Buletinul Ştiinţific al Universităţii "POLITEHNICA" din Timişoara

Seria HIDROTEHNICA
TRANSACTIONS on HYDROTECHNICS

Tom 58(72), Fascicola 2, 2013

Methods to Determining the Tangent Plane to a Sphere

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Abstract: This paper present analytical and graphical determination of a tangent plane to a sphere trough a given point. In the first method, using mathematical methods is determinate the general equations of the tangent plane to the sphere trough a given point situated on the sphere surface in first case and in second case trough a given point situated outside of the sphere surface. In second method the problem is solved using descriptive geometry methods. The sphere is defined by the projections of the center point coordinate and radius in front view and top view, also is presented the axonometric representation.

Keywords: descriptive geometry, tangent plane, matlab representation, plane equation.

1. INTRODUCTION

The aim of this paper is to present the advantages using Descriptive Geometry for solving some mechanical engineering problems. In first part of this paper is presented the mathematical approach for determination of the tangent plane to a sphere through a given point. The point is situated on sphere surface in first case and in second case the point is situated outside of the sphere. For a better representation of a mathematical results, the tangent plane is graphically represented in Matlab software. Second part of this paper solve this problem using a graphical method, without mathematical calculation. The following paragraphs provide an overview of the advantages of the using Descriptive Geometry in technical applications.

Descriptive Geometry is defined as a applied science which treats of the graphical representation of lines, planes, surfaces, and solids, and of the solution of problems concerning size and relative proportions. Thus it lies at the foundation of all architectural and mechanical drafting [1].

While the study of Descriptive Geometry does not require extended mathematical knowledge, and while its operations are not strictly mathematical, the best results cannot be obtained without some acquaintance with the principles of plane and solid geometry.

Solutions of Descriptive Geometry are based on drawings, differs from analytic geometry [2]. By the methods of descriptive geometry the solution of any problem involving three dimensions consists of three distinct processes, as follows :

- Representation of the lines, planes, surfaces, or solids in space by corresponding plane figures.
- Solution of the problem by the use of the plane figures.
- Determination of the relation in space which corresponds to this solution.

For verify the accuracy, the results are plotted in Matlab environement programming software, having a visual check of the problem [4].

This technique can be used in technical applications for determining the tangent plane to a sphere of a rolling track.

2. MATHEMATICAL APPROACH

In this section using the mathematical calculation is presented the way to determining the tangent plane to a sphere through a given point situated on the sphere surface in first case and in second case through a point situated outside of the sphere.

Case 1

Position of the sphere is given by the coordinate of the center point Ω(50, 35, 37) and the radius R=15 mm. All dimensions used in this paper are given in millimeters.

Before starting the calculation is necessary to verify if the given point M(40, 25, 42) is situated on the sphere [5]. The point belongs to the sphere if it satisfies following conditions:

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \quad (1)$$

$$(40-50)^2+(25-35)^2+(42-37)^2=15^2 \quad (2)$$

$$225=15^2$$

This condition is fulfilled.

The radius of the sphere is given by the next relations:

$$R=\sqrt{(x-a)^2+(y-b)^2+(z-c)^2} \quad (3)$$

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$$R=\sqrt{(40-50)^2+(25-35)^2+(42-35)^2} \quad (4)$$
$$R=\sqrt{225}=15$$

The tangent plane must be normal to the radius of the sphere. Cartesian equation of this tangent plane to a sphere through a given point can be written:

$$(x-a)\cdot(x_0-a)+(y-b)\cdot(y_0-b)+(z-c)\cdot \cdot(z_0-c)=R^2 \quad (5)$$

$$(x-x_0)\cdot(x_0-a)+(y-y_0)\cdot(y_0-b)+ +(z-z_0)\cdot(z_0-c)=0 \quad (6)$$

Replacing the values:

$$(x-40)\cdot(40-50)+(y-25)\cdot(25-35)+ +(z-42)\cdot(42-37)=0 \quad (7)$$

It obtains the general equation of a tangent plane through a given point:

$$-10x-10y+5z+440=0 \quad (8)$$

Case 2

In this case position of the sphere are given by the center point Ω(80, 35, 35) and the radius R=15 mm. The M(57, 28, 42) point is situated outside of the sphere surface. This problem has an infinit number of solution. In this case it chooses solution where the tangent plane is normal to the segment line ΩM.

$$(x-57)\cdot(57-50)+(y-28)\cdot(25-35)+ +(z-42)\cdot(42-35)=0 \quad (9)$$

It obtains the general equation of a tangent plane through a given point:

$$-23x-7y+7z+1213=0 \quad (10)$$

Mathematical solution is solved using Matlab programming environement. The sphere and plane are plotted in axonometric view in fig. 1. The plane is tangent to a sphere in the point M (x, y, z) represented on this graphics.

In fig.1 is presented the graphical plot for first case of the mathematical approach. This view allows visualization of the tangent plane between the traces on the projection planes.

Fig. 2 shows the vertical view of the plane and the sphere. Vertical trace of the tangent plane is given by the intersection of the plane and the vertical projection plane. Vertical trace is given by the inclined line, *T`*.

In fig. 3 is presented the horizontal view of the plane and sphere. The horizontal trace of the plane is given by segment line, *T* between the horizontal and vertical piercing-points. Three-dimensional plot generated in Matlab provides an safe visual result, this preventing the errors from the mathematical calulus solved in firs part of this paper.

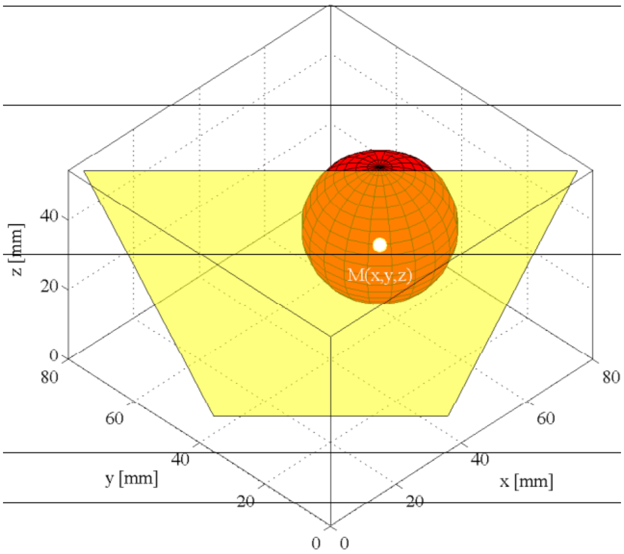

Fig. 1. Matlab representations of the tangent plane through a point situated on the sphere surface

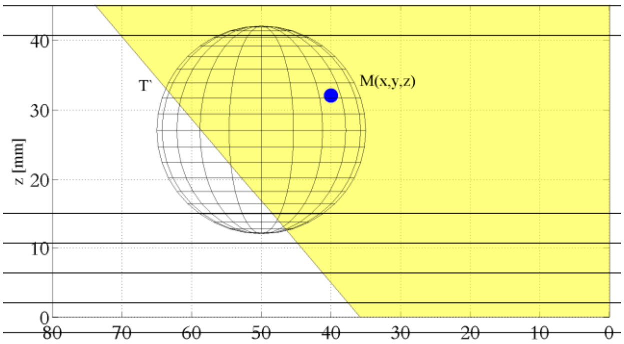

Fig. 2. Vertical view – Matlab representation

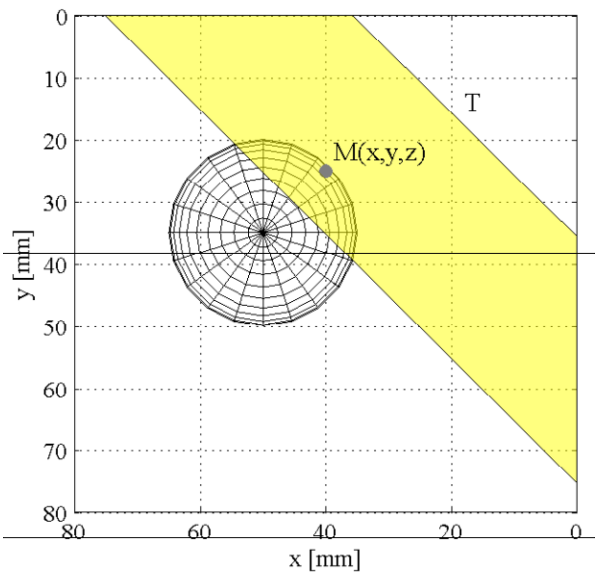

Fig. 3. Horizontal view – Matlab representation

3. GRAPHICAL SOLVE

Graphical solution of this problem is solved using Descriptive Geometry methods. Solving the problems whit Descriptive Geometry method has an important role in the developing of the engineer’s imagination. Graphical representation makes it easy to understand and interpret data at a glance.

Case 1

It consider a sphere determined by the centre point coordinates Ω(50,35,37) and the radius R=15 mm. The point M (40, 25, 42) is placed on the sphere surface. Is intended to represent the tangent plane to the sphere that passes through the point M.

The projection of a sphere may be taken as in fig. 4, being the horizontal and vertical projection. Each projection of the sphere will evidently be equal to a great circle of the sphere, and its center will be one projection of the center of the sphere.

The tangent planes to spheres it must chiefly be noticed that the radius of the sphere, from the tangent point on its surface, is a perpendicular to the tangent plane, and its projections will therefore be perpendicular to the traces of the tangent plane [7].

First step for drawing of this plane is drawing a parallel plane *[N]* to the horizontal plane through the point *m*. Through the point *m`* is traced a horizontal line in vertical view. Horizontal projection of the line *d* is traced perpendicular to the *ωm* segment line and through the horizontal projection of the point *m*.

The vertical trace of the tangent plane pass through the vertical trace of the line *v`* and it is perpendicular to the segment *ω`m`*. The orizontal trace of the tangent plane pass through the *Tx* and it is parallel to the horizontal projection of the line *d`* [8]. In fig. 4 and in fig. 5 is presented the graphical solving of the case 1.

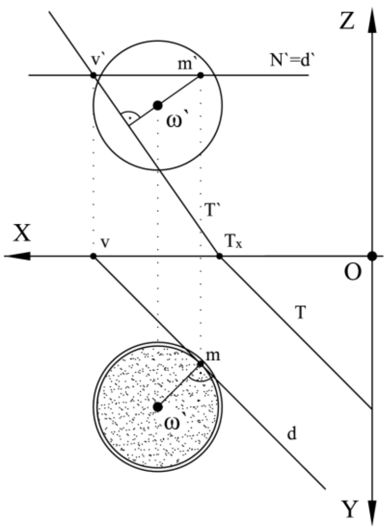

Fig. 4. Graphical representations of the tangent plane through a point situated on the sphere surface

For a better visualization this case is solved in axonometric view in fig. 5. This three-dimensional view is made in Autocad software. In this representation the tangent plane is represented only the part between its trace with projection planes.

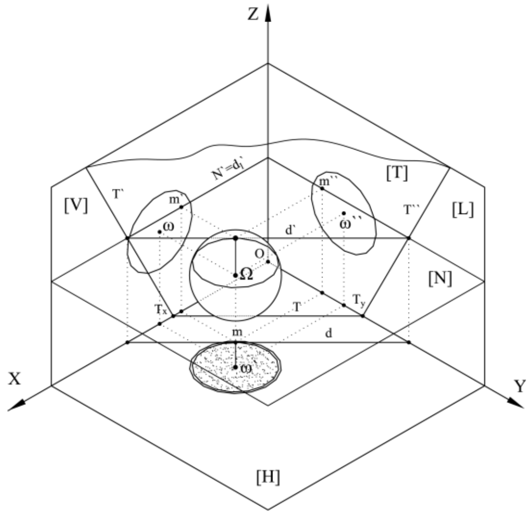

Fig. 5. Three-dimensional representations of the tangent plane through a point situated on the sphere

Case 2

In this case the sphere is determined by the centre point coordinates Ω(80, 35, 35) and the radius R=15 mm. The point M(57, 28, 42) is situated outside of the sphere surface. Is intended to represent the tangent plane to the sphere that passes through the point M. Solution of this problem allows drawing an infinite number of plane through point M.

A tangent plane to a sphere at a given point is the plane whitch contains all the tangent lines to the sphere at that point.

The start point of this solving consist in drawing of a parallel plane to the horizontal plane through the point *m*. Through the point *m`* is traced a horizontal line in vertical view. Horizontal projection of the line d_1 is traced perpendicular to the *ω1* segment line and through the horizontal projection of the point *m*. This line is tangent to a sphere through a point *M*.

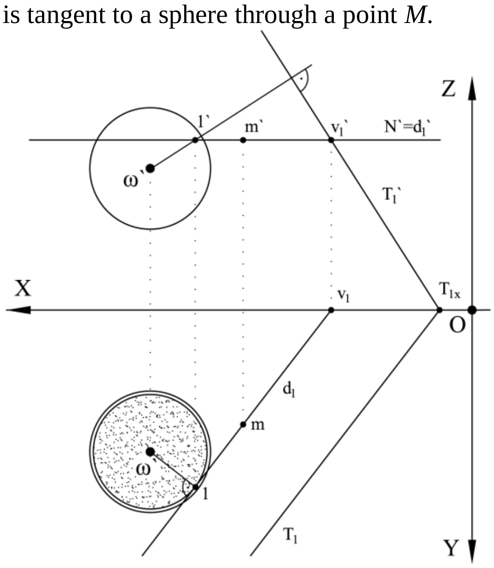

Fig. 6. Graphical representations of the tangent plane through a point situated outside of the sphere surface

The vertical trace of the plane pass through the vertical trace of the line *v`* and it is perpendicular to the segment *ω`1`*. The orizontal trace of the tangent plane pass through the $T_{1x}$ and it is parallel to the horizontal projection of the line $d_1$\`. Graphical solve of this problem is presented in fig. 6.

A plane $[T_1]$ tangent to the surface at a point M will pass through the point M and be perpendicular to the radius of the sphere $\Omega 1$. The tangent plane consist to the meridian section and one to the parallel or circle at the point of contact.

The graphical plot generated in Matlab software of this second case is presented in fig. 7.

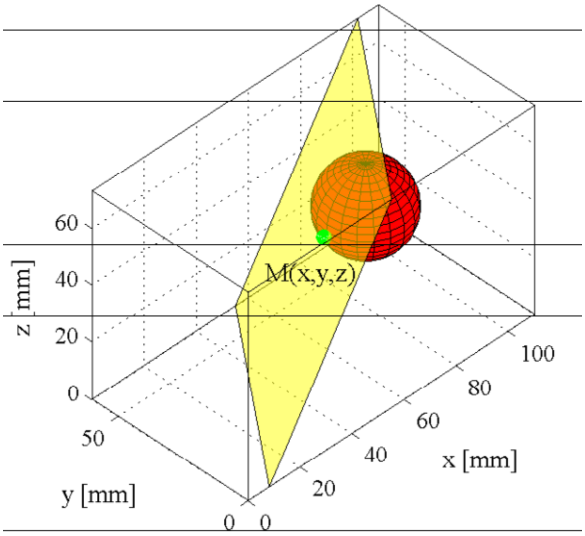

Fig. 7. Matlab representations of case 2

4. CONCLUSIONS

The paper shows the possibilities of Descriptive Geometry to find the tangent plane to a sphere. Solving this problems by the means of analytical geometry it would to lead to a difficult. On the contrary, using the descriptive methods the solve will give a clear and concrete graphic solution. The graphical approach is better suitable for the final goalof this study.

Graphical representation generated by the Matlab software was performed in order to observe the layout of the sphere and the tangent plane in three-dimensional space.

After solving both methods conclude that the graphic method is a simple, fast and with low possibilities of errors.

A requirement of practical drafting is that the construction shall be confined to the limits of the drawing board. There is always a limit beyond which points cannot be made available. This weakness point of Descriptive Geometry can be eliminated using design software, which facilitates the solve of the problems with accuracy and quick.

The following to use of Descriptive Geometry helps to develop technical imagination and three-dimensional vision of the young engineers.

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