Buletinul Ştiinţific al Universităţii „Politehnica“ din Timişoara

Seria HIDROTEHNICA

TRANSACTIONS on HYDROTECHNICS

Tom 58(72), Fascicola 2, 2013

SIMULATION OF THE TWO-DIMENSIONAL LIQUID MOTION WHEN PASSING THROUGH THE BARS OF A RAKING SCREEN

Iosif Anton[1] **Eleş Gabriel**[2]

Abstract – This paper proposes a two-dimensional model of numerical and analitical simulation of the liquid flow which passes through a raking screen with scarce bars, encountered at the entrance of the penstock of a sewage treatment plant or in the approach pipes of a hydropower plant, respectively in channels.The numerical simulation is done for the purpose of obtaining the hydrodinamic field and also the velocity and pressure distributions on the cylindric bars of the screen: the investigated model is the numerical one adequate to the potential flow, that uses the Boundary Element Method (BEM) within which linear elements are utilized. It must be stated that the velocity and pressure distributions obtained with BEM are represented together with the ones computed with the Finit Element Method (FEM).

Keywords: raking screen, potential flow, boundary, streamline, hydrodynamic field.

1. INTRODUCTION

For the purpose of simulating the two-dimensional flow upstream from the screen formed of cylindric bars, we shall acknowledge the following hypotheses: the fluid is perfect and incompressible, the motion is stationary and potential and plane, the screen bars being scarce, we consider as insignificant the influence of the nearby bars, the instalation angle between the bars and the flow direction is 90°. For the example analysed to give a generalisation to the results, we consider the dimensionless treatment in the potential function φ^*. This implies the use in their dimensionless form both the variables x^*, y^*, and the elements that define the analysis domain: the internal between the middle of the distance between two consecutive bars and the axis of one of them is $l^*/2 = 0.6$, the domain length $b^* = 1.$, the radius of the cylindric bar $R^* = 0.2$.

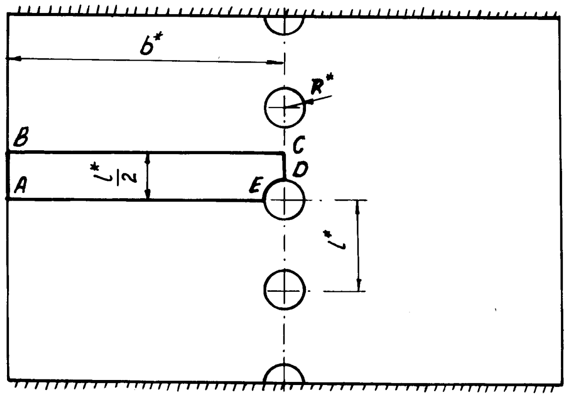

Fig. 1 Sectioning the bars with the motion plane and establishing

2. BOUNDARY ELEMENT METHOD USING LINEAR ELEMENT FOR THE PLANE POTENTIAL FLOW

BEM, as a numerical method proposed for the two-dimensional simulation of the flow upstream the screen of cylindric bars is applied for the numerical solving of the Laplace equation:

$$\nabla^2 \varphi^* = 0 \tag{1}$$

written in potential φ^* of velocity v^* with boundary conditions impose on the boundary Γ^* of the analysis domain Ω^*. The analysis domain together with the boundary conditions illustrated in fig.2 is represented dimensionlessly in the coordinate system ox^*y^*.

[1] Politehnica University of Timisoara, Traian Lalescu Street, no. 2A,300223, Romania, anton.iosif@ct.upt.ro
[2] Politehnica University of Timisoara, George Enescu Street, no. 1/A, 300022, Romania, eles.gabriel@yahoo.com

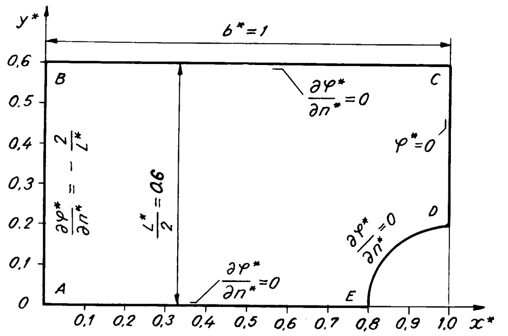

Fig. 2 The analysis domain. Boundary conditions.

It is noticeable that on side AB, BC, DEA of boundary Γ^* are imposable Neumann boundary conditions, respective by Dirichlet on CD. To solve the integral equation [1]:

$$c(\zeta)\varphi^*(\zeta)+\int_{\Gamma^*}\varphi^*(\hat{x})q^*(\zeta,\hat{x})d\Gamma^*(\hat{x})= \int_{\Gamma^*}\frac{\partial\varphi^*(\hat{x})}{\partial n^*}u^*(\zeta,\hat{x})d\Gamma^*(\hat{x}) \qquad (2)$$

on the boundary of the analysis domain this is discretized in a number N = 64 of linear boundary elements as a fig.3.

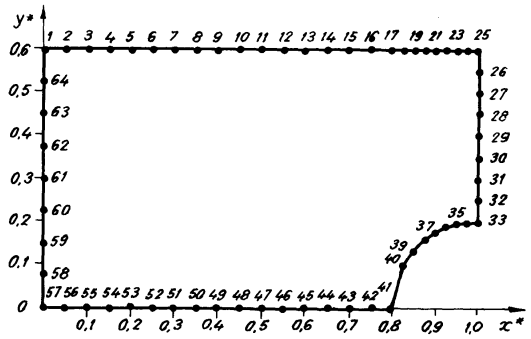

Fig.3 The discretized of the boundary of analysis domain in linear boundary elements

The fundamental solution, $u^*(\zeta,\hat{x})$ from equation (2), for the case of two-dimensional potential problems has the expression [1] :

$$u^*(\zeta,\hat{x})=\frac{1}{2\pi}\ln(1/r(\zeta,\hat{x})) \qquad (3)$$

in which distance $r(\zeta,\hat{x})$ from the source point ζ to a random one $\hat{x}$ from the field is written as such:

$$r(\zeta,\hat{x})= \left[\left(x^*(\hat{x})-x^*(\zeta)\right)^2+\left(y^*(\hat{x})-y^*(\zeta)\right)^2\right]^{\frac{1}{2}} \qquad (4)$$

and $q^*(\zeta,\hat{x})$ is the normal derivative of the fundamental solution. This has the expression [1] :

$$q^*(\zeta,\hat{x})= -\frac{\left[\left(x^*(\hat{x})-x^*(\zeta)\right)n^*_{x^*}+\left(y^*(\hat{x})-y^*(\zeta)\right)n^*_{y^*}\right]}{2\pi[r(\zeta,\hat{x})]^2} \qquad (5)$$

in which : $n^*_{x^*}=\cos\left(n^*,x^*\right)$, $n^*_{y^*}=\cos\left(n^*,y^*\right)$ represents the projection of normal $\vec{n}^*$ after the coordinate axes ox^* and oy^*. We will mention that $c(\zeta)$ represents a coefficient which depends on source point ζ. If we consider on boundary Γ^* a number of N linear elements then for the integral equation (2) corresponds the following discretized form:

$$c_i\varphi^*_i+\sum_{j=1}^{N}\int_{\Gamma^*_j}\varphi^*(\hat{x})q^*d\Gamma^* = \sum_{j=1}^{N}\int_{\Gamma^*_j}\frac{\partial\varphi^*(\hat{x})}{\partial n^*}u^*d\Gamma^* \qquad (6)$$

The values of potential $\varphi^*(\hat{x})$ and its normal derivative $\frac{\partial\varphi^*(\hat{x})}{\partial n^*}$ in any point $\hat{x}$ of a linear element are determined with the help of node values and two interpolation functions [1], [2] :

$$\varphi^*(\hat{x})=\varphi_1\varphi^*_1+\varphi_2\varphi^*_2$$

$$\frac{\partial\varphi^*(\hat{x})}{\partial n^*}=\varphi_1\left(\frac{\partial\varphi^*}{\partial n^*}\right)_1+\varphi_2\left(\frac{\partial\varphi^*}{\partial n^*}\right)_2 \qquad (7)$$

The interpolation functions from (7) have the expressions:

$$\varphi_1=(1-\eta)/2\ ;\varphi_2=(1+\eta)/2 \quad \eta=[-1,1] \qquad (8)$$

According to what has been previously mentioned equation (6) is written as such:

$$c_i\,\varphi^*_i+\sum_{j=1}^{N}\hat{H}_{ij}\varphi^*_j=\sum_{j=1}^{N}G_{ij}\left(\frac{\partial\varphi^*}{\partial n^*}\right)_j \qquad (9)$$

when coefficients $\hat{H}_{ij}$ and G_{ij} have the expressions :

$$\hat{H}_{ij}=h^2_{i\,j-1}+h^1_{ij}\ ;\ G_{ij}=g^2_{i\,j-1}+g^1_{ij} \qquad (10)$$

and $h^2_{i\,j-1}$, h^1_{ij}, $g^2_{i\,j-1}$, g^1_{ij} are computed whith relations:

$$h^2_{i\,j-1}=\int_{\Gamma^*_{j-1}}\varphi_2\,q^*d\Gamma^*\ ;\ h^1_{ij}=\int_{\Gamma^*_j}\varphi_1\,q^*d\Gamma^* \qquad (11)$$

$$g^2_{i\,j-1}=\int_{\Gamma^*_{j-1}}\varphi_2\,u^*d\Gamma^*\ ;\ g^1_{ij}=\int_{\Gamma^*_j}\varphi_1\,u^*d\Gamma^* \qquad (12)$$

From (9) and (10) can be noticed that for writing the discretized equation which corresponds to node i , we must sum up in one term the contribution of two neinghbour elements, thus obtaining the node coefficient. Ecuation (9) can be written under the following matrix form [1]:

$$\boldsymbol{HU}=\boldsymbol{GQ} \qquad (13)$$

where :

$$H_{ij} = \begin{cases} \overline{H}_{ij} & for \quad i \neq j \\ H_{ij} + c_i & for \quad i = j \end{cases} \qquad (14)$$

are elements of matrix $\boldsymbol{H}$, G_{ij} of $\boldsymbol{G}$, and φ_j^* and $\left(\frac{\partial \varphi^*}{\partial n^*}\right)_j$ are elements of vectors $\boldsymbol{U}$ and $\boldsymbol{Q}$.

The diagonal coefficients H_{ii} can be expressed with the help of the off diagonal ones under an easly implementable form on the computer, like such :

$$H_{ii} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} H_{ij} \qquad i = \overline{1, N} \qquad (15)$$

Referring to coefficients G_{ii} we mention that these can be computed analytically and the relation obtained is :

$$G_{ii} = l_{i-1} / 2\pi [0.75 - 0.5 \ln(l_{i-1})] + l_i / 2\pi [0.75 - 0.5 \ln(l_i)] \qquad (16)$$

where l_{i-1} and l_i are the lenghths of the two nearby elements with their contribution summed up in node $\boldsymbol{i}$. After implementing the N boundary conditions, the matrix ecuation (13) can be reorganised in such a way that we obtain a linear system of form:

$$\boldsymbol{AX} = \boldsymbol{B} \qquad (17)$$

in which the vector of unknowns $\boldsymbol{X}$ contains both values φ_j^* as well as $\left(\frac{\partial \varphi^*}{\partial n^*}\right)_j$ in the nodes where these are unknown. For determining the values φ_i^* in point ζ_i from domain Ω^*, we will use the following integral representation written in discretized form [1]:

$$\varphi_i^* = \sum_{j=1}^{N} \overline{G}_{ij} \left(\frac{\partial \varphi^*}{\partial n^*}\right)_j - \sum_{j=1}^{N} \overline{H}_{ij} \varphi_j^* \qquad (18)$$

We shall compute by numerical derivation components $v_{x^*}^*$ and $v_{y^*}^*$ of the dimensionless velocity v^*, because we know both the values of the potential function φ^* in the nodes on the boundary Γ^* as well as in points ζ_i from Ω^*, and then according to the relation:

$$\psi^* = \psi_0^* + \int_c \left(-v_{y^*}^* \, dx^* + v_{x^*}^* \, dy^*\right) \qquad (19)$$

are obtained the values of the stream function ψ^* in point ζ_i from Ω^*.

Further on are determined: the equipotential lines and streamlines, and then are computed the dimensionless velocities v^*, $\overline{v}$ and the dimensionless pressure $\overline{p}$ in the points that define the streamlines using the relations:

$$v^* = \left(v_{x^*}^{*\,2} + v_{y^*}^{*\,2}\right)^{\frac{1}{2}}, \; \overline{v} = \frac{v^*}{v^{*AB}}, \; \overline{p} = 1 - \overline{v}^2 \qquad (20)$$

The inflow dimensionless in the analysis domain is marked with v^{*AB} and has the value 1.667.

3. NUMERICAL REZULTS

For obtaining numerical results were made in FORTRAN lanquage for IBM PC or compatible ones the programms LPLANBEM and LFIMPBEM. The first solves the integral equation (9) on the boundary Γ^* and the second computes according to integral representation (18) the potential function values in points ζ_i from analysis domain and finally determines the hydrodynamic field and the velocities and pressures distributions along the streamlines. În fig. 4 are presented ζ_i from the analysis domain Ω^* and how to number again the nodes on the boundary Γ^*.

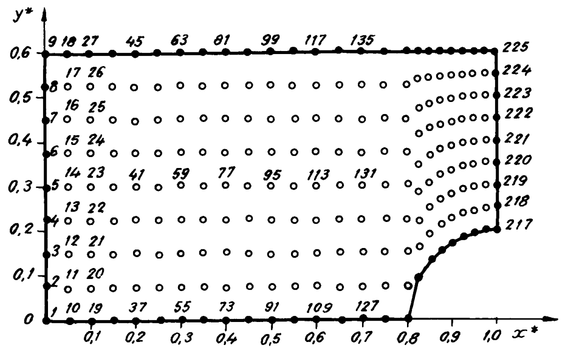

Fig.4 Numbering again the nodes on the boundary and establishing the domain points

The streamlines and the ones forming the hydrodynamic field are presented on the whole in fig.5.

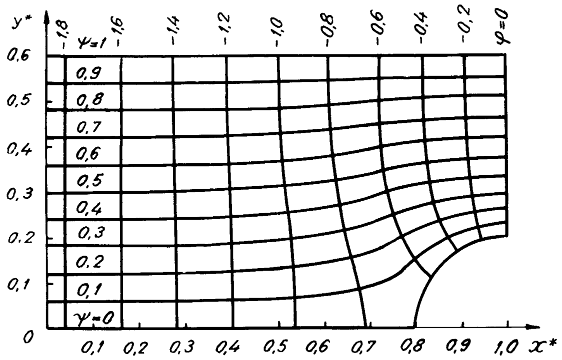

Fig.5 The hydrodynamic field

From the velocities and pressures field from fig.6, fig.7 we interested in the velocities and pressures along the streamline $\psi^* = 0$. This contains also the solid boundary chosen from conditions of symmetry. For compairing these distributions these have been computed with the FEM using the programme PSIELFMP. The components of velocity v^{*e} on the linear isoparametric finite element "e" is computed with relations corresponding to its gravity center [4]:

$$v_{x^*}^{*e} = \alpha_0^{-1} A_{N2} \, \psi_N^{*e} \; ; \; v_{y^*}^{*e} = -\alpha_0^{-1} A_{N1} \, \psi_N^{*e} \qquad (21)$$

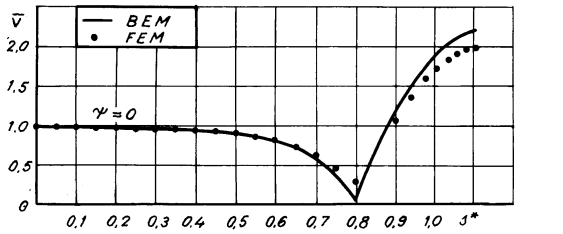

Fig.6 The velocities field along the streamline $\psi^* = 0$.

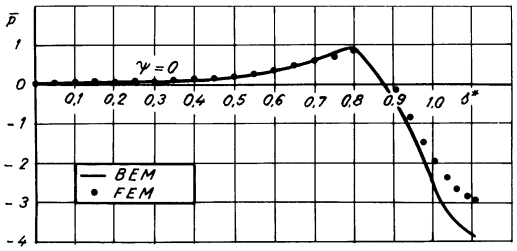

Fig.7 The pressures field along the streamline

4. ANALYTICAL MODEL

The analytical model of the liquid flow motion around a slightly oval cylinder placed in a channel that has parallel walls, is presented in bibliographical references [3], [5], [6]. If we the liquid flow to be uniformed having of v_0 velocity, the channel walls are the flow surfaces as shown in figure 8 in which the system formed by two equal sources with opposite signs has the moment noted with m and is placed in the origin coordinate axes, we shall write the following relations for the complex potential $f(z)$, for m and for the w complex velocity:

$$f(z) = -iv_0 z + im(2l)^{-1} \cot g\left[\pi l^{-1}(z - z_0)\right] \quad (22)$$

$$m = 2\pi^{-1} l^2 v_0 \, sh^2\left(\pi a l^{-1}\right) \quad (23)$$

$$w = -iv_0 - i\,2^{-1} \pi l^{-2} m \sin^{-2}\left(\pi z l^{-1}\right) \quad (24)$$

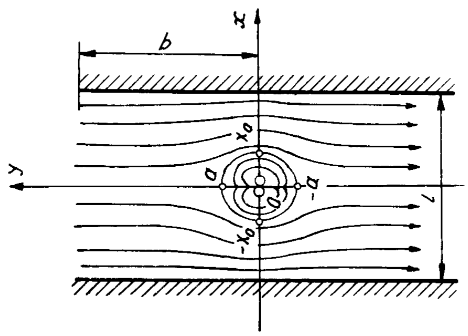

Fig.8 The cylinder placed in a channel with parallel walls

The complex potential of the motion can also be written

$$f(z) = \varphi + i\psi \quad (25)$$

in which $\varphi = \varphi(x,y)$ and $\psi = \psi(x,y)$ are the potential and flow functions. Thein expressions and thos of the v_x and v_y velocity components written both dimensional and dimensionless are explicitely presented in bibliographical reference [6], and for this reason they shall not be mentioned in this paper.

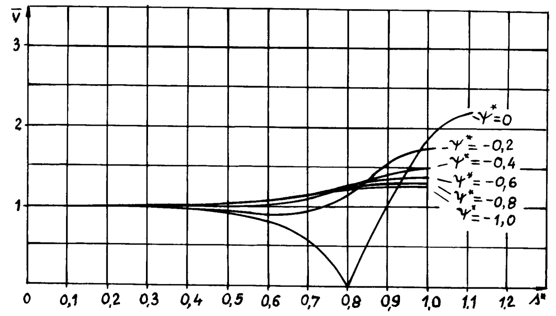

Fig.9 The velocities field along the streamlines

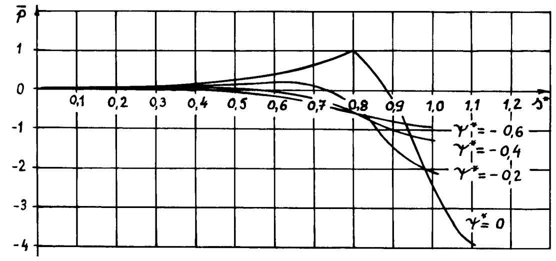

Fig.10 The pressures field along the streamlines

5. CONCLUSIONS

From the velocity and pressure distributions in fig.9 and fig.10, respectively fig.6 and fig. 7 for BEM, taking into account the analysis domain, one can observe that $\bar{v} = 0$ and $\bar{p} = 1$ in point E for $s^* = 0.8$ and $\bar{v} = 2.21$ and $\bar{p} = -3.88$ in point D for $s^* = 1.11$.

The dimensional velocities in the boundary points of the circular section of the raking screen's bases, are obtained by multiplying dimensionless velocity $\bar{v}$ with dimensional velocity v_0 in the case of the analytical model, respectively v^{AB} in the case of the numerical model.

REFERENCES

[1] Brebbia C. A., Telles J. C. F., Wrobel I. C., *Boundary Element Techniques*, Springer Verlag, Berlin, Heidelberg, New York, 1984.

[2] Petrila T., Gheorghiu I. C., *Metode element finit şi aplicaţii,* Editura Academiei R.S. România, Bucureşti, 1987.

[3] Carafoli E., Constantinescu V. N., *Dinamica fluidelor incompresibile,* Editura Academiei R.S. România, Bucureşti, 1981.

[4] Chung T. J., *Finite Element Analysis in Fluids Dynamics,* Mc Graw-Hills, New-York, 1978.

[5] Bărglăzan M., Iosif A., *Modelarea curentului lichid la trecerea prin grătarul unei amenajări hidroenergetice*, Buletinul Ştiinţific al Universităţii " Politehnica " din Timişoara, Seria Mecanică, Tom 44(58) , Fascicola 2, 1999.

[6] Iosif A., Eleş G., *Modelarea analitică a curentului lichid care trece prin grătarul unei amenajări hidroenergetice,* Buletinul Ştiinţific al Universităţii " Politehnica " din Timişoara, Seria Hidrotehnică, Tom 49(63) , Fascicola 1, 2005.