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Analysis of Mapping of Hyperbolical Paraboloid with the Aid of Absolute Conic in General Collinear Spaces

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Abstract: The hyperbolical paraboloid is a 2nd degree rectilinear (quadrics), whose infinitely distant conic is a real 2nd degree curve, on two real stright lines. Absolute conic is imaginary and rest in infinitely distant plane of space. Since that infinitely distant elements cannot be represented graphically, for the constructive processing of quadric is used the general collineation in space, where the infinitely distant elements are associated to finality elements. In this paper we analyze the conditions under which the hyperbolic paraboloid is mapped in the same, and when in the second degree surface.

Keywords: Hyperbolical paraboloid, infinitely distant conic, absolute conic, general collinear spaces

1. INTRODUCTION

2nd degree rectilinear surfaces are divided into: hyperboloid of one sheet, which can be rotating and, elliptical, hyperbolical paraboloid and to singular surfaces 2nd degree, cones and cylinders. [6]

Real 2nd degree rectilinear surfaces (quadrics) intersect the infinitely distant plane along the 2nd degree curve (conic), which can be real (hyperboloid of one sheet and 2nd degree cones), degenerate in two real separated right lines (hyperbolical paraboloid and hyperbolical cylinder), degenerate in two real coinciding straight lines (parabolic cylinder) and degenerate conjugate imaginary conic (circular or elliptical cylinder).

Absolute conic of space is an imaginary conic and it rests in the infinitely distant plane of space. Each plane in space has its infinitely distant straight line on which lie the absolute points of that plane. Geometrical place of all absolute points in infinitely distant plane of the space is absolute conic of space. Its imaginary 2nd degree curve each real straight line in the infinitely distant plane is intersected in two conjugate imaginary (absolute) points. Since in the infinitely distant plane of space there are $\propto^2$ of real straight lines, and on each of them two absolute points, the absolute conic consists of $\propto^2$ pairs of conjugate imaginary absolute points. [9]

2. MAPPING OF QUADRICS IN GENERAL COLLINEAR SPACES

Regarding that infinitely distant elements cannot be graphically presented, for constructive processing of 2nd degree rectilinear surfaces (quadrics), general collineation in space is used, where the finite elements are associated to the infinitely distant elements. General collinear spaces θ^1 and θ^2 are set with five pairs of unequivocally associated points $\mathbf{A_1B_1C_1D_1E_1}$ and $\mathbf{A_2B_2C_2D_2E_2}$. For mapping in general collinear spaces, it is necessary to determine characteristic parameters, those being: vanishing planes, axes and centers of space (fig. 1). [6]

Common elements of the absolute conic of space and infinitely distant conic of any quadric are four-conjugate imaginary intersecting points, which lie on two real double straight line $\mathbf{d_1}$ and $\mathbf{d_2}$ and one common autopolar triangle **PRQ** which is always real. Through the points **P**,**R** and **Q** of autopolar triangle pass the axes of the quadric, which are mutually orthogonal. Through the straight lines $\mathbf{d_1}$ and $\mathbf{d_2}$ pass the planes which intersect the quadric along the circumferences, and that implies there are two systems of parallel planes which intersect the quadric along the circumferences. [2]

To the infinitely distant plane of the first space is associated the vanishing plane of the second space. Conics (2nd degree curve lines) in the infinitely distant plane of the first space have the associated conics in the vanishing plane of the second space. The absolute conic of the first space has its associated conic in the vanishing plane of the second space. The associated conic is an imaginary circumference (fig. 2), which is determined by a real representative. The centers of figures of absolute conics in the vanishing planes coincide with the centers of space (piercing points of space axes through the vanishing planes). [3]

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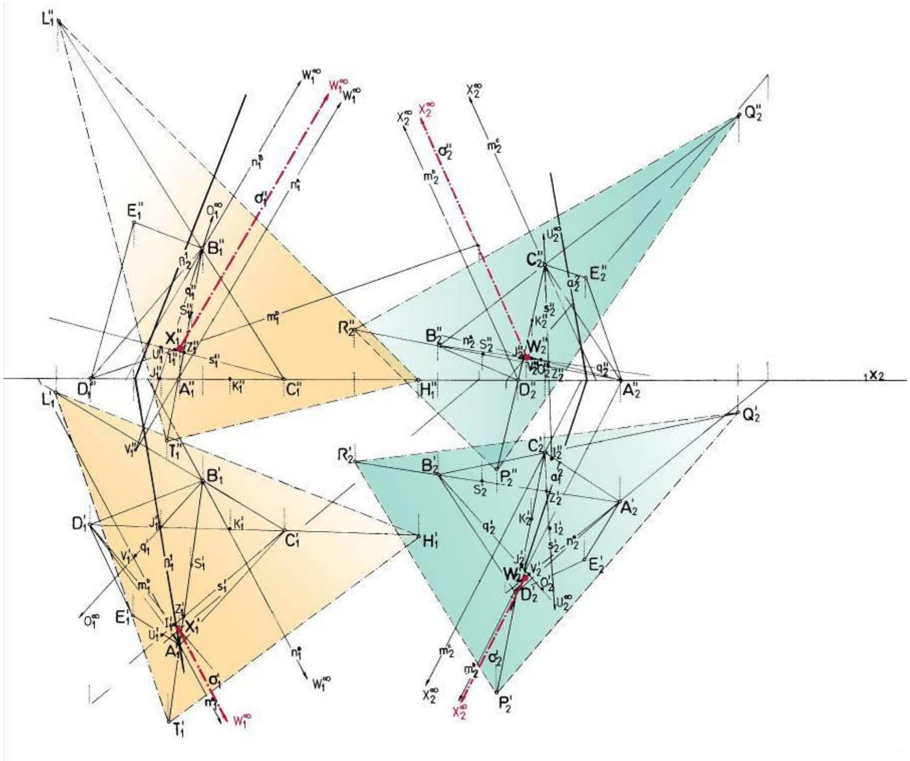

Fig. 1 Determination of the characteristic parameters in general collinear spaces

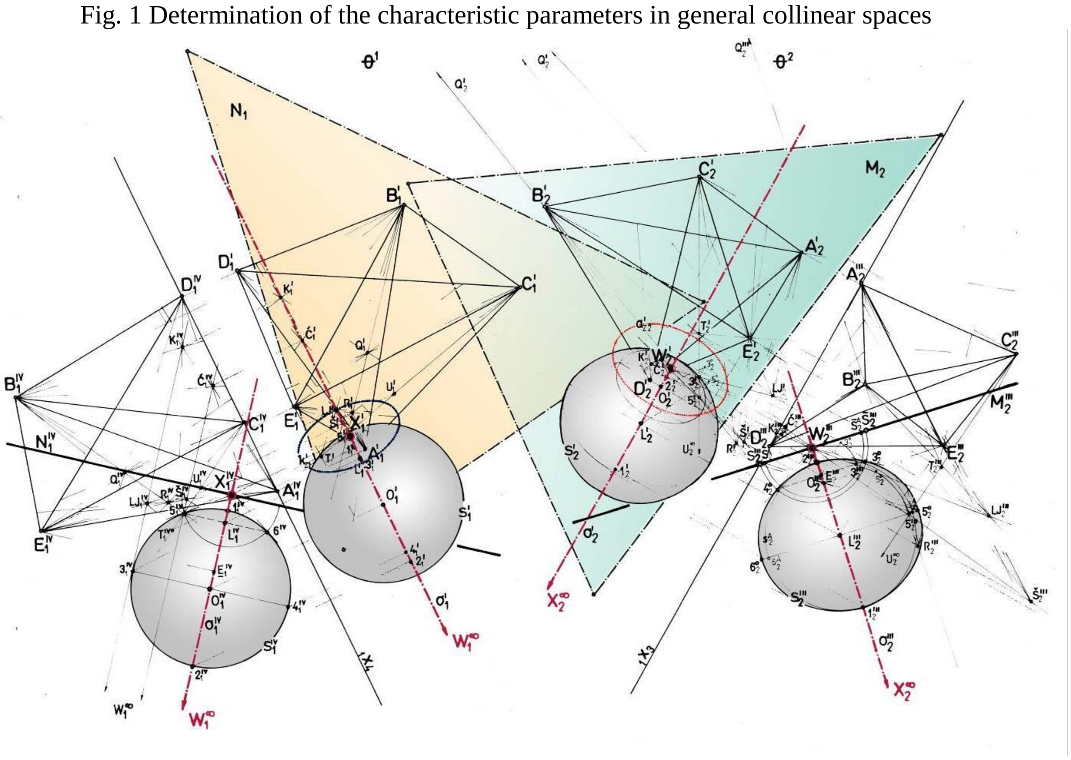

Fig. 2 Determination of figures of absolute conics in general collinear spaces

Depending on what conic is intersecting with the vanishing plane of a chosen quadric in one space and what position of intersecting conic in respect to the figure of the absolute conic is, the associated quadric in the second space is determined. [2]

The intersecting conic of the hyperbolical paraboloid with the vanishing plane can be real 2^{nd} degree conic (parabola or hyperbola) or a conic degenerated to two real separated straight lines.

3. MAPPING OF HYPERBOLICAL PARABOLOID IN GENERAL COLLINEAR SPACES

3.1. Mapping of hyperbolical paraboloid into hyperboloid of one sheet

If the hyperbolical paraboloid intersects the vanishing plane in the first space along the real conic, parabola (fig. 3) or hyperbola (fig. 4), which is in general position with the figure of the absolute conic, the quadric associated to it is the hyperboloid of one sheet in the second space. The intersection of hyperboloid of one sheet with the infinitely distant plane is real conic (ellipse, circumference, parabola or hyperbola).

In this case too, problem determining the points of the common autopolar triangle **P_2**, **R_2** and **Q_2**, for these two conics in the vanishing plane, boils down to intersection of the conic, which is soluble both analytically and graphically. To associated points **P_1^∞**, **R_1^∞** and **Q_1^∞** pass the orthogonal axes of associated hyperboloid of one sheet. [2]

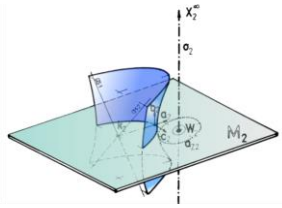

Fig. 3

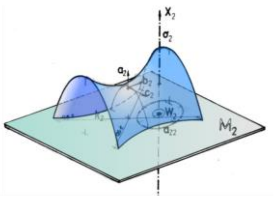

Fig. 4

3.2. Mapping of hyperbolical paraboloid into hyperbolical paraboloid

In order that a hyperbolical paraboloid in the first space,is mapped into the hyperbolical paraboloid in the second space, it is necessary to choose hyperbolical paraboloid to intersect the vanishing plane of the first space along the conic **k_2**, degenerated to two real separated straight lines **b_2** and **c_2**, (fig.5).

Out of three points of the common autopolar triangle **$P_2R_2Q_2$**, for this two conics **k_2** and **a_{I2}** in the vanishing plane, two points **R_2^∞** and **Q_2^∞** are infinitely distant, and one of them **P_2** is in finiteness. Intersecting point **Z_2**, of real straight lines **b_2** and **c_2** coincides with the center **W_2** of circumference **a_{Z2}** real representative imaginary circumference **a_{I2}** and it is vertex **P_2** in finiteness of common autopolar triangle of conics **k_2** and **a_{I2}**.

The other two points **R_2^∞** and **Q_2^∞**of common autopolar triangle, of conics, **k_2** and **a_{I2}** are in the planes of the parabola, having a common axis and are mutually perpendicular. They are called the main planes of hyperbolical paraboloid, which are also its planes of symmetry.

The common axis of parabolas, which is the axis of hyperbolical paraboloid in finiteness, pierce the surface in vertex. A plane perpendicular to the axis is tangent plane of the surface and it intersects along two directrices.

The planes formed by the two generatrices with the axis of the surface are called asymptotic planes and they represent degenerate asymptotic cone of surface. Asymptotic planes are symmetrical in relation to the main plane of hyperbolical paraboloid.

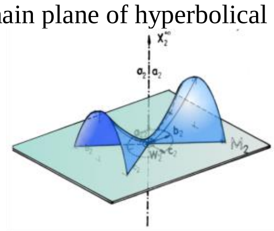

Fig.5

Hyperbolical paraboloid **s_2** in space θ^2 (fig. 6) is set with skew foursquare **$K_2L_2J_2G_2$**, in which the opposite edges are generatrices of one system, and adjacent edges a pair of generatrices of different systems.

Hyperbolical paraboloid's axis **s_2** in space θ^2, passing through infinitely distant center **X_2^∞**, and piercing the vertex **Z_2**=**W_2**, coincides with the axis **o_2** of the space θ^1, and it was associated with axes **o_1**, hyperbolical paraboloid **s_1**, of space θ^1, which passes through the center, the point **Z_1^∞**=**W_1^∞**, and pierces hyperbolical paraboloid **s_1** in vertex, the point **X_1**.

Vanishing plane **M_2** is a hyperbolical paraboloid **s_2** tangent plane at the vertex **W_2**, which intersects it along two straight lines **b_2** and **c_2**, which are called the main generatrices.

Main planes of hyperbolical paraboloid **s_2** are determined with vertex point **W_2**, center **X_2^∞** and opposite vertices of skew foursquare, which set the hyperbolical paraboloid in the space θ^2. These main planes in space θ^2 are associated to the main planes of hyperbolical paraboloid **s_1** in space θ^1. Their intersection with hyperbolical paraboloid **s_1** is two parabolas.

Infinitely distant plane **M_1^∞** of space θ^1, intersects hyperbolical paraboloid **s_1** by two infinitely distant real generatrices **b_1^∞** and **c_1^∞**.

In order for the hyperbolical paraboloid **s_2** in space θ^2 to be mapped into hyperbolical paraboloid **s_1** in space θ^1, it is necessary to map the vertices of skew foursquare **$K_2L_2J_2G_2$**. In this way the main planes are determined, and thus the other two axes of hyperbolic paraboloid **s_1**.

Point **K_1**, vertex of skew foursquare in space θ^1 was determined by dual relationship λ =(**$K_2X_2^\infty\acute{C}_2\check{Z}_2$**)=(**$K_1X_1\acute{C}_1\check{Z}_1$**). Since the point **$J_2$** is symmetric to the point **K_2** to the axis **o_2** and located in the plane through the point **E_2**, it follows that the point **J_1** is symmetric to the point **K_1** to the axis **o_1** and is located in a plane parallel to the vanishing point **N_1**, through the point **E_1**.

Points **K_1**, **J_1** and vertex **X_1** determine one main plane of hyperbolical paraboloid **s_1**, whose axis **$X_1R_1^\infty$** is parallel to connector **K_1J_1**.

In the same way, the points **G_1** and **L_1** of skew foursquare **$K_1L_1J_1G_1$**, are determined in the space θ^1.

Points **L_1**, **G_1** and vertex **X_1** are determined second main plane of hyperbolical paraboloid **s_1**, whose axis **X_1 Q_1^∞** is parallel to connector **L_1G_1**. In this way, all of three axes of hyperbolical paraboloid in space θ^1 are determined.

Circumferential intersections in the case of hyperbolical paraboloid degenerate into two real straight lines and there are no real circular points, all points are hyperbolic.

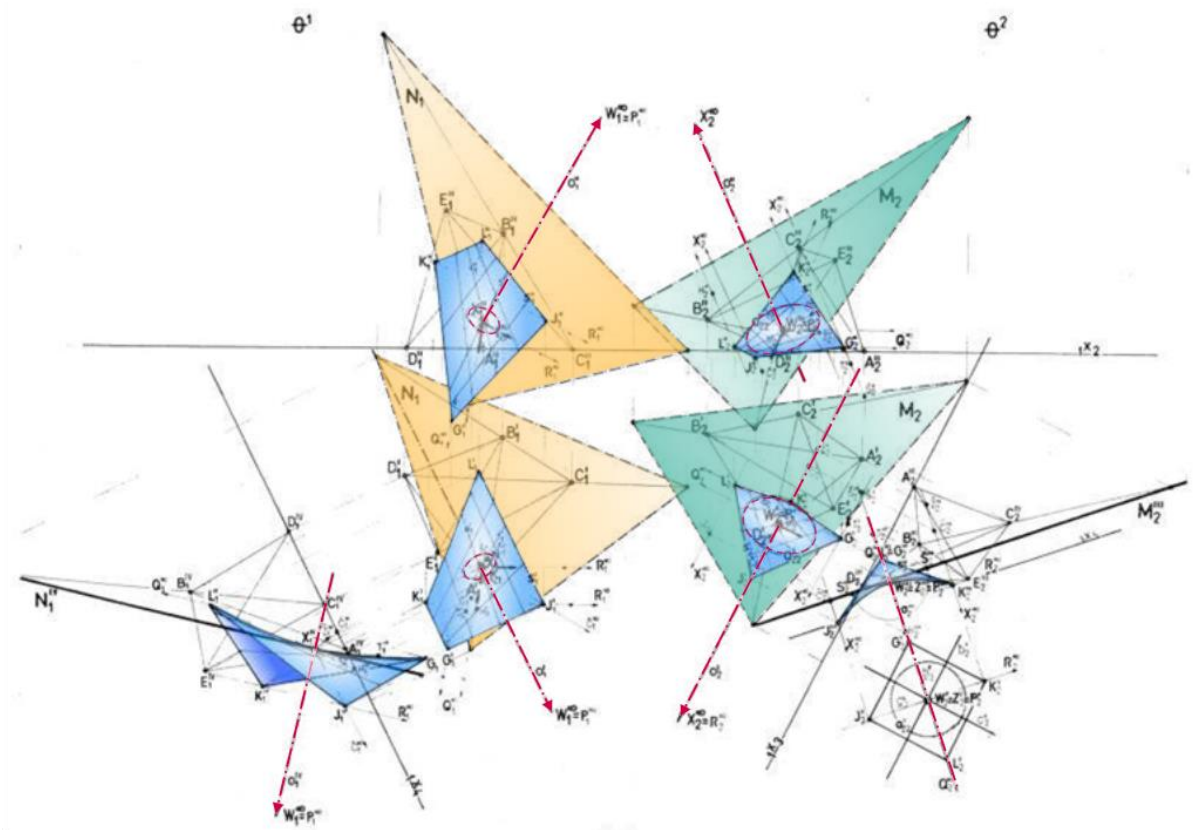

Fig. 6 Mapping of hyperbolical paraboloid into a hyperbolical paraboloid

5. CONCLUSION

Hyperbolical paraboloid is a 2nd degree rectilinear surface, which can be mapped into the same surface or in hyperboloid of one sheet in general collinear spaces.

If the intersection of hyperbolical paraboloid with the vanishing plane in the first space lies along a real curve (parabolic or hyperbolic), the quadric associated to it in the second space is hyperboloid of one sheet, whose infinitely distant conic is real. If the intersection of hyperbolical paraboloid with the vanishing plane in the first space lies along two real separate straight lines, then its associated surface is hyperbolical paraboloid, whose infinitely distant conic degenerate in two real separate straight lines.

The absolute conic of a space and infinitely distant conic of quadrics cannot be graphically represented, and to determine the axis and circular sections of hyperbolical paraboloid general collineation in space is used. The finite elements are associated to the infinitely distant elements. Common elements of the figure of the absolute conic and intersecting conic of hyperbolical paraboloid with the vanishing plane in the first space, autopolar triangle and the double straight lines can be constructively determined. Using general collineations they are mapped into another space.

Through the vertices of autopolar triangle in the infinitely distant plane the axis of mapped surfaces passes (hyperboloid of one sheet or hyperbolical paraboloid). Through the double straight lines in the infinitely distant plane pass the planes which intersect associated surface by circumferences, which in the case of hyperbolical paraboloid degenerate into two real separate straight lines.

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