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Ruled surface forms

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Abstract: This paper deals with the field of forms consisting of ruled surfaces or scrolls, widely used in the creation of contemporary architecture, emphasizing the class of non-developable surfaces. Various formal categories that can occur and methods of spatial composition and planar and axonometric representation are highlighted.

Ruled surfaces have numerous applications in construction techniques, allowing simpler and easier ways of execution, in addition to easier building load calculation. In this respect, the paper illustrates some of the international successful achievements.

In the end, the paper presents some teaching methods used for some practical applications of subjects taught in first years of study at the Faculty of Architecture in Timisoara, aiming to find interesting forms adapted to different functions.

Keywords: architecture, spatial forms, geometry, generatrix, directrix

1. INTRODUCTION

Ruled surfaces are part of the formal language of architecture, found in different styles and contemporary trends, along with the polyhedral forms, thin curved shells, folded plates, helix forms, tensile and pneumatic membranes or various forms resulting transformations or combinations of the above.

Spatial forms defined by ruled surfaces are usually volumes defined by moving a line (the generatrix) along three curves (the directrices). The shapes of these surfaces are infinite, easily understood from the geometry point of view and rational from the structural point of view, favoring a constructive relationship between the architect and the engineer.[1]

Ruled surfaces are divided into two main categories: developable surfaces and non-developable surfaces (skew ruled surfaces). This paper focuses on the latter category due to their wide formal variety and, at the same time, to the fact that they are less studied and harder to be constructed because of technological or economic limitations.

Therefore we will present, using orthogonal projections and axonometric images for a better understanding of their spatiality, seven formal categories resulted from different relational possibilities between the generatrix and the curves, the lines or the director surfaces, concluding with some examples of building structures that make use of scrolls and some applications of architect students in early years of study.

The hyperboloids, paraboloids, conoids, cylindroids or surfaces defined by parametrical translation of the generatrix, are presented as form givers with an infinite variation of forms. Within each formal family there are presented both general variants that define a certain surface and particular ones, when one of the directrices grows to infinity and is replaced with a directrix plan.

Thus, this paper highlights the diversity of forms using ruled surfaces, their classification by geometric and structural criteria, as well as methods for obtaining variations, both within the same formal category or combinations.

Nature is an inexhaustible source of examples for architectural creation and these surfaces can be found in the form of crystals, minerals and rocks, especially in the way they metamorphose at high pressures and temperatures.[2] They also appear in the morphology of tree or plants leaves and in the infinite variety of living organisms in the terrestrial environment, especially marine, either in the general form or in certain parts, as shown in Fig.1.

Fig. 1

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2. NON-DEVELOPABLE RULED SURFACES

They are also called *skew surfaces* and are characterized by the variation of the tangent plane, along with the changing position of the point of tangency on the generatrix. Hence, to every new position of the point on the generatrix there is a corresponding new plane tangent to the surface.

The tangent plane at a point at infinity of a generatrix is called the *asymptotic plane*. The *central plane* of a generatrix is the tangent plane at a point of that generatrix (*center point*), perpendicular to the asymptotic plane corresponding to the same generatrix. In a central point, the corresponding tangent planes have an equal angle with respect to the central plane. The *striction line* of a surface is the locus of the central points of the generatrix of that surface.

The variation of the tangent plane can be studied using *Chasles's formula*: $\mathbf{tg\theta = x / k}$, expressing that the trigonometric tangent of the angle formed by a tangent plane at a point with the central plane equals the quotient of the distance of this point to the central point (x) by the distribution parameter of the generatrix (k). [3]

For an easier understanding and spatial representation, the generatrices of the ruled surfaces are generally found in a beam of vertical planes.

Ruled non-developable surfaces generated by a line which is moving along:

2.1 – Three curves as directrices represented in Fig. 2.1 and 2.2 by curves contained in vertical and horizontal planes. One can easily see that the position of two curves is random, but the third is a result of the intersection of the generatrices with the horizontal or vertical traces of the vertical planes that contain them.

Each generatrix must move along all three directrices, thus the condition that one of them is to be obtained at the intersection of the plane that contains it with various particular planes that include the generatrices.

2.2 – Two directrices and a core surface, which is a surface of revolution. In Fig.2.3 the core surface is a cylinder, one directrix curve is given and the other is spatially determined, as in the previous case, at the intersection of the vertical planes containing the generatrices (tangent to the core surface) with the plane that contains it. The position of the elements defining this surface is random, therefore there is an infinity of spatial variations.

In the case where the core surface is a cone, the beam of vertical planes will be tangent to the cone, containing one of its generatrices and in the case where the core surface is a sphere, one of the directrix is grows to infinity and a director plane is formed, the generatrices tangent to the sphere being parallel to the latter. Due to lack of space, these two cases were not figured.

2.3 – Two directrices and is parallel to the generatrices of a director cone

In Fig. 2.4 the directrix curves are the sections of the cone parallel to the director cone, hence conical, and the generatrices are obtained by sectioning the same cone by planes containing its vertex. The director cone provides the certitude that the generatrices of the ruled surface are moving along the directrix curves, while under certain conditions it may be transformed into a director plane.

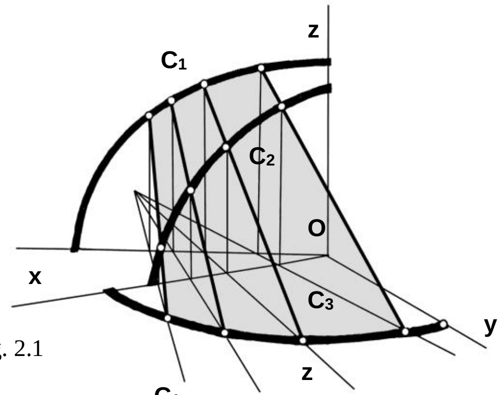


Fig. 2.1

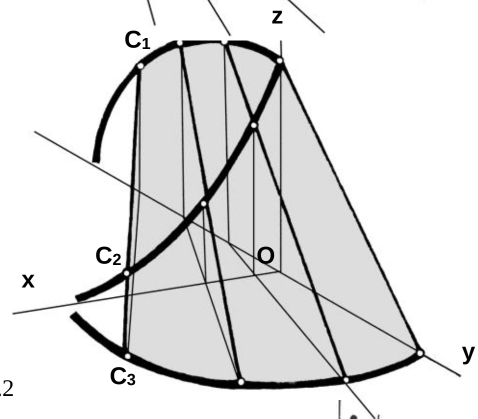


Fig. 2.2

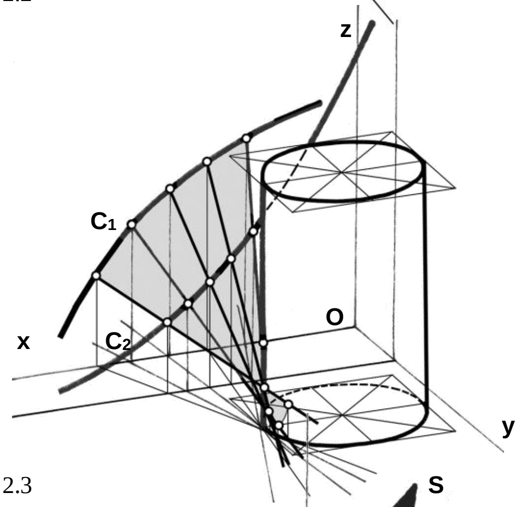


Fig. 2.3

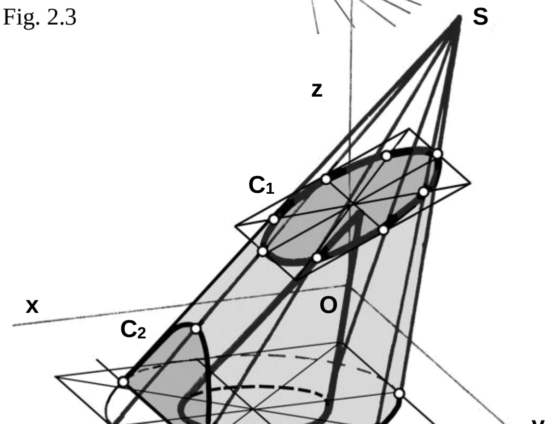


Fig. 2.4

2.4 – Two curves and a rectilinear line as directrices, when the surface is called a *cylindroid.*

In the case where the directrix is at a finite distance we have the case of a *general cylindroid*, while when it is at infinity, we have the *cylindroid with a director plane*. [4]

Therefore, in Fig. 2.5 we have the representation of a general cylindroid, the generatrices are moving along a directrix curve $\mathbf{C_1}$ situated in the vertical plane, another curve $\mathbf{C_2}$ situated in the horizontal plane and finally, a directrix line **D** situated in the profile plane of reference. In order to represent the generatices moving along the three directrices, a fascicle of guiding intersecting vertical planes is used. One directrix curves results from the intersection of these generators ruling on the other directrices with the traces of the vertical planes containing them.

Likewise, in Fig. 2.6 we represented a cylindroid with a director plane parallel to the profile plane of reference, since the directrix line **D** contained in this plan was at infinity. In this case, all generatrices are parallel to this plane. In the case of the general cylindroid, if the planes containing the generatrices are carried out through the directrix line, we are dealing with the *axis cylindroid*, widely used in architecture. Some of the frequent applications of cylindroids in the field of construction and architecture are: the cylindroid vault (which makes the transition between two different openings on the same wall thickness), the skew arch passage (often used at the intersection of two highways at different levels at an angle different from 90^0) or the roofing of freeforms, when directrix curves $\mathbf{C_1}$ and $\mathbf{C_2}$ materialized in concrete or metal structural elements may present extremely interesting paths. [5]

2.5 – A curve and two rectilinear lines as directrices, when the surface is called a *conoid.*

When the two lines are to be found at a finite distance we have the case of the *general conoid* (Fig.2.7 with a directrix curve **C** situated in the horizontal plane and two rectilinear directrices $\mathbf{D_1}$ and $\mathbf{D_2}$ situated in vertical and profile planes of reference), while when one of the lines is at infinity, we talk about the *conoid with a director plane* (Fig. 2.8 with a director plane parallel to the profile plane of reference and all generators parallel to it).

For its part, the conoid with a director plane may be *straight*, when the straight directrix is perpendicular to the plane (particular cases: the helicoid with a director plane, Plüker or Viviani) or *oblique* when it does not have this position (particular case - the Kuper conoid).[6] In every particular case bearing the name of the person who used it for the first time, the directrix elements have different shapes and spatial positions. In the alternative where the two rectilinear directrices are lines at infinity, the surface becomes a *cylinder*.

In axonometry, one of the directrices is always resulting from the intersection of the generatrices moving along the other two with the traces of the fascicle of guiding planes.

Among the numerous applications of conoids in architecture we can mention: the conoidal vault (with a vertical rectilinear line as a directrix, a frontal semicircle as the directrix curve and the horizontal plane of reference as the director plane), different thin curved shells roofs (with a sinusoidal directix, a frontal-horizontal line as directrix and the profile plane as the director plane), different shed roofs with clerestory windows (for lighting and ventilation).

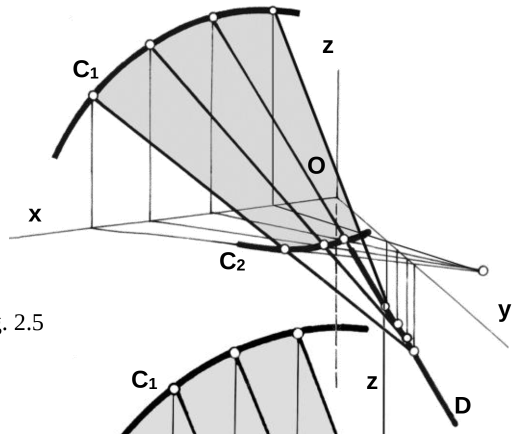


Fig. 2.5

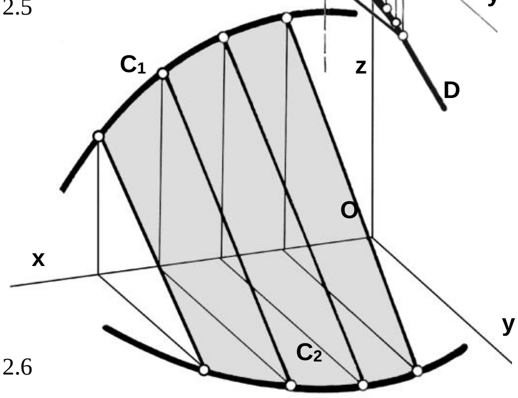


Fig. 2.6

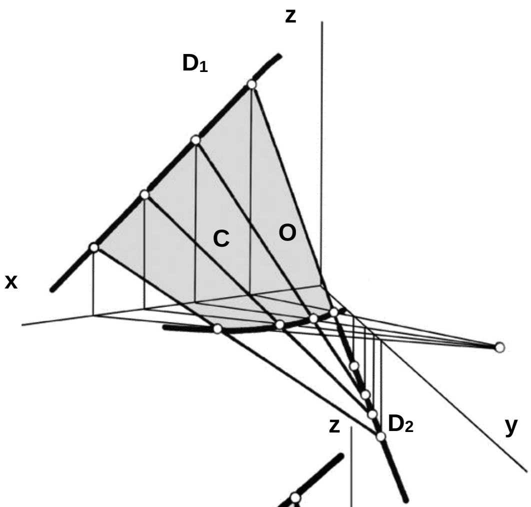


Fig. 2.7

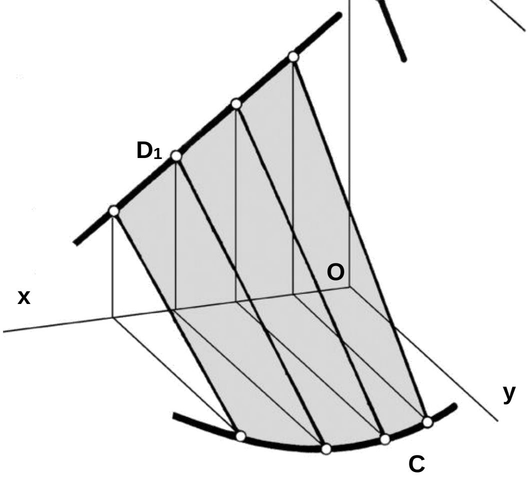


Fig. 2.8

2.5 – Three rectilinear lines at a finite distance as directrices, resulting a ruled surface called *the general hyperboloid of one-sheet*, when the directrices are at a finite distance.

Thus, in fig.2.9 a general hyperboloid is represented, where directrix line **D_1** is the axis **Oz**, in order to simplify the axonometric construction, **D_2** directrix is an end vertical line and line **D_3** is contained in the horizontal plane. For finding the generatrices, a fascicle of guiding vertical planes contains one of the directrices, in our case the line **D_1**.

The image in Fig. 2.11 represents another spatial variation of a general hyperboloid, where the directrix **D_1** is contained in the vertical plane, the directrix line **D_2** is horizontal and the directrix line **D_3** and the axis **Oy** are the same, while the fascicle of guiding vertical planes contains the directrix line **D_3**. There are two ways in which a general hyperboloid can be determined or represented orthogonally.

Thus, in fig.2.9 and 2.11 two variants are presented where the surfaces are determined by generatrices moving along three rectilinear lines **D_1**, **D_2** and **D_3**. Sectioning these surfaces with the planes of reference or with the faces of a circumscribed parallelepiped defines either rectilinear lines (directrices or generatrices) or hyperbolic segments, which are determined by points.

The second way that allows viewing and defining the surface of a general hyperboloid is to border it by different sections (ellipses or hyperboles) drawn on the faces of a parallelepiped frame.

2.6 - Three rectilinear lines, two at a finite distance and one at infinity as directrices, resulting a ruled surface called the *hyperbolic paraboloid (HP)*.

Thus, in Fig. 2.10 **D_2** and **D_3** directrices are taken from the previous figure, while **D_1** is at infinity and a director plane parallel to the vertical plane of reference is developed, so the surface becomes a hyperbolic paraboloid. In this case, all the generatrices are parallel to the director plane and determine equal rapports on the directrix lines.

Likewise, in Fig. 2.12 directrices from Figure. 2.11 are reproduced, but the directrix **D_3** is a line at infinity and a director plane parallel to the profile plane is developed, while the generatrices will be parallel to this plan.

We should point out that the hyperbolic paraboloid is the only ruled surface with two director planes, being a double ruled surface and there are two families of rectilinear generatrices that can generate the same surface. Due to the ease of planar representation and spatial materializing of these surfaces, they represent the most frequently used applications of ruled surface in buildings.

The roofing possibilities of planar outlines depends on:

• the surface shape • the number of hyperbolic paraboloids (HP) and their combination (adjacency or intersection) • the load bearing outlines (on the sides or points). [7]

A HP can be generated by a parabola that performs a translational motion, moving parallel to itself and moving along another parallel axis parabola in one point, but facing backwards. In this case the hyperbolic paraboloid surface is bounded by four parabolic arcs.

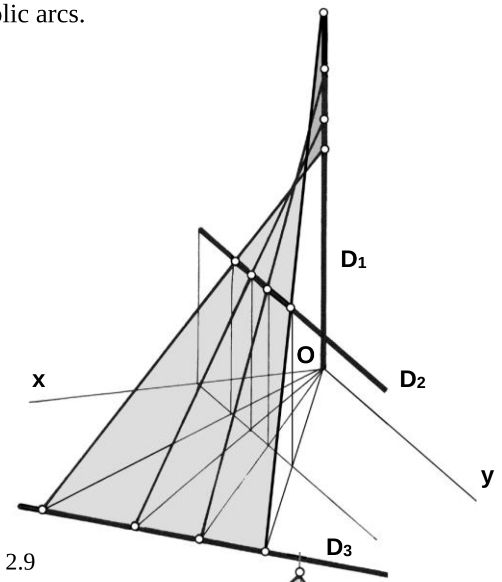


Fig. 2.9

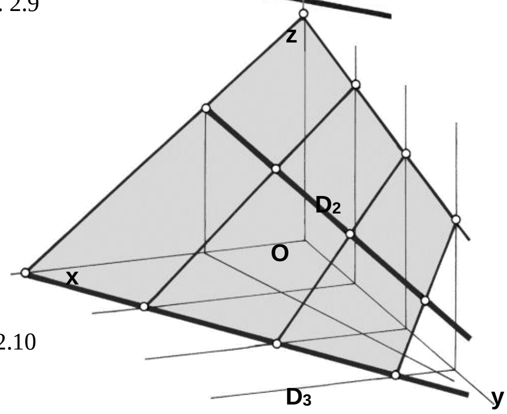


Fig. 2.10

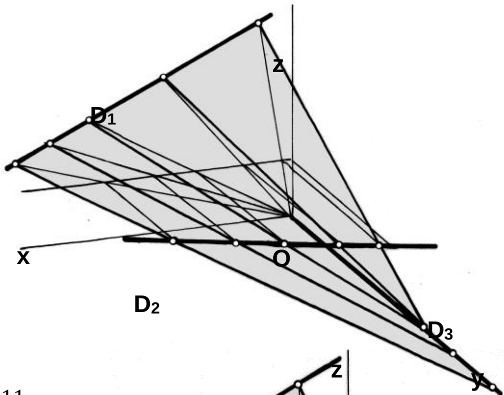


Fig. 2.11

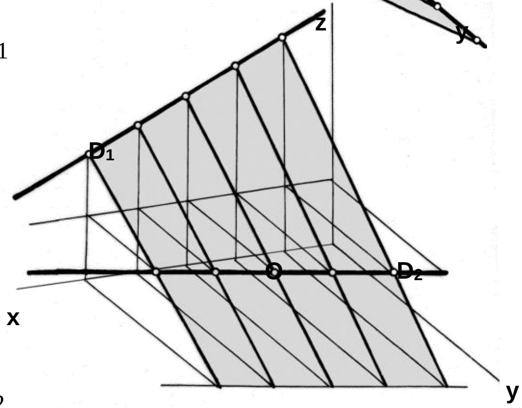


Fig. 2.12

3. EXAMPLES

In the following images we present some remarkable buildings using ruled surfaces (cylindroids in Fig. 3.1, Fig. 3.2; conoids in Fig. 3.3, Fig. 5.4, hyperboloids in Fig. 3.5 and paraboloids in Fig. 5.6) in order to illustrate the richeness of these categories of spatial forms designed to accommodate different types of architecture programs. [8]

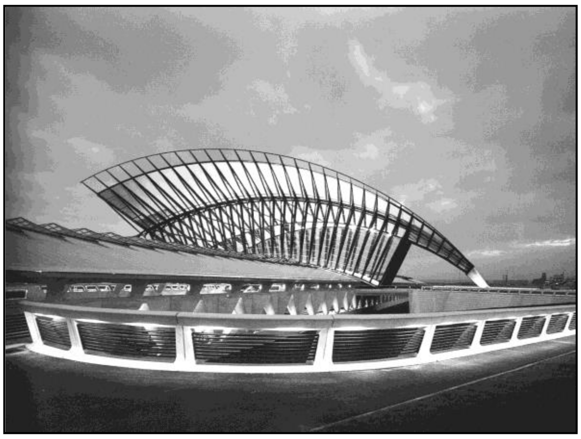


Fig. 3.1 Cylindroid - Santiago Calatrava
TGV train station,
Lyon– Satelas, France, 1989 – 1994

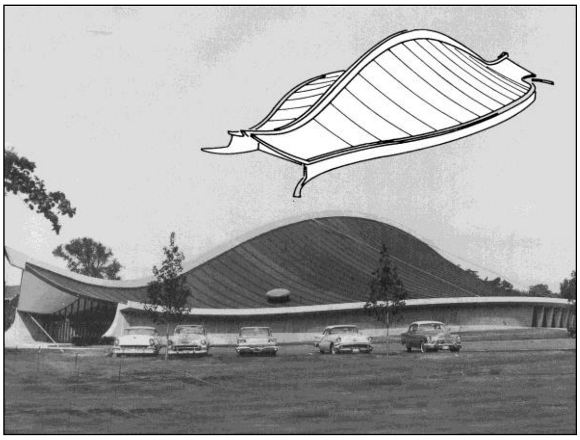


Fig. 3.2 Cylindroid - Eero Saarinen
Yale Univeristy Ingalls Ice Arena
New Haven, Connecticut, 1959

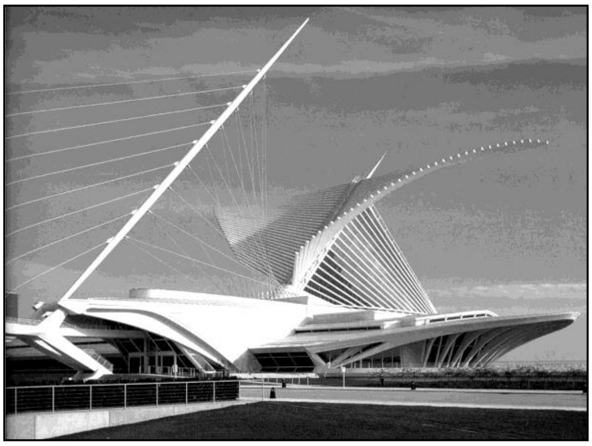


Fig. 3.3 Conoid - Santiago Calatrava
Art Museum
Milwaukee,Wisconsin, USA, 2001

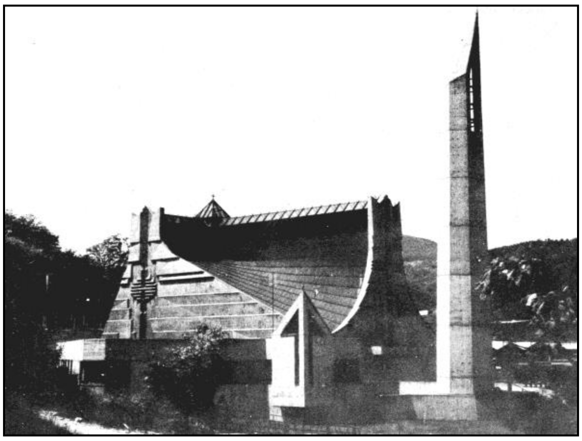


Fig. 3.4 Conoid - Hans Fackelman
Reformed Church
Orşova, România, 1975

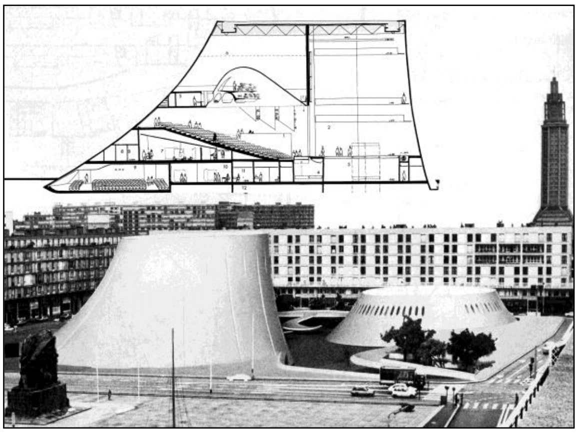


Fig. 3.5 Hyperboloid - Oscar Niemeyer
Maison de la Culture
Le Havre, Franţa, 1978

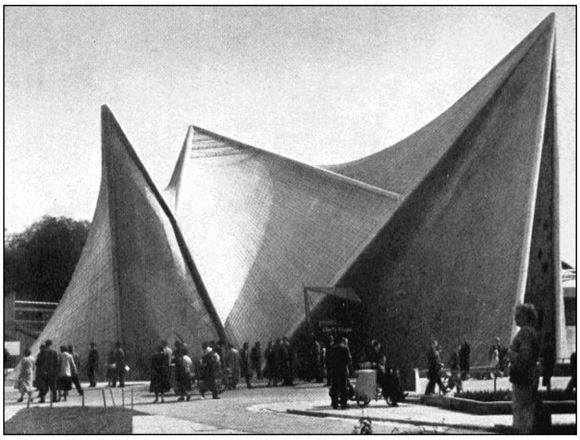


Fig. 3.6 Paraboloid - Le Corbusier
Philips Pavillion
Expo Bruxelles, 1956

4. CONCLUSIONS

Based on the aforementioned facts, we are trying to convince that ruled surfaces, especially the non-developable ones, offer infinite spatial variations and numerous practical applications in architecture and choosing them as a topic of study for this paper is motivated by the following facts:

• they have a clearly defined geometry, which allows a coherent calculation of structural components, hence foreshadowing a good working relationship between the architect and the structural engineer;

• recently remarkable achievements emerged in this formal category, supported by current technical development (eg. achievements of Santiago Calatrava);

• in many cases their execution is easier than that of other formal categories, due to the rectilinear generatrices;

• based on a teaching approach that shows students of architecture the architectural diversity derived from spatial relation of primary geometric elements (lines, surfaces, simple volumes).

As practical applications of the Geometry of Architectural Forms subject taught in the first year of study and the Study of Forms, taught in the second year, the students seek to answer spatial variations requirements of different functionalities (eg exhibition buildings, sports centers, religion, etc.), making use of 2D or 3D representation techniques: a series of drawings (in orthogonal projections or axonometry) and models. Once they've chosen the function for a given area bordered by a certain perimeter, they have to find a spatial answer to these types of ruled surfaces.

The aim is both finding a graphical ways of highlighting the geometrical elements (directrices and generators) that define each surface, in drawings as well as in the built models, and also deciding the possibilities of placing the ruled surface on load-bearing elements. Some of the second year of study students' applications are presented on this page in Fig. 4.1, Fig. 4.2, Fig. 4.3, Fig. 4.4.

Fig. 4.1

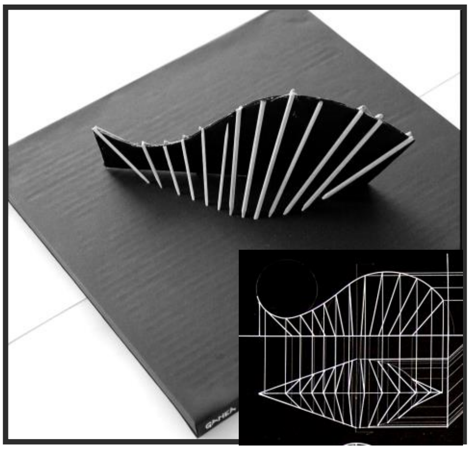


Fig. 4.2

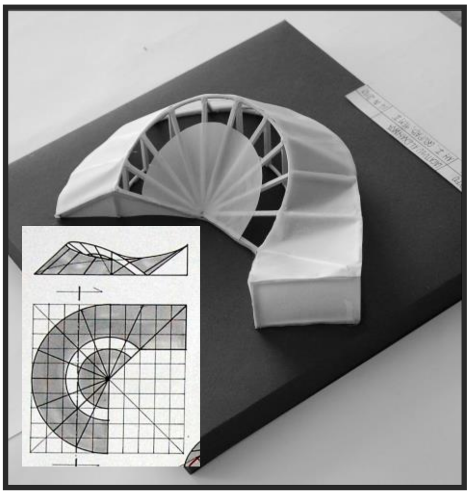


Fig. 4.3

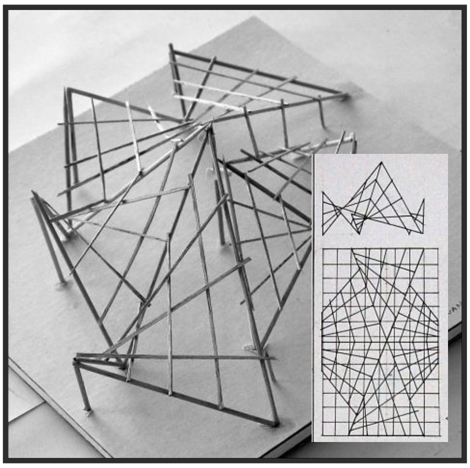
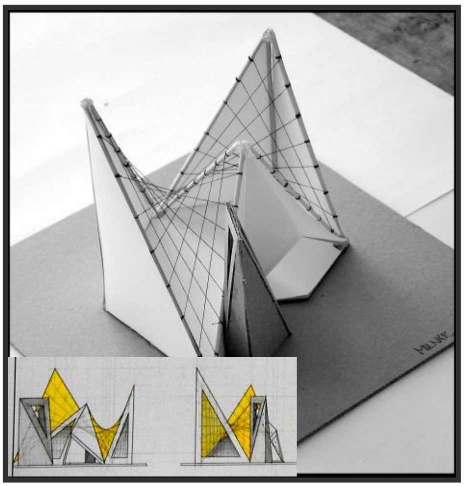


Fig. 4.4



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