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# Possibilities of Application of Transcendent Cones in the Form of Architectural Structures

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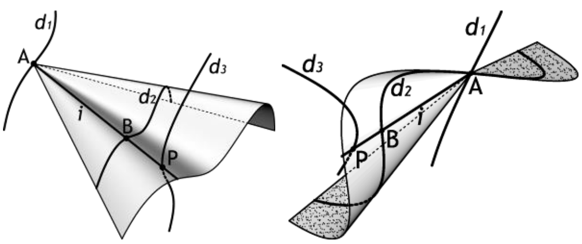
**Abstract: The paper considers potential for application of transcendent cones with an apex on the directrix, helix, in the form of architectonic structures, and of cones with a torus knot as a directrix and with an apex on it. Acting as the helix directrix were cylindrical helix, conical helix, spherical helix and hyperboloid helix. The cones, as rectilinear deriving single curved surfaces are considered because of their rationality and attractive forms which can be obtained by applying them. Such spatial structures are based on segments, that is, cutouts of these segments of the cones and they are formed both by their mutual combination, or by the combination with other elements in space.**

**Keywords: transcendent cones, helices, torus knot, spatial structures.**

## 1. INTRODUCTION

The cone is a deriving singly curved rectilinear surface. A randomly chosen point **A** on the directrix $d_1$ will, along with the directrix $d_2$, determine the totally defined conical surface $k$, (Fig. 1a). If directrix $d_3$ penetrates through this conical surface in the point **P**, then the connection line **AP**, as it intersects all three directrices ($d_1$, $d_2$ and $d_3$), will be the generatrix of the rectilinear surface. If the directrix $d_3$ penetrates through the mentioned conical surface in two, three or more points, then through point **A** will pass two, three or more generatrices of the rectilinear surface./7/ The directrix of a rectilinear surface can be any planar or spatial curve.




a) b)

Fig. 1: Conical surface

By changing the form and mutual position of the directrices, various types of rectilinear surfaces can be obtained. If the directrices $d_1$ and $d_2$ intersect, and the intersection point is designated with **A**, then the top of the created surface will occur on the directrix $d_2$, (Fig. 1b). The cones formed in this manner are rectilinear surfaces which form bisecants of one spatial curve. In this paper are considered some of possible variants of cones formed with the top on the directrix which is a cylindrical helix, as well as the characteristic sections of the cone formed in this way.

Every planar or spatial curve of n-th order connected with a point in space, produces an n-th order surface, a cone. If this point, the vertex of the cone, is one of the points of the spatial curve of n-th order, directrix, then the bisectors of the directrix forms a cone of n-1-th order. If the top of the cone in this process coincides with the double, triple, point of the spatial directrix curve of n-th order, then the formed cone is of the (n-2)-th, (n-3)-th,… order./9/

If a transcendent spatial curve is taken for the directrix of the cone, then a transcendent conical surface is formed.

Using segments and cut outs of transcendent cones, as well as of the cones created on torus knots, it is possible to form complex and attractive spatial structures of architectonic structures. As they are rectilinear single curved surfaces, a relatively simply, practical derivation is possible from the spatial structures created in this way.

The paper discusses and presents examples of structures created in this manner, based on using the cone with directrices, helices on several different rotating surfaces and an apex on these directrices.

## 2. APPLICATION OF THE CONE WITH A CYLINDRICAL HELIX AS DIRECTRIX

If a cylindrical helix which is a transcendent spatial curve, is taken for the directrix, and a point on it, $V$, for the vertex, then a transcendent conical surface is formed. In the (Fig. 2) is presented a conical surface formed in this way. Due to the easier assessment, only a segment of such surface bounded by the cylinder along whose surface runs the helix, directrix of the formed cone, is displayed. All three

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orthogonal projections are given, as well as a perspective presentation.

Every intersection of such cone perpendicular to the axis of the helix is a cochleoid, thus it can also be called a cochleoid cone.

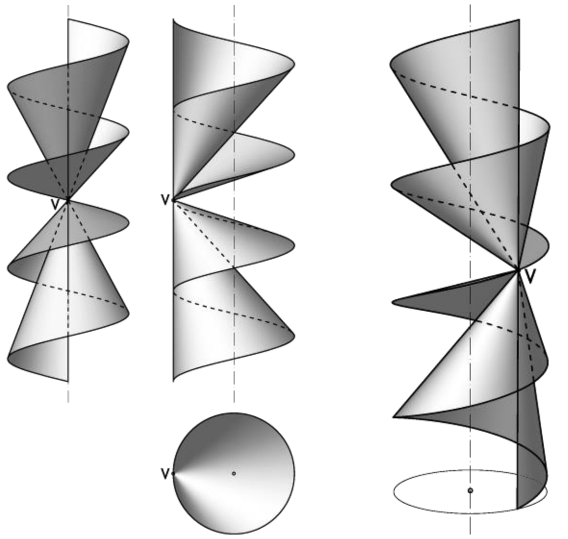

Fig. 2: A part of cochleoid cone

By using the segments of a cochleoid cone as well as the cut-outs of these segments, it is possible to form attractive and complex spatial structures. An example of a spatial structure based on using a cochleoid cone is presented in (Fig. 3). The spatial structure is created by combining cut-outs of one segment of a cochleoid cone, in combination with a cylindrical surface.

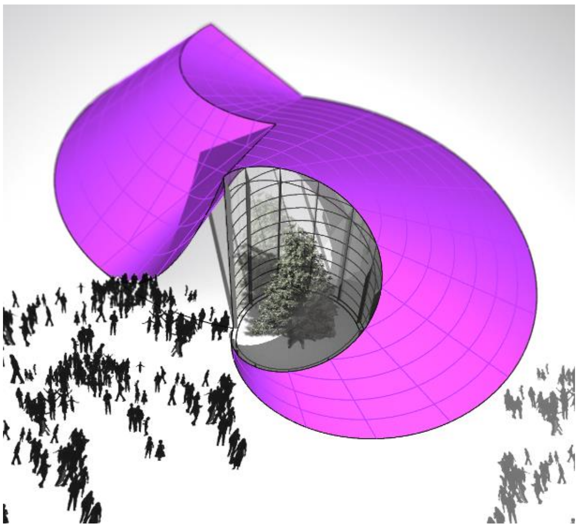

Fig. 3: Spatial structure based on using cut-outs of a cochleoid cone

3. APPLICATION OF THE CONE WITH A CONICAL HELIX AS DIRECTRIX

If for deriving a cone is used the cone helix (Fig. 4), as a directrix, and if for an apex is taken a point on it that does not coincide with the apex of the cone which is a carrier of the helix, a transcendent conical surface is created.

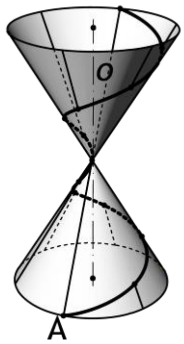


Fig. 4: Cone helix

By using such cones with the apices $\mathbf{K_1}$, $\mathbf{K_2}$ and $\mathbf{K_3}$ on the conical helix, that is cut-outs of their segments, the spatial structure presented in (Fig. 5) is formed.

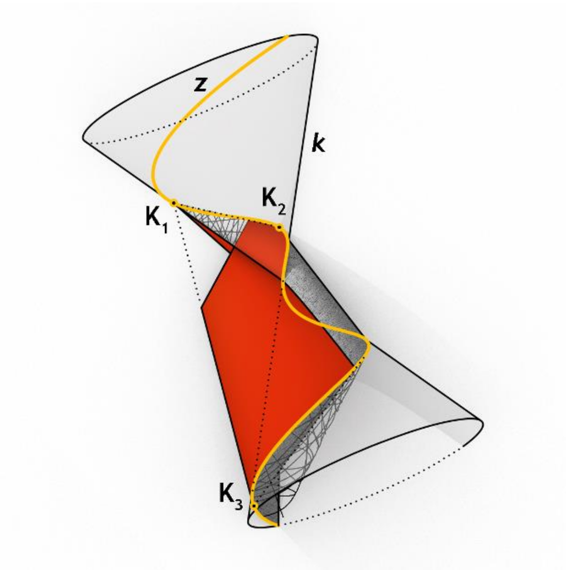


Fig. 5: Formed structure formed from the cone by apices $K_1$, $K_2$ and $K_3$ (layout)

In (Fig. 6), a structure is presented in a perspective that was created in a previously described way, by joining the segments of the cone with a conical helix as directrix and apices on it, and by cutting out those segments. Attractiveness and dynamism of the form is achieved by mutual intersection of conical surfaces in a complex way. The surfaces of a spatial structure are rectilinear, deriving and single curved.

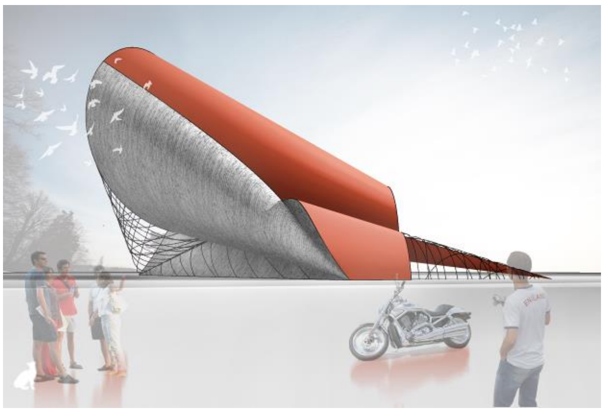

Fig. 6: Spatial structure formed by three cones with a conical helix as a directrix and apices on it

4. APPLICATION OF THE CONE WITH A SPHERICAL HELIX AS DIRECTRIX

A spherical helix is a spatial curve traveling from one to another pole of the sphere, while keeping a fixed (but not right) angle with respect to the meridians. A special case of spherical helix is a loxodrome, transcendent spatial curve.

Using a loxodrome as a directrix, and a point on it as an apex, a cone is formed whose cut-out is applied in the spatial structure presented in (Fig. 7). The apex of that cone is an intersection of the loxodrome and the equator of the sphere which is the carrier of the loxodrome. Three orthogonal projections and the perspective of the object formed in this way are presented.

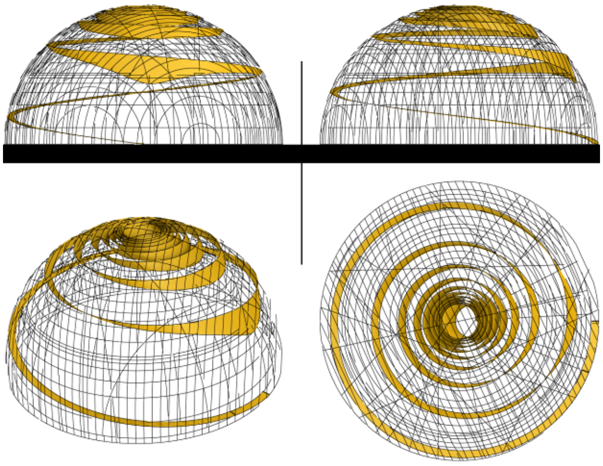

Fig. 7: Spatial structure formed from the cone with a loxodrome as a directrix and the apex on it

The perspective, a view into a previously mentioned spatial structure is presented in (Fig. 8). The cut-out of one segment of the cone is obtained by cutting out with two concentric spheres. The spatial structure is carried by linear elements created by the planar intersection of the sphere, carrier of the loxodrome, which is a directrix of the cone.

Fig. 8: Perspective display of a spatial structure

5. APPLICATION OF THE CONE WITH A HYPERBOLOID OF ONE SHEET HELIX AS DIRECTRIX

The helix can be derived on each rotating surface provided that this spatial curve has a fixed angle with all the meridians of the rotating surface. Such curve, hyperboloid of one sheet is presented in (Fig. 9), and the meridians of the surface are the branches of the hyperbola. The helix is derived on the rotating hyperboloid of one sheet.

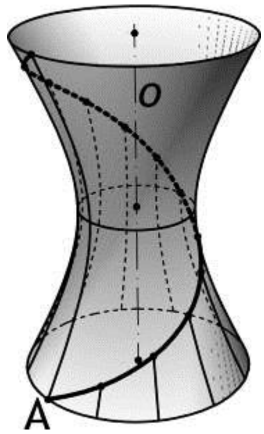


Fig. 9: Hyperboloid of one sheet helix

By using the previously presented helix as a directrix and the apex on it, in the intersection of the helix and the smallest circle section of hyperboloid and transcendent cone is formed. The segment of such cone is applied in formation of the spatial structure of the structure presented in (Fig. 10).

Fig. 10: Perspective of spatial structure

6. APPLICATION OF THE CONE WITH A TORUS HELIX AS DIRECTRIX

The torus knot (***p***, ***q***) is obtained by looping a string through the hole of a torus ***p*** times with ***q*** revolutions before joining its ends, where ***p*** and ***q*** are relatively prime. Each torus knot can be a directrix for the derivation of a surface. By assigning an apex on the directrix, a conical surface is formed. A cone formed in this way will be of ***(n-1)*-th** order. In (Fig. 11) are presented the variants of the cone formed on the torus knot (1,1), with the directrix and the apices taken in several characteristic points on it.

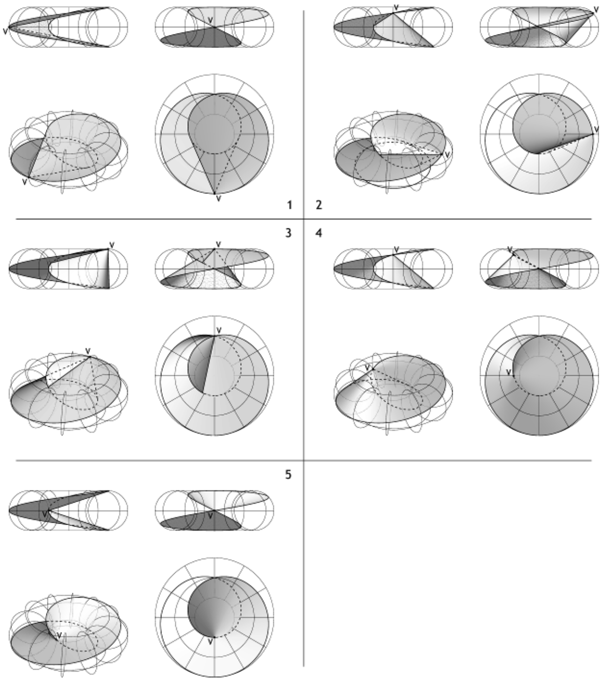


Fig. 11: The variants of the cone with a torus knot as a directrix and the apex on it

Using the cone presented in (Fig. 11-5) a structure presented in (Fig. 12) is formed in three projections and the perspective. The surface is used for formation of the belvedere ramp, with a linear beams created on the surface of the torus.

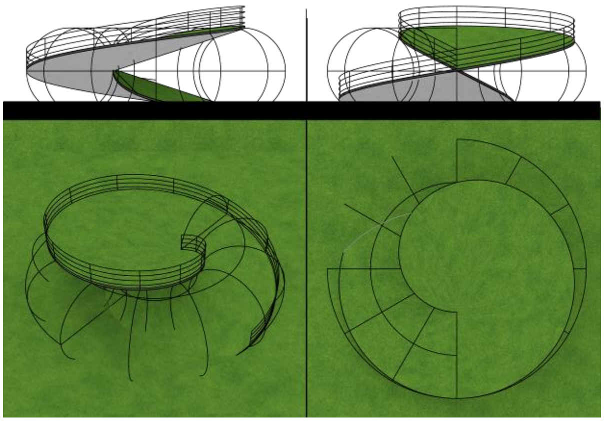

Fig. 12: The structure based on using the cone with torus knot as a directrix and an apex on it

In (Fig. 13) is presented the perspective of a structure created in this way. A special quality of the created form is the fluency of the surface and the light-weightiness upper part of the ramp used for a belvedere.

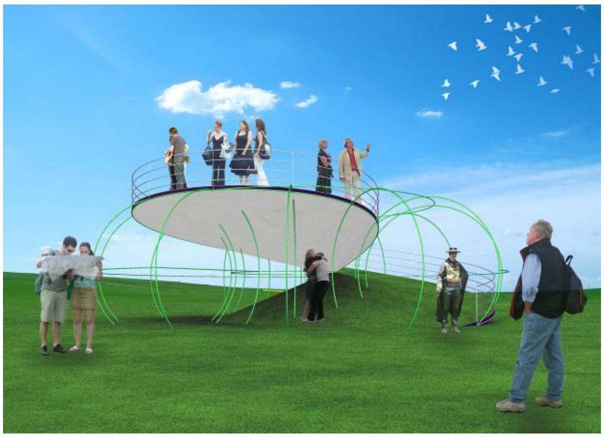

Fig. 13: Perspective presentation of the structure

## 7. CONCLUSION

By considering the presented variants of spatial structures based on the cones with helices as directrices and with apexes on the directrices, attractive and complex forms of architectonic structures can be formed. A special potential is a large number of possible constructions of conical surfaces and their mutual combinations.

As all the parts of the presented structures are rectilinear deriving single curved surfaces, their simpler derivation is possible, with respect to the non-derivational or double curved surfaces. The presented examples prove that the attractiveness of the form of such spatial structures is not reduced to the structures based on much more complex surfaces, both for designing and for construction. The application of such spatial structures in modern architecture, based on the application of computers in all segments of designing and construction, provide a lot of potential for further visual research.

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