. DESCRIPTIVE GEOMETRY METHOD

The connections, as those shown in Figure 1 are common in industrial installations. The problem is reduced to the intersection of two cylinders, of different diameters and axis inclined and concurrent, and for the two cylinders with equal diameters to the intersection of a cylinder with a plan.

a) To determine the points of intersection between the cylinder with the vertical axis and the cylinder with inclined axis (other intersection is similar), the cylinder bases are equally divided. Thus, the large cylinder base is divided into 12 $1,2,\ldots,1$ equal parts, and the lower cylinder in 8 $m, n,\ldots,m$ equal parts. Through these points, the $\overline{f_1}, \overline{f_2},\ldots,\overline{f_6}$, respectively $\overline{f_m}, \overline{f_n},\ldots,\overline{f_s}$ auxiliary planes will be constructed.

At the intersection of the generators contained of the f_6 and f_m planes, the $m7$ point results, belonging to the curve of intersection, and similarly, the $q6$, $v13$ and $p14$ points are obtained.

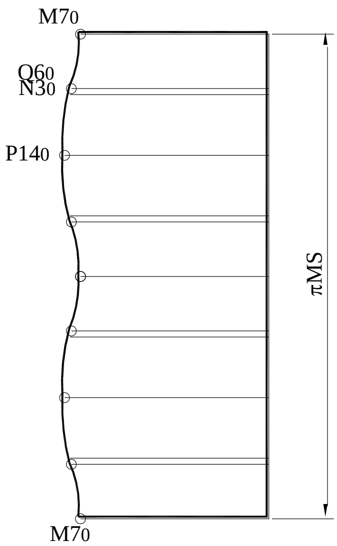

Fig. 2. Unfolding of the cylinder with vertical axis

b) To determine the points of intersection of the two cylinders with equal axes, the f'_m intersection of a cylinder and an auxiliary plane is considered. Thus, at the intersection of this plane with the generators, belonging to the $f_1, f_{2,\ldots}$ planes, the $p_1, p_2,\ldots,p_5, p_{14}$ points will be obtained.

The unfolding of the small cylinder, with vertical axis (Fig. 2) will be a rectangle, with a side equal to the $\pi\overline{MS}$ base circle length, and the other dimension equal with the $l_1, l_2,\ldots,l_4$ generators lengths. It is obvious that the true size of the generators is measured in the vertical represented plane. Thus, the points of the intersection curve will be $M7_0, Q6_0, V13_0, P14_0,\ldots,M7_0$.

The unfolding of the inclined cylinder is obtained similarly. It is a rectangle having the length $\pi\overline{17}$, and the other dimension equal with the $l_5, l_6,\ldots,l_{14}$ lengths (Fig. 3). The points of the intersection curve will be $M7_0, V13_0, P14_0, P1_0,\ldots,M7_0$.

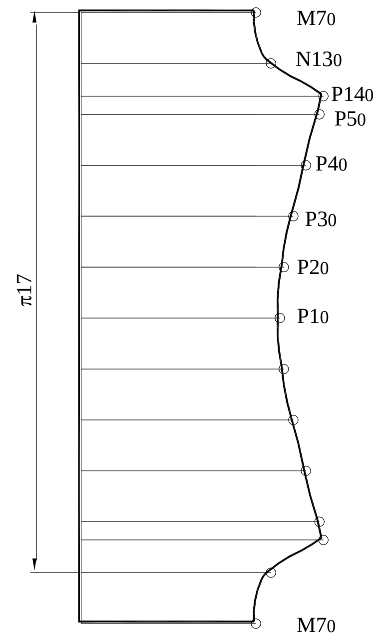

Fig. 3. Unfolding of the cylinder with inclined axis

3. THE MATHEMATICAL METHODS

In accordance with the Fig. 4 we take the cylinder C, of diameter D, and its reference system $Oxyz$ and the cylinder C_1, of diameter D_1, and its reference system $O_1x_1y_1z_1$, where $y \equiv y_1$ and $O \equiv O_1$.

The cylinders equations expressed in the chosen reference systems are:

$$x^2 + y^2 = R^2 \quad (1)$$

$$y_1^2 + z_1^2 = R_1^2 \quad (2)$$

The two reference system are rotated, one given another, by the angle φ. The transformation formula of the coordinates, to passing from the system $Oxyz$ into $O_1x_1y_1z_1$ and vice versa is:

$$x_1 = x\cos\varphi + z\sin\varphi \quad (3)$$

$$z_1 = z\cos\varphi - x\sin\varphi \quad (4)$$

$$x = x_1\cos\varphi - z_1\sin\varphi \quad (5)$$

$$z = x_1\sin\varphi + z_1\cos\varphi \quad (6)$$

We relate the equations of the both cylinders to system $Oxyz$ and by eliminating the variable y, we obtain the equation of the vertical projection of the intersection:

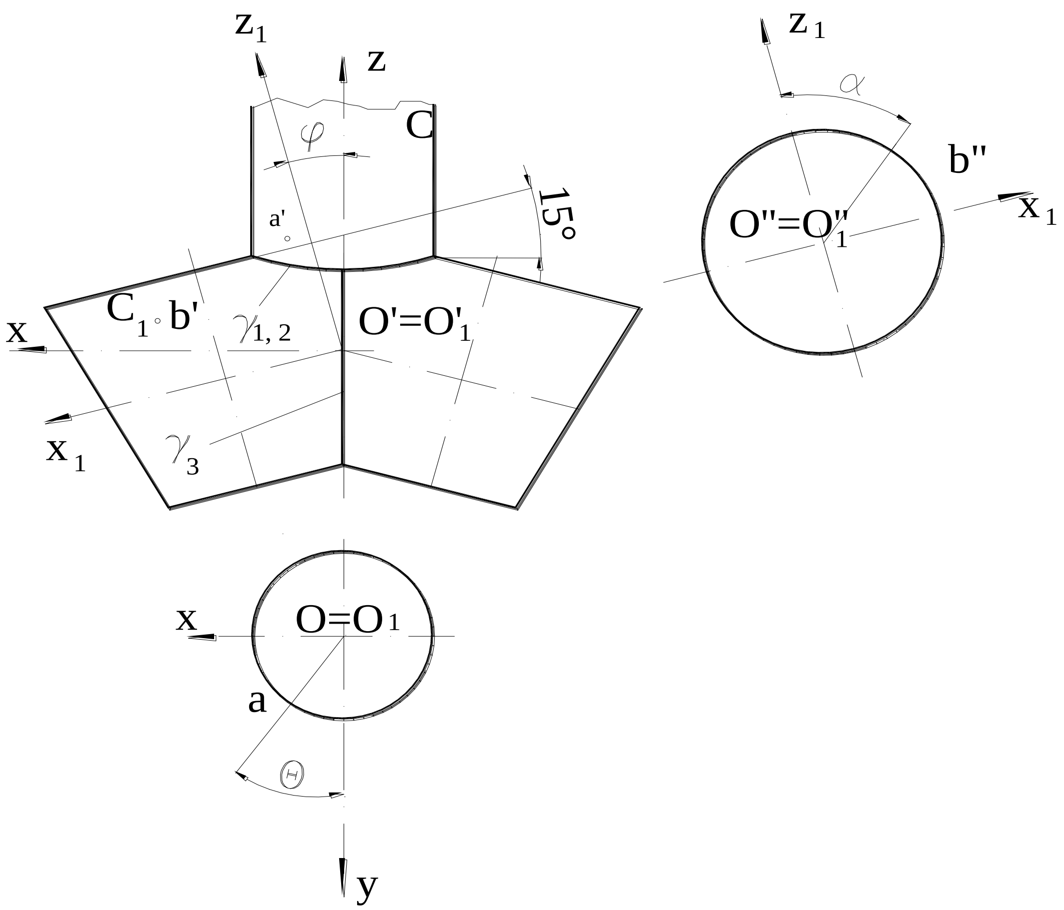

Fig. 4. The geometrical elements of the cylinders

$$z^2 - 2x \cdot \mathrm{tg}\varphi\, z + \frac{R^2 - R_1^2}{\cos^2\varphi} - x^2 = 0 \qquad (7)$$

The equation of the transformation curve γ_1, border of the cylinder C, is obtained by applying the transformations (8, 9) to the equation (7).

$$x = R\cos\theta = R\cos\frac{x_d}{R} \qquad (8)$$

$$z = z_d \qquad (9)$$

where x_d and z_d are the coordinates of the point A in unfolding. This point A is indicated by its projections a' and a''.

In this case the following equation is obtained:

$$z_d^2 - 2Rz_d\cos\frac{x_d}{R}\cdot \mathrm{tg}\varphi + \left[\frac{R^2 - R_1^2}{\cos^2\varphi} - R^2\cos^2\frac{x_d}{R}\right] = 0$$

Then:

$$z_{d1,2} = R\cos\frac{x_d}{R}\cdot \mathrm{tg}\varphi \pm \frac{1}{\cos\varphi}\sqrt{R_1^2 - R^2\sin^2\frac{x_d}{R}} \qquad (11)$$

$$x_d \in [-R\arcsin\frac{R_1}{R},\ R\arcsin\frac{R_1}{R}]$$

We obtain the Fig. 5, by introducing the relations (11,) into Mathematica program.

The equation of the transformation curve γ_1, border of the cylinder C_1, is obtained by applying the transformations (12, 13) to the equation (7):

$$x_1 = x_{d1} \qquad (12)$$

$$z_1 = R_1\sin\alpha = R_1\sin\frac{z_{d1}}{R} \qquad (13)$$

where x_{d1} and z_{d1} are the coordinates of the point $B(b, b')$ in unfolding.

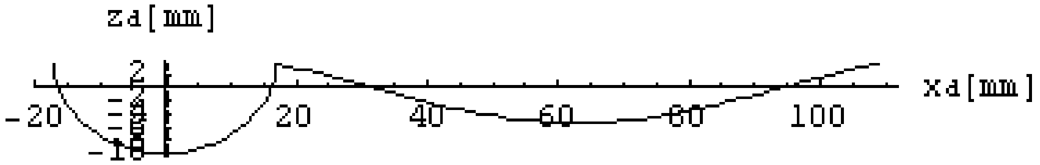

Fig. 5. The unfolding of the intersection curve γ_2

The following equation is obtained:

$$x_{d1}^2 + 2R_1 \sin\frac{z_{d1}}{R_1} x_{d1} - R_1^2 \sin^2\frac{z_{d1}}{R_1} - \frac{R^2 - R_1^2}{\cos^2\varphi} = 0 \quad (14)$$

$$x_{d1} = -R_1 \sin\frac{z_{d1}}{R_1} \pm \frac{1}{\cos\varphi}\sqrt{R^2 - R_1^2 \cos^2\frac{z_{d1}}{R_1}} \quad (15)$$

$$z_{d1} \in [0, 2\pi R_1] \quad (16)$$

The Fig. 6 is obtained by introducing the relations (15, 16) into Mathematica program.

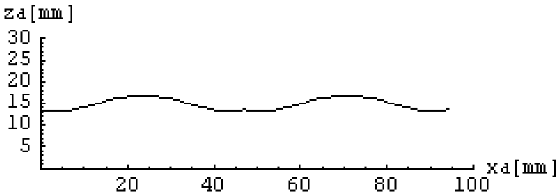

Fig. 6. The unfolding of the intersection curve γ_1

4.CONCLUSIONS

For the correct execution of some pieces or subassemblies with complex form, which meet the requirements, the methods of descriptive geometry are absolutely necessary. Resolve the difficulties of producing patterns, by determining the types of surfaces that are part of that is very necessary.The presented method is very speedy and exactly and using the program we can obtain the cylinders unfoldings for any other dimensions. The two methods have the same results.

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