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Pencils of Curves of the 4th and 3rd Order Obtained as Harmonic Equivalents of Hyperbolic-Elliptic Pencil of Conics

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Abstract: In this paper hyperbolic-elliptic pencils of conics are analyzed by the use of descriptive and synthetic geometric methods and than the pencils of conics are mapped into pencils of curves of the 3rd and 4th order using the harmonic symmetry (inversion). The basic transformation is inversion, which will be interpreted in two ways: as quadratic transformation in classical projective geometry and as pure symmetry in relativistic geometry.

Key words: pencil of conics, pencil of curves of the 3rd and 4th order, inversion, symmetry.

1. INTRODUCTION

This research deals with transformation of hyperbolic-elliptic (HE) pencils of conics and their specific features into pencils of curves of the 3rd and 4th order and their specific features and vice versa. The basic transformation will be inversion. The projection model was used to create the Lisp routine and the AutoCAD software was used for the purposes of computer drawing the pencils of conics, pencils of curves of the 3rd and 4th order

The graphic of transformation (inversion), which has the same drawing representation in both geometric systems, is interpreted using both classical concepts (which include infinity) and relativistic concepts (which include an observer and his antipodal point). This is done because substantial differences between the two systems can be found in those `infinitely distant` or `antipodal` areas, which are equally out of observer’s sight. It is essential to compare non-circularity of conics, circularity of the equivalent curve of the 3rd order and bicircularity of the equivalent curve of the fourth order in projective geometry with completely corresponding correlation systems of these curves in relativistic geometry, in which they are consequently all of the same 4th order.

Relativistic geometry, in short, has arisen from demonstrating that following the principle of continuity, branches of hyperboles together with their asymptotic tangents really intersect in infinity. Since asymptotes intersect in the center of the hyperbole, as well, it turns out that these straight lines intersect two times, which means that they cannot be just circles on the sphere of incalculable dimensions (there is no `swelling` which could turn it into classic plane). The perception that `straight lines` and `planes` are actually circles and spheres, seen through the antipodal point of an arbitrary viewer has brought the theory of relativity into geometry (what is seen as a straight line by one viewer, can be seen as a simple circle by another). The farther these lines are from the viewer, the smaller they become. Finally they shrink into infinitesimal antipodal `straight lines` (i.e. small circles through the antipodal point). If relativistic geometry as a new theory is to be consistent, apart from accepting some general principles, it must follow only its own concepts and findings, no matter what concepts and findings have been discovered by other geometric systems. However, the fundamental change of the concept of `straight line` has changed the meaning of all other geometric concepts. Therefore, it is crucial to make constant comparisons between the old and the new findings. For example, inversion as square transformation has become pure symmetry (in relation to a circle or a sphere in general or in relation to `straight line` or `plane` in particular cases). This has prompted another important discovery in the field of this new geometry – discovery that certain curves and areas of different shapes and orders, which used to be disparate in non-relativistic geometry, are now seen as different forms of one single curve or area, which can finally make the dream of absolute classification of all curves and areas come true.

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2. CREATING A HE PENCIL OF CONICS

In this paper, a HE pencil of conics is mapped into a pencil of curves of the 4th and 3rd order by using inversion. We have analyzed the case of the one-sided hyperboloid whose two parallel generating lines (from both systems) are in a projecting position. The pencil of the intersecting planes is positioned through the passant – $\boldsymbol{p_s}$ of the one-sided hyperboloid. All the conics in the intersecting planes have to pass through these generating lines (a and b). In the first projection, these are two real and separate fundamental points of the pencil of conics S_1 and S_2. The other two fundamental points of the pencil of conics are conjugatively imaginary, and their real representatives stand on the passant of the conic – ***p.*** Hence, a hyperbolic-elliptic (HE) pencil of conics is created. It is shown in Fig. 1.

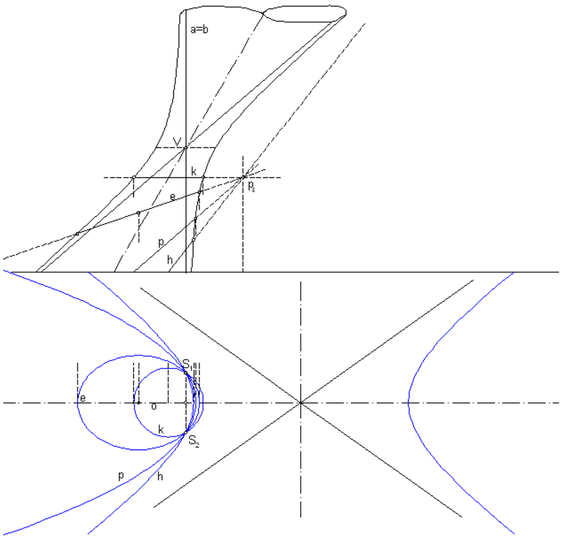

Fig. 1 HE pencil of conics created by slicing the one-sided hyperboloid through its passant

3. MAPPING THE HE PENCIL OF CONICS INTO THE PENCIL OF CURVES OF THE 4TH AND 3RD ORDER

Points are mapped by the central inversion and it is very simple. It is based on mapping the points P and $\bar{P}$ by the polarity with respect to the circle of inversion s. Inversion is a quadratic transformation obtained by means of reciprocal radius vectors of inversion. Coordinates of the mapped points $\bar{x}$ and $\bar{y}$ are obtained using the following equation:

$$\bar{x} = \frac{r^2 x}{x^2 + y^2} \qquad \bar{y} = \frac{r^2 y}{x^2 + y^2}$$

Fig. 2 illustrates the equations.

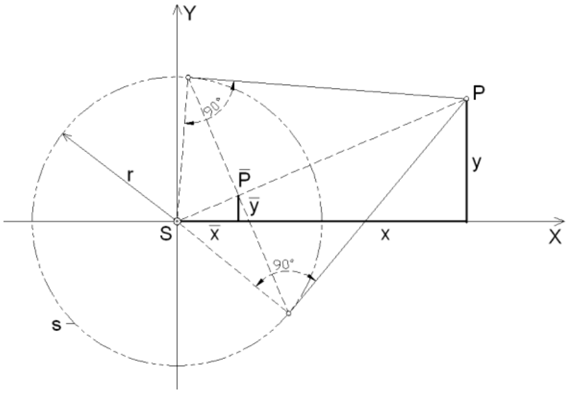

Fig. 2 Reciprocal radius vector of inversion

If we choose the center of inversion (***S, s***) to be on the axis of symmetry of all conics from the pencil, a pencil of curves of the 4th order is obtained – Fig. 3. We have further singled out all the conics from the pencil and their harmonic equivalents are shown (Fig. 4, Fig. 5 and Fig. 6).

Mapping of the ellipse from the HE pencil of conics into the egg-shaped curve of the 4th order is presented in Fig. 4. The obtained curve has one circular - $\bar{a}_e$ and one linear - $\bar{b}_e$ axis of symmetry and an isolated point that coincides with ***S***. Mutual symmetry of the hyperbola and the curve of the 4th order with a self-intersected point at the center of inversion ***S*** is shown in Fig. 5. The parabola from the HE pencil of conics is mapped into the curve of the 4th order with a spire at ***S***. It has one axis of linear symmetry - $\bar{a}_p$, which is shown in Fig. 6.

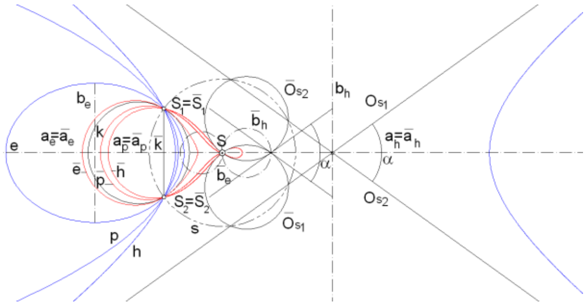

Fig. 3 HE pencil of conics and its harmonic equivalent, a pencil of curves of the 4th order

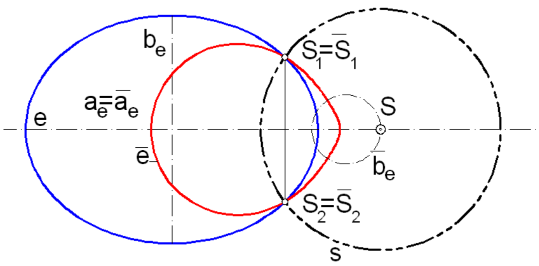

Fig. 4 The ellipse mapped from the HE pencil of conics into the egg-shaped curve of the 4th order for the selected center of harmonic inversion S

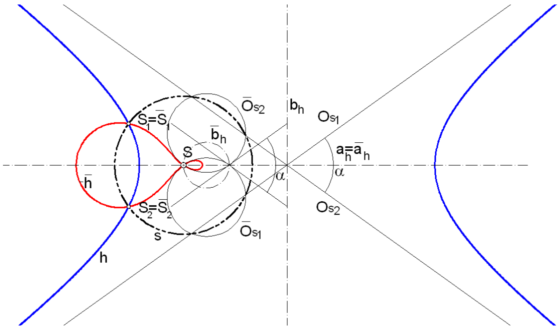

Fig. 5 The hyperbola mapped from the HE pencil of conics into the curve of the 4th order for the selected center of the harmonic inversion S

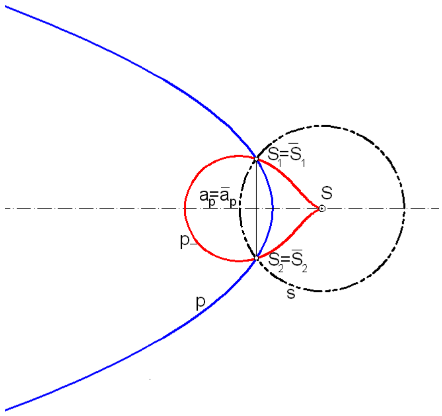

Fig. 6 The parabola mapped from the HE pencil of conics into the curve of the 4th order for the selected center of harmonic symmetry S

When the center of inversion lies at point ***S*** of the ellipse, the pencil of conics is mapped into the HE pencil of curves of the 4th and 3rd order. It can be seen in Fig. 7. The ellipse is mapped into the curve of the 3rd order with two circular axes of harmonic symmetry - $\bar{a}_e$ and $\bar{b}_e$ – Fig. 8. The osculatory circle of the ellipse at point S is mapped into a straight line - the asymptote of the curve of the 3rd order.

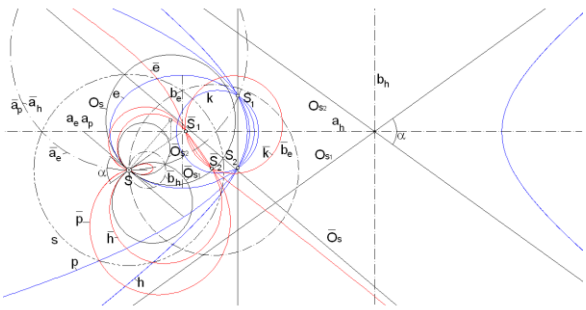

Fig. 7 HE pencil of conics and its harmonic equivalent, the pencil of curves of the 4th order whose center of inversion is at a point of the ellipse

Mutual harmonic symmetry of the hyperbola (*h*) from the HE pencil of conics and the curve of the 4th order is shown in Fig. 9. The obtained curve has two circular axes of harmonic symmetry or $\bar{a}_h$ and $\bar{b}_h$. It can be seen that the angle of self-intersection of the hyperbola`s assymptotes equals the angle of self-intersection of the two branches of the curve of the 4th order that pass through point ***S.*** This angle is marked α in Fig. 9.

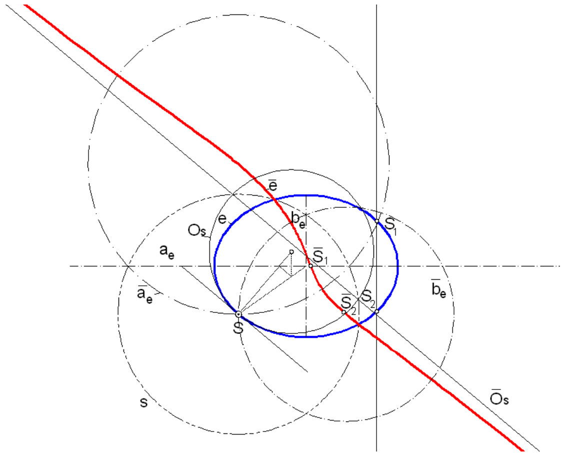

Fig. 8 The ellipse mapped from the HE pencil of conics into the curve of the 3rd order with the center of harmonic inversion S at a point of the ellipse

Fig. 10 shows the parabola mapped from the HE pencil of conics into the curve of the 4th order with a spire which is mapped from point $\bar{S}$ into point *S*. The harmonic symmetry thus proves that the parabola has a spire in the antipode. It reveals the area that is not within sight of the observer.

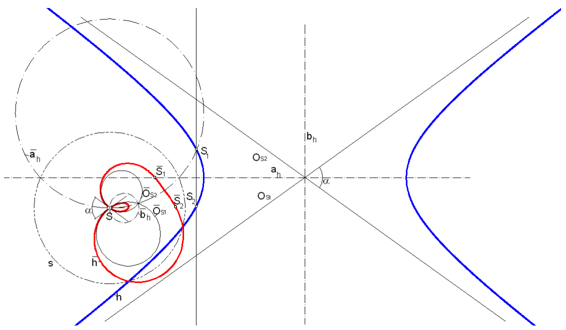

Fig. 9 The hyperbola mapped from the HE pencil of conics into the curve of the 4th order with the selected center of harmonic inversion S at a point of the ellipse

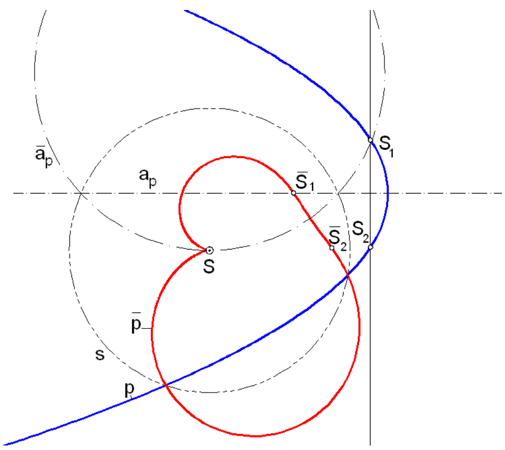

Fig. 10 The parabola mapped from the HE pencil of conics into the curve of the 4th order with the selected center of harmonic inversion S at a point of the ellipse

4. CONCLUSIONS

Inversion is in relativistic geometry a completely bijective transformation. Therefore, a pencil of curves of the 4th order has the same number of intersection points as a pencil of conics. The mapping of conics into curves of the 3rd and 4th order is best observed through their `crossing` on the absolute.

In the projective-Euclidean space, the pole of inversion in the Eucledean center of the sphere of inversion is in different directions inverted into different points of a fictively infinitely distant planes, which disturbs the bijective projection.

If we introduce the relativistic terms of the straight line and plane (circle and sphere) into the classical central inversion, we get a transformation on the sphere – harmonic symmetry. All the points from the "spheric plane" are inverted into the equivalent points on the sphere, while the point of inversion pole is in all directions inverted into one finite antipodal point of the observer. In this way, complete bijectivity and conformity, i.e. typological isomorphism of inversion is achieved, so it becomes harmonic symmetry.

The paper proves that various classical constructions of `different` curves can be replaced with one conformal symmetry (inversion). It can transform a conic into a large number of its conformally equivalent forms which are composed of circular curves of the 4th and (classical) 3rd order. The main objective of the study was to apply the findings about the pencils of the curves of the 2nd order (conics) on the related pencils of curves of the 3rd and 4th order. The variety of the obtained curve forms indicates the possibility of widespread use of curves of the 3rd, 4th and higher orders in the practice of modern architecture.

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