Buletinul Ştiinţific al Universităţii "POLITEHNICA" din Timişoara

Seria HIDROTEHNICA

TRANSACTIONS on HYDROTECHNICS

Tom 58(72), Fascicola 2, 2013

Graapho-analytical studies of a conical surface and its applications in industrial design

Dumitru MARIN[1] **Ligia PETRESCU[1]**

Abstract: The paper presents a complete study regarding section, development, combination of the conical and cylindrical surfaces. This study is necessary in order to achieve different useful applications: joining pipes, reducing pieces, tubulatures etc. Analytical formulas can be used for a programming algorithm conceiving and automatic drawing of different conical shapes: oblique circular cone, rotation cone, truncated or sectioned cone etc.

Keywords: oblique circular cone, rotation cone, antiparallel section, inflexion point, development.

1. INTRODUCTION

Cylindrical and conical shapes are frequent used in industrial products design. Moreover, these shapes are the most technological ones,that is they can be produced easier than other ones: spherical, toroid or helical surfaces, etc.

Conical shapes can be full parts (axles, shafts, spindles, spigots, etc) or parts made of iron sheet (pipes, reducing pieces, tubulatures, decorative objects etc.)

We propose a complete study about conical surface. This study presets representation, section, development of conical surfaces, as well as their combination with other surfaces in order to achieve a lot of useful applications.

2. GRAPHIC STUDY OF CONICAL SURFACE

Oblique circular cone in figure 1 has radius R and vertex S. A level plane *[N]* cuts the oblique circular cone in a radius r circle.

The angle α between the plane *[P]* $\perp$ *[V]* and the shortest generatrix *SB* is the same with the angle between horizontal plane and the longest generatrix *SA*. The plane *[P]* cuts the cone in a circle (antiparalle section) with the same radius r as level plane *[N]*.

S vertex cones and radius *r* base circles, one of them in level plane *[N]*, the other in plane *[P]* are equivalent, therefore they have the same development.

S vertex cone and radius *r* base circle in the plane *[P]* can be considered as well as base cone in the level plane *[N]*, which was 180^0 turned round angle $\emptyset$ bisector between generatrix *SA* and *SB* (see fig. 1).

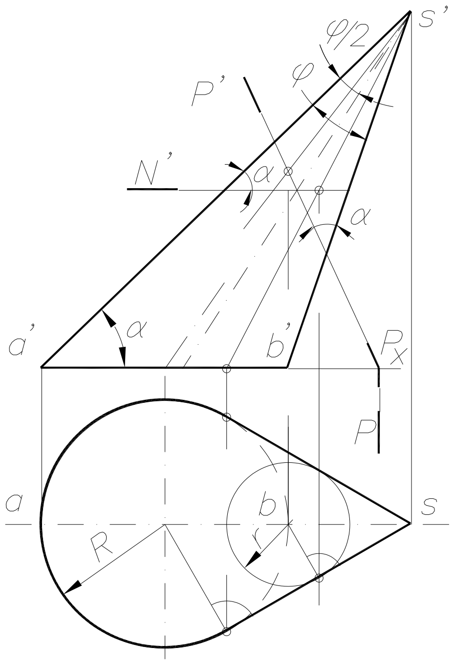


Fig.1. Sections in oblique cone

As a result of the above- mentioned observations there are three conclusions to be considered:

-the development of the r radius circle in level plane *[N]* is a congruent radius *R* circle development curve, the similitude ratio is *r/R* (see fig. 4)

- *r* radius circle development in plane *[P]* is the same with *r* radius circle in level plane *[N]*, which was turned in 2γ angle between generatrix *SA* and *SB* on cone development (see fig. 4)

-Conical portions as those included between horizontal plane and planes *[N]* or *[P]* can join two different diameters pipes, in coplanar or non-coplanar axis (see fig. 9 and 10)

3. ANALYTIC STUDY OF CONICAL SURFACE

The oblique circular cone in figure 2 has its base circle in horizontal plane *[H]* and its vertex *S*.

We have:

R- base circle radius,

O- base circle center,

h - the cone height,

[1] POLITEHNICA University of Bucharest, Engineering Graphics and Industrial Design Department, Romania, Phone: 0722708047, e-mail: dumarpub@yahoo.com; ligiapetrescu@yahoo.com.

s - S projection on base plane (X_s size),

β - the base circle angle corresponding to SM usual generatrix.

An usual generatrix length can be calculated in *SNM* triangle:

$$SM = \sqrt{MN^2 + SN^2} = \sqrt{MN^2 + h^2 + sN^2} \tag{1}$$

$$SM = \sqrt{R^2 \sin^2 \beta + h^2 + \left(X_s - R\cos\beta\right)^2} \tag{2}$$

In the end,

$$SM = G = \sqrt{h^2 + R^2 + X_s{}^2 - 2RX_s \cos\beta} \tag{3}$$

Formula (3) can be verify for different β values (see fig. 3):

If $\beta = 0$, the cone minimum generatrix is;

$$G_{\min} = \sqrt{h^2 + \left(X_s - R\right)^2} \tag{4}$$

According to Olivier theorem, the inflexion points on base circle development are placed on apparent outline generatrix (generatrix placed in cone tangent planes and at the same time perpendicular on the secant plane).

In conclusion, if

$$\beta = \arccos R/X_s\ ,$$

the generatrix corresponding to inflexion point on the base circle development is:

$$G_{\inf} = \sqrt{h^2 + X_s{}^2 - R^2} \tag{5}$$

If $\beta = \pi/2$,

$$G = \sqrt{h^2 + X_s{}^2 + R^2} \tag{6}$$

If $\beta = \pi$, the cone maximum generatrix is:

$$G_{\max} = \sqrt{h^2 + \left(X_s + R\right)^2} \tag{7}$$

If $X_{s=}R$,

$$G_{\min} = G_{\inf} = h \tag{8}$$

According to Olivier theorem, the base circle development has not inflexion points.

If $X_{s=}0$

$$G = \sqrt{h^2 + R^2} = constant \tag{9}$$

and the oblique cone turns into a rotation one.

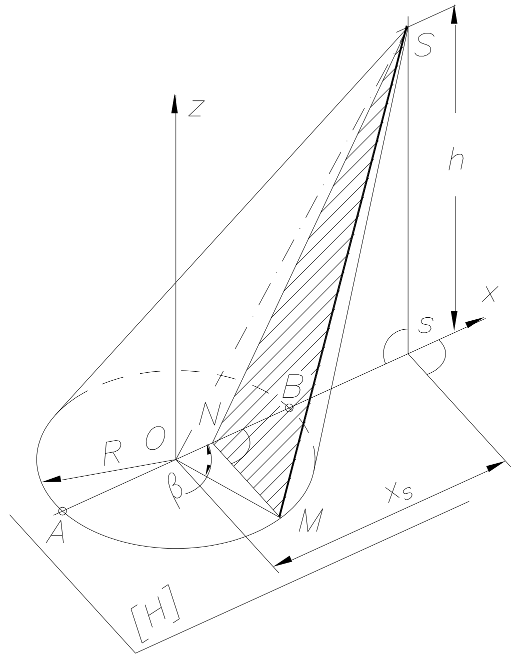


Fig. 2. Usual generatrix length

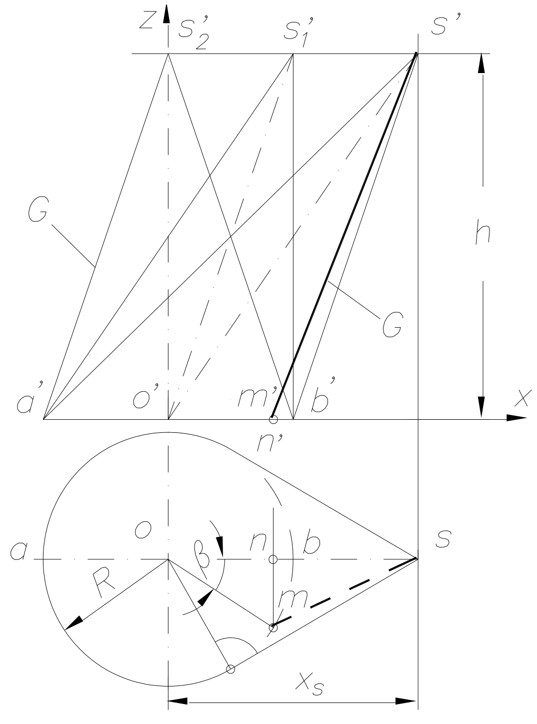


Fig. 3. The cone different positions.

4. CONE DEVELOPMENT

The oblique cone in figure 1(*R=18, h=50, X_s=36*) is presented in figure 4 in its development.

The inflexion points on the *R* radius circle as well as on the *[N]* and *[P]* planes *r* radius circles are marked on the cone development.

In figure 5 is presented oblique cone development with vertex S_1 in figure 3 (X_s= *R=18, h=50*).

The inflexion points join into the angular point which is marked on the development.

In figure 6 is presented rotation cone development with vertex S_2 in figure 3 ($X_s=0$, $R=18$, $h=54$).

The base circle development is a vertex angle circle arc:

$$\Phi = 2\pi R / G = 120^0 \tag{10}$$

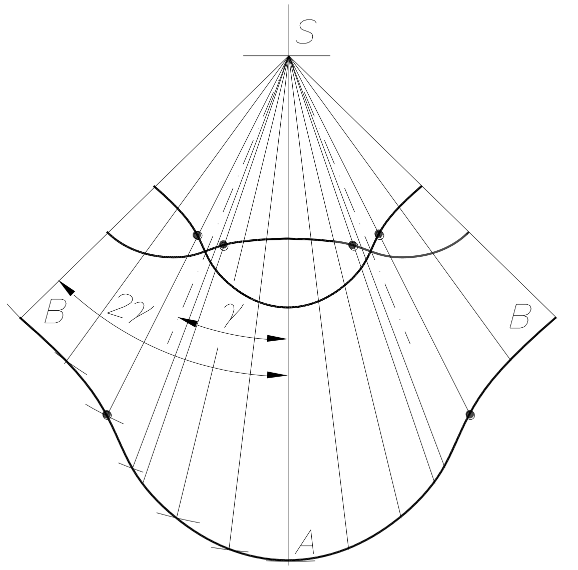


Fig. 4. The oblique cone development

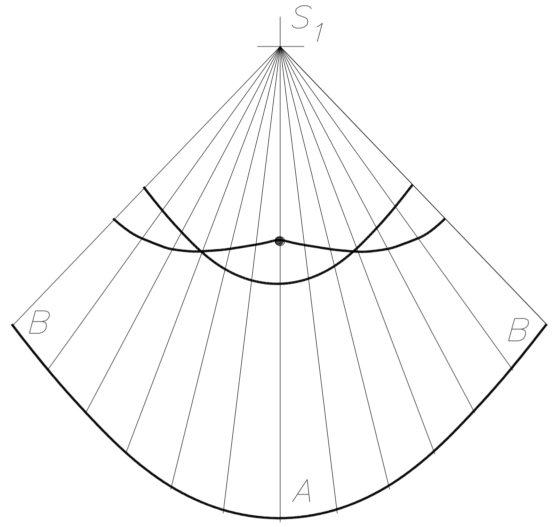


Fig. 5. The S_1 vertex oblique cone development.

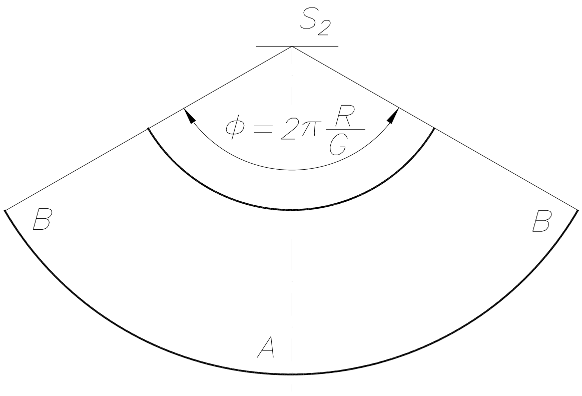


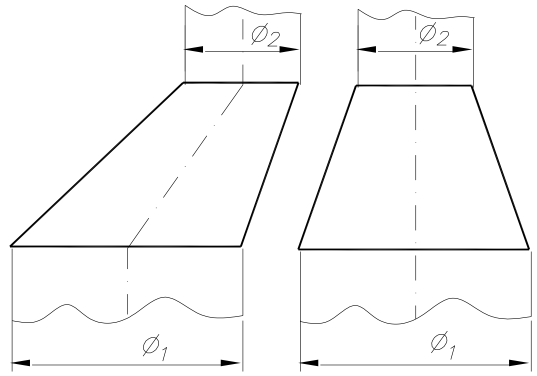
Fig. 6. The rotation cone development

5. PRACTICAL APPLICATIONS

In figure 7, *a* and *b* are presented two among the most useful reducing pieces. $\varnothing_1$ and $\varnothing_2$ diameter reducing pieces is achieved with conic trunks whose developments are presented in figures 4 and 6.

In figure 8, the same two $\varnothing_2$ diameter pipes are joint with the $\varnothing_1$ diameter pipe using two identical conic trunks which are intersected in an ellipse.

The bases of the conic trunk in figure 9a are antiparalell section circles, its development being presented in figure 4.



a b

Fig. 7. Two unequal diameter pipes joining: a- with parallel axis b- coaxial pipes

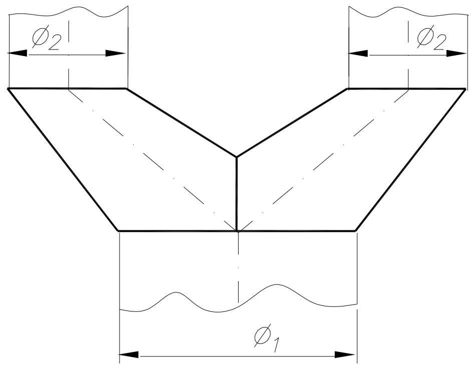
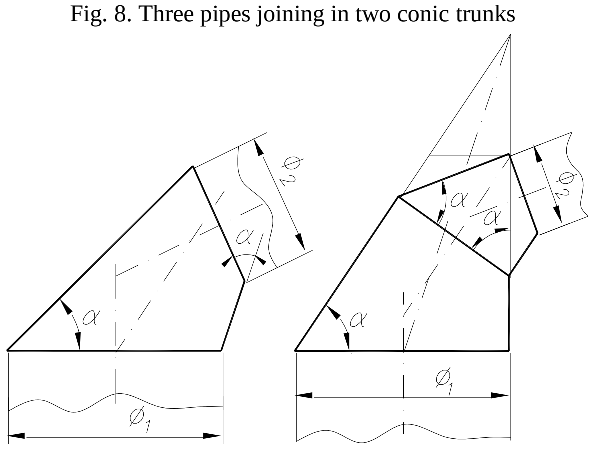


Fig. 8. Three pipes joining in two conic trunks



a b

Fig. 9. Two cross axis pipes joining a-with cross axis, with a conic joining trunk b-with cross axis, with two conic joining trunks

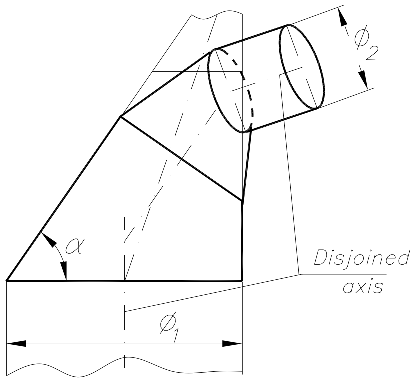


Fig. 10. Two skew axis pipes joining.

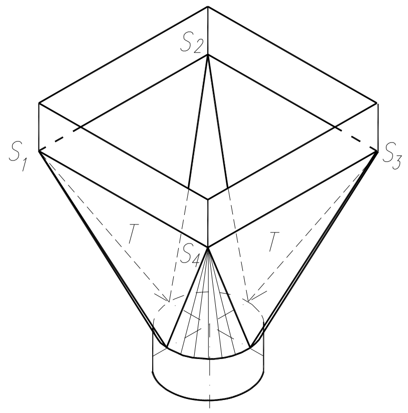


Fig. 11. Reducing piece from prism to a cylinder.

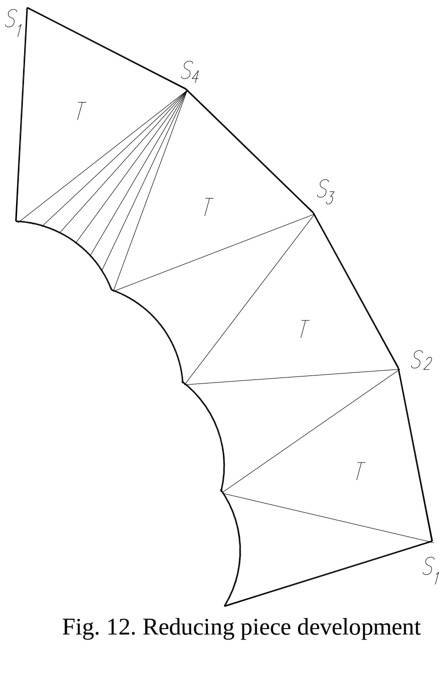


Fig. 12. Reducing piece development

The bases of the conic trunk in figure 9a are antiparalell section circles , its development being presented in figure 4.

In figure 9b it is observed that conic trunks are component parts of the same cone. The second conic trunk is 180^0 turned on antiparalell section circle, so the two pipes have cross axis.

In figure 10, the conic trunks are component parts of the same cone but the second conic trunk is turned whit a smaller than180^0 angle on antiparalell section circle. Therefore, the two pipes do not have cross axis (the two axis are disjoined lines).

Reducing piece in figure 11, is composed of 4 conic sheets (portions) with S_1, S_2, S_3, S_4 vertex and 4 *T* isosceles triangles . The symmetry axis of the conic sheet portions is the shorter generatrix of the cone (see the generatrix *SB* in figures 1, 2 and 3).

Figure 12 represents reducing piece development in figure 11. The development thin lines are bend lines, thick lines are cutting up lines

6. CONCLUSION

- The analytical study of conical surface proves its validity both the generally case (oblique circular cone) and particularly case(rotation cone).
- Different diameter pipes joining along of any spatial route can be achieved with conic trunks. The conic trunk bases are antiparalell section circles.
- A programming algorithm can be conceived by using formula (3) for automatic developing of conic surface; *h*, *R* and X_s parameters can be considered constant but β parameter, a variable one.
- This paper can be used both in industrial design applications and with a view to complete didactic study of the conic surface.

7. REFERENCES

[1] *S., Aldea,* Geometrie Descriptivă. Studiul suprafeţelor şi corpurilor, Ed. I.P.B., Bucureşti, 1979.

[2] *D., Marin, A., Pavel,* Studii si aplicatii privind desfasurarea suprafetelor conice, A VII-A editie a Conferintei Nationale GRAFICA 2000 **,** Craiova, 2000, Romania**.**

[3] *D., Marin,* Studiul secţiunilor antiparalele în conul şi cilindrul circular oblic, International Conference „Engineering Graphic and methodics”, pp. 29-40, 3-4 October 1995, Chişinău.

[4] *D., Marin,* Geometrie descriptivă. Noţiuni teoretice şi aplicaţii, Ed. Bren, ISBN 973 648 192 1, Bucureşti, 2008.

[5] *D., Marin,* Design industrial- Designul formei, Editura BREN, ISBN 978-973-648-875-7, Bucuresti, 2011.

[6] *L., Petrescu, s.a.*, Geometrie descriptiva si grafica inginereasca, Editura BREN, Bucuresti, 1998.

[7] *L., Petrescu,* Engineering Graphics, Editura BREN, ISBN 973-648-107-7, Bucureşti, 2003.