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3-D Curves Intersection between cylindres: Projection on curved surfaces Tangent to the curve in particular points

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Abstract: The solving proposed for intersections between cylinders implies finding the tangent to the curve in particular points determined by their position. In the same time, the curve movement in space is determined through projection parallel to cylinder's axis of the circle inscribed in the square.

The metode has mainly an educational aspect, with the role of improving the students' 3-D view. It is faster, more precise and more intuitive.

Keywords: 3-D CURVES , INTERSECTIONS CYLINDERS, EDUCATION

1. INTRODUCTION

In case of intersection between two cylinders, the classical solving methods are:

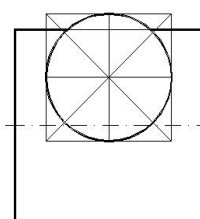
- sections with auxiliary planes, parallel to both axes: intersections of rectangulars or parallelogram result from it [1] [2]
- sections with auxiliary planes, parallel to one axis and perpendicular to the other: intersections between rectangulars and circles result from it, less productive especially in axonometry.

To simplify the presentation and to fulfil the educational aim of stimulating the 3-D view through average complex exercises, both cylinders with horizontal axes, perpendicular in horizontal projection are taken into account, especially because the architectural applications are closer to it.

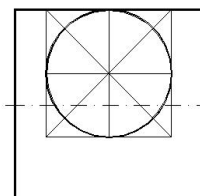
In the following, the three possible positions (that have a meaning for the present topic) of the two cylinders are presented:

(Fig. 1)

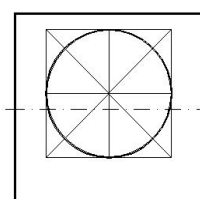
- the cylinder with smaller diameter has the superior generatrix above the other cylinder
- the cylinder with smaller diameter has the superior generatrix intersected by the superior generatrix of the other cylinder
- the cylinder with smaller diameter has all its generatrices intersected by the other cylinder.



There is a single 3-D curve.



There are two 3-D curves that touch each other in a point placed at the intersection of the two superior generatrices



There are two separated 3-D curves

Fig. 1

A starting remark must be done, that is for the two cylinders there are two vertical symmetrical planes passing through the cylinder axes. Once a point is found, this thing allows us to find by symmetry the other three points, thus helping a fast way of solving the problem (Fig. 2). For a cylinder intersected by a quarter of a cylinder there is a vertical symmetry plane through the cylinder axis (Fig. 3).

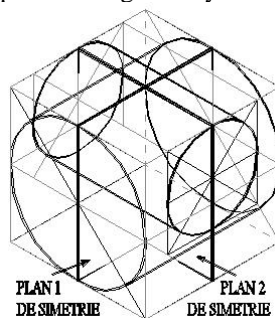


Fig. 2

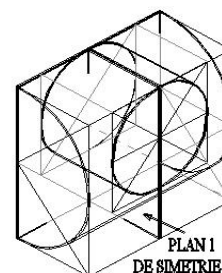


Fig. 3

2. PROJECTION ON CURVED SURFACES

Analogical to the anamorphoses [3] used in the wall painting on the churches and palaces ceilings to correct the visual deformations that occur when paintings do not take the support geometry into account, the projection on a cylindrical surface of the ABCD rectangle along the OY axis results into a cylindrical surface ApBpCD where each point on the side and inside of the rectangle has a correspondent at the intersection between a parallel to the OY axis and the cylindrical surface.

In this case, the vertical lines generate arcs of circle, the horizontal lines generate horizontal lines on the cylinder generatrix. (Fig. 4).

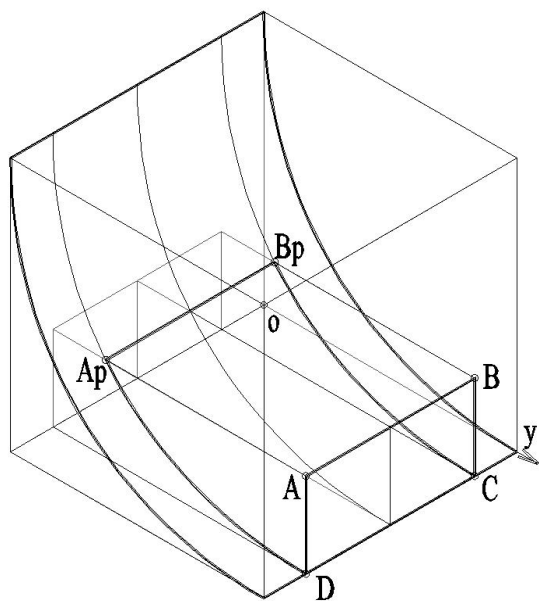


Fig. 4

Diagonals generate parts of an ellipse; and in case of O1Ap, it is always seen as a straight line, making the solving very easy.

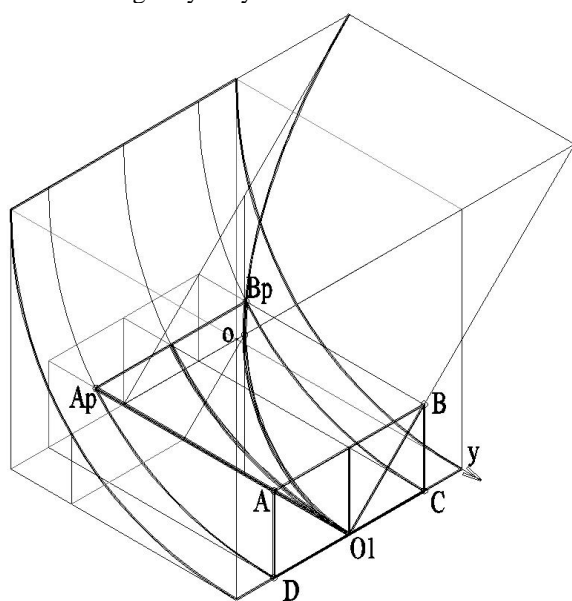


Fig. 5

3. 3-D CURVES AND THEIR TANGENTS IN PARTICULAR POINTS

A circle can be approximated as being a regulated geometrical polygon with N sides, similarly a cylinder can be approximated as being formed by N rectangulars with two sides equal to the generatrix and two sides determined by N.

The bigger the N, the smaller the other two variable sides tending towards zero.

Taking into account a rectangle with one side equal to DD2 and another short side, variable, with one end in D and the other in L1, on the arc of circle, the more the second end L1 tends towards D the more the rectangle tends to become vertical.

As limits in math, when the variable side of the rectangle D L1 tends towards 0 it is a vertical rectangle, tangent to the cylinder according to DD2. Similar for the rectangle determined by L2FC it is a horizontal rectangle, for the one determined by L3GH it is a rectangle included in the tangent plane to the cylinder according to the GH line, for the rectangle formed around the EEp straight line it is a horizontal rectangle tangent to the cylinder according to EEp (Fig. 6).

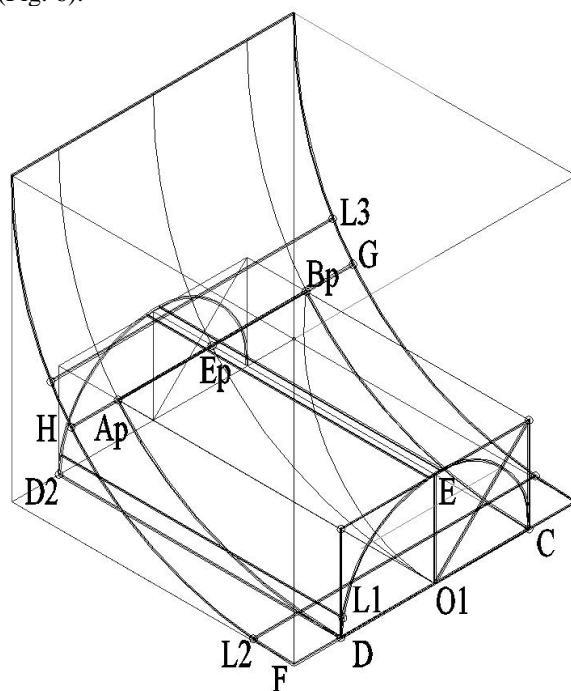


Fig. 6

Following the reasoning of approximated cylinders to N and M rectangles, so that the short sides are comparable, the intersection between them is a curve approximated by P segments, determined by the intersection between two by two rectangles.

Extending further, the tangent in a point to the intersection curve between two cylinders is given by the intersection line between the two planes tangent to the cylinders in that point.

The DD2 tangent to the intersection curve in D and ApBp tangent to the intersection curve in Ep result from it.

The solving can be done both in the triple projection and in axonometry. Generally, tangents to the curve on the generatrices that start from the middle of the square sides and from the square diagonals are enough.

If a too long distance of the curve stays uncontrolled, the tangent to the curve on one or two intermediary generatrices are found, like in the next case. (Fig. 7)

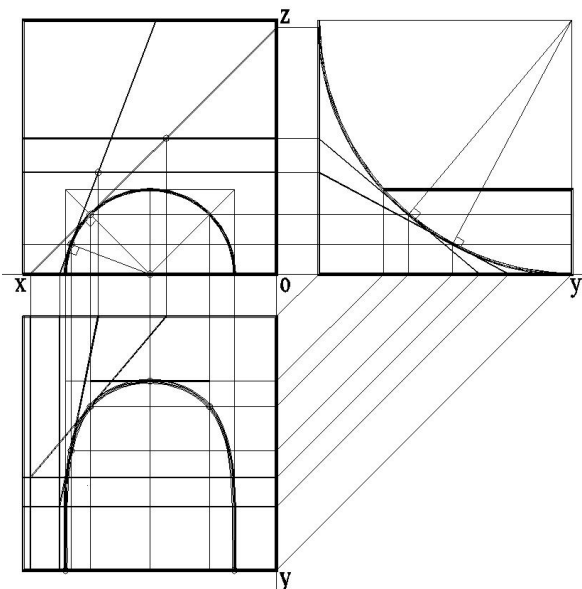


Fig. 7

In fig. 8 the intersection between the two planes that determine the tangent to the curve in the Ip point for the generatrix passing through the square diagonal (point I) and its symmetrical for the symmetrical point Ips is presented.

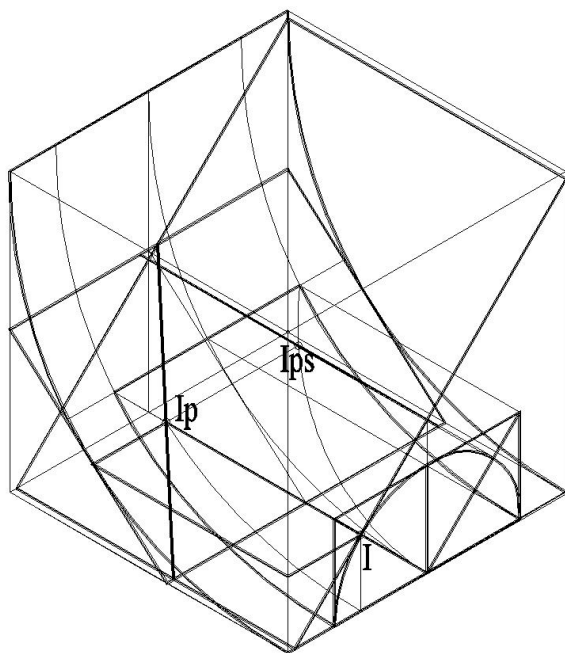


Fig. 8

Because a larger part of the curve would remain uncontrolled, a supplementary generatrix passing through a point J chosen between points E and D (fig. 6), on which the tangent to the curve and its symmetrical lay, is drawn (Fig 9).

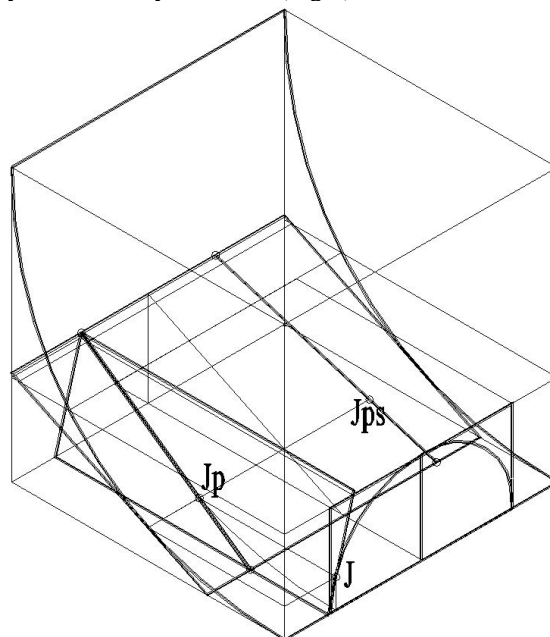


Fig. 9

After finding points through which the curve passes, out of which 5 particular and 2 supplementary, after tangents to the curve in these points were found, the intersection curve is represented minding the way it passes through the tangency points and the way it curves or straightens. Once it is freely drawn, the areas that match best the curve movement are searched on the French curve and they are carefully drawn in the racording zones (Fig. 10).

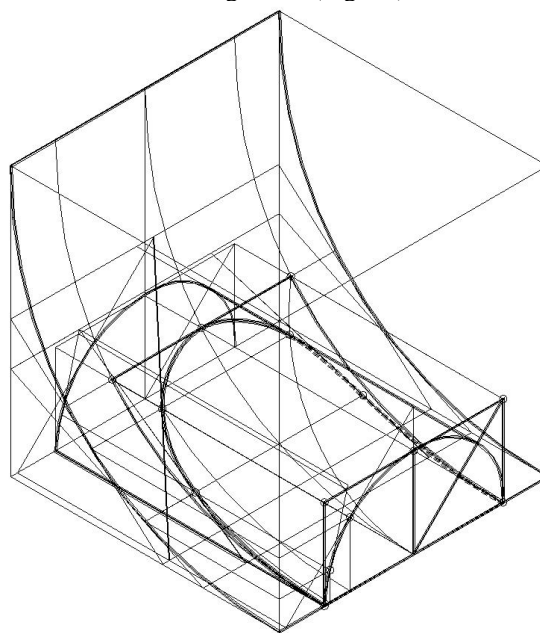


Fig. 10

In figures 11 and 12 the curve movement in space can be noticed, it depends on the two cylindrical surfaces that generate it.

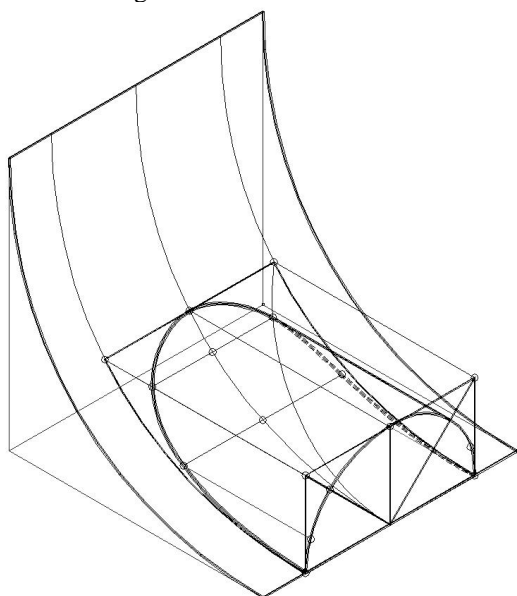


Fig. 11

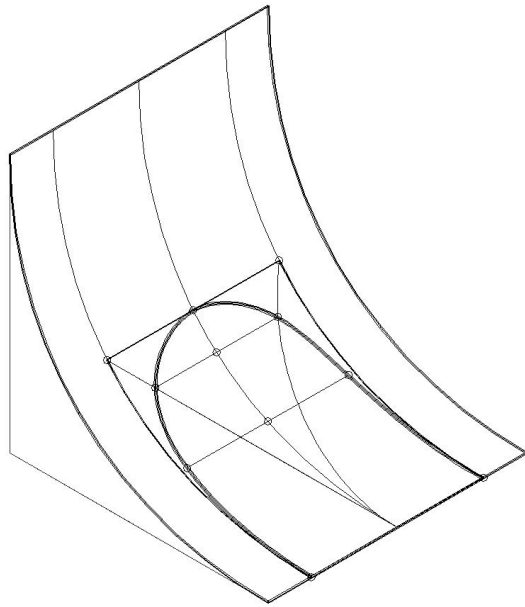


Fig. 12

4. CONCLUSIONS

The method is applicable in problems of cylinder intersections, when these have parallel axes to a projection plane. The methodology does not differ in cases when the axes projections form an angle

different from 90 degrees then from the point of view of taking the tangent symmetricals.

It has the advantage of being very fast, intuitive and in the same time, due to introducing the tangent, more precise than the other methods because by simplification there are less errors to be accumulated.

It addresses firstly to the architecture students due to the free drawing skills .

After understanding the method, students were able to guess before solving the problem the way in which the curve unfolds, proof of the development of the 3-D view capacity. This is important because descriptive geometry, as it is seen in the architecture faculties, it has firstly the role of developing the 3-D view.

5. CLAIMS

Crossing from an analytic approach to a synthetic one, where the curve features in specific points and its dynamics are studied.

Becoming aware of the existence of the tangents to the curve and finding them.

Rapidity and especially the precision of the method.

Developing the 3-D view.

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