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Sections in Cylinders by the Projection on Inclined Planes.

Tangent to the Sectioning Curve in Particular Points. Tangent Generatrix Method.

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Abstract: The proposed method to solve sections with inclined planes through cylinders can be also used in the case of the intersection of a cylinder with a prism that has inclined faces related to the cylinder generatrix. It has mainly an educational aspect, with the role of improving the students' 3-D view. In the same time, the method has the advantage of being faster, more precise and more intuitive and, once learned, there is no room for errors. The method involves using the characteristics of the inclined projections on planes, respectively finding some particular points and the tangent to the curve in those points.

Keywords: CYLINDER, INCLINED SECTIONS, EDUCATION

1. INTRODUCTION

Nowadays, more methods can be used to solve inclined sections through cylinders or the intersection of a cylinder with an inclined faces geometrical body (prisms, pyramids, complex compositions), such as:

- auxiliary particular planes parallel to the cylinder axis – the most frequent used methods [1] [2], (Fig. 1)
- auxiliary planes perpendicular to the cylinder axis – more laborious and uneconomic methods
- auxiliary planes through the cylinder axis - in particular cases
- changing the projection planes

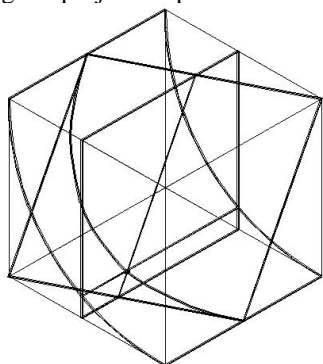


Fig. 1

More planes are necessary for a more correct solving of the curve. The solving becomes more complicated when the problem complexity grows, e.g. a cylinder empty inside in the shape of a cylinder cut by a plane (1), or intersected by an inclined prism

generates 4, 6 or 8 points for each auxiliary plane taken (Fig. 2).

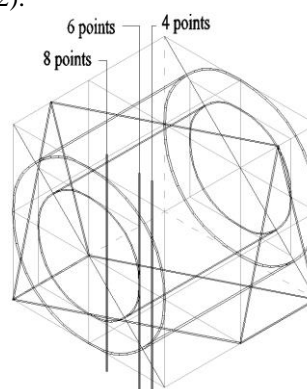


Fig. 2

The points can be used to draw curves only after all planes were taken. A very loaded drawing results from it, where it is very difficult to follow all the points through which the intersection curve must be drawn.

On the other hand, because of the errors, the points do not align on the curve, but they have deviations that are corrected by drawing.

To exemplify a quarter of a cylinder was taken into account. Problems are similarly solved both in isometrical axonometry and in the triple projection. (Fig. 3) (Fig. 4).

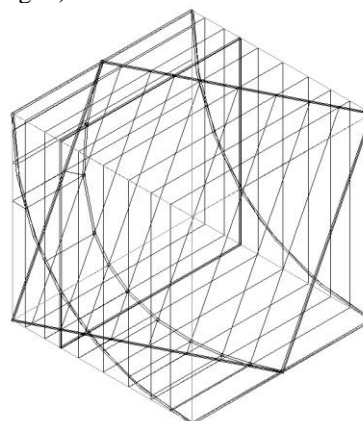


Fig. 3

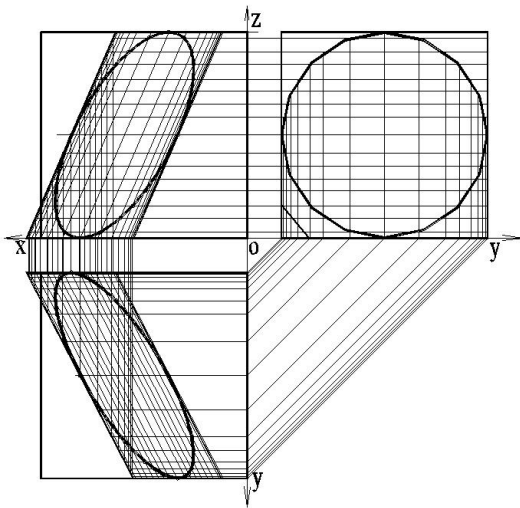


Fig. 4

In addition, planes drawing and construction lines following errors occur. It is not the ideal case that we talk about, but students' solving and inaccuracy originate from a lower precision, lack of attention, lack of focus on longer term, practical experience accumulated, lack of drawing abilities when the curve is prefigured before being thickened with the French curve. On the other side, the solving becomes automatic and tests the student's patience, moving him/her from the pleasure of drawing.

2. TANGENT GENERATRIX METHOD

2.1 INCLINED PROJECTION

Taken into consideration a geometrical figure formed by the lateral square MNPQ and the quarter of a circle inscribed into it, by projecting parallelly to the Ox axis on the inclined plane $MpNpPpQp$ its deformed figure comes out. Nevertheless, this deformed figure still keeps features of the initial figure, such as parallelism, linear proportions, tangency (Fig. 5).

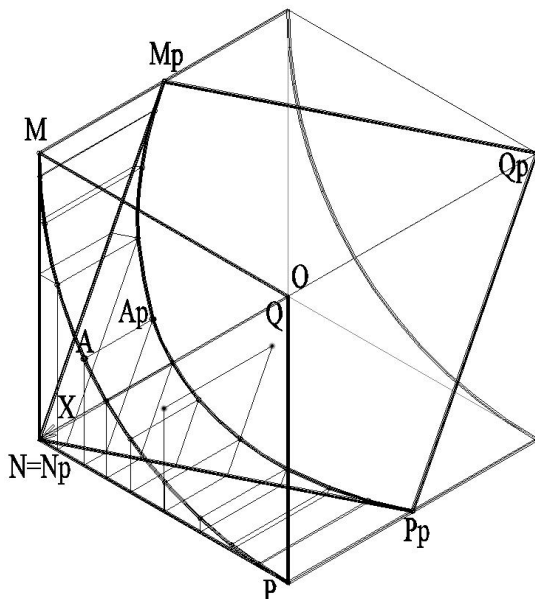


Fig. 5

Conveniently, the plane materialization within the cube, in this phase, is named $MpNpPpQp$ (where Mp = projected M), in order to highlight the fact that it is a projection of the MNPQ square.

In the same time, each point in the lateral plane has a projection on the inclined plane to a parallel to OX axis. It means that each A_p type point has the same relationship with the points on the inclined plane like the A type point in connection to the points on the lateral plane, including in the equation, of course, the deformation of distances and angles resulted from the plane slope.

2.2 SOLVING METHOD

Next, to avoid redundancy and to keep the drawing as simple as possible, we will discuss the solving method for a quarter of a cylinder.

1 – the plane is prolonged until it cuts all the edges parallel to the quarter of a cylinder (the cylinder), edges of the parallelepiped in which it is inscribed, resulting into a $MpNpPpQp$ parallelogram. This parallelogram is the deformation of MNPQ square on the inclined plane.

2 – the NQ and $NpQp$ diagonals are drawn and with the help of a plane (in this case the frontal plane) the projection of D, (D being the point where the diagonal cuts the arc of circle), is determined on $NpQp$ diagonal included in the sectioning plane resulting the point Dp .

3 – the tangent to the curve in point D is drawn; it is parallel to the $MpPp$ diagonal in the same way as the tangent to the initial curve passes through D and it is parallel to MP.

4 – if the drawing is bigger or if the student does not have a developed ability to draw freely, two intermediary points could come out (Rp și Sp).

5 – next, the curve is drawn freely passing tangently through Mp to $MpPp$, through Dp to the tangent determined parallel to $MpPp$ and through Pp tangently to $NpPp$.

6 – the curve is drawn with the French curve searching on it the areas that correspond best to the freely suggested curve. (Fig. 6)

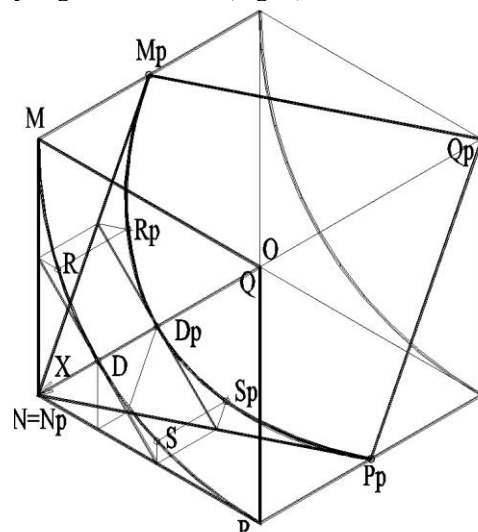


Fig. 6

Form the previous solving, one notices that generatrix MMp and PPp of the quarter of a cylinder according to which it (the quarter) is tangent to the paralleliped in which it is inscribed, include the points where the curve is tangent to the sectioning parallelogram, hence the name of the method. In the same time, the generatrix DDp drawn through the point where the circle cuts the diagonal will include the point Dp that has to be found, but through which a tangent to the curve can be drawn, parallel to the other diagonal of the MpPp parallelogram, therefore it will also include tangency points.

In the following examples, the notes were left aside for a better understanding of the drawing.

The problem in the triple projection is solved similarly. (Fig. 7).

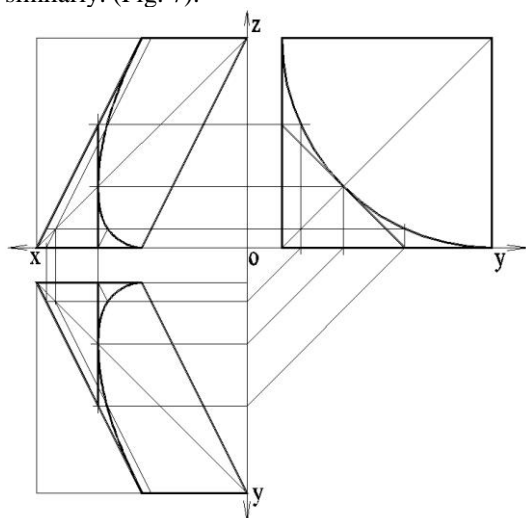


Fig. 7

Next comes a solving for a cylinder (Fig. 8) (Fig. 9). In the left - down corner the plane was chosen to be prolonged outside the cub to form the parallelogram.

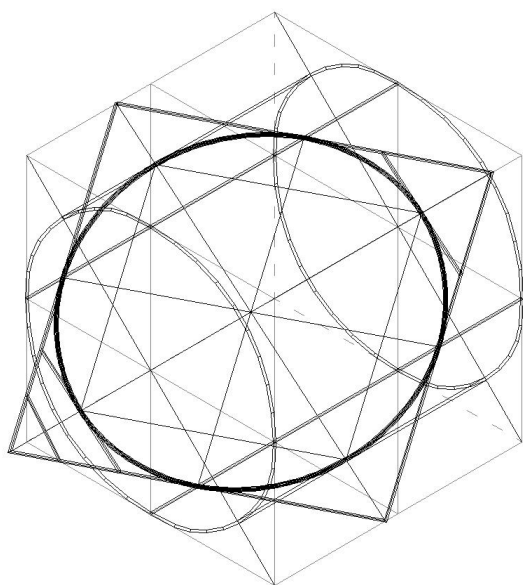


Fig. 8

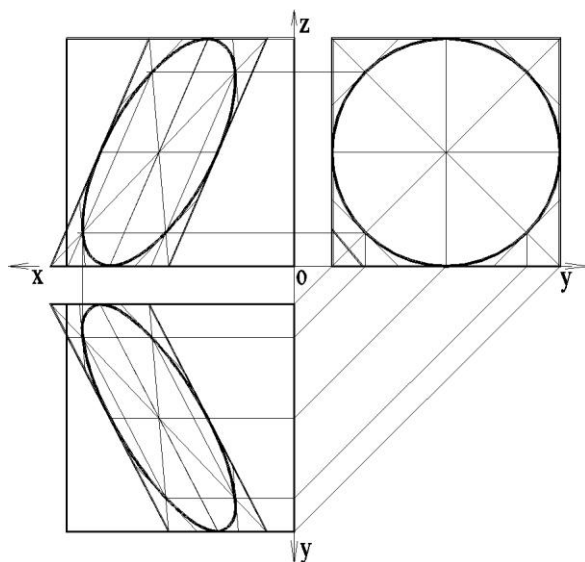


Fig. 9

Solving in the version where the plane cuts the cylinder discs (Fig. 10 , Fig. 11)

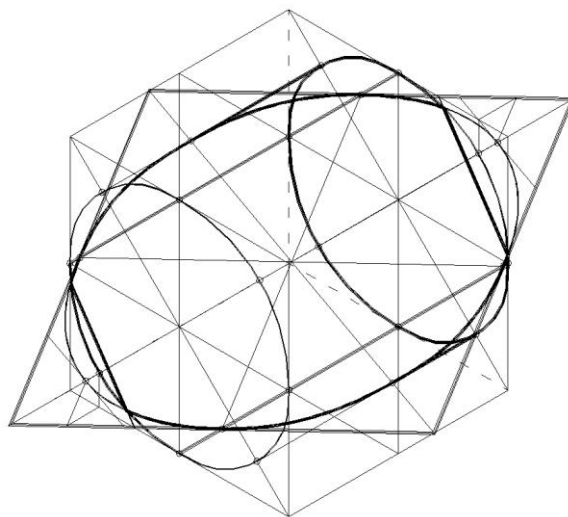


Fig. 10

T1 , T2 , T3 , T4 , T5 , T6 – tangent points
T1B, BT3, GJ (GJ II BF) , DT5, KLT6 – tangents
ABCDE – sectioning plane

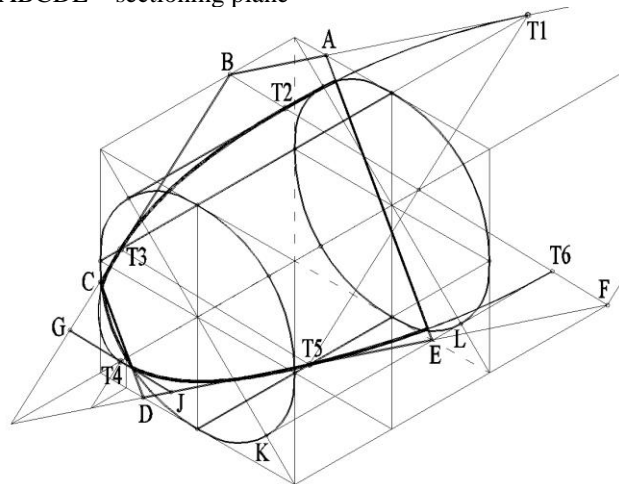


Fig. 11

There are many ways to pass through 3 points.
The tangents stabilizes the path to the right trajectory.
(Fig. 12).

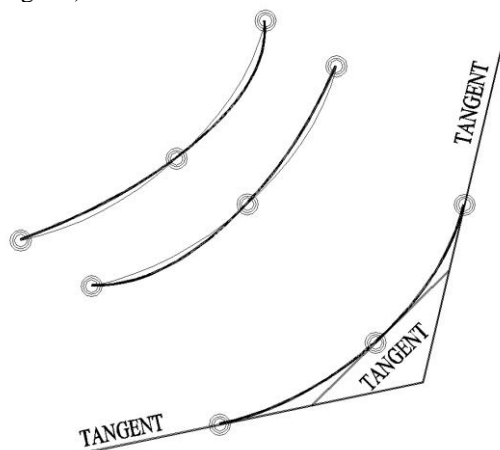


Fig. 12

The bending of the curve depends of AB/BC proportion and ABC angle (Fig. 13).

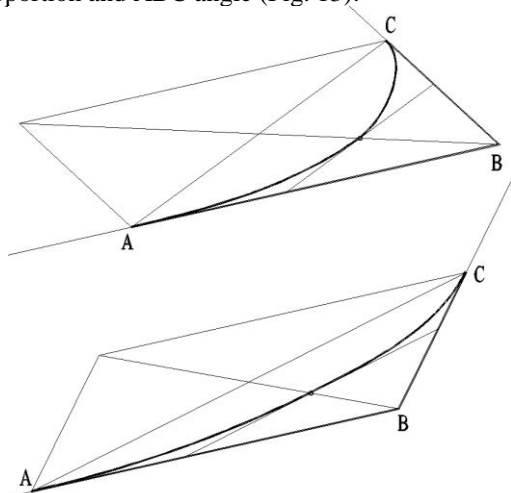


Fig. 13

The FE/ED proportion is $1/\sqrt{2}$ (Fig. 14)

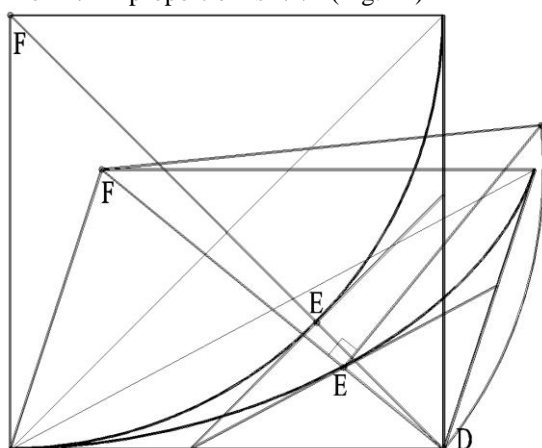


Fig. 14

3. CONCLUSIONS

The method is applicable in any problem of sections or intersections connected to cylinders.

It has the advantage of being very fast, intuitive and in the same time, due to introducing the tangent, more precise because it controls the slope of the curve and by simplification there are less errors to be accumulated.

It addresses firstly to the architecture students due to the free drawing skills and to the type of problem it solves, but it works with visible results to beginners, to cases where the same problem was solved by both methods; in the previous case the curve was more correct and faster solved.

After understanding the method, students were able to foresee before solving the problem the way in which the curve unfolds, proof of the 3-D view capacity development. This is important because descriptive geometry, as it is regarded in the architecture faculties, it has firstly the role of developing the 3-D view.

4. CLAIMS

Crossing from an analytic approach to a synthetic one, where the curve features in specific points and its dynamics are studied.

Becoming aware of the existence of the generatrix where the curve is tangent to both the parallelogram and the parallelepiped where the cylinder (or a fraction of it) is inscribed.

Becoming aware of the existence of the generatrix through the points of the diagonal, through which tangents to the curve can be easily drawn.

Rapidity and precision of the method.

- Developing the 3-D view.

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