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Study Concerning Various Representations of the Küper Conoid Surface

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#### Abstract

In the family of warped surfaces used in civil engineering, conoids occupy an important position. This paper presents a study concerning the representation of the Küper conoid in orthogonal projection on two planes of projection, axonometric projection and in the projection with elevation. Every type of representation is exemplified with drawings, and advantages and disadvantages related to the representations mentioned are highlighted.


Keywords: Küper conoid, axonometric projection, orthogonal projection on two planes of projection, projection with elevation.

## 1. INTRODUCTION

The way in which complex surfaces are generated and the selection of the most appropriate and suggestive system of representation of these surfaces

CONOID IN AXONOMETRIC PROJECTION

represents a challenge for the specialists. In the course of our research, we have presented various ways of representing conoid type surfaces. In this respect, we have shown the representation of the oblique conoid, circumscribed to the sphere and of the Plücker conoid.

The Küper conoid is part of the warped surfaces having a directrix plan. References [1],[2] show that the generation of the Küper conoid contains the following elements: the directrix curve which is a circle belonging to a horizontal plane, the directrix line, that is a vertical line belonging to the vertical projection plane and the directrix plane which is the bisector plane in this case [B II-IV].

The generatrices of the Küper conoid bear on one end on the directrix circle, and at the other end on the directix line, keeping a parallel position to the bisecting plane [B II-IV].
2. REPRESENTATION OF THE KÜPER

In Fig. 1, the Küper conoid is represented in isometric orthogonal axonometric projection.

The construction of the axonometric projection took into consideration the data taken from the orthogonal projection on two planes of projection.

The representation in axonometric projection is illustrative and easily understood even by nonspecialists.

However, this type of representation deforms, i.e. it does not allow the measurement in true size of the surfaces, angles or length of the straight lines that are not parallel to the axes of coordination.
3. REPRESENTATION OF THE KÜPER CONOID IN ORTHOGONAL PROJECTION ON TWO PLANES OF PROJECTION
3.1. The orthogonal projection on two planes of projection of the Küper conoid

In Fig. 2, a Küper conoid is represented, whose curve directrix is a circle $\Gamma\left(\gamma, \gamma^{\prime}\right)$ situated in a horizontal plane, and the straight directrix is a vertical line $D\left(d, d^{\prime}\right)$ belonging to plane [V]. All the eneratrices are parallel to the directrix plane $\left[\mathrm{B}_{\mathrm{II}-\mathrm{VV}}\right]$.

Fig. 1. Küper conoid. The isometric orthogonal axonometric projection.

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Fig. 2. The Küper conoid. Orthogonal projection on two planes of projection.

### 3.2. Section with an oblique plane in the Küper conoid

Fig. 3 represents the section with an oblique plane [P] in a Küper conoid.

In order to find the section curve, auxiliary planes parallel to Ox (perpendicular to [W]) were used.


Fig. 3. Section with an oblique plane in the Küper conoid. Orthogonal projection on two planes of
3.3. Section with a vertical plane in the Küper conoid

In Fig. 4 one can see the section curve resulting from the intersection of the Küper conoid with [Q] a vertical plane (perpendicular to $[\mathrm{H}]$ ). The cutting
curve is drawn through points found in horizontal projection and elevated with lines of recall, in vertical projection, onto the corresponding generatrices.
3.4. Intersection of the Küper conoid with a straight line

In order to determine the intersection pints between the Küper conoid and the straight line $\Delta\left(\delta, \delta^{\prime}\right)$, the auxiliary plane [R] was used and drawn through the straight line.


Fig. 4. Section with a vertical plane in the Küper conoid. Orthogonal projection on two planes of projection.

Following the intersection of plane [R] with the conoid surface, a curve was found and its horizontal projection intersects in a and b , with the horizontal projection $\delta$ of the straight line. With the help of lines of recall, we found $\mathrm{a}^{\prime}$ and $\mathrm{b}^{\prime}$ onto $\delta^{\prime}$ (Fig. 5).


Fig. 5. Intersection with a straight line of the Küper conoid. Orthogonal projection on two planes of projection.

## 4. REPRESENTATION OF THE KÜPER

 CONOID IN PROJECTION WITH ELEVATION. PLANE SECTIONS. INTERSECTION WITH A STRAIGHT LINE.4.1. Finding the projection with elevations of the Küper conoid

In order to find an advantageous projection with elevations of the Küper conoid, the conoid was turned around an axis so that generatrices reached the position of horizontal lines (parallel to plane [H]) Fig. 6.


Fig. 6. Orthogonal projection on two planes of projection of the rotated Küper conoid

The generatrices of the conoid were chosen so that they belong to equidistant horizontal planes (parallel to $[\mathrm{H}]$ ), where the equidistance is equal to a unit.

Fig. 7 shows the projection with elevations of the Küper conoid.


$$
\begin{array}{lllllll}
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
& 1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}
$$

Fig. 7. The Küper conoid represented in the projection with elevations.
4.2. Section with an oblique plane in the Küper conoid

In Fig. 8 it is given the intersection between the oblique plane $[\mathrm{P}]$ and a Küper conoid having generatrices as horizontal lines (parallel to plane [H].

The cutting curve was found in the points at the intersection between the conoid generatrices and the horizontal lines of the cutting plane.


Fig. 8. Section with an oblique plane in the Küper conoid. Projection with elevations

### 4.3. Section with a vertical plane in the Küper conoid

In Fig. 9, it is shown the intersection of the Küper conoid with the vertical plane [Q]. It is noticed that the cutting curve overlaps the plane trace. The true length of the curve (C) was defined by bringing it into coincidence in the plane of elevation +26 .


Fig. 9. Section with a vertical plane in the Küper conoid. Projection with elevations.

### 4.4. Intersection of the Küper conoid with a straight line

In order to determine points $\alpha$ and $\beta$ in which the straight line ( $\Delta$ ) intersects the conoid, we used an auxiliary plane drawn through the straight line. Mention should be made that the straight line $(\Delta)$ is not the line of maximum inclination or the steepest line) for this auxiliary plane. Points $\alpha$ and $\beta$ appeared at the intersection of the projection with elevation $\delta$
for the straight line with curve c . The visibility of the straight line was then investigated - Fig 10.


$$
\begin{array}{llllll}
1 & 0 & 1 & 2 & 3 & 4 \\
1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}
$$

Fig. 10. Intersection with a straight line of the Küper conoid. Projection with elevations.

## 5. CONCLUSIONS

The representation in projection with elevations of the Küper conoid placed in the particular position shown in this paper is an original one.

It is noticed that after the projection with elevations is determined, it becomes relatively easy to solve the various problems regarding the intersections with planes and straight lines.

Both the representation in orthogonal projection on two planes of projection and in the projection with elevations is dedicated to specialists in this domain.

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